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PHYSICS REPORTS
[illegible] 435 (2006) 157–182
[illegible]

[illegible]

[illegible]*, D.[illegible], A.[illegible]

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

A[illegible] 20 [illegible] 2006

[illegible]: [illegible].A. B[illegible]

Abstract

[illegible] $(U(1)_A)$ [illegible] CD, [illegible]. A [illegible] $SU(2)_L \times SU(2)_R$ [illegible] *chirally invariant* [illegible] N_c [illegible]

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PACS: 11.30.[illegible]; 11.40.[illegible]; 12.39.[illegible]

Contents

* C[illegible].
E-mail address: [illegible]@[illegible] ([illegible]).

0370-1573/$ - [illegible] © 2006 [illegible] B.[illegible]. A[illegible]
[illegible]10.1016/[illegible].2006.09.004

1. Introduction

1960 . [1] . 2,3 . CD . $SU(2)_L \times SU(2)_R$ CD [4–8] [9,10] .

$u, d,$ s .

? CD ? $SU(2)_L \times SU(2)_R$ CD, ? Δ), (Σ Λ) (S) Ξ ($S = -2$) Ω^- ($S = -3$). B

CD : $SU(2)_L \times SU(2)_R$ $U(1)_A$. Q_5^a ($a = 1, 2, 3$), Q_5

$U(1)_A$ x x CD
[17], x , Q_5,
CD [17]. x x CD , C
[18], $Q_5|0\rangle \neq 0$.
does not $U(1)_A$ x . CD
$\langle \bar{q}q \rangle$ x x
x *a matter of degree*. A , $SU(2)_L \times SU(2)_R$
x x . A D. $m_e \neq 0$, $U(1)_A$
$\bar{e}\gamma_\mu\gamma_5 e$ x x D.
$\bar{e}e$ x.
$\overline{MS}$ $\langle \bar{e}e\rangle_\mu = m^3/(4\pi^2) f(\alpha)$, $f(\alpha) = 1 + \ell + \alpha/(2\pi)(5 + 5\ell + 3\ell^2) +$ (α^2), $\ell =$ (μ^2/m^2)
μ [19]. D $\langle \bar{e}e\rangle \neq 0$, x x x *explicitly broken*.
CD *explicit* $U(1)_A$ x x x
η' x x x
x x , f_π. x [17], $U(1)_A$ x x x.
not N_c CD.
$U(1)_A$ x x η'
($m_{u,d,s} = 0$). $U(1)_A$ x x x .
$N_c = 3$ $U(1)_A$.
explicitly broken x x, *that violate* $U(1)_A$ *symmetry*
x
x x
[20]. x x
x $U(1)_A$. x,
$U(1)_A$ x x . $U(1)_A$ x x .[1]
x x $U(1)_A$ x
$SU(2)_L \times SU(2)_R$, *chirally invariant*
x x . $U(1)_A$
x *violate the symmetry*
x x x x x
x x x x x x ,
x, x
x x
[1,2,22] x x J
x x x $J = 1/2$
3/2, x [8]
CD,
x $N_c \to \infty$ (N_c CD) x x
x .
x , $SU(2)_L \times SU(2)_R$
x $SU(2)_L \times SU(2)_R$
x x
x x [23,24]. x x
x x
$SU(2)_L \times SU(2)_R$
x
x x N_c [25,26].

