The Standard Model of Particle Physics

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The Standard Model of Particle Physics

The Standard Model:

- is a mathematical model for describing the behaviour of the elementary particles: quarks, leptons, gauge bosons and the Higgs boson
- is the most precisely tested scientific model every constructed – it has passed thousands of tests and most of its predictions have now been verified
- is based on **symmetry principles**: gauge invariance and Lorentz invariance (special relativity)
- is based on wave equations: Dirac, Maxwell, Yang-Mills, Klein-Gordon which all generalise the Schrodinger equation
- each of the particles obeys one of these equations and particle interactions are "potential terms" in the equations
- All interactions are determined from the symmetries except their strengths (coupling constants) and masses which are determined from experimental data e.g. α the fine structure constant or m_e the mass of the electron.

In these lectures we will.....

- Show how the basic symmetry principles determine all the properties of elementary particles
- Derive some predictions from the Standard Model and test them against experimental data
- This should serve as a good introduction to the rest of the school

Disclaimer!

- This is NOT a complete course on the Standard Model!
- that would require all of the three weeks of the school and more
- This is just an introduction. You can consult my lecture notes for a more detailed introduction and can read the books suggested there for an even more detailed understanding.

Natural Units and Dimensional Analysis

- Natural units are so-called because they intuitively express natural physical quantities in terms of mass scales or length scales.
- Natural units are defined by viewing the fundamaental constants $\hbar$ and c as conversion factors
- $\hbar = 1$ converts 1 Joule into about 10^{34} seconds^{-1}. Equivalantly 1 eV $= 1.6{\times}10^{15}$s^{-1}
- $c = 1$ converts 1 second into approximately 3×10^{8} metres.
- In natural units $[E] = [M] = [L]^{-1} = [T]^{-1}$.
- With these units **any** quantity can be converted into (some power of) eV. This provides it with a mass scale.
- Excercise: Convert Newton's constant G_N into eV^{-2}.

Natural Units and Dimensional Analysis

$$\hbar = 1 \sim 10^{-34}\text{J s} \sim 3\times 10^{-26}\text{J m} \sim 3\times 10^{-26}\frac{10^{19}}{1.6..}\text{eV m} \sim 2\times 10^{-7}\text{eV m} \quad (1)$$

$$1 \ \text{m} \sim \frac{1}{2\times 10^{-7} \ \text{eV}} \quad (2)$$

which means that a distance of one metre is equivalent to an energy scale of 2×10^{-7} eV. Larger distances correspond to smaller energies and vice-versa.

In natural units, the mass of the electron is about 0.5 MeV. This is the inverse of a distance of $\frac{1}{0.5\text{MeV}} \sim 4 \times 10^{-13}$ m This is the Compton wavelength of the electron. The mass of proton is about 0.94 GeV, corresponding to a length scale of order 2×10^{-16} m. This is the characteristic size of a nucleus. The masses of the W-bosons, the Z-boson, the top quark and the Higgs boson are of order 100 GeV – corresponding to a distance of around 2×10^{-18} m. The LHC collisions in 2012 took place at energies of 8 TeV = 8 thousand GeV – a distance scale of almost 10^{-20} m. This makes the LHC the world's most powerful microscope.

Particle Interactions and Gauge Symmetries

- The SM describes three forces: the strong nuclear force, the weak nuclear force and the electromagnetic force
- Associated with each of these is a symmetry called a gauge symmetry
- In electromagnetism, this symmetry leads to the interaction between photons and charged particles like electrons.
- In the strong nuclear force this leads to the interaction between gluons and particles "charged" under the strong gauge symmetry – the quarks.
- In the weak nuclear force this leads to the interactions between W and Z-bosons and particles charged under the weak gauge symmetry.

Wave equations – a reminder

- Schrodingers equation derives from $E = \frac{p_j^2}{2m}$ by replacing $E \to i\partial_t$ and $p_j \to -i\partial_j$
- The same exercise with $E^2 = p_j^2 + m^2$ gives the Klein-Gordon equn: $\partial_\mu \partial^\mu \Psi - m^2 \Psi = 0$.
- The K-G equation describes **relativistic** spin-0 particles.
- Dirac derived a relativistic equation which is first order in ∂_t and ∂_i.
- Dirac's ansatz: $i\partial_t \Psi = (-i\alpha_j \partial_j + \beta m)\Psi$ with α_i and β **operators** ie matrices
- $\alpha_i^2 = \beta^2 = \mathbf{1}$ and $\{\alpha_i, \alpha_j\} = 0 = \{\alpha_i, \beta\}$ ensures $H^2 \Psi = (p_i^2 + m^2)\Psi$
- Dirac equation in covariant form: $(i\gamma^\mu \partial_\mu + m)\Psi = 0$ with $(\gamma_0, \gamma_i) = (\beta, \alpha_i \beta)$

Lagrangian Formulation

In classical mechanics one considers generalised coordinates $q_i(t)$ of a particle. Then the Lagrangian

$$L = T - V \tag{3}$$

which is the difference between Kinetic and Potential energy leads to the Euler-Lagrange equations of motion

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}_i}\right) - \frac{dL}{dq_i} = 0 \tag{4}$$

We can use this formalism to obtain the relativistic wave equations such as the Klein-Gordon equation, the Maxwell equations and the Dirac equation.

Instead of considering L to be a function of discrete coordinates q_i, we consider Lagrangians which are functions of the **fields** which are continuous functions of both x_i and t i.e. of x_μ.

Lagrangian Formulation

For example, for the Klein-Gordon equation L is a function of $\phi(x_\mu)$ as well as the derivatives $\frac{\partial \phi}{\partial x_\mu} \equiv \partial_\mu \phi$:

$$L(q_i, \dot{q}_i, t) \rightarrow L(\phi, \partial_\mu \phi, x_\mu) \tag{5}$$

L is obtained from a Lagrangian density $\mathcal{L}$ integrated over space

$$L = \int d^3x \mathcal{L}(\phi, \partial_\mu \phi) \tag{6}$$

Integrating over time gives the action, usually called S:

$$S = \int dt L = \int d^4x \mathcal{L} \tag{7}$$

By varying S wrt ϕ and $\partial_\mu \phi$ and $\partial_\mu \phi^*$ we obtain the Euler-Lagrange equations (this is derived at the end of the notes in the section on Noethers theorem):

Lagrangian Formulation

$$\partial_\mu \left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \right) - \frac{\delta \mathcal{L}}{\delta \phi} = 0 \tag{8}$$

The Lagrangian density for the KG equation is

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \tag{9}$$

Substituting this into the Euler-Lagrange equations gives

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0 \tag{10}$$

Note:

$$\left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \right) = \partial^\mu \phi^* \tag{11}$$

The Lagrangian density for Maxwells equations in vacuum is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{12}$$

Lagrangian Formulation

Here we consider $\mathcal{L}$ as a function(al) of fields A_μ and derivatives $\partial_\mu A_\nu$. ie since A_μ has four components, we treat each component as a separate field.

In the presence of a current j_μ there is an additional interaction term

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu \tag{13}$$

Lagrangian Formulation

Consider a small transformation in a field which leaves the Lagrangian invariant

$$\Psi \rightarrow \Psi + i\alpha\Psi \tag{14}$$

$$0 = \delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\Psi}\delta\Psi + \left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\right)\delta(\partial_\mu\Psi) + c.c. \tag{15}$$

so

$$\begin{aligned} 0 &= i\alpha\Psi\frac{\delta\mathcal{L}}{\delta\Psi} + i\alpha\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\right)(\partial_\mu\Psi) + .. \\ &= i\alpha\left[\frac{\delta\mathcal{L}}{\delta\Psi} - \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\right)\right]\Psi + i\alpha\partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\Psi\right) + ... \end{aligned} \tag{16}$$

where, to get to the last line from the previous one we use that:

$$\partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\Psi\right) = \left(\partial_\mu\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\right)\Psi + \left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\right)\partial_\mu\Psi \tag{17}$$

Going back to the previous expression, the equation before the one above, there are several key points:

