Zeno dynamics yields ordinary constraints

P. Facchi,[1] S. Pascazio,[2] A. Scardicchio,[2] and L. S. Schulman[3]
[1]*Atominstitut der Österreichischen Universitäten, Stadionallee 2, A-1020, Wien, Austria*
[2]*Dipartimento di Fisica, Università di Bari, I-70126 Bari, Italy*
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, I-70126 Bari, Italy
[3]*Physics Department, Clarkson University, Potsdam, New York 13699-5820*
(Received 8 January 2001; published 13 December 2001)

The dynamics of a quantum system undergoing frequent measurements (quantum Zeno effect) is investigated. Using asymptotic analysis, the system is found to evolve unitarily in a proper subspace of the total Hilbert space. For measurements represented by spatial projections, the generator of the ''Zeno dynamics'' is the Hamiltonian with Dirichlet boundary conditions.

DOI: 10.1103/PhysRevA.65.012108 PACS number(s): 03.65.Xp, 03.65.Db, 02.30.Mv

Frequent measurement can slow the time evolution of a quantum system, hindering transitions to states different from the initial one [1,2]. This phenomenon, known as the quantum Zeno effect (QZE), follows from general features of the Schrödinger equation that yield quadratic behavior of the survival probability at short times [3,4]. Interfering with a transition at a later stage in its progress leads to a second non-Markovian phenomenon, known as the inverse or anti-Zeno effect [5–8], in which decay is accelerated. Both effects have recently been seen in the same experimental setup [9].

However, the QZE does not necessarily freeze everything. On the contrary, for a projection onto a multidimensional subspace, the system may evolve away from its initial state, although it remains in the subspace defined by the ''measurement.'' This continuing time evolution *within* the projected subspace we call *quantum Zeno dynamics*. It is often overlooked, although it is readily understandable in terms of a theorem on the QZE [2] that we will recall below.

The aim of this paper is to show that Zeno dynamics yields ordinary constraints. In particular, suppose a system has Hamiltonian H and the measurement (that will be made frequently) is checking that the system is within a particular spatial region. Then the Zeno dynamics that results is governed by the same Hamiltonian, but with Dirichlet boundary conditions on the boundary of the spatial region associated with the projection. Moreover, the Hamiltonian with these boundary conditions is self adjoint and remains reversible within the Zeno subspace. This shows that irreversibility is not compulsory, as noted in [10].

Some of our results are already known in the mathematical literature [11], namely, the case of a free particle. However, our method of proof is completely different (and perhaps more transparent to the physicist) and extends easily to the case of nonzero potential.

At the experimental level, besides [9], the QZE has been tested on oscillating systems [12]. Although these experiments have invigorated studies on this issue, they deal with one-dimensional projectors (and therefore one-dimensional Zeno subspaces): the system is forced to remain in its initial state. This is also true for interesting quantum optical applications [13]. The present paper therefore enters an experimentally uncharted area, although the property of being a multidimensional measurement is not at all exotic, and in particular, applies to the most basic quantum measurement: position. The latter is the subject of the present paper.

We introduce notation. Consider a quantum system Q, whose states belong to the Hilbert space $\mathcal{H}$ and whose evolution is described by the unitary operator $U(t)=\exp(-iHt)$, where H is a time-independent lower-bounded Hamiltonian. Let E be a projection operator that does not commute with the Hamiltonian, $[E,H]\neq 0$, and $E\mathcal{H}=\mathcal{H}_E$ the subspace defined by it. The initial density matrix ρ_0 of system Q is taken to belong to $\mathcal{H}_E$

$$\rho_0=E\rho_0E, \quad \mathrm{Tr}\,\rho_0=1. \tag{1}$$

The state of Q after a series of E-observations at times $t_j=jT/N$ $(j=1,\dots,N)$ is

$$\rho^{(N)}(T)=V_N(T)\rho_0V_N^\dagger(T), \quad V_N(T)\equiv[EU(T/N)E]^N \tag{2}$$

and the probability to find the system in $\mathcal{H}_E$ (''survival probability'') is

$$P^{(N)}(T)=\mathrm{Tr}[V_N(T)\rho_0V_N^\dagger(T)]. \tag{3}$$

Our attention is focused on the limiting operator

$$\mathcal{V}(T)\equiv\lim_{N\to\infty}V_N(T). \tag{4}$$

Misra and Sudarshan [2] proved that if the limit exists, then the operators $\mathcal{V}(T)$ form a one-parameter semigroup, and the final state is

$$\rho(T)=\lim_{N\to\infty}\rho_N(T)=\mathcal{V}(T)\rho_0\mathcal{V}^\dagger(T). \tag{5}$$

