A perspective on the status of mathematics in India

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My intention here is to comment briefly on the present status of mathematics in India. I approach the task with some diffidence because my own mathematical knowledge is limited and I have lived for some thirty-three years outside India. Thus, the perspective can at best be partial. Please bear with me if you see flagrant errors of detail or substance.

Keywords: Expatriate Indians, mathematics, science in democracy, university system.

A few historical remarks

I am no expert on the history of mathematics in India and am aware of the intrinsic difficulties in being precise. My brief remarks, such as they are, have been culled entirely from secondary though serious sources, and are meant to set the stage for comments to follow.

It appears even to a casual observer that India has good reasons to be proud of its heritage in mathematics. It can even be said that the sustained efforts in India led, through the Arabs, to the revival of mathematics in Europe. Scholars have noted that the decimal system was already in place in the Harappan period, though a few centuries had to lapse for its importance to be appreciated fully. Arithmetic operations and sequences, fractions and certain geometric rules were known some one thousand years before Christ, including results such as the Pythagoras theorem. Jains were fascinated by large numbers and had an advanced knowledge of infinity (though it fell far short of our present understanding), and maintained the tradition of mathematics for centuries. Buddhists knew infinite and indeterminate numbers. Important developments came through astronomy. Because the tradition of scholarship in ancient India was to reduce everything to aphorisms and mnemonics while obliterating intermediate steps, it is hard to ascertain the reliability of modern-day assessments of the past. The best I can do is to refer to some source material[1] where lively discussion can be found; there is, in fact, a well-articulated view, even in India, that the claims of its mathematical past have been exaggerated. There is, however, no controversy to my knowledge about the legendary contributions of the mathematician–astronomers, Aryabhata (476–550), Varahamihira (505–587), Brahmagupta (598–670), Bhaskara (600–680) and Bhaskaracharya (1114–1185).

It is interesting to trace how and why the centres of excellence in mathematics moved gradually to the south of the country along its west coast: Rajasthan, Gujarat, Karnataka and Kerala. The Kerala School blossomed well into the late 16th century, with prominent mathematicians such as Madhavan, Parameshvaran, Nilakanthan, Jyeshtadevan and Achyutan. It has been generally recognized at least since the 1940s that the Kerala School had obtained a number of important mathematical results in analysis, infinite series, calculus, integration, trigonometry, geometry and algebra[2]. Several concepts developed later in Europe were known to this school[3]. However, as noted by Divakaran[4], the results on series expansions and infinitesimals, which were devised for special cases, did not contain the generality and abstraction of the later inventions of the same concepts in Europe. Among other things, they did not use mathematical symbols effectively. Divakaran notes that in India by that time, 'mathematics was seen as no more than a handmaiden of astronomy', and adds that 'Bhaskara's algebraic legacy seems to have better nourished in the Arabic lands than among his own mathematical heirs in India'. Bhaskara here is Bhaskaracharya of the 12th century.

Even though the main centres of excellence moved southward, there was a continuity of tradition. For example, the Kerala School, though removed from Aryabhata by six or seven centuries in time and some fifteen hundred miles in space, built itself on the heritage of Aryabhata. However, the continuity with the past ended abruptly with the Portuguese incursion into Kerala and after the British occupation of the subcontinent. One can speculate why the mathematics of the land, which survived Muslim invasions after a fashion, could not survive the changes that accompanied the British occupation, but I will leave this interesting discussion for mathematical historians.

The advent of the early Christian era drew to a close the glorious days of Greek mathematics: Plato's academy was closed and the library of Alexandria was destroyed. This resulted in shifting – perhaps 'reverting' is the better word – the centre of gravity of mathematics to the East. In particular, for several centuries during that period, India played a dominant role. Over time, disruptive interference from religion and astrology seems to have caused the de-

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cline of mathematics of the land, and people who claimed to be creating new mathematics while pursuing its recreational aspects of no general significance seem to have increased in number; and the decline seems to have accelerated with Islamic invasions. The contact with Islam, however, had the benefit that Indian mathematics texts were translated into Arabic and Persian, and disseminated to the Western world. Arab scholars[5] relied on a variety of sources, but the Indian sources were particularly important. (In the reverse direction, some of the Greek texts, such as Euclid's Geometry, were translated into Indian languages, though much later, from Arabic or Farsi[6].)