Lagrangian Formulation

- The last term in (14) is proportional to a total derivative. Hence, it only contributes to the action at the boundary of space-time ie at infinity. Requiring this term to vanish at infinity implies that: **the action is extremised ($\delta S = 0$) exactly when the Euler-Lagrange equations are satisfied (the term in square brackets)** . We have thus **derived** the Euler-Lagrange equations.
- In the case that we require that $\Psi \to \Psi + i\alpha\Psi$ is a **symmetry** of the action, then, because the terms in square brackets vanish due to the Euler-Lagrange equations, the total derivative term must vanish **everywhere** not just at infinity. Hence,
- **When a variation of the fields is a SYMMETRY of the action, there exists a conserved quantity**: $j^\mu \equiv \left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \Psi)}\Psi\right)$ which obeys:

$$\partial_\mu \left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \Psi)}\Psi\right) = 0 \quad (18)$$

Lagrangian Formulation

Recall that to consider the motion of a particle of charge $-e$ in an electromagnetic field generated by a vector potential A^μ we replace the derivative ∂_μ by

$$\partial_\mu \to \partial_\mu - ieA_\mu \tag{19}$$

We call the rhs of this expression a **covariant derivative**. This is usually denoted by D_μ:

$$D_\mu \equiv \partial_\mu - ieA_\mu \tag{20}$$

By making the above replacement in the Dirac equation equation we obtain

$$i\gamma^\mu \partial_\mu \Psi + m\Psi = -e\gamma^\mu A_\mu \Psi \tag{21}$$

i.e. we get a "potential" for the field Ψ. Since the modified equation of motion was obtained by replacing ∂_μ with D_μ, the Lagrangian density for a charged fermion with mass m is

$$\begin{aligned} \mathcal{L} &= i\bar{\Psi}\gamma^\mu D_\mu \Psi + m\bar{\Psi}\Psi \\ &= i\bar{\Psi}\gamma^\mu \partial_\mu \Psi + m\bar{\Psi}\Psi + e\bar{\Psi}\gamma^\mu A_\mu \Psi \end{aligned} \tag{22}$$

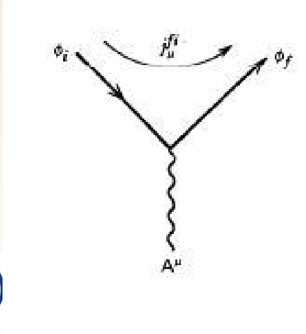
This is *exactly*:

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi + m\bar{\Psi}\Psi + j^\mu A_\mu \tag{23}$$

Lagrangian Formulation

The fact that $j^\mu A_\mu$ *appears in* $\mathcal{L}$ *suggests that we can just "read off" the vertices allowed in Feynman diagrams from* $\mathcal{L}$. This is a general rule for any Lagrangian! **We just read off the Feynman rules from** $\mathcal{L}$.

In this example, the three point vertex between the photon and two charged particles is represented in $\mathcal{L}$ by the presence of the $j^\mu A_\mu$ term.



Lagrangian Formulation

The Lagrangian for Quantum Electrodynamics, QED has various symmetries.

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\gamma^{\mu} D_{\mu}\Psi + m\bar{\Psi}\Psi \tag{24}$$

Lorentz Invariance. Since all the Lorentz indices are contracted ($\mathcal{L}$ is a scalar), the Lagrangian is invariant under Lorentz transformations.

Internal Symmetry. In addition to this "spacetime symmetry" it is invariant under an internal symmetry i.e. one which does not act on the coordinates, but just on the fields. This is intrinsic to electromagnetism and the other forces as we will see.

Gauge Symmetries

We are going to consider a transformation of ϕ by a **unitary, 1-by-1** matrix, a $U(1)$ transformation.
Any such matrix U is of the form $U(\alpha) = e^{i\alpha}$. α can take any continuous value between zero and 2π.
Clearly, under

$$\Psi \to U\Psi \quad (25)$$

we have that

$$D_\mu \Psi \to U D_\mu \Psi \quad (26)$$

L is clearly invariant under this transfomation since it is of the form $D_\mu \Psi$ times the complex conjugate of Ψ, and $UU^* = 1$. Similarly $\bar{\Psi}\Psi$ is invariant, so the mass term is invariant. Therefore $\mathcal{L}$ is invariant under this transformation of Ψ.
Noethers theorem proves that there is a conserved quantity when a Lagrangian is invariant under a symmetry transformation. In fact, when α is small i.e. when $U \approx 1 + i\alpha$ we see that the conserved current j^μ is precisely that which we derived before.

Gauge Symmetry

Now, we would like to consider when α is different from point to point in spacetime. i.e. we make $\alpha = \alpha(x_\nu)$ – a function of the coordinates. Now, we will get terms proportional to derivatives of α.

$$D_\mu \Psi \rightarrow U\partial_\mu \Psi + iU\partial_\mu \alpha \Psi - ieUA_\mu \Psi \quad (27)$$

$$= UD_\mu \Psi + iU\partial_\mu \alpha \Psi \quad (28)$$

Because of this, the Lagrangian is no longer invariant. However, the unwanted term in the transformation of $D_\mu \Psi$ can be removed if A_μ also transforms:

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha \quad (29)$$

which can be verified by replacing this transformed A_μ in the "unwanted" term. Thus:

$$D_\mu \Psi \rightarrow e^{i\alpha(x)} D_\mu \Psi \quad (30)$$

and the Dirac terms Lagrangian are invariant under this **gauge transformation** (this is the name for transformations whose parameters depend on the coordinates. What about the Maxwell term in the Lagrangian?

Electromagnetism and $U(1)$ Gauge Symmetry

- Maxwells equations can be written in terms of a gauge potential A_μ. $A_0 =$ scalar potential and $A_i =$ the vector potential
- The electric field has components $E_i = \partial_0 A_i - \partial_i A_0$
- The magnetic field has components $B_1 = \partial_2 A_3 - \partial_3 A_2$ plus $1 \to 2 \to 3 \to 1$
- They are unified into the EM field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Maxwells equations in vacuum are then:
- $\partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0 \to \dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$
- $\partial^\mu F_{\mu\nu} = 0 \to \dot{\mathbf{E}} - \nabla \times \mathbf{B} = 0$
- $\partial^\mu F_{\mu\nu} = 0 \to \nabla \cdot \mathbf{E} = 0$

Electromagnetism and $U(1)$ Gauge Symmetry

- Maxwells equations posess a large, infinite dimensional symmetry.
- Consider an **arbitrary** function $\lambda(x, y, z, t)$. (So $\lambda = \frac{1}{e}\alpha$)
- Then replace $A_\mu \longrightarrow A'_\mu = A_\mu + \partial_\mu \lambda$
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \rightarrow \partial_\mu A_\nu + \partial_\mu \partial_\nu \lambda - \partial_\nu A_\mu - \partial_\nu \partial_\mu \lambda$
- Thus, since $\partial_\mu \partial_\nu \lambda = \partial_\nu \partial_\mu \lambda$, $F'_{\mu\nu} = F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- So, the gauge potentials A_μ and A'_μ describe **identical** $\mathbf{E}$ and $\mathbf{B}$ fields!
- These are the equations required by all electronic devices and technology and they have an infinite dimensional symmetry!
- **So QED is invariant under $U(1)$ gauge transformations**

Gauge Symmetry as a Principle

If we use gauge symmetry as a principle then it has far reaching consequences.

- The covariant derivative **must** be introduced otherwise the kinetic energy term would not be gauge invariant.
- This requires the introduction of a vector field A_μ which couples to the matter current. A_μ is usually called the gauge field.
- If we consider the kinetic energy of the gauge field, then gauge invariance requires it to be of the form $F_{\mu\nu}F^{\mu\nu}$ (or more generally a function thereof). Thus, Maxwell's equations follow from **gauge symmetry plus Lorentz symmetry**
- The photon is massless

Gauge Symmetry as a Principle

This last point is crucial. It provides an **explanation** for why the photon essentially behaves as a massless particle. (Experimentally of course one cannot prove that the photon is exactly massless. Rather, one obtains an upper limit on its mass. The current upper limit is about 10^{-18} eV.) To see why the photon is massless, we ask: **what would a mass term look like?**:

$$\partial_\nu \partial^\nu A_\mu + m^2 A_\mu = 0 \tag{31}$$

This mass term would arise from a term in the Lagrangian of the form

$$\Delta\mathcal{L} \sim -m^2 A_\mu A^\mu \tag{32}$$

Such a term is clearly not invariant under the gauge transformation of A_μ, which is

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha \tag{33}$$

Gauge Symmetry as a Principle

In fact, combining all of these points, the most general gauge and Lorentz invariant Lagrangian which is up to quadratic in the fields and their derivatives is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi}\gamma^{\mu} D_{\mu}\Psi + m\bar{\Psi}\Psi \tag{34}$$

Here we used the fact that $[A_\mu] = [\partial_\mu] = [M]$ and $[\Psi] = [M]^{3/2}$ so that $[\mathcal{L}] = [M]^4$ (energy density)

Thus: the symmetries determine the Lagrangian and, hence, the physics

This is a key point in particle physics. The Lagrangian for the Standard Model is essentially determined by its symmetries. In other words, symmetries determine the physics of all elementary particles!