The probability to find the system in $\mathcal{H}_E$ is

$$\mathcal{P}(T)\equiv\lim_{N\to\infty}P^{(N)}(T)=1. \tag{6}$$

This is the QZE. If the particle is constantly checked for whether it has remained in $\mathcal{H}_E$, it never makes a transition to $(\mathcal{H}_E)^\perp$.

A few comments are in order. First, the final state $\rho(T)$ depends on the characteristics of the model investigated and on the measurement performed (the specific forms of V_N and $\mathcal{V}$ depend on E). Moreover, the physical mechanism that ensures the conservation of probabilities within the relevant subspace hinges on the short-time behavior of the survival probability: probability leaks out of the subspace $\mathcal{H}_E$ like t^2 for short times. Since the infinite-N limit suppresses this loss, one may inquire under what circumstances $\mathcal{V}(T)$ actually forms a group, yielding reversible dynamics within the Zeno subspace.

In this paper, we show that Zeno dynamics for a position measurement yields a particular kind of dynamics within the subspace defined by that measurement, namely, unitary evolution with the restricted Hamiltonian, and with the domain of that (self-adjoint) operator defined by Dirichlet boundary conditions. This elucidates the reversible features of the evolution for a wide class of physical models. As a spinoff, our proof provides a rigorous regularization of the example considered in [10] (where it was suggested that the Trotter product formula could be used to demonstrate the result).

We start with the simplest spatial projection. Q is a free particle of mass m on the real line, and the measurement is a determination of whether or not it is in the interval $A=[0,L]\subset\mathbb{R}$. The Hamiltonian and the corresponding evolution operator are

$$H=\frac{p^2}{2m},\quad U(t)=\exp(-itH). \tag{7}$$

H is a positive-definite self-adjoint operator on $L^2(\mathbb{R})$ and $U(t)$ is unitary. We study the evolution of the particle when it undergoes frequent measurements defined by the projector

$$E_A=\int dx\,\chi_A(x)|x\rangle\langle x|, \tag{8}$$

where χ_A is the characteristic function

$$\chi_A(x)=\begin{cases}1 & \text{for} \quad x\in A=[0,L]\\ 0 & \text{otherwise}\end{cases}. \tag{9}$$

Thus, E_A is the multiplication operator by the function χ_A. We study the following process. We prepare a particle in a state with support in A, let it evolve under the action of its Hamiltonian, perform frequent E_A measurements during the time interval $[0,T]$, and study the evolution of the system *within* the subspace $\mathcal{H}_{E_A}=E_A\mathcal{H}$. We will show that the dynamics in $\mathcal{H}_{E_A}$ is governed by the evolution operator

$$\mathcal{V}(T)=\exp(-iTH_Z)E_A, \tag{10}$$

with

$$H_Z\equiv\frac{p^2}{2m}+V_A(x),\quad V_A(x)=\begin{cases}0 & \text{for} \quad x\in A\\ +\infty & \text{otherwise}\end{cases}. \tag{11}$$

This is the operator obtained in the limit (4). In other words, the system behaves as if it were confined in A by rigid walls, inducing the wave function to vanish on the boundaries $x=0, L$ (Dirichlet boundary conditions).

We now prove our assertion. Let the particle be initially $(t=0)$ in A. We recall the propagator in the position representation [14,15]

$$\begin{aligned}G(x,t;y)&\equiv\langle x|E_AU(t)E_A|y\rangle=\chi_A(x)\langle x|U(t)|y\rangle\chi_A(y)\\ &=\chi_A(x)\sqrt{\frac{m}{2\pi it\hbar}}\exp\left[\frac{im(x-y)^2}{2\hbar t}\right]\chi_A(y),\end{aligned} \tag{12}$$

where $t=T/N$ is the time when the first measurement is carried out and the particle found in E_A. To study the properties of G we choose a complete basis in $L^2(A)$

$$u_n(x)=\langle x|u_n\rangle=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right),\quad n=1,2,\dots, \tag{13}$$

