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Standard Model of Particle Physics
Exercises Two

1. What are the defining properties of a Unitary $N \times N$ matrix, M.

2. If the unitary matrix M is generated by a matrix T i.e. $M = e^{iT}$ what properties must T have?

3. Show that the kinetic term of N complex scalar fields $\mathbf{\Phi} \equiv \phi_i$, $(\partial^\mu \mathbf{\Phi})^\dagger \partial_\mu \mathbf{\Phi} \equiv (\partial^\mu \phi_i)^\dagger (\partial_\mu \phi_i)$ is invariant under global $U(N)$ transformations.

4. How many gauge fields are required for $U(1)$ gauge invariance? For $SU(2)$ gauge invariance? How many gauge coupling constants are there in the Standard Model?

5. Write explicitly the two-by-two matrix which is the $SU(2)$ gauge covariant derivative.

6. Now we want to examine how this covariant derivative acts on the u and d quark 'doublet' $\begin{pmatrix} u \\ d \end{pmatrix}$ which transform under $SU(2)$ gauge transformations. That is, we want to write explicitly $D_\mu \begin{pmatrix} u \\ d \end{pmatrix}$ where D_μ is the $SU(2)$ part of the gauge covariant derivative in the Standard Model. This calculation is a crucial step towards determining the interactions between the quarks and the W bosons which occur in the Standard Model Lagrangian.