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[1] $U(1)_A$ x x

2. What is the experimental evidence?

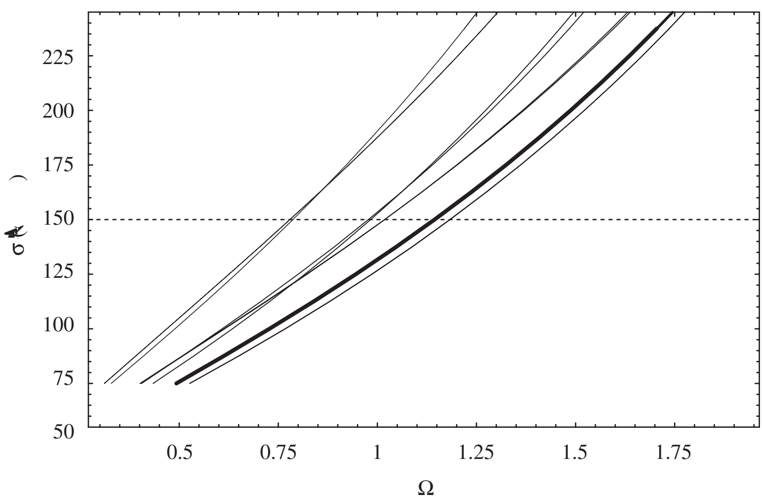


Fig. 1. Ω (I = 1/2, S = 0) $J = 1/2$ σ. Ω

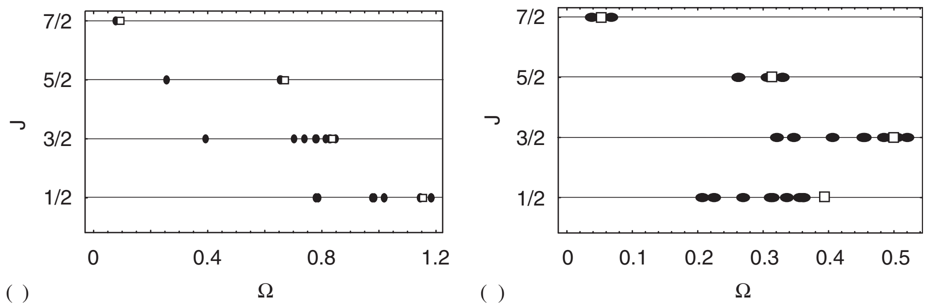


Fig. 2. Ω $\sigma = 125$ ( ) Δ ( ) J. Ω

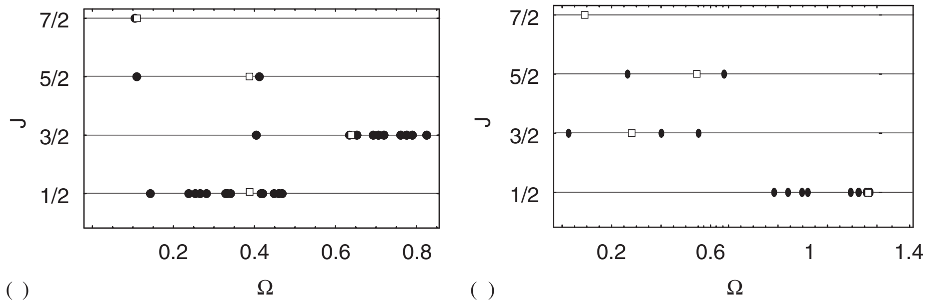
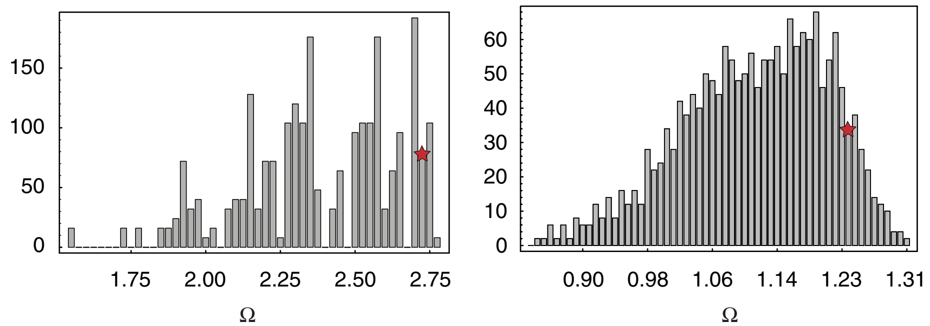
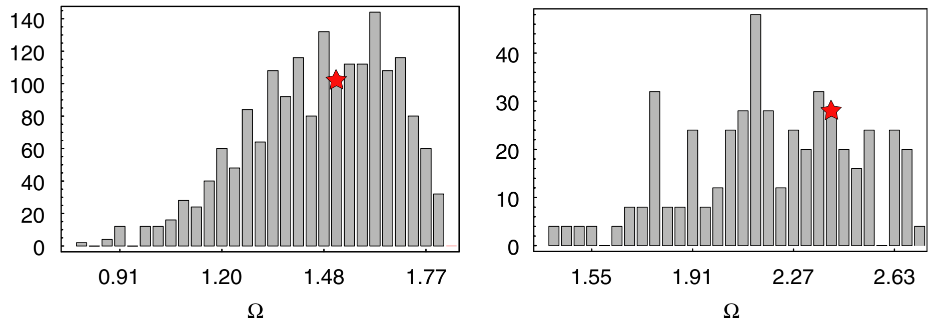


Fig. 3. Ω $\sigma = 125$ Σ ( ) Λ ( ) J. Ω

D ( ) fl


150
100
50
0
1.75
2.00
2.25
2.50
2.75
Ω
60
50
40
30
20
10
0
0.90
0.98
1.06
1.14
1.23
1.31
Ω


140
120
100
80
60
40
20
0
0.91
1.20
1.48
1.77
Ω
40
30
20
10
0
1.55
1.91
2.27
2.63
Ω

$J, I,$ and S ... J, ... I, and ... S ... $\rho^{\pm}_{JIS}(m)$ ... D ...[2] ... Δ ... 30 ... W_j ... D ...

$$\rho^{+}_{IJS}(m,\{C\}) = \sum_j \frac{W_j}{2\pi}\frac{\Gamma_j}{(m-m_j)^2+\Gamma_j^2/4}C_j,$$

$$\rho^{-}_{IJS}(m,\{C\}) = \sum_j \frac{W_j}{2\pi}\frac{\Gamma_j}{(m-m_j)^2+\Gamma_j^2/4}(1-C_j). \tag{1}$$

... *all* ... $J, I,$ and S, and $W_j = 1.0, 0.75, 0.50, 0.25$ for $4^*, 3^*, 2^*,$ and 1^* ... $W_j = 1.0, 0.50, 0.25, 0.125,$ ... Σ $(I=1, S=-1)$, ... C_j ... j ... : $C_j = +1$ ... and $C_j = 0$ ... C_j ... $\rho^{\pm}_{IJS}$ ...

$$\int_{-\infty}^{\infty} dm\,\rho^{\pm}_{IJS}(m,\{C\}) \equiv N^{\pm}_{IJS}(\{C\}) = \sum_j W_j \begin{cases} C_j & \text{for } \rho^{+}, \\ (1-C_j) & \text{for } \rho^{-}, \end{cases} \tag{2}$$

... QCD. ... σ, ... 50–200 ... σ. ...

$$\Omega_{IJS}(\{C\}) = \int dm_1\,dm_2\,\rho^{+}_{IJS}(m_1,\{C\})\,\mathrm{erfc}\left(\frac{|m_1-m_2|}{\sigma}\right)\rho^{-}_{IJS}(m_2,\{C\}), \tag{3}$$

... $\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} dt\, e^{-t^2}$ ... $\mathrm{erfc}(0) = 1$ and $\mathrm{erfc}(z) \to 0$ as $z \to \infty$. ... (3) ... m_1 and m_2, ... $\mathrm{erfc}(|m_1-m_2|/\sigma)$ ... μ ... m_1 and ... σ ... $|\mu - m_1| \geqslant |m_2 - m_1|$, ... m_2 ... m_1 ... σ). ... σ ... $m_2 \leftrightarrow m_1$). ... m_1 and m_2 ... $\rho^{\mp}_{IJS}(m_{1,2},\{C\})$, ... $N^{\pm}_{IJS}(\{C\})$. $\Omega_{IJS}(\{C\})$ ... σ. Ω ... $\mathrm{erfc}(x) \leqslant 1$ ... Ω_{IJS} ... $|m_i - m_j|$ and ... Γ_i ...

[2] ...