At this point, one should not confuse the selection of a basis for the space with the selection of a domain for the Hamiltonian. For the free particle on the interval with Dirichlet boundary conditions, $\{u_n\}$ provides a basis of eigenfunctions, but even if one took Neumann (or whatever) boundary conditions (leading to a different self-adjoint operator) the basis (13) could still be used. The functions $\{u_n\}$ would simply not be eigenfunctions. Even though this last observation is well known, correspondence prior to publication has convinced us of the need to emphasize these facts of functional analysis for clarity. In this basis,

$$H_Z|u_n\rangle=E_n|u_n\rangle,\quad E_n=\frac{\hbar^2n^2\pi^2}{2mL^2}, \tag{14}$$

and the matrix elements of G are

$$\begin{aligned}G_{mn}(t)&\equiv\langle u_m|E_AU(t)E_A|u_n\rangle\\ &=\int_0^L dx\int_0^L dy\,u_m(x)\sqrt{\frac{m}{2\pi it\hbar}}\exp\left[\frac{im(x-y)^2}{2\hbar t}\right]u_n(y).\end{aligned} \tag{15}$$

Let $r=x-y$, $R=(x+y)/2$, and $\lambda=m/2\hbar t$, so that

$$\begin{aligned}G_{mn}(\lambda)=\sqrt{\frac{\lambda}{i\pi}}\int_0^L dR\int_{-r_0(R)}^{r_0(R)}dr\,u_m(R+r/2)\\ \times u_n(R-r/2)\exp[i\lambda r^2],\end{aligned} \tag{16}$$

where $r_0(R)=L-|L-2R|$. We now use the asymptotic expansion

$$g(\lambda)=\sqrt{\frac{\lambda}{i\pi}}\int_{-a}^{a}dx f(x)e^{i\lambda x^2}=g_{\rm stat}(\lambda)+g_{\rm bound}(\lambda), \tag{17}$$

where

$$g_{\rm stat}(\lambda)=f(0)+\frac{i}{4\lambda}f''(0)+O(\lambda^{-2}) \tag{18}$$

and

$$g_{\rm bound}(\lambda)=\frac{e^{i\lambda a^2}}{2ia\sqrt{i\pi\lambda}}[f(a)+f(-a)]+O(\lambda^{-3/2}) \quad (19)$$

are the contributions of the stationary point $x=0$ and of the boundary, respectively. By expanding the inner integral in Eq. (16) as in Eqs. (17)–(19), one gets

$$\sqrt{\frac{\lambda}{i\pi}}\int_{-r_0(R)}^{r_0(R)} dr\, u_m(R+r/2)u_n(R-r/2)\exp[i\lambda r^2]$$
$$=u_m(R)u_n(R)+\frac{i}{4\lambda}\frac{d^2}{dr^2}[u_m(R+r/2)u_n(R-r/2)]_{r=0}$$
$$+O(\lambda^{-3/2}). \quad (20)$$

(Note that the contribution of the boundary vanishes identically.) Using this result, we integrate by parts and after a straightforward calculation obtain

$$G_{mn}(t)=\int_0^L dR\left[u_m(R)u_n(R)-\frac{it}{\hbar}u_m(R)\frac{-\hbar^2}{2m}\frac{d^2}{dR^2}u_n(R)\right]+O(t^{3/2})=\langle u_m|u_n\rangle-\frac{it}{\hbar}\left\langle u_m\left|\frac{p^2}{2m}\right|u_n\right\rangle+O(t^{3/2})$$
$$=\delta_{mn}\left(1-\frac{it}{\hbar}E_n\right)+O(t^{3/2}). \quad (21)$$

With this formula, we may carry out the limit required in Eq. (4). At time T, in the representation (13), the propagator becomes

$$\mathcal{G}_{mn}(T)=\langle m|\mathcal{V}(T)|n\rangle=\lim_{N\to\infty}\sum_{n_1\ldots n_{N-1}}G_{mn_1}(T/N)G_{n_1n_2}(T/N)\cdots G_{n_{N-1}n}(T/N)=\delta_{mn}e^{-iTE_n/\hbar}. \quad (22)$$

This is precisely the propagator of a particle in a square well with Dirichlet boundary conditions. This in turn proves that $H_{\rm Z}$ is given in Eq. (11) and has eigenbasis (13). Note also that the $t^{3/2}$ contribution in Eq. (21) drops out in the $N\to\infty$ limit since it appears as $N\times O(1/N^{3/2})$.