Indian mathematics in recent centuries

For the past four or so centuries, most major developments have come from Europe and, lately, also from the US. Descartes, Newton, Euler, Gauss, Riemann, Hilbert and Poincaré are some names that come to mind immediately. In that class belongs Ramanujan, who represents the transition between the traditional and the modern mathematics in India. For instance, Hardy[7] remarks that the concept of mathematical proof was somewhat alien to Ramanujan, yet one can argue that his work was more influenced by the Western tradition than by his Indian background. Ramanujan's story[8] is known too well to need recounting here, but I wish to call attention to Narasimha's analysis[9] of Ramanujan's way of doing mathematics. It suffices here to say that while Ramanujan was singular in most respects – and the first Indian mathematician to gain recognition from the West in his lifetime – the country did have several other less well-known mathematicians in his time[10]. All of them explicitly followed the traditions of the West[11].

When higher education in the modern sense started in 1857, the three universities in Calcutta, Bombay and Madras were the main centres, and all of them taught mathematics. The Mathematics Department of Madras University, now known as the Ramanujan Institute of Mathematics, had an advanced centre in fundamental mathematics. Allahabad, Agra, Annamalai University and Andhra University had some good mathematics activities as well. Later, Banaras Hindu University and the Central College of the Mysore University nurtured talented mathematicians.

Andre Weil, later the major figure in the Bourbaki group, describes the mathematics of the early 1930s in India[12]. As is well known, Weil was Professor of Mathematics in Aligarh Muslim University during 1930–32. He remarks that the only Indian mathematician whose name he knew before coming to the country was Ramanujan. Though he did learn about some of them later and spoke about them in positive terms, he noted that 'none of them had attained a reputation which would make it possible to start developing a school' of international repute around them. He comments as follows on a mathematics meeting he attended:

'At the meeting I was struck, as I had already been elsewhere, not by the level of mathematics – which was mediocre at best – but by the eagerness and openness of mind evident among the younger generations, a sharp contrast with the routine in which their elders were mired. I judged this to be a good omen for the future of mathematics in India. This optimism was actually somewhat premature, but later developments showed that it was not entirely unfounded'. These later developments will take us to more recent times, whose threads I will pick up subsequently in this essay.

The status of mathematics in modern India

After independence, many new universities were created, as were several National Laboratories and five Indian Institutes of Technology (IIT). There already was the Indian Institute of Science (IISc) in Bangalore – which, by the way, will be celebrating its 100th anniversary soon, and about which I will say more later on. A particular mention must be made of the Tata Institute of Fundamental Research (TIFR), Mumbai, which has been a centre of sustained excellence in mathematics, rejuvenating the tradition, though perhaps inadvertently, of the mathematics schools of the last fifteen centuries which thrived along the western coast of India. Let me first comment on these special institutions before returning to the universities.

When Andre Weil made his remark, he had in mind fundamental, or pure, mathematics. In the following, I will treat pure and applied mathematics somewhat separately, though it is unclear if this separation is logical; after all, pure mathematics of the past has sometimes become applicable mathematics of the present. Nevertheless, it is a practically useful distinction for present purposes. Mathematics departments in the US have recently recognized that some branches of pure mathematics have become formal and sterile for having lost connection with reality. Special efforts are being made to nurture applied mathematics sections within mathematics departments. Applied mathematics is not esoteric meteorology, fluid dynamics or control theory, but a subject in which mathematical ideas are used to solve practical problems. Applied mathematicians should be able to borrow tools readily from existing mathematics or generate new ones as required, but the motivation should come from practical problems. Applied mathematicians are not engineers or biologists who know some asymptotic theory or group theory, but are master mathematicians who willingly engage in applications and draw on their reserves of mathematical knowledge; or, when none is adequate, are capable of creating new mathematical tools. I must also call attention to the close link that now exists between applied mathematics and large-scale computations.