Beyond $U(1)$ gauge symmetry

We would now like to consider generalising $U(1)$ gauge theory (i.e. QED) to $U(N)$ gauge theory. That is to say that U – the transformation matrix – will become a *Unitary* $N \times N$ matrix U^i_j where i, j run from 1 to N each. An $N \times N$ matrix acts naturally on N component vectors v_i. Hence we should introduce N complex fields Ψ_i on which these act. Under

$$\Psi_i \to U^j_i \Psi_j \tag{35}$$

we would like to impose a condition that a suitable covariant derivative transforms acting on Ψ_i transforms in the same way $\to U^\dagger U = \mathbf{1_N}$ (Identity matrix) then would guarantee that

$$\bar{\Psi}^j \gamma^\mu D_\mu \Psi_j \longrightarrow \bar{\Psi}^k {U^\dagger}^m_k U^n_m \gamma^\mu D_\mu \mathbf{\Psi} = \bar{\Psi}^k \gamma^\mu D_\mu \Psi_k \tag{36}$$

This would be the $N \times N$ generalisation of the $U(1)$ case.

Beyond $U(1)$ gauge symmetry

But what is this covariant derivative?
If we try to introduce a gauge field, in general it is **a matrix of gauge fields** i.e. we can have up to $N \times N$ gauge fields:

$$(D_\mu \phi)_i = \partial_\mu \phi_i - ig(G_\mu)_i^j \phi_j \tag{37}$$

that is, for each of the values of i and j, $(G_\mu)_j^i$ is a different gauge field.
So if $U(N)$ gauge symmetry is a principle it requires N matter particles Ψ_j and N^2 gauge bosons $G_\mu)_i^j$
How is this matrix of gauge fields defined?

Unitary matrices, exponentials, group generators and all that

$$\exp i\theta = 1 + i\theta - \frac{\theta^2}{2} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + ... = \cos\theta + i\sin\theta \quad (38)$$

Now consider the rotation matrix

$$R(\theta) \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (39)$$

We want to Taylor expand this matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} + ... & \theta - \frac{\theta^3}{3!} + ... \\ -\theta + \frac{\theta^3}{3!} + ... & 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4!} + ... \end{pmatrix} \quad (40)$$

Just as $e^{i\theta}$ is the exponential of a 1-by-1 matrix, the rotation matrix above is the exponential of a two-by-two constant matrix:

$$R(\theta) = \exp(i\theta T) = \mathbf{1} + i\theta T - \frac{\theta^2}{2}T^2 - i\frac{\theta^3}{3!}T^3 + .. :\rightarrow T = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (41)$$

and T^2 is the matrix product of T with itself.

Beyond $U(1)$ gauge symmetry

So, since any rotation can be written as $\exp i\theta T$ we say that T **generates** the rotations. Now consider some other examples of this, because we will need matrices like T to define the covariant derivative properly.
Let U be a N-by-N unitary matrix ie

$$U^{\dagger}.U = \mathbf{1} \tag{42}$$

Now assume that

$$U = \exp iM \tag{43}$$

What properties does M have? unitarity implies that

$$M = M^{\dagger} \quad so \quad M \quad is \quad \textbf{Hermitian}. \tag{44}$$

If, additionally, we require that det$(U) = 1$ i.e. that U is **special unitary**, then one can show that the trace of M is zero

$$\det U = 1 \leftrightarrow \mathrm{Tr} M = 0 \tag{45}$$

$SU(2)$ gauge symmetry

When $N = 2$, one can show that M is a linear combination of the Pauli matrices:

$$M = \alpha^a \sigma_a \tag{46}$$

where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{47}$$

That is to say that the Pauli matrices **generate** $SU(2)$ transformation matrices! An important fact about the Pauli matrices is that they obey an algebra:

$$[\sigma_a, \sigma_b] = 2i\epsilon_{ab}{}^c \sigma_c \tag{48}$$

ϵ_{abc} is totally antisymmetric i.e. if we interchange neighbouring indices then we get a minus sign:

$$\epsilon_{abc} = -\epsilon_{bac} = \epsilon_{bca} = -\epsilon_{cba} \;\; etc \tag{49}$$

and $\epsilon_{123} = 1$

$SU(N)$ gauge symmetry

In general, for an $SU(N)$ matrix

$$U = \exp iM \tag{50}$$

where M is traceless and Hermitian, there are $N^2 - 1$ generators T_a such that

$$M = \alpha^a T_a \tag{51}$$

and the T_a 's obey an algebra

$$[T_a, T_b] = i f_{ab}{}^c T_c \tag{52}$$

where the $f_{ab}{}^c$ are constants called the structure constants. For $SU(N)$ the *algebra* defined by the above equation is called the Lie Algebra of $SU(N)$. If you choose a basis for this algebra, you can explicitly calculate the structure constants in that basis. They are totally antisymmetric, like ϵ_{abc}. For $SU(3)$ there is a basis for the eight 3-by-3 T_a matrices which is used a lot in particle physics called the Gell-Mann basis. You can find these eight matrices in the textbooks.

$SU(N)$ gauge symmetry

Finally(!)

$$D_\mu = \partial_\mu + igT_a G^a_\mu \tag{53}$$

where the second term is an N-by-N matrix since T_a is a matrix. There are $N^2 - 1$ gauge fields G^a_μ.

The Standard Model is a gauge theory with $SU(3)$ gauge symmetry, $SU(2)$ gauge symmetry and $U(1)$ gauge symmetry. There are $8 + 3 + 1 = 12$ gauge bosons. These are the eight gluons, the two W-bosons (W^+ and W^-), the neutral Z boson and the photon.

The full covariant derivative is thus

$$D_\mu = \partial_\mu - i\frac{Y}{2}g_1 B_\mu - ig_2\frac{\sigma_j}{2}W^j_\mu - ig_3\frac{\lambda_a}{2}G^a_\mu \tag{54}$$

B_μ is the gauge boson of the $U(1)$. The photon is a linear combination of B_μ and W^3_μ. The Z-boson is the opposite linear combination. Y is called the hypercharge. The charge under $SU(2)$ is called isospin I. The proton has isospin $1/2$ and the neutron $-1/2$.

$SU(N)$ gauge symmetry

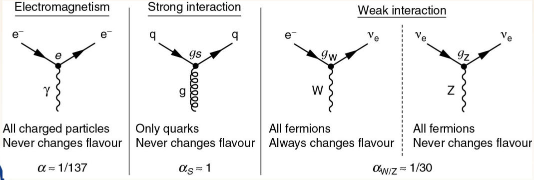
Most of the physics of all non-dark matter comes from this covariant derivative!

Putting this covariant derivative into the Dirac Lagrangian gives

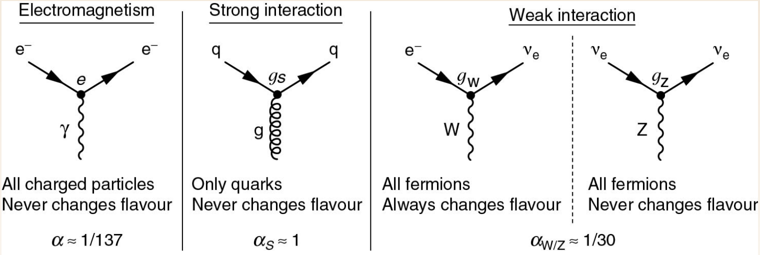
$$\mathcal{L} = i\bar{\Psi}\gamma^\mu D_\mu \Psi = i\bar{\Psi}\gamma^\mu \partial_\mu \Psi + j_1^\mu B_\mu + j_i^\mu W_\mu^i + j_a^\mu G_\mu^a \quad (55)$$

where we have three currents, one for each symmetry group.

