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Stretching the boundaries in turbulent convection

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Abstract.

Laboratory experiments investigating turbulent Rayleigh-Benard convection (RBC) have nearly always been confined to cylindrical cells of small diameter-to-height aspect ratio Γ (typically 1/2). The motivation for this has been that the governing control parameter, the Rayleigh Number Ra, depends on the cube of the height, and so laboratory experiments have been made as tall as possible, which for technical reasons has precluded them from having lateral dimensions substantially larger than this. Obtaining large Γ simply through a reduction in cell height, at least for for ordinary fluids and/or cryogenic apparatus which are small, has the unfortunate consequence of also moving the experiment substantially out of the turbulent regime. The work reviewed here achieves concurrently large aspect ratio and high Ra, taking advantage of an apparatus of relatively large diameter and the use of an optimum test fluid, namely low temperature helium gas. This allows the experiment to approach more closely geophysically relevant conditions as well as theoretical analysis which typically assumes an infinite horizontal extent.

1. Introduction

In part because it is the paradigm for many diverse natural phenomena, and important to many industrial applications, turbulent Rayleigh-Bénard convection (RBC) has received much attention in recent years. RBC occurs in a fluid layer heated from below and cooled from above, creating a sustained, unstable density difference which drives a macroscopic flow. This flow, when turbulent, can increase the effective heat transport over that due to molecular conduction by orders of magnitude[1].

The measure of convective turbulence is given by the Rayleigh number defined as $Ra \equiv \alpha\Delta T g H^3/\nu\kappa$, where α is the isobaric thermal expansion coefficient of the fluid, ΔT the temperature difference between the bottom and top walls, g the acceleration due to gravity and H the vertical dimension of the fluid layer; ν and κ are, respectively, the kinematic viscosity and the thermal diffusivity of the fluid. It is desirable to push Ra as large as possible, since most natural flows have large values of this parameter, mainly due to the large system length scales. An additional control parameter is the Prandtl number, $Pr = \nu/\kappa$ which measures the ratio of time scales based on momentum and thermal diffusion. For these experiments it is of order unity except near the critical point where the working fluid, cryogenic helium gas, is no longer ideal.

It is the use of cryogenic helium that makes it possible to approach dynamical similarity with geophysical flows in small apparatus with well-controlled boundary conditions. Achievement of this goal is aided dramatically by making the sample space tall, since it is the vertical length scale that directly enters into Ra. Technical and economic considerations have then led to experiments performed in tall, but narrow width containers, which differ both from many natural flows, which often have a laterally extended structure, and from analytic treatments which for simplicity typically assume infinite lateral extent. Experimentally, these two conditions–large Ra and large horizontal to vertical length ratio–typically are mutually exclusive.

Considering the usual case of cylindrical apparatus, a small diameter-to-height aspect ratio, $\Gamma = 1/2$ allowed Rayleigh numbers up to 10^{17} to be reached in the laboratory[1], equivalent in order of magnitude to the convectively driven flows naturally occurring in the atmosphere. However, it has not been clear what influence the severe confinement in experiments has had on turbulent flows, which was the motivation for the present work at larger Γ.

The simplest method of increasing Γ, whether in a low temperature apparatus or otherwise, is of course to lower the vertical height of the sample space keeping the diameter fixed. However, by lowering the sample height H, Ra– which depends on its cube– is significantly reduced. In the work reviewed here, Γ was increased to 4, precisely by lowering the sample height form 100 cm to 12.5 cm, with an accompanying reduction in Ra; however, the apparatus still allows Ra greater than 10^{12} within strict Boussinesq conditions[2]. For more details, the reader is referred to an earlier manuscript[3].