Focusing first on pure mathematics, there have indeed been substantial contributions from Indians recently. This is the essence of Weil's remark mentioned earlier, to which I can add only superficially. Raghunathan[13] has pointed out that the international community has recognized the quality of Indian mathematics by inviting, since the 1950s, at least one Indian mathematician to present an invited talk at each of the meetings of the International Congress of Mathematics. This meeting takes place once every four years and to give an invited talk is regarded as an honour. It is worth pointing out that many – if not all – of these invited lecturers are from TIFR, or have had a TIFR connection. Today, some 45 active mathematicians work in TIFR – mainly in algebra, algebraic geometry, Lie groups and number theory, with a few working in combinatorics, PDEs and geometric topology. In its School of Technology and Computer Science several people are now working on stochastic processes.

Traditionally, probability and stochastic processes have been the province of the Indian Statistical Institutes (ISIs; Delhi, Kolkata and Bangalore), and this continues to be so even now. Together, the ISIs house around 70 mathematicians working on these subjects as well as operator theory and harmonic analysis. There seems to be some activity, among younger people, in non-commutative geometry. Of course, statistics has always been an important subject there. The two centres in Chennai (Matscience and Chennai Mathematics Institute) hold around 25 people between them, mainly working in algebraic geometry, operator theory and number theory.

Among the IITs, the one in Mumbai has an active department, especially in commutative algebra. Theoretical computer science with a mathematical flavour is pursued at IIT Kanpur, perhaps also at IIT Madras and other places. It is unfortunate that the IITs have not been more successful in creating schools of mathematics. The Mathematics Department at IISc, where pure mathematics is of relatively recent origin, has recruited a few young geometers. I hope that the subject will thrive there.

Let me now turn to applied mathematics. For some time, IISc was the strongest centre of applied mathematics, mostly centred on fluid dynamics and special solutions of the governing nonlinear PDEs. I know some of those mathematicians well. P. L. Bhatnagar and his students created a vibrant fluid dynamics activity in the Applied Mathematics Department (as it was then known); and, in the Aeronautics Department (as it was then known), R. Narasimha and his students made significant contributions to applied mathematical aspects in fluid dynamics. The IITs in Delhi and Chennai had some good people.

Now, at IISc as well as IITs, most of the fluid dynamics work is being done in engineering departments. This evolution produces applied mathematics of a different flavour: the focus is rarely on the deep mathematical structure often inherent in the problem. At IISc, in the Mathematics Department (as it is now called), a new generation of applied mathematicians has expanded its interests to coding theory, cryptography, image analysis, communication networks and neuroscience, mathematical finance, systems biology, data mining and dynamical systems. It is yet to grow to the level of recognition that the fluid dynamics activity once enjoyed. I myself have high hopes for their future.

The TIFR Centre in Bangalore has an established group of mathematicians working on PDEs, both pure and applied, with good connections to France. It would do well to expand to other areas. IIT Bombay is another good centre for applied mathematics. The work there is on PDEs, finite element methods, financial mathematics, reliability theory, etc. There is also a recent move to have an inter-disciplinary initiative (based in IIT Bombay but involving faculty from IISc, IIT Kanpur, IIT Gauhati, etc.) in numerical analysis and PDEs. This involves both research and training components.

IIT Kanpur has a few faculty members working in computational fluid dynamics, numerical analysis, coding theory, signal processing, finite element methods and parallel computing. They have integrated mathematics and computations in their Master's programme and produce a number of good students. The IITs in Chennai, Delhi, Kharagpur and Gauwahati also have some experts in fluid dynamics and probability.