So we have three interaction terms which lead to the Feynman diagrams for the quarks and leptons interacting with the gauge bosons



Standard Model Interactions



By putting these together we can describe **any physical process** involving Standard Model particles!

Calculating the decay properties of W-bosons

We need the formula

$$-ig_2\frac{\sigma_j}{2}W_\mu^j = -i\frac{g_2}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} \tag{56}$$

which we re-write as:

$$-i\frac{g_2}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} = -i\frac{g_2}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix} \tag{57}$$

we have introduced $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$ which are the actual positive and negative charge eigenstates. All the left-handed fermions pair up as doublets under the $SU(2)$ gauge symmetry i.e.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \tag{58}$$

SM Interactions

Remember that each of the entries of these doublets are 4-component Dirac spinors. We have suppressed the spinor labels in the last equation. Similarly the Lagrangian depends on the "bars" of all of these multiplets and hence the couplings to the W-bosons also involves the right-handed anti-particles to all of the above e.g. right handed-positrons also couple to W's, but left-handed positrons do not. Let's write the $SU(2)$ gauge interactions for the muon doublet.

Muon and Muon-neutrino Interactions with W bosons

It is

$$\left(\bar{\nu}_\mu \;\; \bar{\mu}^-\right)\gamma^\nu \times g_2 \begin{pmatrix} W_\nu^3 & \sqrt{2}W_\nu^+ \\ \sqrt{2}W_\nu^- & -W_\nu^3 \end{pmatrix} \frac{1}{2}(\mathbf{1}-\gamma^5) \times \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \tag{59}$$

The "i"'s have multiplied to 1. The $\times$'s are there to remind us we have to multiply the matrices together. The $\frac{1}{2}(\mathbf{1}-\gamma^5)$ is there to project onto the left-handed component of the fermion doublet to the right of the matrix of W-bosons. The expression above is equivalent to

$$\begin{aligned} \sqrt{2}\frac{g_2}{2}\bar{\mu}_L^-\gamma^\nu\nu_{\mu L}W_\nu^- &\quad + \quad \sqrt{2}\frac{g_2}{2}\bar{\nu}_{\mu L}\gamma^\nu\mu_L^-W_\nu^+ \\ -\frac{g_2}{2}\bar{\mu}_L^-\gamma^\nu\mu_L^-W_\nu^3 &\quad + \quad \frac{g_2}{2}\bar{\nu}_{\mu L}\gamma^\nu\nu_{\mu L}W_\nu^3 \end{aligned} \tag{60}$$

The first line above is the charged current interaction. The second is a neutral current interaction. If we were to change any of the four coefficients, the Lagrangian would no longer be $SU(2)$ gauge invariant. The relative strengths of these interactions $\sqrt{2}:1$ is thus fixed by symmetry.

SM Interactions

For all of the six fermion doublets, there is a similar expression for the $SU(2)$ interaction terms in the Lagrangian. The interactions we have derived lead to some remarkable consequences:

- The couplings of W^+ and W^- are **universal** for both quarks and leptons. e.g. the interaction between W^+, u and $\bar{d}$ is the *same* as the interaction between W^+, μ and $\bar{\nu}$.
- W^+ can decay into $(e^+, \bar{\nu}_e), (\mu^+, \bar{\nu}_\mu), (\tau^+, \bar{\nu}_\tau), (u, \bar{d}), (c, \bar{s})$
- Similarly for W^-.
- W bosons cannot decay into $(t, \bar{b})$ because $m_t \sim 173$GeV± 1GeV and $m_W \sim 80.4$GeV.
- Since m_W is much larger than $m_e, m_\mu, m_\tau, m_u, m_d, m_c, m_s$ the decay width of the $W^\pm$ doesn't "care" about the fermion masses
- So the partial decay widths

$$\Gamma(W^+ \to e^+\bar{\nu}) = \Gamma(W^+ \to \mu^+\bar{\nu}) \quad (61)$$

$$= \Gamma(W^+ \to \tau^+\bar{\nu}) \quad (62)$$

- The decays into quarks are not just two decay channels: $(u\bar{d})$ and $(c, \bar{s})$,

W-boson decays

- The decays into quarks are not just two decay channels: $(u\bar{d})$ and $(c, \bar{s})$, but three each, since there are three u quarks, three d quarks, three c quarks and three s quarks. This is because the quarks transform under the $SU(3)$ gauge symmetry (the leptons do not). So the decays into quarks are actually six channels.
- This gives a total of three leptonic and six hadronic decay channels, nine in total.
- If $\Gamma(W^+ \to all)$ is the total W decay width, the Standard Model predicts that

$$\frac{\Gamma(W^+ \to e^+\bar{\nu})}{\Gamma(W^+ \to all)} = \frac{\Gamma(W^+ \to \mu^+\bar{\nu})}{\Gamma(W^+ \to all)} \tag{67}$$

$$= \frac{\Gamma(W^+ \to \tau^+\bar{\nu})}{\Gamma(W^+ \to all)} \tag{68}$$

$$= 1/9 \tag{69}$$

$$\frac{\Gamma(W^+ \to hadrons)}{\Gamma(W^+ \to all)} = 6/9 = 2/3 \tag{70}$$

W-boson decays – what the data says

http://pdg.lbl.gov is where we "record" data about all known particles "The Particle Data Book"

W-boson decays

W-boson decays

W^+ DECAY MODES

W^- modes are charge conjugates of the modes below.

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\ell^+\nu$	[a]	(10.80 ± 0.09) %	
Γ_2	$e^+\nu$		(10.75 ± 0.13) %	
Γ_3	$\mu^+\nu$		(10.57 ± 0.15) %	
Γ_4	$\tau^+\nu$		(11.25 ± 0.20) %	
Γ_5	hadrons		(67.60 ± 0.27) %	
Γ_6	$\pi^+\gamma$		$< 8 \times 10^{-5}$	95%
Γ_7	$D_s^+\gamma$		$< 1.3 \times 10^{-3}$	95%
Γ_8	cX		(33.4 ± 2.6) %	
Γ_9	$c\bar{s}$		$(31 {}^{+13}_{-11})$ %	
Γ_{10}	invisible	[b]	(1.4 ± 2.9) %	

[a] ℓ indicates each type of lepton (e, μ, and τ), not sum over them.

[b] This represents the width for the decay of the W boson into a charged particle with momentum below detectability, $p < 200$ MeV.

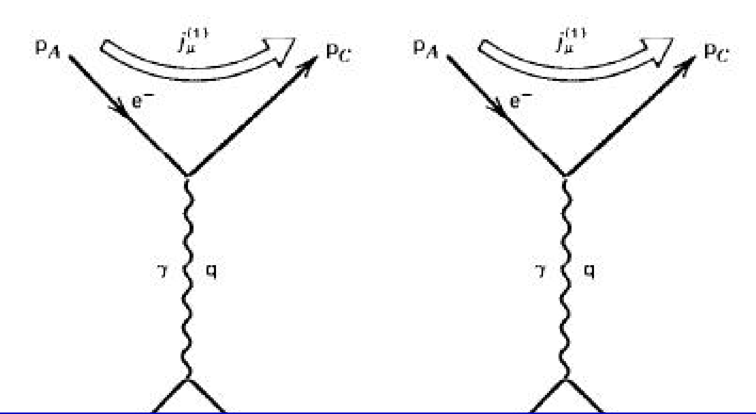
W-boson decays

W-boson decays

- These branching fractions have been measured and agree with the Standard Model predictions to within 1%. Have a look at the PDG.
- This excellent agreement between theory and experiment represents a thorough check of the structure of the Standard Model. The interactions are derived entirely from symmetry principles. If we changed the number of leptons the result would change. If there were four u quarks instead of three (which would be the case if $SU(3)$ were replaced with $SU(4)$) the result would change.
- Note: gauge symmetry, Lorentz invariance and charge conservation allows the possibility of **flavour-changing decays** that involve different quark families e.g. $W^+ \to c\bar{b}$. In fact these decays also occur, but are suppressed by so-called CKM-mixing (Cabibbo-Kobayashi-Maskawa). We will not discuss this in any detail.