At this point (and for reasons similar to those mentioned earlier) it is worth emphasizing that the basis given in Eq. (13) is only one of many (infinite in fact) possibilities for a basis for the interval. Any one of these would be valid, but not all would be equally convenient. Thus, with a basis whose functions did not vanish at the endpoints, the dominant contribution of order $\lambda^{-1/2}$ in $g_{\rm bound}(\lambda)$ would have given a nondiagonal term in Eqs. (20)–(22). The matrix representation of G (in this basis) would in that case still need to be diagonalized, leading back to the matrix we have found using a more convenient basis. Our point is that one may always choose to use the basis $\{u_n\}$ of Eq. (13). For that choice, the calculation is easiest and the resulting interpretation transparent.

At this point, we have recovered, using rather different techniques, the result of Friedman [11]. Continuing to use our approach of asymptotic analysis, the result may be generalized to a wide class of systems. Let

$$H=\frac{p^2}{2m}+V, \quad U(t)=\exp(-itH), \quad (23)$$

where V is a regular potential. (It may be unbounded from below, for example $V(x)=Fx$, although within the projected region A the total Hamiltonian H should be lower bounded.) The measurement performed is again application of the projector (8) and we study the short-time propagator

$$G(x,t;y)=\chi_A(x)\sqrt{\frac{m}{2\pi it\hbar}}\exp\left[\frac{im(x-y)^2}{2\hbar t}\right]$$
$$\exp\left[-\frac{it(V(x)+V(y))}{2\hbar}\right]\chi_A(y). \quad (24)$$

The basis to be used for representing the propagator is again that of the Hamiltonian with Dirichlet boundary conditions in $[0,L]$

$$H_{\rm Z}|u_n\rangle=\left(\frac{p^2}{2m}+V\right)|u_n\rangle=E_n|u_n\rangle, \quad u_n(x)|_{x=0,L}=0. \quad (25)$$

As before $(r=x-y,\ R=(x+y)/2,\ \lambda=m/2\hbar t)$,

$$G_{mn}(\lambda)=\sqrt{\frac{\lambda}{i\pi}}\int_0^L dR\int_{-r_0(R)}^{r_0(R)} dr\, u_m\left(R+\frac{r}{2}\right)$$
$$\times e^{-itV(R+r/2)/2\hbar}u_n\left(R-\frac{r}{2}\right)e^{-itV(R-r/2)/2\hbar}e^{i\lambda r^2}. \quad (26)$$

Using the asymptotic expansion (17)–(19), a calculation identical to the previous one yields

$$G_{mn}(t)=\int_0^L dR\left[u_n(R)u_m(R)-\frac{it}{\hbar}u_n(R)\left(\frac{-\hbar^2}{2m}\frac{d^2}{dR^2}+V(R)\right)u_m(R)\right]+O(t^{3/2})$$

$$=\langle u_n|u_m\rangle-\frac{it}{\hbar}\left\langle u_n\left|\left(\frac{p^2}{2m}+V\right)\right|u_m\right\rangle+O(t^{3/2})$$

$$=\delta_{nm}\left(1-\frac{it}{\hbar}E_n\right)+O(t^{3/2}) \tag{27}$$

and the limiting propagator at time T again reads

$$\mathcal{G}_{mn}(T)=\delta_{nm}e^{-iTE_n/\hbar}. \tag{28}$$

Again, the simplicity of the proof is due to the choice of the basis (25), satisfying Dirichlet boundary conditions.

We have also obtained an improvement with respect to earlier approaches to this problem. The aforementioned theorem by Misra and Sudarshan [2] requires that the Hamiltonian be lower bounded from the outset. However, we need only require that the Hamiltonian be lower bounded in the Zeno subspace. Despite the fact that for unbounded potentials (such as $V=Fx)H$ may not be lower bounded on the real line, the evolution in the Zeno subspace is governed by the Hamiltonian

$$H_Z=\frac{p^2}{2m}+V_A(x),\qquad V_A(x)=\begin{cases}V(x) & \text{for}\quad x\in A\\ +\infty & \text{otherwise}\end{cases} \tag{29}$$

that can be lower bounded in A, yielding a *bona fide* group for the evolution operators.