There are a few interdisciplinary initiatives in the country in which mathematics plays a central role. One is the Mathematics Initiative involving faculty members from many IISc departments and the TIFR centre at Bangalore. Though the initiative is based in IISc, its activities are geared towards researchers from the entire country. Every year, a theme is chosen at the interface between mathematics and an other discipline and an intensive set of activities is planned around this theme. The Department of Science and Technology, New Delhi, has begun a National Mathematical Sciences Initiative; a mathematical biology group has also been formed to work in areas of neuroscience, proteomics and genomics, and of mechanics applied to biological systems. Finally, there is the DRDO–IISc programme on advanced research in mathematical engineering. This programme focuses on the areas of communication, control and signal processing (though it is not clear whether it is functioning well at the moment).

The languishing university system

In the 1980s, the University Grants Commission (UGC), New Delhi tried to build centres of excellence in Pune, Bangalore, Coimbatore, as well as other places. I cite two examples merely for specificity: the UGC Centre Advanced Fluid Mechanics in the Central College, Bangalore, built around N. Rudraiah, and the Centre for Nonlinear Dynamics in the Bharathidasan Universrity, Tiruchirapalli, built around M. Lakshmanan. These and other existing groups are generally centred on a single leader, and work against many odds. There are active researchers elsewhere, but few schools engaged in adjacent or overlapping activities at a high level, over a sustained period of time.

There are some 350 or so universities in the country and, to a first approximation, most of them work in poor conditions. Some of them have illustrious histories but have fallen on hard times; many were created by upgrading smaller colleges without, however, providing adequate support. Politics often looms large, and research culture is a hit-and-miss affair; those few faculty members who are engaged in it cannot perform at competitive levels because of their heavy teaching loads and poor incentives. Thus, the Indian system as a whole is a mix of unequal opportunities and unbalanced accomplishments.

If, despite these odds, students from the universities make it in the world – as many indeed do – it is because even this poor system has not succeeded in vanquishing the drive of these students, often the desire of their parents, to succeed in education – which is regarded quite often as a means of upward social mobility.

I cannot overstate the fact that one of the tragedies of the Indian higher educational system has been the steady decline of its university system. The situation has been discussed at length within the country and various remedial measures are being contemplated[14]. The problem is the massive amount of infrastructure and the large number of creative people needed to set the situation right; this can only get worse as new universities come into being without adequate preparation. Without rectifying the university system, even India with its vast reservoir of human potential will deplete its pool of talented people. This is not the occasion for us to dwell on the university system as a whole, so I move on.

Brief remarks on doing elitist science in democracy

As I have implied already, the focus in science and technology shifted to a small number of elite institutions after independence. Especially in the last few years, these institutions have enjoyed a special status of relative affluence, while universities have stagnated. I submit that this dichotomy of relative plenty in elite institutions and the poverty in the university system was probably a conscious choice of policy makers in independent India. Elitism in science and democracy in practice have to be balanced somehow, and the balance was achieved essentially by singling out some institutions for elitist treatment, while others served the egalitarian ethos. This may remind some of us of a new class system, but a benign interpretation seems more accurate.

In a totalitarian system, such as the former USSR, or even present-day China, the elite few can pursue their interests without paying a heavy price. In democratic societies, there is always a tension between the pursuits of the few and the interests of the many. The solution that India found to resolve this issue with respect to science is not that different from what one has in the US, about which I know well. (A more apt comparison here is perhaps to the great Ecoles in France, but I cannot undertake that comparison.) There are the elite private universities in the US, in one of which I taught for some 22 years, which take the best students possible and recruit the best faculty available. They have no special obligation to admit students from a particular state or minority groups (although they strive to be responsive to such issues).

On the other hand, the State Universities do have an obligation to admit students from the state that funds them and, in general, cannot opt for the ‘merit-only’ philosophy. The difference between India and the US is, however, that the average level of the faculty in the US is higher, at least in premier State Universities, and there is reasonable support for research in nearly all of them. The superior material wealth of the country enables it to compete for the best talent from anywhere. I know Indians who have opted to remain in a US university of lower quality than join an Indian institution of higher intellectual standing. This may be seen as the power of the dollar against the rupee, but the truth is that perceptions and realities meld in complex ways that cannot be generalized.