Cross-section estimates for particle collisions

In particle physics experiments we measure quantities like the lifetime of a decaying particle or the rate of production of particles from a collider experiment. We need to be able to calculate quantities like these, starting with the invariant amplitude $\mathcal{M}$ for such a process.



Collider cross-sections

In a particle collider experiment, we collide beams of particles with some given **flux** (also called **luminosity** and then we measure the particles that come out of the collisions. The total number of events is obviously proportional to the flux since e.g. if we increase the number of protons in the LHC beams we will increase the total number of collision events that we get. Thus, the luminosity is something that we control as experimenters in a laboratory. It is not an **intrinsically** physical entity.

However, the proportionality "constant" in the relation

$$N_{events} \propto \text{Luminosity} \equiv \mathcal{L} \tag{71}$$

is an **intrinsically physical** quantity. This is called the **cross-section** and is usually labeled by σ.

$$N_{events} = \mathcal{L}\sigma \tag{72}$$

Cross-section

The left hand side is dimensionless, a whole number if we count the number of events (or collisions) after a given time interval. Or we could consider the number of events per second (or any other unit of time). The luminosity is thus a flux of particles per unit time. This has dimensions of $[\mathcal{L}] = [L]^{-2}[T]^{-1} = [M]^3$, where the last equality is because we use natural units. The dimensions of luminosity are like this because it is essentially the number of particles going through a given area i.e. number of particles per unit area per second.
So, by dimensional analysis, the cross-section σ has dimensions of AREA.
In fact, that is where its name comes from. It is, effectively, the area over which the interaction takes place.

Fermi's Golden Rule (post-modernist view)

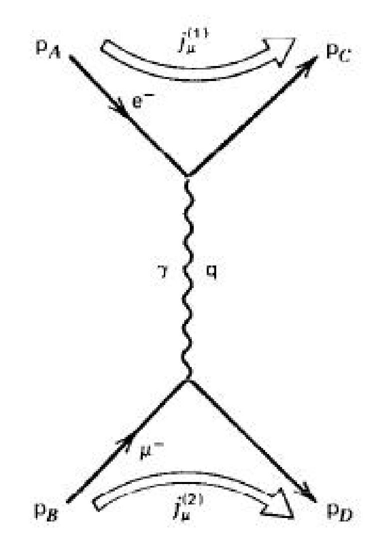
- Take a process with initial and final states
- Draw the Feynman diagram for this
- Use the Feynman rules to convert the diagram into a "matrix element" $\mathcal{M}$
- The probability density for this process is proportional to "$P \sim |\mathcal{M}|^2 \rho(E)$
- $\rho(E)$ represents the phase space density of states subject to energy/momentum conservation

Cross-section estimates

Lets get a feel for σ by estimating it for various processes. Our estimates are based on Fermi's Golden Rule plus dimensional analysis in natural units. We will also compare our results to the "properly calculated" results as well as the actual experimental observations.

1. $\sigma(e^+e^- \to \mu^+\mu-)$ at high energies. 2. $\sigma(\nu N \to X)$ 3. $\sigma(pp \to X)$

$\sigma(e^+e^- \rightarrow \mu^+\mu-)$



$\sigma(e^+e^- \to \mu^+\mu-)$

We would like to estimate the cross-section for this at high centre of mass energies $\sqrt{s}$ much greater than m_μ.
Fermi's Golden Rule tells us this is proportional to the "square of the Feynman diagram" for this process:

$$\sigma \propto e^4 \sim \alpha^2 \quad (73)$$

because we have a factor of the charge at each vertex. $\alpha \equiv \frac{e^2}{4\pi}$ is dimensionless, a number with no units. But σ has dimension $[M]^{-2}$. The only other scale in this problem is the center of mass energy $\sqrt{s}$. Hence we expect that

$$\sigma \sim \frac{\alpha^2}{s} \quad (74)$$

$\sigma(e^+e^- \to \mu^+\mu-)$

The actual "properly calculated" leading order result is

$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{s} \tag{75}$$

so, we were off by a factor of four or so. Not bad!

The cross-section for this process has been measured at various energies, as shown in the figure. The smooth line on this graph shows the theoretical prediction from the above equation and the different 'markers' are actual, experimentally measured values. The agreement between the theory and experiment is very good. Notice the vertical lines which eminate from the different experimental points.

Since no experiment is **perfect**, every reported measurement is **uncertain** by a certain amount depending on either the limitations of the experimental apparatus and/or the size of the data sample available. The latter, statistical uncertainty, is reduced when a larger data sample becomes available. The vertical lines on the graph are the total uncertainty on the measurement made, so the actual cross-section is somewhere in between the "band" represented by the line.

$\sigma(e^+e^- \to \mu^+\mu-)$

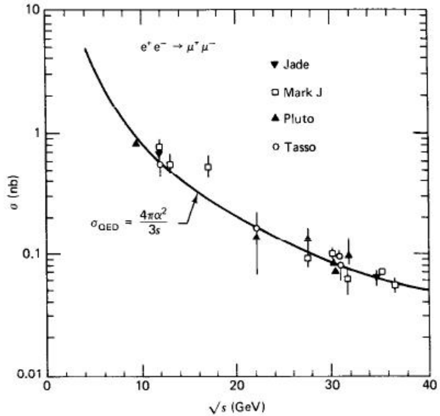


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

$\sigma(e^+e^- \rightarrow \mu^+\mu-)$

Putting actual numbers to the cross-section we get:

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) \sim \frac{4 \times 10^{-32}}{s/\mathrm{GeV}^2}\mathrm{cm}^2 \tag{76}$$

So in order to produce a few muon pairs in electron-positron scattering at a centre-of-mass energy of one GeV, you need to have a luminosity of order 10^{32}cm^{-2}. At higher energies, since the cross-section decreases quadratically with energy, you need much higher luminosities to produce the same number of muons.

For comparison, the luminosity of the proton-proton beams at full luminosity is 10^{34}cm^{-2} **per second!**

$\sigma(\nu N \to X)$

Here we consider neutrinos interacting with nucleons in matter. Neutrinos are not electrically charged so they don't couple to photons. Nor do they participate in the strong nuclear interactions. But they do undergo weak interactions, eg via exchanging W-bosons and Z-bosons.

Centre-of-Mass Frame and Laboratory Frame

The centre of mass frame for a collision of two particles is one in which the two particles have equal and opposite momentum and equal energies. The Lorentz 4-vectors for the two particles are

$$p_1^\mu = (1/2E_{cm}, p^i) \quad (77)$$
$$p_2^\mu = (1/2E_{cm}, -p^i) \quad (78)$$

We can calculate the Lorentz invariant quantity s:

$$s = (p_1 + p_2)_\mu (p_1 + p_2)^\mu = E_{cm}^2 \quad (79)$$

In the lab frame one of the particles is at rest and the other is moving.

$$p_1^\mu = (M, 0) \quad (80)$$
$$p_2^\mu = (E_{lab}, p_{lab}^i) \quad (81)$$

In the lab frame

$\sigma(\nu N \to X)$

Neutrinos do not interact with photons and gluons, but they do interact with W and Z bosons. Hence, to interact with other particles, they must **exchange** W or Z bosons. Since quarks also interact with W's and Z's, neutrinos can interact with atoms.

This "costs" energy and as a result, the Feynman diagram (for neutrino energies small compared to m_W) will have a factor of $\frac{1}{m_W^2}$ or $\frac{1}{m_Z^2}$. Hence, the cross-section will have a factor of $\frac{1}{m_W^4}$. If we add the coupling constants at the vertices as well, these combine with this factor to give what is usually called G_F^2.

G_F is the Fermi constant – a number of order 10^{-5}GeV^{-2}.

$\sigma(\nu N \to X)$

$$\sigma(\nu N) \sim G_F^2 \tag{83}$$

But σ must have mass dimension minus two. At high energies, the only other scale in the problem is s which has mass dimension two.
Therefore, we expect that

$$\sigma(\nu N) \sim G_F^2 s \tag{84}$$

In most experimental situations with neutrinos we are normally in the lab frame, scattering a beam of neutrinos off a fixed target. e.g. the nucleons could be a "block" of matter and the neutrino beam is "fired" into it.
Therefore $s \sim 2E_\nu m_N$

$\sigma(\nu N \to X)$

Using the fact that $m_N \sim 1$ GeV and that $G_F \sim 10^{-5}$ GeV^{-2} we get

$$\sigma(\nu N) \sim \text{afew} \times 10^{-38} \frac{E_\nu}{\text{GeV}} \text{cm}^2 \tag{85}$$

which is again in agreement with the "proper calculation" to within a factor of 10.