2. The apparatus

The apparatus has been described previously [1, 3] and here we recount the most salient features. The sample space was cylindrical, with a fixed diameter of 50 cm diameter, and variable height ranging from 12.5 cm ($\Gamma = 4$) to 100 cm ($\Gamma = 1/2$). The sides were made of thin-wall stainless steel, with a thickness of 0.267 cm. The top and bottom plates were made of annealed OFHC copper, 3.8 cm in thickness, and with thermal conductivity of the order of 1 kW $\mathrm{m}^{-1}\mathrm{K}^{-1}$ at the measurement temperature. The plates heated uniformly, using distributed thin film heater elements. At the top, temperature-controlled surface a so-called soft vacuum space filled with dilute helium gas allowed the heat to be uniformly removed. At the bottom plate a constant heat flux was applied, but measurements were initiated only after the resulting plate temperatures reached steady values. Thermal radiation protection was provided by three outer conducting shields at various graded temperatures and residing in a common vacuum space. The working fluid was cryogenic helium gas, at a nominal temperature of about 5K.

3. Experimental results

The heat transport in turbulent convection is a global observable that is often the quantity of interest. It represents an equivalent "black-box" resistance which integrates all dynamical and static characteristics of the fluid system. In dimensionless form it is given in terms of the Nusselt number Nu, where Nu is the ratio of the measured total heat flux to that which would occur solely by molecular conduction for the *same* ΔT and H. Nu must be corrected for various effects related to the confinement, i.e. to the non-zero conductivity of the sidewalls, as well as those the effect of having heated plates of finite conductivity. These latter corrections, e.g., are nearly negligible in helium experiments due to the large conductivity of the copper plates at low temperature. The former effects become negligible at the highest Ra which are afforded by the cryogenic fluid due to the fact that the fluid conductivity is so large ($Nu >> 1$). These corrections in helium experiments are described in more detail in an earlier manuscript[3].

Figure 1c shows Nu as a function of Ra, and over a range of Ra in which strict Boussinesq conditions are adhered to[2]. The data have been corrected[3] in the usual way for effects of non-infinite plate conductivity and non-zero wall conduction.

At high Ra, there is a clear region of about two decades in which the $\log Nu$-$\log Ra$ slope attains a value within 2% of 1/3, as enhanced by the normalized plot. The 1/3 slope is the theoretical expectation for RBC in the limit as Ra tends to infinity, for which the bulk is mixed