The above calculation and conclusions may readily be generalized to higher dimensions, so long as the measurement projects onto a set in $\mathbb{R}^n$ with a smooth boundary (except, at most, a finite number of points). We again take $\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^n$ and let the measurement projection be defined by $A\subset\mathbb{R}^n$, which is not necessarily bounded. Again setting $\boldsymbol{r}=\boldsymbol{x}-\boldsymbol{y},\quad \boldsymbol{R}=(\boldsymbol{x}+\boldsymbol{y})/2$, Eq. (26) becomes

$$G_{mn}(\lambda)=\left(\frac{\lambda}{i\pi}\right)^{n/2}\int_A d\boldsymbol{R}\int_{D(\boldsymbol{R})}d\boldsymbol{r}\,u_m(\boldsymbol{R}+\boldsymbol{r}/2)\times e^{-itV(\boldsymbol{R}+\boldsymbol{r}/2)/2\hbar}u_n(\boldsymbol{R}-\boldsymbol{r}/2)\times e^{-itV(\boldsymbol{R}-\boldsymbol{r}/2)/2\hbar}e^{i\lambda\boldsymbol{r}^2}, \tag{30}$$

where $D(\boldsymbol{R})$ is the transformed integration domain for $\boldsymbol{r}$. The n-dimensional asymptotic expansions (17)–(19) read [16]

$$g_{\text{stat}}(\lambda)=f(\boldsymbol{0})+\frac{i}{4\lambda}\triangle f(\boldsymbol{0})+O(\lambda^{-2}), \tag{31}$$

$$g_{\text{bound}}(\lambda)=O(\lambda^{-1/2})\times f(\text{boundary})+O(\lambda^{-3/2}), \tag{32}$$

and the theorem follows again because f vanishes on the boundary (Dirichlet). The proof is readily generalized to nonconvex and/or multiply-connected projection domains, the only difficulty being that the integration domain in Eq. (30) must be broken up. It is interesting to notice that at those points at which the boundary fails to have a continuously turning tangent plane, the asymptotic contribution of the discontinuity in the boundary in Eq. (32) would be of yet higher order in λ.

In conclusion, for traditional position measurements, namely projections onto spatial regions, we have shown that Zeno dynamics uniquely determines the boundary conditions, and that they turn out to be of Dirichlet type. This is also relevant for problems related to the consistent histories approach [17–19], where different boundary conditions were proposed. For us, the frequent imposition of a projection, the traditional idealization of a measurement, provides all the decohering of interfering alternatives that is needed. On the other hand, in the works just cited, one seeks a restricted propagator (using the path decomposition expansion [20]) and such interference can occur.

A second issue discussed in these works (especially [18]) is the validity of the Trotter product formula in certain cases. Again, our implicit use of this formula [in Eq. (24), etc.] is nothing more than its use for a particle in an ordinary potential (in particular, the Trotter formula is *not* used in connection with the potentially singular projection operation by "E"). This is because the propagator of Eq. (24) provides time evolution under a *sequence* of operations: the particle evolves under the Hamiltonian (23) (on the entire line) for a time t, and *then* one applies the projection (left and right multiplication by the operator E_A). Our results are also relevant for understanding the physical features of "decoherent free" subspaces, which are of great interest in quantum computation [21]. The Zeno mechanism not only forces the system to remain in a given subspace, it also constrains its (sub)dynamics in this space, determining the behavior of the wave function on the boundary and yielding a unitary, *decoherence free* evolution. Besides its theoretical interest, this feature might lead to potential applications and practical implementations of the Zeno constraints.

The present paper has implications for the notion of "hard wall," as used for example in elementary quantum mechanics. Everyone would agree (we expect) that this notion is an idealization. However, in many cases where this idealization is useful the "wall" is dynamic rather than static, the result of some fluctuating atomic presence. In this article, we have a sufficient condition for the validity of this notion in a dynamic situation. Moreover, there is a quantitative framework (arising from our asymptotic analysis and finite-time-interval QZE effects) for gauging the effects of less than perfect hard walls.

