Standard Model of Particle Physics

Exercises 1

1. My child came home from school having learned that a "google" was the number 10^{100}. She asked me: are there more than a google grains of sand on the Earth? Answer this question and explain your answer.

2. In natural units, every quantity has dimensions of mass to some power. Use natural units to express Newtons constant G_N in powers of eV. Define the Planck scale. Express it in (powers of) seconds.

3. Compare the relative strength of the electromagnetic force between an electron and a positron with that of the gravitational force between them. Use natural units for this and write the answer as a function of α, the electron mass and the Planck scale.

4. Estimate the number of protons in the Atlantic ocean. Estimate, in GeV^2 units the gravitational force between the Atlantic ocean and the moon. If the moon had the same charge as a positron, what charge would the Earth have to have in order that the electromagnetic force between it and the moon exactly balanced the gravitational force?

5. Estimate how many 100g bars of chocalate you would need to consume to generate the same amount of energy as in one of the proton beams at the LHC?

6. Starting with the classical energy-momentum relation, derive the Schrodinger equation.

7. Show explicitly that the QED Lagrangian is invariant under $U(1)$ gauge symmetry transformations.

8. Energy (E) and the three components of momentum (p^i) can be combined into a four component, Lorentz 4-vector $p^\mu = (E, p^i)$. The Lorentz invariant scalar product of p^μ with itself is $p^\mu p_\mu = E^2 - (p^1)^2 - (p^2)^2 - (p^3)^2$. For a particle of mass M, $p^\mu p_\mu = M^2$. Use this relation to derive a wave equation describing relativistic particles (the Klein-Gordon equation).

9. Show that $\phi = Ne^{-ip^\mu x_\mu}$ solves the Klein-Gordon equation.

10. The known properties of the Standard Model particles are listed in the *Particle Data Group* book: `http://pdg.lbl.gov/2012/listings/contents_listings.html`

Using the information about the quarks and leptons and gauge bosons there, this question concerns the decays of some of these particles. For each of the particles below and using the fact that energy, momentum, spin and charge have to be conserved, give a plausible set of other particles that the given one might decay into:

a) top quark
b) bottom quark
c) positron
d) Z boson
Next,
e) Give a reason why a W-boson cannot decay into a muon plus an anti-muon.
f) Why can't a top quark decay into two W bosons?
g) Can a Z boson decay into a top/anti-top quark pair?

11. Consider a top quark which has been produced in a proton-proton collision. The three components of its momentum are $(p^x, p^y, p^z) = (245, 302, 90)$ in units of GeV. What is the energy of this top quark?

12. A neutral pion decays to two photons almost 100% of the time, $\pi^0 \to \gamma\gamma$. Assuming the pion is at rest, what is the energy of each photon?