Notice that this is a much smaller cross-section than the previous one we estimated for a fixed centre-of-mass.

How far can neutrinos propagate through matter?

Imagine a neutrino which has been emitted by the sun and arrives at the Earth. Lets estimate how far a neutrino can propagate in the Earth before it actually interacts with a proton or neutron in the Earth.

Obviously the rate is proportional to both the cross-section for the reaction per nucleon (as estimated above) and the density of nucleons i.e. the density of the Earth, ρ. The greater the reaction rate, the shorter the distance a neutrino can propagate before interacting. Thus, we have, the average propagation distance L before an interaction takes place is:

$$L \propto \frac{1}{\rho\sigma} \tag{86}$$

where σ is calculated above for neutrinos interacting with nucleons and ρ is the mass per unit volume of the matter through which the neutrino propagates. Now use dimensional analysis. TL has dimensions of $[L] = [M]^{-1}$. The RHS has to have the same dimensions. This will help us to fix the proportionality constant in the above. ρ has dimensions of $[M]^4$ and $[\sigma] = [M]^{-2}$. Thus, if

$$L = \frac{C}{\rho\sigma} \tag{87}$$

$\sigma(\nu N \to X)$

Thus, if

$$L = \frac{C}{\rho\sigma} \tag{88}$$

the dimension of C is $[C] = [M]$. Therefore we are looking for a quantity which plays an important role in the interaction between a neutrino and a nucleon with the dimensions of mass. The obvious candidate is the nucleon mass, $m_N \sim 1$ GeV. We therefore find:

$$L = \frac{m_N}{\rho\sigma} \tag{89}$$

Notice that $\frac{\rho}{m_N}$ is essentially the number of atoms per unit volume in the Earth. Let us call this N. Hence we see that

$$L = \frac{1}{N\sigma} \tag{90}$$

$\sigma(\nu N \to X)$

If we take $\rho \sim 10^3 kg/m^3 \sim 10^{30}$ GeV$/m^3$. Since $m_N \sim$GeV we have that $N \sim 10^{30}/m^3$. We have calculated σ above. For a neutrino with energy of order 1 GeV

$$\sigma \sim 10^{-38} cm^2 = 10^{-42} m^2 \tag{91}$$

Hence, neutrinos with energies of order a GeV propagate roughly $10^{12}m$ through water before interacting! This is a billion kilometres. Most of the neutrinos from the sun have energies which are one hundred or more times less than a GeV and, hence they propagate much further.
But, there are a lot of neutrinos from the sun, so we can still study their interactions!!

$\sigma(pp \to X)$

This cross-section is different to those above because hadrons are **not** point particles. Rather, they are bound states of quarks, anti-quarks and gluons, bound together by the strong nuclear force. The strong nuclear force is the $SU(3)$ part of the Standard Model. The remarkable thing about the strong nuclear force is that all hadrons have masses which are of order a GeV. In fact, most of the particles described in the PDG are hadrons (either mesons or baryons). If you look at their masses, they are all within one order of magnitude of the proton mass. This reflects the fact that the strong nuclear force is characterised by a scale $\Lambda \sim$ GeV. This is known as the QCD scale since the underlying theoretical description of the strong nuclear interaction is called Quantum Chromodynamics.

$\sigma(pp \to X)$

Λ is essentially the **binding energy** of the quarks, anti-quarks and gluons inside any hadron. Since the u d and s quarks have masses which are much smaller than Λ, the masses of hadrons made of these quarks are mostly binding energy. Therefore your mass, and the masses of all the stars in the Universe is binding energy of the strong nuclear force. The b and c quarks have masses of order Λ itself, so c and b hadrons have masses which are not just binding energy. The t quark, which is the most massive known elementary particle ($m_t \sim 173\text{GeV} \pm 1$ GeV) actually decays before it has time to "hadronise" and form a hadron. This is because $\tau_t = \frac{1}{\Gamma_t} < \frac{1}{\Lambda}$.

$\sigma(pp \to X)$

We want to calculate the cross-section for scattering two hadrons which interact via the strong nuclear interaction. We have just seen that everything about the strong nuclear force is characterised by a single scale Λ. Hence, we expect that the effective cross-section for strong nuclear interactions is also determined by Λ. Hence,

$$\sigma(pp \to X) \sim \frac{1}{\Lambda^2} \tag{92}$$

It is Λ^{-2} on dimensional grounds. This has the dimensions of a cross-section. Since GeV$^{-1} \sim 10^{-15}$m,

$$\sigma(pp \to X) \sim 10^{-30} m^2 = 10^{-26} cm^2 \tag{93}$$

$\sigma(pp \to X)$

Now, the in 2012, the LHC was running with an instantaneous luminosity of about $\mathcal{L} = 10^{33} cm^{-2} s^{-1}$ at a centre of mass energy of 8 TeV. Hence, with our rough estimate, we expect $\mathcal{L}\sigma \sim 10^7$ events every second! Actually, our estimate is around a factor of 10 smaller than the actual answer so we are producing even more collisions than that.
The greater the number of LHC collisions, the greater the probability of creating a "rare" event such as the production of a Higgs boson. The cross-section for producing a Higgs boson with a mass of around 126 GeV at the LHC is about $10^{-35} cm^2$. This means that we have to "sift through" around a billion events for every Higgs boson produced. The search for the Higgs is thus very much like looking for a needle in a haystack.

Exercise: The Higgs boson mass is approximately 126GeV. How many Higgs bosons were produced in the 2012 run of the LHC? For this you need to find out how much data was recorded i.e. the total integrated luminosity.

Helicity: fermions are left or right handed

The following 4-by-4 matrix:

$$\Lambda \equiv \frac{1}{2} \begin{pmatrix} \sigma_i \hat{p}^i & 0 \\ 0 & \sigma_i \hat{p}^i \end{pmatrix} \tag{94}$$

where $\hat{p}^i = \frac{p^i}{|p|}$ satisfies $\hat{p}^i \hat{p}_i = 1$ i.e. is the unit vector which points in the same direction as p^i. Λ commutes with the Hamiltonian. Therefore, the eigenvalues of Λ are conserved.

$\frac{1}{2}\sigma_i \hat{p}^i$ is clearly the spin projected in the direction of motion. We call this the **helicity** of the state. The possible eigenvalues of $\frac{1}{2}\sigma_i \hat{p}^i$ are just $\lambda = \pm\frac{1}{2}$.

We say that fermions with helicity $-1/2$ are left-handed and those with helicity $1/2$ are right-handed.

Parity Violation

- The left and right handed fermions have **different** interactions with W and Z bosons.
- In fact, only left handed quarks and leptons couple to W-bosons
- Equivalently, only right handed anti-quarks and anti-leptons couple to W-bosons
- This violates parity (maximally).
- The weak interactions are not the same if you look at them in a "mirror"
- This is the only known violation of parity in the fundamental laws of physics

Chiral Fermions have no mass terms

A fermion mass would be

$$\mathcal{L} \sim m\bar{\Psi}\Psi \tag{95}$$

One can show (see exercises) that

$$\bar{\Psi}\Psi = \bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L \tag{96}$$

so a mass term couples the left-handed and right-handed components of the fermion together.

But in the Standard model, the left and right handed fermions transform differently under $SU(2)$ and $U(1)_Y$. Therefore:

Mass terms in the Standard Model are forbidden by gauge invariance!

This is a deep and significant point.

Chirality and mass

One can see this by looking at how $\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L$ transforms under $U(1)_Y$ for the case the $\Psi = e^-$, the Dirac spinor for the electron. It is important to remember that $\Psi = e = \begin{pmatrix} e_L^- \\ e_R^- \end{pmatrix}$ ie the Dirac spinor is the 4-component spinor containing both the left and right handed fermions.

So, in order for a mass term to be generated, the gauge symmetry must be broken. This is consistent with the fact that the W and Z bosons must also be massive – something which is not possible unless the gauge symmetry is actually broken. The gauge symmetry breaking and mass generation is accomplished by introducing the Higgs boson. The Higgs boson is associated with a Higgs field – this is a scalar field which transforms under $SU(2)$ exactly as the left-handed quarks and leptons and has hypercharge $Y = +1$. The remarkable thing is that the Lagrangian including the Higgs is gauge invariant, but the vacuum state of the Standard Model is not!