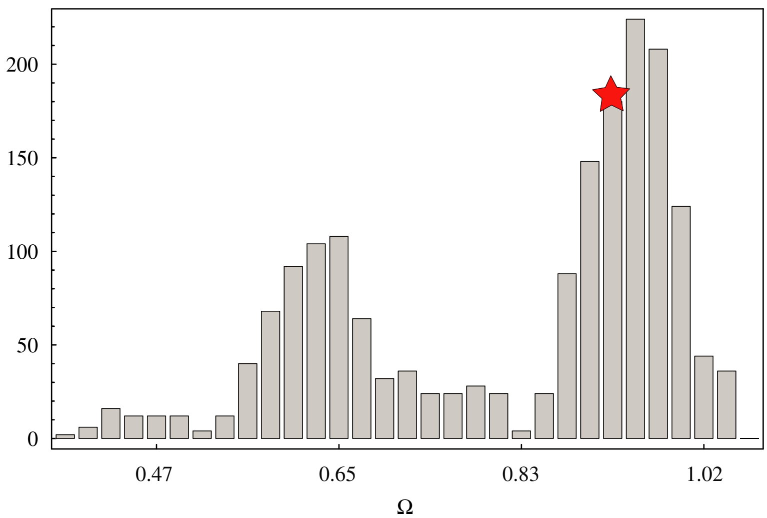


Fig. 6.

3. Possible origins of parity doubling

$$SU(n_f)_L \times SU(n_f)_R \times U(1)_A \times U(1)_V. \tag{4}$$

$SU(n_f)_L \times SU(n_f)_R$, $U(1)_A$, $U(1)_V$. $n_f = 2$.

[11] $SU(2)_L \times SU(2)_R$ $SU(2)_L \times SU(2)_R$ *invariant*

4. Effective restoration of $SU(2)_L$ $SU(2)_R$

n_f $SU(n_f)_L \times SU(n_f)_R$ T^a X^a

$$[T^a, T^b] = i f_{abc} T^c, \quad [T^a, X^b] = i f_{abc} X^c, \quad [X^a, X^b] = i f_{abc} T^c, \tag{5}$$

$a, b, c = 1, \ldots, n_f^2 - 1$. $n_f = 2$ $f_{abc} = \varepsilon_{abc}$. $\partial^\mu j^a_\mu = \partial^\mu j^a_{5\,\mu} = 0$. T^a, $\{X^a, P\} = 0$.

1/2 $j^a_{5\,\mu}$

$$\langle B_+ | j^a_{5\,\mu} | B_- \rangle = \bar{u}(p, s)(\gamma_\mu g_A(q^2) + q_\mu g_P(q^2) + \sigma_{\mu\nu} q^\nu g_M(q^2)) t^a u(p', s'), \tag{6}$$

$q^\mu = p^\mu - p'^\mu$. $q^\mu \langle B_+ | j^a_{5\,\mu} | B_- \rangle = 0$,

$$\Delta m g_A(q^2) + q^2 g_P(q^2) = 0, \tag{7}$$

$g_P(q^2)$ $q^2 = 0$ $f_\pi g_{\pi B_+ B_-}$, $g_{\pi B_+ B_-}$ s- $B_+ B_- \pi$ q^2 $q^2 \to 0$. $(\vec{q} = 0)$, $q^0 = \Delta m = m_+ - m_-$ Δm,

$$\Delta m g_A + f_\pi g_{\pi B_+ B_-} + \mathcal{O}(\Delta m^2) = 0, \tag{8}$$

$g_A = g_A(0)$.

B_+ … B_- … $\Delta m = 0$ … (8) … $g_{\pi B_+ B_-} = 0$ … $\Delta I = 1$, … s- …

… 9,10,4–6 … 1.5–3 …

… $SU(2)_L \times SU(2)_R$ … (I_L, I_R) … $I^a_{L,R} = T^a \mp X^a$. … (I_L, I_R) … $(2I_L+1)(2I_R+1)$ … $|I_L - I_R| \leqslant I \leqslant I_L + I_R$. … $I_L \leftrightarrow I_R$, … $I_L = I_R$. … N, Δ … 1/2 … 3/2. … (1/2, 0), (1/2, 1), (3/2, 0), (3/2, 1), …,

$$H_+ \sim (I_L, I_R) \oplus (I_R, I_L), \quad H_- \sim (I_L, I_R) \ominus (I_R, I_L). \tag{9}$$

… $H_\pm$ … $SU(2)_L \times SU(2)_R$ …

… $SU(2)_L \times SU(2)_R$ … 11 … (7) … $SU(2)_L \times SU(2)_R$ … *completely decouple from pions*. … 4–6, … s- … (7), … 4–6 … 27,28 … $SU(2)_L \times SU(2)_R$ … *cannot* … $SU(2)_L \times SU(2)_R$, …

… 11 … $SU(2)_L \times SU(2)_R$ … CD … (9). … CD, …

… $SU(2)_L \times SU(2)_R$ … $SU(n_f)_L \times SU(n_f)_R$ … $SU(n_f)_V \times Z^A_{n_f}$, … $Z^A_{n_f}$ … $SU(n_f)$ [32]. …

... -1/2 ... (B_+) ... (B_-) ... I.[3] ...

$$[T^a, B_+^i] = -t_{ij}^a B_+^j, \quad [T^a, B_-^i] = -t_{ij}^a B_-^j, \tag{10}$$

$$[X^a, B_+^i] = t_{ij}^a B_-^j, \quad [X^a, B_-^i] = t_{ij}^a B_+^j, \tag{11}$$

... i, j ... $SU(2)_L \times SU(2)_R$ ... $B_+ \sim (I,0) + (0,I)$ ... $B_- \sim (I,0) - (0,I)$. ... *containing only* $B_\pm$, ...

$$\mathscr{L} = \bar{B}_+ \partial\!\!\!/ B_+ + \bar{B}_- \partial\!\!\!/ B_- - m_0(\bar{B}_+ B_+ + \bar{B}_- B_-) + \cdots, \tag{12}$$

... 6 ... B_+ ... B_- ... $B_\pm$ ... π^a, ... (12). ... 13,14 ...

$$[X^a, \pi^b] = -f^{ab}(\pi) = -(\delta^{ab}\tfrac{1}{2}(1 - \vec{\pi}^2) + \pi^a \pi^b), \tag{13}$$

... $f_\pi = 13$ ... (12), ...

$$\delta\mathscr{L} = m_1\left(\bar{B}_+ \frac{1-\pi^2}{1+\pi^2} B_+ - \bar{B}_- \frac{4\pi^a t^a}{1+\pi^2} B_+ - (B_+ \leftrightarrow B_-)\right).$$