ACKNOWLEDGMENTS

This work was supported in part by the TMR Network of the European Union "Perfect Crystal Neutron Optics" ERB-FMRX-CT96-0057 and by the U.S. NSF under Grant Nos. PHY 97 21459 and PHY 00 99471.

[1] A. Beskow and J. Nilsson, Ark. Fys. **34**, 561 (1967); L. A. Khalfin, Zh. Éksp. Teor. Fiz. Pis. Red. **8**, 106 (1968) [JETP Lett. **8**, 65 (1968)].
[2] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1977).
[3] For a review, see H. Nakazato, M. Namiki, and S. Pascazio, Int. J. Mod. Phys. B **10**, 247 (1996); D. Home and M. A. B. Whitaker, Ann. Phys. (Leipzig) **258**, 237 (1997); P. Facchi and S. Pascazio, *Progress in Optics 42* (North-Holland, Amsterdam, 2001), Chap. 3.
[4] The presence of a short-time quadratic region has been experimentally demonstrated by S. R. Wilkinson, C. F. Bharucha, M. C. Fischer, K. W. Madison, P. R. Morrow, Q. Niu, B. Sundaram, and M. G. Raizen, Nature (London) **387**, 575 (1997).
[5] A. M. Lane, Phys. Lett. A **99**, 359 (1983).
[6] W. C. Schieve, L. P. Horwitz, and J. Levitan, Phys. Lett. A **136**, 264 (1989).
[7] A. G. Kofman and G. Kurizki, Nature (London) **405**, 546 (2000).
[8] P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. **86**, 2699 (2001).
[9] M. C. Fischer, B. Gutiérrez-Medina, and M. G. Raizen, Phys. Rev. Lett. **87**, 040402 (2001).
[10] P. Facchi, V. Gorini, G. Marmo, S. Pascazio, and E. C. G. Sudarshan, Phys. Lett. A **275**, 12 (2000).
[11] C. N. Friedman, Indiana Univ. Math. J. **21**, 1001 (1972).
[12] R. J. Cook, Phys. Scr. T **T21,** 49 (1988); W. H. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990); T. Petrosky, S. Tasaki, and I. Prigogine, Phys. Lett. A **151**, 109 (1990); Physica A **170**, 306 (1991); A. Peres and A. Ron, Phys. Rev. A **42**, 5720 (1990); S. Pascazio, M. Namiki, G. Badurek, and H. Rauch, Phys. Lett. A **179**, 155 (1993); T. P. Altenmüller, and A. Schenzle, Phys. Rev. A **49**, 2016 (1994); S. Pascazio and M. Namiki, *ibid.* **50**, 4582 (1994); L. S. A. Beige and G. Hegerfeldt, *ibid.* **53**, 53 (1996); A. G. Kofman and G. Kurizki, *ibid.* **54**, R3750 (1996); L. S. Schulman, *ibid.* **57**, 1509 (1998).
[13] A. Luis and J. Periňa, Phys. Rev. Lett. **76**, 4340 (1996); M. B. Plenio, P. L. Knight, and R. C. Thompson, Opt. Commun. **123**, 278 (1996); A. Luis and L. L. Sánchez-Soto, Phys. Rev. A **57**, 781 (1998); K. Thun and J. Peřina, Phys. Lett. A **249**, 363 (1998); J. Řeháček *et al.*, Phys. Rev. A **62**, 013804 (2000).
[14] R. P. Feynman and G. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
[15] L. S. Schulman, *Techniques and Applications of Path Integration* (Wiley, New York, 1981) (reissued in 1996).
[16] N. Bleistein and R. A. Handelsman, *Asymptotic Expansions of Integrals* (Dover, New York, 1986).
[17] N. Yamada and S. Takagi, Prog. Theor. Phys. **85**, 985 (1991); **86**, 599 (1991); **87**, 77 (1992); N. Yamada, Phys. Rev. A **54**, 182 (1996).
[18] J. B. Hartle, Phys. Rev. D **44**, 3173 (1991).
[19] I. L. Egusquiza and J. G. Muga, Phys. Rev. A **62**, 032103 (2000).
[20] A. Auerbach, S. Kivelson, and D. Nicole, Phys. Rev. Lett. **53**, 411 (1984).
[21] A. Beige, D. Braun, B. Tregenna, and P. L. Knight, Phys. Rev. Lett. **85**, 1762 (2000).