Mass and The Higgs Boson

The Higgs boson is described by a scalar field in the Standard Model. It is therefore described by a Klein-Gordon equation with a covariant derivative. We will use a simplified model to begin:a $U(1)$ gauge theory with a Higgs boson field ϕ of charge q. The Lagrangian is:

$$\begin{aligned}\mathcal{L} &= D_\mu\phi(D^\mu\phi)^* - m^2\phi\phi^* \qquad (97)\\ &= \partial_\mu\phi\partial^\mu\phi^* - iqA_\mu\phi\partial^\mu\phi^* + iq\partial_\mu\phi A^\mu\phi^* - q^2A_\mu A^\mu\phi\phi^* - m^2\phi\phi^*\end{aligned}$$

Since ϕ has mass dimension $[M]^1$ and $|\phi|^2 = \phi * \phi$ is gauge invariant under $\phi \to e^{iq\theta}\phi$, we can add one more gauge invariant interaction to $\mathcal{L}$:

$$\Delta\mathcal{L} = -\lambda|\phi|^4 \qquad (98)$$

which represents a self-interaction of the Higgs with strength given by the dimensionless coupling λ. So, the potential for ϕ is

$$V(\phi) = m^2|\phi|^2 + \lambda|\phi|^4 \qquad (99)$$

Mass and the Higgs Boson

If ϕ were real, instead of complex, V looks like

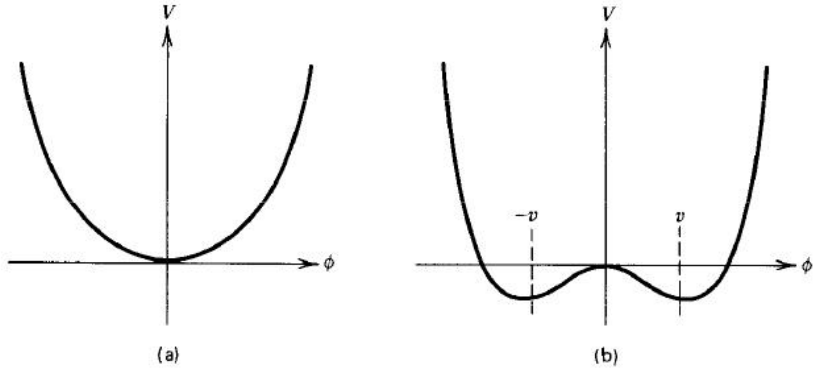


Fig. 14.3 The potential $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$ for (a) $\mu^2 > 0$ and (b) $\mu^2 < 0$, and $\lambda > 0$.

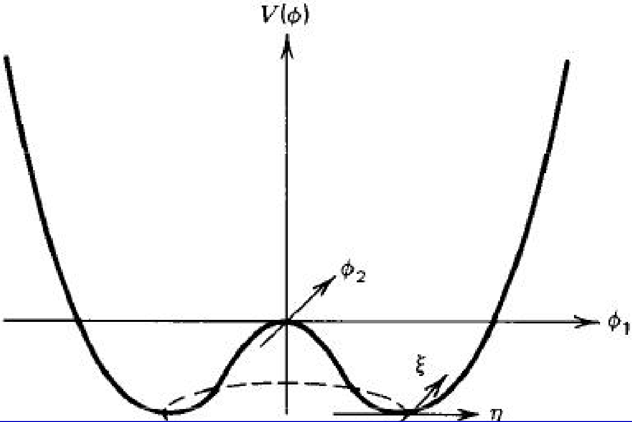
Note: some books use $\mu^2/2$ instead of m^2 and $\lambda/4$ instead of λ

Spontaneous Symmetry Breaking

- The shape of the potential takes a very different form depending on whether or not m^2 is $+$ve or -ve.
- When $m^2 > 0$ there is one minimum at $\phi = 0$
- When $m^2 < 0$ there are two minima, both at $\mathbf{non-zero}$ values of ϕ.
- $V(\phi)$ is invariant under the symmetry $\phi \to -\phi$.
- When $m^2 > 0$, the minimum is still invariant under the symmetry
- When $m^2 < 0$ any of the two minima break the symmetry!

Spontaneous Symmetry Breaking

In gauge theory, however, ϕ is actually complex. The potential, for $m^2 < 0$ looks like



Spontaneous Symmetry Breaking

- When $m^2 > 0$ there is one minimum at $\phi = 0$
- When $m^2 < 0$ there is a whole circle of minima, all at $\mathbf{non-zero}$ values of ϕ.
- $V(\phi)$ is invariant under the gauge symmetry $\phi \to e^{iq\theta}\phi$.
- When $m^2 > 0$, the minimum is still invariant under the gauge symmetry
- When $m^2 < 0$ any of the minima **break the gauge symmetry** since a non-zero value of ϕ transforms to a non-zero, different value!
- This is called spontaneous symmetry breaking

The Higgs Mechanism

Going back to the original Lagrangian, there is an interesting term:

$$\mathcal{L} = -q^2 A^\mu A_\mu |\phi|^2 \tag{100}$$

Now, since the value of ϕ in the minimum is $<|\phi|>= v = \sqrt{-m^2/2\lambda}$ this leads to

$$\mathcal{L} = -q^2 A^\mu A_\mu v^2 \tag{101}$$

This is a mass term for the gauge field and it arises from a gauge invariant Lagrangian! The mass of the $U(1)$ gauge boson is

$$M = \sqrt{2}qv = q\frac{|m|}{\sqrt{2\lambda}} \tag{102}$$

The factor of $\sqrt{2}$ is present since a mass term for a real boson, such as A_μ is of the form

$$\mathcal{L} = \frac{M^2}{2} A^\mu A_\mu \tag{103}$$

Therefore the Higgs mechanism as described in this simple model can give masses to gauge bosons without the Lagrangian violating gauge symmetry.

What about Fermion Masses?

Since left and right handed fermions have different charges, mass terms are **forbidden** by gauge invariance.
E.g. under $U(1)$-hypercharge of the SM, left-handed electrons e_L have charge -1 bt right-handed electrons e_R have charge -2
Without the Higgs field, the bare mass term

$$m_e \bar{\Psi}\Psi = m_e \bar{\Psi}_L \Psi_R + m_e \bar{\Psi}_R \Psi_L \tag{104}$$

is not gauge invariant.

Yukawa Couplings

However, if the charge of the Higgs field ϕ is equal to one, then

$$\mathcal{L}_{yukawa} = y_e \phi \bar{\Psi}_L \Psi_R + y_e \phi^* \bar{\Psi}_R \Psi_L \tag{105}$$

with a dimensionless coupling constant y_e, is gauge invariant (0 = 1 + 1 -2 = -1 + 2 -1)! Since it is gauge invariant and has mass dimension $[M]^4$ we should include it. Such terms are called Yukawa interactions and y_e the Yukawa coupling.
Remarkably, in the vacuum we get

$$\mathcal{L}_{yukawa} = y_e v \bar{\Psi}_L \Psi_R + y_e v \bar{\Psi}_R \Psi_L \tag{106}$$

which is a mass-term for the electron. The electron mass is

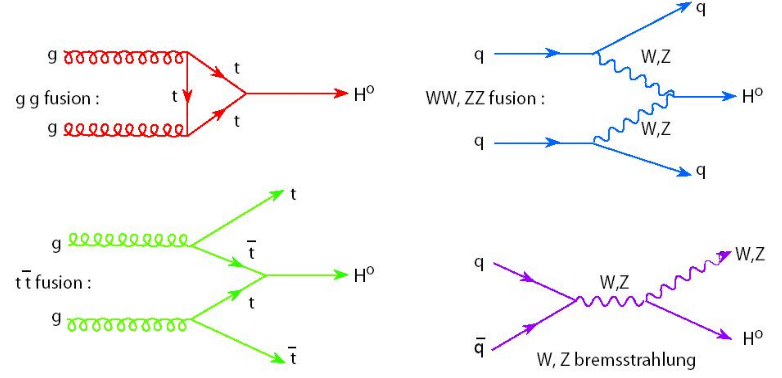
$$m_e = y_e v \tag{107}$$

Fermion masses

In the Standard Model, v is of order 200GeV in order that the W and Z bosons have the correct masses. Therefore $y_e \sim 0.25 \times 10^{-5}$. Mass terms for all the other fermions arise in the same way by introducing y_μ, y_t, y_d etc. By fixing these parameters appropriately the model correctly describes all the particle masses.