... $B_\pm$ ... $m_0 \pm m_1$. m_1 ... B_+ ... B_- ...

$$\tilde{B}_+ = \frac{B_+ - 2\pi^a t^a B_-}{\sqrt{1+\pi^2}}, \quad \tilde{B}_- = \frac{B_- - 2\pi^a t^a B_+}{\sqrt{1+\pi^2}}. \tag{14}$$

... $(\tilde{B}_+)$ ... $(\tilde{B}_-)$ ...

$$[X^a, \tilde{B}_{\pm i}] = v_0(\pi^2)\varepsilon_{abc}\pi^c t_{ij}^b \tilde{B}_{\pm j}, \tag{15}$$

... $v_0(\pi^2)$ ... 14, ... $v_0(x) = 1$ ... $\tilde{B}_\pm$ ... $B_+ \leftrightarrow B_-$ ... (10), ... (14) ... $\tilde{B}_i$ ...

... $\tilde{B}_\pm$ ... 12 14. ... $B_\pm$, ... (12). ... $\tilde{B}_\pm$ ...

$$\begin{aligned}\mathscr{L} = {} & \bar{\tilde{B}}_+(\partial\!\!\!/ - \varepsilon^{abc}\pi^a \partial\!\!\!/\pi^b t^c)\tilde{B}_+ - \bar{\tilde{B}}_+(\partial\!\!\!/\pi^a)t^a \tilde{B}_- \\ & + (\tilde{B}_+ \leftrightarrow \tilde{B}_-) - (m_0 - m_1)\bar{\tilde{B}}_+\tilde{B}_+ - (m_0 + m_1)\bar{\tilde{B}}_-\tilde{B}_- + \delta\mathscr{L}_2,\end{aligned} \tag{16}$$

---

[3] ... 16 ...

[illegible] $D_\mu\pi^a = 2\partial_\mu\pi^a/(1+\pi^2)$ [illegible] $\delta\mathscr{L}_2$ [illegible]

$$\delta\mathscr{L}_2 = c_2[\bar{\tilde{B}}_+(\not{D}\pi^a)\gamma_5 t^a\tilde{B}_+ + \bar{\tilde{B}}_-(\not{D}\pi^a)\gamma_5 t^a\tilde{B}_-] + c_3[\bar{\tilde{B}}_+(\not{D}\pi^a)t^a\tilde{B}_- + \bar{B}_-(\not{D}\pi^a)t^a\tilde{B}_+]$$
$$+ c_4[\bar{\tilde{B}}_+(\not{D}\pi^a)\gamma_5 t^a\tilde{B}_+ - \bar{\tilde{B}}_-(\not{D}\pi^a)\gamma_5 t^a\tilde{B}_-]. \tag{17}$$

[illegible] $\tilde{B}_\pm$ [illegible] $2m_1$. [illegible]

$$A^a_\mu = (c_2+c_4)\bar{\tilde{B}}_+\gamma_\mu\gamma_5 t^a\tilde{B}_+ + (c_2-c_4)\bar{\tilde{B}}_-\gamma_\mu\gamma_5 t^a\tilde{B}_-$$
$$+ (1-c_3)(\bar{\tilde{B}}_+\gamma_\mu t^a\tilde{B}_- + \ldots) + (\ldots). \tag{18}$$

[illegible] [6] [illegible] m_1, c_{2-4} [illegible] $SU(2)_L\times SU(2)_R$ [illegible] m_1 [illegible] c_{2-4}. [illegible] $SU(2)_L\times SU(2)_R$ [illegible] [11]. [illegible] $B_\pm$ [illegible] $(0,\frac{1}{2}), (\frac{1}{2},0)$ [illegible]

[illegible] 1969 [illegible] [32] [illegible] [33] [illegible] $\{T^a\}$ [illegible] $Z^A_{n_f}$ [illegible] $SU(n_f)_A$). [illegible] $n_f\geqslant 3$, [illegible] $n_f = 3$. [illegible] $SU(3)$ [illegible] $Z = \exp((2\pi i/3)X_8)$ [illegible] Z^2 [illegible] Z [illegible] $D_3 = (1, Z, Z^2, P, PZ, PZ^2)$. [illegible] [18] [illegible] D_3 [illegible] [4] [illegible]

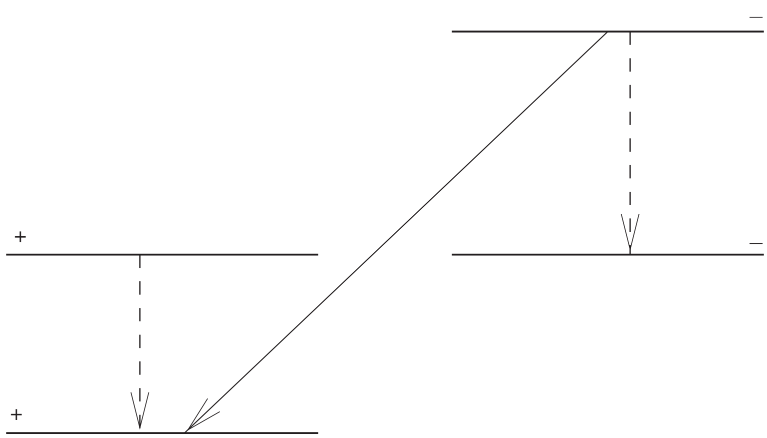


Fig. 7. [illegible] $h_1\to h_2\pi$ [illegible]

[4] [illegible] $Z_2\times$ [illegible]

... $h^* \to h\pi$ ... [7]. ... D ... [32] ... D ... $\langle \bar{q}q \rangle$ ... $f_\pi \sim m_q$ ... [33,34] ... CD ... [35] ... D ...