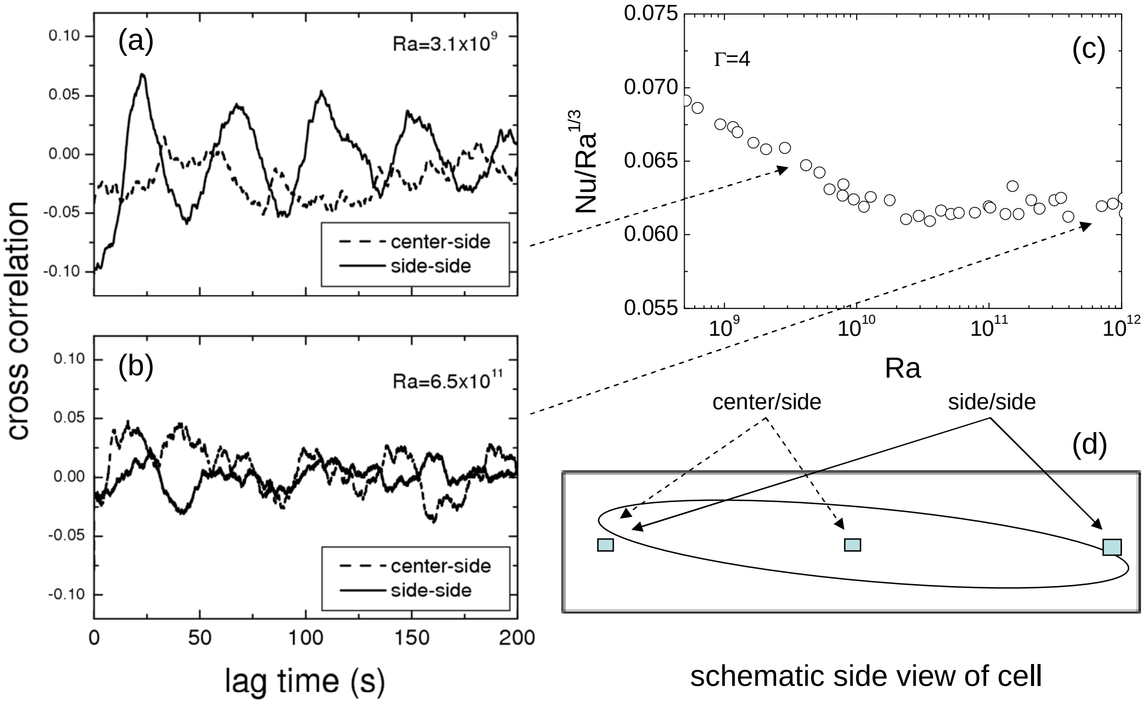

Figure 1. Heat transfer and temperature correlations. (a) cross-correlation between pairs of sensors as shown in (d). $Ra = 3x10^9$. (b) same as in (a) but for $Ra = 6.5x10^{11}$. (c) $Nu/Ra^{1/3}$ vs Ra showing the attainment of a large range of Ra in which the logNu-logRa slope is very close to the asymptotic theoretical value 1/3 in the limit as Ra tends to infinity. The dashed lines from parts (a) and (b) indicate the approximate location along the Ra line where the cross-correlations were taken. (d) schematic side view of the $\Gamma = 4$ cell showing the sensor positions along a diameter at the cell mid-height.

by strong turbulence to such an extent that there is no direct communication between the two fluctuating boundary regions near the top and bottom walls[4, 5].

To shed further light on the significance of the heat transfer scaling, we show in figure 1(a) and (b) the cross-correlation between pairs of temperature sensors situated at the cell mid-height, and at various positions along a diameter. Two of the sensors–250 micrometer cubes of bare NTD germanium–were placed near the sidewall and at opposite ends of a diameter, (i.e., π radians apart along the circumference). A third sensor was places at the precise cell center. The plotted correlations between the two side sensors (solid lines) and between a side and the center sensor (dashed line) are shown for two values of Ra: In (a), $Ra = 3.1 \times 10^9$, and corresponds to the region of logNu-logRa slope 0.31. It is clear that there is a strong long-time correlation between opposite sidewall sensors, indicative of a robust large scale circulation (known as the "mean wind"), while no clear correlation exists between the sidewall fluctuations and those of the center. Thus we may conclude that on average there is a single circulation around the entire cell periphery. Part (b) shows the same correlations for a higher $Ra = 6.5 \times 10^{11}$, well into the region of approximate logNu-logRa slope 1/3. Clearly, here there is no correlation between any set of temperature signals, indicating that the coherence of the large scale circulation has been lost in the bulk region, which itself is consistent with the assumptions outlined above for deriving a 1/3 power law exponent in the relation between Nu and Ra.

4. Discussion

We have reviewed heat transfer and temperature fluctuation data for relatively large aspect ratio 4 at high Ra. As seen in figure 1, for $10^8 < Ra < 10^{10}$, the logNu-logRa slope is nearly constant and of magnitude 0.31. Here there is a robust large scale circulation. For higher Ra within the Boussinesq limit the slope saturates around 1/3. Consistent with this observation is the loss of correlation between temperature signals, indicating that the bulk mixing is relatively strong. This observation is also consistent with a recently published investigation of bulk turbulence[**?**] and also discussed separately within these proceedings.

We further note that from the temperature fluctuations used to obtain the correlations in figure 1, we can also compute a Γ-independent Reynolds number, Re_f, based on the frequency of the peak in the power spectral density of the temperature fluctuations. In particular, we observe $Re_f^* Pr^{2/3} = 0.44 Ra^{0.453}$ which compares favorably with a recent theoretical prediction $Re \sim Ra^{4/9} Pr^{-2/3}$ in the Pr-Ra phase space region corresponding to these experiments[6].

Finally, we note that no other experiment to date has combined both similarly high Ra and large aspect ratio. This unique advantage is afforded by the use of cryogenic helium gas, which also has allowed a range of Ra to be investigated in the same apparatus.

4.1. Acknowledgments

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