So, according to the Standard Model, all elementary particles get their masses via their interactions with the Higgs boson. The stronger the coupling between a particle and the Higgs, the more massive that particle is. e.g. the top quark Yukawa coupling is of order one, but the coupling with the muon is of order 0.5×10^{-3}.

Producing Higgs Bosons with the LHC



Once produced, the Higgs decays to lighter Standard Model Particles.

Two photons, two tau leptons, two Z bosons,...

We then measure these decay products and "reconstruct" the

Higgs

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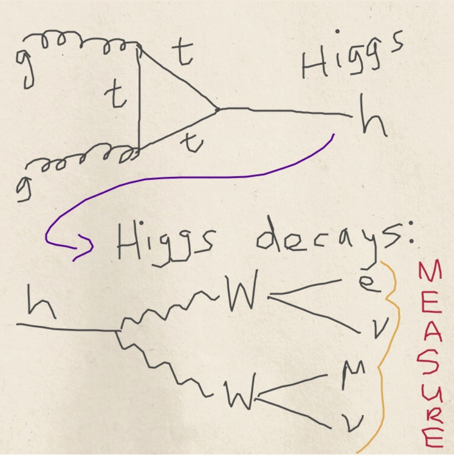
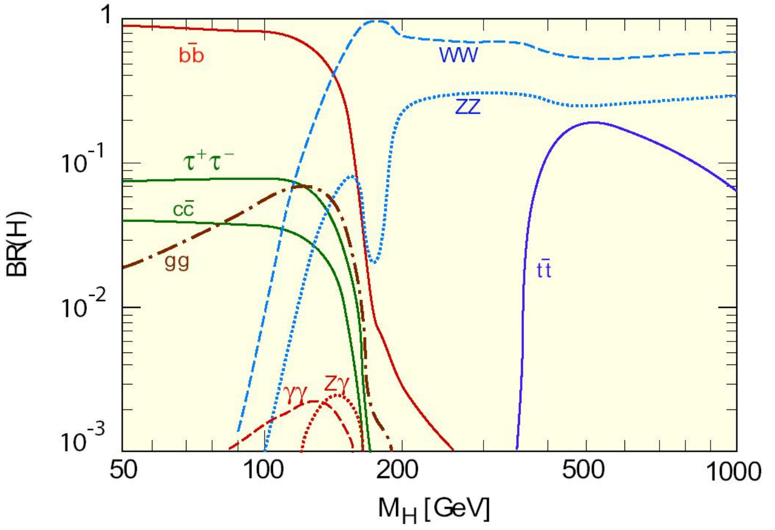


Higgs Decay Probabilities



Higgs

The above figure shows the branching ratios of the Higgs boson as a function of its mass. These are the probablities for the Higgs to decay into a certain set of particles as a function of its mass. Consisder the region from 100 to 130 GeV. Here the decays are dominated by the $b\bar{b}$ final state. This is because the mass of the Higgs is not large enough to decay into two Z bosons or W bosons or two top quarks. But it can decay into any of the other fermion/anti-fermion pairs. It decays into $b\bar{b}$ most of the time as its coupling to bottom quarks is larger than the other fermions (except the top quark, but it can't decay to $t\bar{t}$) because it doesn't have enough mass.
Beyond 130 GeV, decays into W's become available (initially with one virtual W) and start to dominate until 180 GeV when Z's also enter. Finally at twice the top quark mass, decays into $t\bar{t}$ are allowed.
The actual Higgs mass as measured by the LHC experiments is around 125 GeV $\pm$ 2 GeV or so. This region is interesting because, with enough data many of the final states are available and many have actually been observed. If the Higgs mass had turned out to be 190 GeV, only WW and ZZ would have been observable.

Higgs decays

Note: even though the Higgs is neutral, decays to two photons i.e. $\gamma\gamma$ appear on the graph. These decays actually occur due to higher order (one loop) processes in the Standard Model. This is why the $\gamma\gamma$ channel has a small branching fraction.

BUT: even though it is small, experimentally it is easier to study than $b\bar{b}$ at a hadron collider.

This is because the background to $b\bar{b}$ is much larger than that of $\gamma\gamma$. In fact, the calorimeters of ATLAS and CMS were designed to find the Higgs boson in the $\gamma\gamma$ final state. These instruments turned out to perform even better than anticipated and the Higgs was first seen in the $\gamma\gamma$ channel.

Higgs Observation

How is the Higgs seen in the diphoton channel?
First, we select events with two photons and nothing else.
We require that these photons are "very clean" and have energies/momenta of at least 15 GeV or so.
Since we measure the energies and momenta of the photons, for each such event we have a Lorentz 4-vector for each photon: P_1^μ and P_2^μ.
From these we construct the invariant mass squared:

$$M^2 = (P_1^\mu + P_2^\mu)(P_{1\mu} + P_{2\mu}) = (E_1 + E_2)^2 - (p_{1x} + p_{2x})^2 - \ldots \quad (108)$$

The reason for this is that, if the Higgs decays to two photons then Lorentz invariance implies that

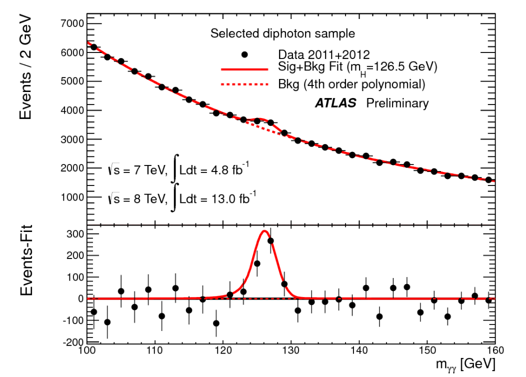
$$P_h^\mu = P_1^\mu + P_2^\mu \quad (109)$$

i.e. that the Higgs' Lorentz vector equals the sum of the Lorentz vectors of the two photons. This implies:

$$P_h^\mu P_{h\mu} = m_h^2 \quad (110)$$

Higgs Discovery

Events with diphotons can be produced by many other processes leading to a large background to this search. But, if we understand precisely enough the background and the signal-to-background rate is large enough then we should be able to see the "Higgs peak" above the background. This is an example of what has been seen:



You can see a clear peak in the data around 126 GeV.

Conclusions

The Higgs has subsequently been observed in the ZZ, WW^* and $\tau\tau$ channels as well, in agreement with the Standard Model predictions

This has been a truly monumental, remarkable set of discoveries which completes the story of the Standard Model.

The Standard Model is actually very simple: just based upon gauge and Lorentz symmetry

Combining this with the fact that all of its predictions have been verified represents one of the greatest achievements of human civilization

There is Much To Do

Thankfully, there are still many questions:
The Standard Model does not explain Dark Matter
The Standard Model does not explain Dark Energy
The masses of the Standard Model particles are not explained
In particular, why is m_h so small compared to, say, the Planck mass $m_{pl} = \frac{1}{G_N^{1/2}}$?
Why is there so much more matter compared to anti-matter? Baryogenesis.

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Thank you and Good Luck with your Studies!

Notation

- We use natural units:$\hbar = c = 1$
- V^μ is a 4-component vector with components V_0 and $V_x, V_y, V_z \equiv V_i$.
- V_μ is a 4-component co-vector with components V_0 and $-V_x, -V_y, -V_z$.
- $\partial_t \equiv \frac{\partial}{\partial t}$. $\partial_i \equiv \nabla \equiv \frac{\partial}{\partial x_i}$
- $V^\nu V_\nu \equiv V_0^2 - V_x^2 - V_y^2 - V_z^2 \equiv V_0^2 - V_i^2 \equiv V_0^2 - \mathbf{V}^2$
- $\partial_\mu \equiv (\partial_t, \nabla)$ and $\partial^\mu = (\partial_t, -\nabla)$
- 4-momentum can be represented as $p_\mu \rightarrow i\partial_\mu$ and $p^\mu \rightarrow i(\partial_t, -\nabla)$