5. Dynamical suppression of $U(1)_A$ symmetry violation

... $U(1)_A$ ... CD ... [17]. $U(1)_A$ ... $u, d,$ ... s ... CD ... j_5^μ ... $Q_5(t) = \int d^3x\, j_5^0(t, \vec{x})$, ...

$$[H, Q_5] = -i\frac{d}{dt} Q_5 = -i\int d^3x\,\partial_\mu j_5^\mu, \quad \partial_\mu j_5^\mu = \sum_{q=u,d,s} 2m_q \bar{q}\gamma_5 q - \frac{3g^2}{16\pi^2} \tilde{F}^{\mu\nu} F_{\mu\nu}. \tag{19}$$

B ... j_5^μ ... $Q_5^a,\ a = 1, 2, 3,$ ... $U(1)_A$ ... $[H, Q_5]$ ... Q_5 ... η' ... 1 ... $U(1)_A$ ... *strong*, ... *weak* ... CD ... $\partial_\mu j_5^\mu$ ... Q_5 ... Q_5 ... $|B_+\rangle$ ... $|B_-\rangle$ ... B_+ ... B_- ... $\pm$ ... (19) ... $\partial_\mu j_5^\mu$ ...

$$\langle B_+|[Q_5, H]|B_-\rangle = (m(B_-) - m(B_+))\langle B_+|Q_5|B_-\rangle = 0. \tag{20}$$

$\langle B_+|Q_5|B_-\rangle \neq 0,$ ... $m(B_+) = m(B_-)$. ... $J, I,$ ... $S,$ ... Q_5 ... Q_5 ...

. $[H, Q_5] = 0$ ... $Q_5|0\rangle = 0$.
... CD ...

. $[H, Q_5]|B\rangle = 0$ ... $B \in$ ...
... $|B\rangle$ ... $Q_5|B\rangle$ ... H ... $|B\rangle$. ... Q_5 ... η' ... $\langle B'\eta'|Q_5|B\rangle$, ... $|B'\rangle$ ... $|B\rangle$ ... η'.

	$U(1)_A$	$U(1)_A$ B
$Q_5\|0\rangle = 0$		
Q_5		

$\partial_\mu j_5^\mu$ … $U(1)_A$ … $SU(2)_L \times SU(2)_R$ [5] … $B_\pm$ … -1/2 …

$$\langle B_+|j_{5\,\mu}(0)|B_-\rangle = \bar{u}(p_+)(\gamma_\mu g_A(q^2) + q_\mu g_P(q^2) + \sigma_{\mu\nu}q^\nu g_M(q^2))u(p_-),$$

$$\langle B_+|\partial^\mu j_{5\,\mu}|B_-\rangle = \bar{u}(p_+)d_A(q^2)u(p_-). \tag{21}$$

$q^2 = (q^0)^2 = \Delta m^2$. … q^μ … (21) …

$$\Delta m(g_A(\Delta m^2) + \Delta m g_P(\Delta m^2)) = d_A(\Delta m^2). \tag{22}$$

… : $\langle B_+|Q_5|B_-\rangle \propto g_A(\Delta m^2) + \Delta m g_P(\Delta m^2)$. … g_P … η' … $\partial_\mu j_5^\mu$ … Δm^2), …

$$\Delta m g_A(0) \approx d_A(0). \tag{23}$$

(23) … $U(1)_A$ … : A … $d_A(0)$ … $g_A(0)$ … ? … CD … $d_A(q^2)$. … $U(1)_A$ … $U(1)_A$ … $U(1)_A$ … [1] … $U(1)_A$ …

… j … j … $\Delta m \sim d_A(0)$, … $g_A(0)$ … $d_A(0)$ … $\tilde{F}F$ … $m_s\bar{s}\gamma_5 s$ … ?

[5] $U(1)_A$ … [36].

n.[6] $F\tilde{F}$ … $U(1)_A$ … $m_s = 0$ $SU(3)_V$ … $\langle F\tilde{F}\rangle$ … $m_s\bar{s}\gamma_5 s$. … u … d …, $\bar{J}_5^{\mu} \equiv \bar{u}\gamma^{\mu}\gamma_5 u + \bar{d}\gamma^{\mu}\gamma_5 d$, …

$$\partial_{\mu}\bar{J}_5^{\mu} = -\frac{2g^2}{16\pi^2}\tilde{F}^{\mu\nu}F_{\mu\nu}.$$

… $\bar{J}^{\mu}$ … j_5^{μ}. … $\langle N^{\pm}, \Delta^{\pm}|F\tilde{F}|N^{\mp}, \Delta^{\mp}\rangle \approx 0$ … $F\tilde{F} \approx 0$ … $SU(3)$ … $\bar{q}_R q_L +$ … $\bar{q}_L q_R$, … $\bar{u}\bar{d}\bar{s}uds$ [41]. … Λ … Σ, … $F\tilde{F}$, …

… *explicit* … *spontaneous* … $\delta H_{SB} \propto e(r_i r_j - \frac{1}{3}\delta_{ij})Q^{ij}$. … $j = 1/2$ … $\langle\psi_{\frac{1}{2},m}|[\delta H_{SB}, \vec{J}]|\psi'_{\frac{1}{2},m'}\rangle = 0$ … $[\delta H_{SB}, \vec{J}]|\psi_{\frac{1}{2},m}\rangle \neq 0$ …

6. Parity doubling and chiral symmetry restoration at short distances

… $SU(2)_L \times SU(2)_R$ … CD … 1967 … [8,47], … CD … [11], … [8,47] … N_c. … [48] … N_c … 1.0 … 2.5 … [47] … [8] …

… $\Pi(Q^2)$, … $\tilde{\Pi}(Q^2)$, … $Q^2 = -q^2 > 0$. … $\Delta\Pi(Q^2) \equiv \Pi(Q^2) - \tilde{\Pi}(Q^2)$

---

[6] … [37] … [8] … [40]. … $F\tilde{F}$ … $J, I,$ … $S,$ … [38].

x n n n, n,

$$\Delta\Pi(Q^2) \equiv \Pi(Q^2) - \tilde{\Pi}(Q^2) = -\int d\sigma^2 \frac{\Delta\rho(\sigma^2)}{Q^2+\sigma^2}, \tag{24}$$

n $\rho(\sigma^2)$ and $\tilde{\rho}(\sigma^2)$ ($\Delta\rho \equiv \rho - \tilde{\rho}$) n n x n σ x n n CD 49 $\Delta\Pi(Q^2)$ $1/Q^4$ $Q^2 \to \infty$,

$$\Delta\Pi(Q^2) \underset{Q^2\to\infty}{\sim} \frac{\langle\overline{\Psi}\Psi\rangle^2}{Q^4}, \tag{25}$$

$\langle\overline{\Psi}\Psi\rangle$ x x n n , Π and $\tilde{\Pi}$, x Q^2 Q^2 $Q^2 \to \infty$, n $1/Q^4$ n n x x *at short distances* n CD. C n n n, n n n $\sim 1/Q^2$ n $\Delta\Pi(Q^2)$ n n n,

$$\int d\sigma^2 \Delta\rho(\sigma^2) = 0. \tag{26}$$

. (26) *only* n x x n n 8 . C n n σ n n n , σ^2 x σ. x n n n x x , n n n n n ρ and ρ. n n n n n x n n CD x x n n , N_c, n n x *before* n n n (x n n *n*).

n n x fi N_c n n n 8 , n $N_c \to \infty$ fi n, n ρ ($\tilde{\rho}$) n n x *zero width* ( ) n n n , M_n ($\tilde{M}_n$) n n f_n ($\tilde{f}_n$). C n n n n x N_c. n , , n n n n n N_c:

$$M_n^2, \tilde{M}_n^2 \sim \Lambda^2 n,$$
$$\Gamma_n^2, \tilde{\Gamma}_n^2 \sim \Lambda^2 n/N_c^2,$$
$$f_n, \tilde{f}_n \sim N_c \Lambda^4 n, \tag{27}$$

Λ x n CD. *n* n x n n , n n 8,51 . N_c n -N_c n n . n n x , n $N_c \to \infty$ *n*, n n n ,

$$\Delta\rho(\sigma^2) = \sum_n (f_n\delta(\sigma^2 - M_n^2) - \tilde{f}_n\delta(\sigma^2 - \tilde{M}_n^2)), \tag{28}$$

n n n ,

$$N_c\Lambda^2 \sum_n (\alpha_n M_n^2 - \tilde{\alpha}_n \tilde{M}_n^2) = 0, \tag{29}$$

fin $f_n = N_c\Lambda^2\alpha_n$, n *n*. n $\alpha_n = \tilde{\alpha}_n = \alpha$, n n n (n n 8) *n* n n n n . (29)

$$M_n^2 - \tilde{M}_n^2 \underset{n\to\infty}{\sim} \frac{1}{n},$$

$M_n \sim \tilde{M}_n \sim \sqrt{n}$ n, $\Delta M_n \equiv M_n - \tilde{M}_n \sim n^{-3/2}$, n.[7]

N_c (. (27)), $n \to \infty$ N_c. $1/N_c$ n [52]. q^2- n. [52] . (29)

$$N_c \Lambda^2 \sum_n \left(\alpha_n (M_n^2 - \frac{1}{\pi}\Gamma_n M_n) - \tilde{\alpha}_n \left(\tilde{M}_n^2 - \frac{1}{\pi}\tilde{\Gamma}_n \tilde{M}_n \right) \right) = 0. \quad (30)$$

. (29),

$$\Delta M_n - \frac{1}{\pi}\Delta\Gamma_n \underset{n\to\infty}{\sim} \frac{1}{n^{3/2}},$$

$\Delta\Gamma_n \equiv \Gamma_n - \tilde{\Gamma}_n$. . (27) $\Gamma, \tilde{\Gamma} \sim \sqrt{n}/N_c$ N_c. n *the difference* $\Delta\Gamma_n$. n, n $n^{-3/2}$. β, 1/2, $1/N_c$ [8],

$$\Delta M_n \underset{n\to\infty}{\sim} \frac{1}{n^{3/2}} + C\frac{n^{\beta}}{N_c},$$

β C . $1/N_c$ ($N_c = 3$) $1/N_c$ CD *violating* . [8] . (27) N_c.

7. Parity doubling and intrinsic deformation

( ) ( ) ( ) [225] ( ) T- [42,44]. $|\psi\rangle$ $\hat{P}|\psi\rangle$, $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle \pm \hat{P}|\psi\rangle)$. $|\psi\rangle$ $\hat{P}|\psi\rangle$

[2], CD.

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[7] [50], $N_c \to \infty$, $\Delta\Pi(Q^2)$, $\Delta\rho(\sigma^2)$ N_c N_c ( ), δ- $\Delta\rho$ ([52]), n N_c, [50] N_c .

Table 2
[illegible] $I=1/2$ [illegible] $3/2$ [illegible] $5/2$ [illegible] [29]

$B(J^P)$	[illegible]	[illegible]
$N(5/2^-)$	∗∗∗∗	1675
$N(5/2^+)$	∗∗∗∗	1680
$N(7/2^+)$	∗∗	1990
$N(7/2^-)$	∗∗∗∗	2190
$N(9/2^+)$	∗∗∗∗	2220
$N(9/2^-)$	∗∗∗∗	2250
$\Delta(5/2^+)$	∗∗∗∗	1905
$\Delta(5/2^-)$	∗∗∗	1930
$\Delta(7/2^+)$	∗∗∗∗	1950
$\Delta(7/2^-)$	∗	2200
$\Delta(9/2^+)$	∗∗	2300
$\Delta(9/2^-)$	∗∗	2400

[illegible] [45] [illegible] $(I=1/2)$ [illegible] Δ $(I=3/2)$ [illegible] Table 2). [illegible] (Λ [illegible] Σ) [illegible] Λ [illegible] Σ [illegible] Table 2 removed. [illegible]

8. Implications from mended chiral symmetry

[illegible] [23,24] [illegible] [24] [illegible] [23,24] [illegible] [23,24] [illegible]

[illegible] [23] [illegible] $J^P=0^\pm,1^\pm$ [illegible] (σ,π,ρ,a_1), [illegible] (N,Δ,N_1) [illegible]

$$A(h_\alpha(p,\lambda) \to h_\beta(p',\lambda')\pi^a) \equiv \mathscr{M}^a_{\alpha\beta} = \frac{1}{f_\pi}(p-p')_\mu \langle h_\beta(p',\lambda')|A^{a\mu}|h_\alpha(p,\lambda)\rangle. \tag{31}$$

$$\mathscr{M}^a_{\alpha\beta} = \frac{1}{f_\pi}|\vec{q}|\langle h_\beta(p',\lambda')|A^{a0}-A^{a3}|h_\alpha(p,\lambda)\rangle \equiv [X^a(\lambda)]_{\beta\alpha}\delta_{\lambda\lambda'}. \tag{32}$$

$$[T^a, T^b] = i\epsilon^{abc}T^c, \quad [T^a, X^b(\lambda)] = i\epsilon^{abc}X^c(\lambda). \tag{33}$$

$$[X^a(\lambda), X^b(\lambda)] = i\epsilon^{abc}T^c. \tag{34}$$

$$[X^a(\lambda), [X^b(\lambda), m^2]] \propto \delta_{ab}. \tag{35}$$

$$\hat{m}^2(\lambda) = \hat{m}_0^2(\lambda) + \hat{m}_4^2(\lambda) \tag{36}$$

3. $\hat{m}_4^2$ …

… [56–59].

… N_c … [25,26] … N_c … CD. … $SU(2n_f)_c$, … n_f … J^i, … I^a, … G_0^{ia} … p- … N_c … $(G^{ia} \to G_0^{ia})$

$$\mathscr{M}^a_{\alpha\beta} = \frac{1}{f_\pi} q^i N_c [G^{ia}]_{\beta\alpha}. \tag{37}$$

… (32) … $X^a(\lambda) \sim [G^{3a}]_{\lambda\lambda}$. … J^i, T^a, G_0^{ia} … $SU(2n_f)$ …

$$[J^i, J^j] = i\epsilon_{ijk} J^k, \quad [T^a, T^b] = i\epsilon_{abc} T^c, \tag{38}$$

$$[G_0^{ia}, J^j] = i\epsilon_{ijk} G_0^{ka}, \quad [G_0^{ia}, T^b] = i\epsilon_{abc} G_0^{ic}, \quad [G_0^{ia}, G_0^{jb}] = 0. \tag{39}$$

… N_c … m_0^2 … $SU(2n_f)$ … N … Δ … $SU(4)_c$, … $J = I = 1/2, 3/2, 5/2, \ldots$ … [25,26].

B… N_c … (35). … N_c … $X^a(\lambda)$. … N_c … λ … (I_L, I_R) … $SU(2)_L \times SU(2)_R$, … (35) … N_c …

An… N_c …

… λ … (j_L, j_R) … $SU(2)_L \times SU(2)_R$

$$|h(\lambda)\rangle \sim \sum_{j_L, j_R} c_{j_L j_R}(\lambda) |j_L, j_R\rangle, \tag{40}$$

$SU(2)_R \times SU(2)_L$ ... $\hat{m}_4^2$ ... (...), ... $|h(\lambda)\rangle$ ... [8] A ... $\hat{P}$ ... $h_\alpha(\vec{p}, \lambda)$ ... $\hat{P}|h_\alpha(\vec{p}, \lambda)\rangle = \pi_h |h_\alpha(-\vec{p}, -\lambda)\rangle$, ... π_h ... h. ... *normality* ... $\Pi = \hat{P} R_\pi^y$, ... $180°$ ... $\Pi |h_\alpha(\vec{p}, \lambda)\rangle = \pi_h (-)^{j_h} |h_\alpha(\vec{p}, -\lambda)\rangle$. ... Π

$$\Pi X^a(\lambda) \Pi = -X^a(-\lambda), \quad \Pi T^a \Pi = T^a. \tag{41}$$

... $|h(-\lambda)\rangle$ ... (j_R, j_L) ... (40) ... $j_L \leftrightarrow j_R$

$$|h(-\lambda)\rangle \sim \sum_{j_R, j_L} c_{j_R j_L}(\lambda) |j_L, j_R\rangle. \tag{42}$$

... N, Δ ... $N N\pi$, $N\Delta\pi$, $\Delta\Delta\pi$ ... 1/2 ...

Scenario ... 23 ... 55 ... $SU(4)$ ... N_c ... $1/N_c$. ... $SU(2)_L \times SU(2)_R$

$$\lambda = +\tfrac{1}{2} : (1_L, \tfrac{1}{2}{}_R) \supset N, \Delta, \tag{43}$$

$$\lambda = +\tfrac{3}{2} : (\tfrac{3}{2}{}_L, 0_R) \supset \Delta. \tag{44}$$

... (42)

$$\lambda = -\tfrac{1}{2} : (\tfrac{1}{2}{}_L, 1_R) \supset N, \Delta, \tag{45}$$

$$\lambda = -\tfrac{3}{2} : (0_L, \tfrac{3}{2}{}_R) \supset \Delta. \tag{46}$$

... 8 ... $\lambda = \pm\frac{1}{2}$, ...

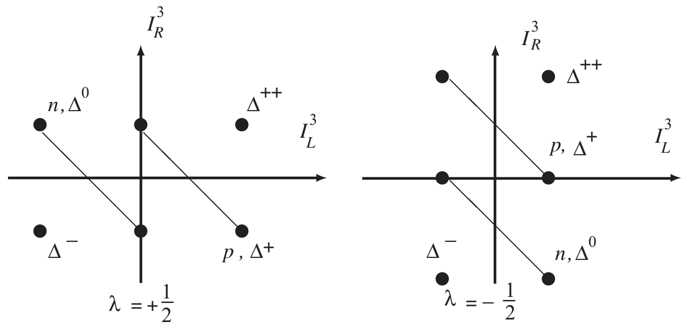
... (36)) ... $\lambda = +1/2$ ... $(N_{+1/2}, \Delta_{+1/2})$, ... $\hat{m}_4^2$ ... (I_L, I_R). ...

... $X^a = \frac{1}{2}(I_R^a - I_L^a)$. ... *p*- ...

$$\langle J' I'; \lambda I_3' | X^a | J I; \lambda I_3 \rangle = \frac{1}{2J'+1} X(J', J) \langle J' \lambda | J 1; \lambda 0 \rangle \langle I' I_3' | I 1; I_3 a \rangle, \tag{47}$$

... $X(1/2, 1/2) = -5/\sqrt{2}$, $X(1/2, 3/2) = -4$, $X(3/2, 3/2) = -5\sqrt{2}$, ... $SU(4)$ ... $X(1/2, 1/2)$ ...

[8] ... $|\lambda|$ ...



. 8. $(1,\frac{1}{2})$ $(\frac{1}{2},1)$ $\lambda=+\frac{1}{2}$ ( ) $\lambda=-\frac{1}{2}$ ( ), x . $I^3=\frac{1}{2}(I_L^3+I_R^3)$, Δ n.

$g_A/g_V=5/3$. $g_A/g_V=1.2695\pm0.0029$ x $(\frac{1}{2}{}_L,0_R)$ $\lambda=+\frac{1}{2}$. x g_A/g_V [23] . , [23] .

Scenario . C $(0,\frac{1}{2})$ $(\frac{1}{2},0)$. +1/2 , -1/2 N_A, N_B , θ

$$\lambda=+\tfrac{1}{2}: N_A = \quad \theta(0_L,\tfrac{1}{2}{}_R) + \quad \theta(\tfrac{1}{2}{}_L,0_R), \tag{48}$$

$$N_B = - \quad \theta(0_L,\tfrac{1}{2}{}_R) + \quad \theta(\tfrac{1}{2}{}_L,0_R). \tag{49}$$

, m_A, m_B. x . C $(\frac{1}{2},0)$ $(0,\frac{1}{2})$

$$m^2_{\lambda=+1/2} = \begin{pmatrix} \mu_L^2 & \alpha \\ \alpha & \mu_R^2 \end{pmatrix}, \tag{50}$$

$\mu^2_{L,R}$ $\hat{m}_0^2$; α - . (36).

$$\mu_L^2 = m_A^2 \quad {}^2\theta + m_B^2 \quad {}^2\theta,\quad \mu_R^2 = m_A^2 \quad {}^2\theta + m_B^2 \quad {}^2\theta,\quad \alpha=\tfrac{1}{2}(m_A^2-m_B^2) \quad 2\theta. \tag{51}$$

x $-1/2$ x x , . C $\pi_{N_A}=+1$, $\pi_{N_B}=-1$, x $-\frac{1}{2}$

$$\lambda=-\frac{1}{2}: \Pi N_A = \quad \theta(0_L,\tfrac{1}{2}{}_R) + \quad \theta(\tfrac{1}{2}{}_L,0_R), \tag{52}$$

$$\Pi N_B = \quad \theta(0_L,\tfrac{1}{2}{}_R) - \quad \theta(\tfrac{1}{2}{}_L,0_R). \tag{53}$$

w , . $X^a=1/2(I_R^a-I_L^a)$ x x , nx . x $p_{i\uparrow}\to n_{j\uparrow}\pi^+$, x $X^-=X^1 - X^2$.

$N_A \rightarrow N_A\pi$

$$
\begin{aligned}
A(p_{A\uparrow} \rightarrow n_{A\uparrow}\pi^{+}) \\
&\sim (\quad \theta\langle(1,0)_L,(\tfrac{1}{2},-\tfrac{1}{2})_R| + \quad \theta\langle(\tfrac{1}{2},-\tfrac{1}{2})_L,(1,0)_R|) \\
&\quad \times X^{-}(\quad \theta|(1,0)_L,(\tfrac{1}{2},+\tfrac{1}{2})_R\rangle + \quad \theta|(\tfrac{1}{2},+\tfrac{1}{2})_L,(1,0)_R\rangle) \\
&= \ {}^2\theta\langle(\tfrac{1}{2},-\tfrac{1}{2})_R|I_R^{-}|(\tfrac{1}{2},+\tfrac{1}{2})_R\rangle - \ {}^2\theta\langle(\tfrac{1}{2},-\tfrac{1}{2})_L|I_L^{-}|(\tfrac{1}{2},+\tfrac{1}{2})_L\rangle = \quad 2\theta,
\end{aligned} \tag{54}
$$

$I^{-}_{L,R}$ ... (16), ... $c_2 + c_4$. ...

$$c_2 + c_4 \sim \quad 2\theta, \tag{55}$$

$$c_2 - c_4 \sim - \quad 2\theta, \tag{56}$$

$$1 - c_3 \sim - \quad 2\theta. \tag{57}$$

... $\theta = 3\pi/4$ ... $(c_{2-4} = 0)$, ... $\mu_L^2 = \mu_R^2 = (m_A^2 + m_B^2)/2$ ... $\alpha = -(m_A^2 - m_B^2)/2$, ... $m_A^2 - m_B^2 \neq 0$.

... $(I_L, I_R) = (1, 1/2), (1/2, 1)$ ... 1. ... CD ... 60 .

9. Conclusion and discussion

A ... 35 ... $U(1)_A$ ... $SU(2)_L \times SU(2)_R$ ...[9] ... $\Omega_{IJS}(\{C\})$ ... J, ... I ... S, ... $\{C\}$), ... Ω_{IJS} ... (p-) ... (N ... Δ), ... $J = 1/2$ ... 7/2. ... 2. ... *strange* ...

[9] ... $U(1)_A$ ...

(n) ... (j_L, j_R), $SU(2)_L \times SU(2)_R$ ... $(1/2, 1) \oplus (1, 1/2)$ ... 54 ... $(N_J^+, N_J^-, \Delta_J^+, \Delta_J^-)$ ... J. ... (3). ...

$SU(2)_L \times SU(2)_R$ ... CD ... u ... d ...) $SU(2)_L \times SU(2)_R$ ... $SU(2) \times SU(2)$ ... ? ... 11 ... $SU(2)_L \times SU(2)_R$ *invariant* ... *two independent* $SU(2)_L \times SU(2)_R$ ... *non-linearly*, ... *linearly*. A ... C ... 61 ... ? ... 27 ... A ...

... 3A, ... $U(1)_A$ ... CD. ... $U(1)_A$ ... N_c CD. ... η, η' ... $\langle B_1|\partial_\mu j_5^\mu|B_2\rangle \approx 0$, ... $B_1, B_2 \in$ [10] ... [11] ... $\tilde{F}F$ ... CD.

A ... $U(1)_A$ ... $U(1)_A$ ... $U(1)_A$ ... CD. ... (26). ... N_c ... $N_c \to \infty$ ... $(1/N_c)$ ...

[10] ...

[11] ...

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