A broad-brush assessment

It is clear that the lifting up of the entire mathematics system will be difficult, and so one will have to be selective by singling out some universities or centres for specific actions. This choice should be guided by the internal drive from the institutions themselves, accompanied by proper evaluation and funding mechanisms. Here, the intellectual leadership of the leading institutions in the country, such as the TIFR, has a large role to play, but at least a few universities have to be brought on-board.

A great department in any branch of science always consists of some stars and a number of up-and-coming young people. Star departments or centres, are essential for upholding the excitement of the field. Such schools have been few in India, and TIFR has been an exception in mathematics.

Since the TIFR model has been one of unapologetic elitism, the fact that major recognitions such as the Fields Medal have eluded it so far has been held against it occasionally. One should keep this issue in the background, but not agonize over it unduly. Let me relate an exchange that took place in Beijing some three years ago at one of its high-profile meetings. Bruce Alberts, then the President of the US National Academy of Sciences, made a presentation on international science, at the end of which a few Chinese students persistently questioned him as to when a Chinese scientist was likely to get a Nobel Prize. Alberts’ response was that the Chinese scientists should continue doing good science of the sort that they were already doing, and one day sooner than later there will be a Nobelist among them. Of course, there is such a thing as doing mathematics for the sheer pleasure of it without seeking external rewards.

One of the failures of TIFR is that it has not strived hard to raise the level of mathematics elsewhere in the country. Lest I should be regarded as being harsh on TIFR, let me remind you of the admiring words I have already said, and further add that the closed attitude of the mathematicians in the other universities has been part of the problem. A few centres connected to TIFR have now grown up elsewhere and young mathematicians have a choice of several departments with reasonable mathematical culture. Nevertheless, the number of Indian graduate students of reasonable competence continues to be small. For a country that has prided itself on its abstract thinking for millennia – as I have illustrated in my introductory remarks – this is a highly unsatisfactory situation. I suppose that just to draw the attention of the world to the strength of mathematics in a group, a department, or a university – or even a country – there should be a threshold strength of mathematics, at least some of which must be outstanding. Quality and quantity do not necessarily go together, but from the perspective of long-term sustainability, one cannot expect that quality can be pursued exclusively through a chosen few – even in mathematics[15]. The ground has to be fertile, and the challenge is to increase the number of people of quality. I would like to see Weil's words[12] come true – even if many years after they were spoken – that 'the intellectual potentialities of the Indian nation are unlimited, and not many years would perhaps be needed before India can take a worthy place in world mathematics'.

I submit that applied mathematics in the country is in transition. For instance, IISc was excellent in fluid dynamics but its strengths in that subject have declined, at least in part because the problems became too difficult or because the mathematics of special solutions became less relevant with the advent of large-scale computing. The practical activities moved to engineering departments and serious mathematical problems have become the domain of pure mathematics. The connection between scientific computation and numerical analysis did not take-off well in India, even though both these issues are intimately linked with fluid dynamics. And work in the new areas in which the country has embarked has yet to reach the threshold levels of strength.

The NIAS workshop

Improving the status of mathematics in the country was the subject of a two-day workshop held in October 2006 in the National Institute of Advanced Studies (NIAS), Bangalore. The workshop entitled 'Perspectives and Future Prospects in Higher Mathematics', was attended by prominent mathematicians of the country as well as other distinguished scientists. The stated objectives of the meeting were to create conditions by which careers in mathematics in India can be made attractive for talented young men and women. The workshop produced a set of recommendations[16], which I shall summarize below.

The recommendations concerned institutions as well as people. With respect to institutions, they included the establishment of a centre for computational mathematics, one or two inter-university centres patterned after the Inter-University Centre for Astronomy and Astrophysics in Pune, an ICTP-like centre to foster interdisciplinary work, an institute for training mathematicians to meet the strategic needs of the country and upgrading several existing research centres. The estimated cost is about three crore rupees per year.

With respect to people, the recommendations were to support promising researchers through special fellowships (5 lakh rupees per year per person for three years, say), dedicated teachers through awards, and good teaching departments in universities. The cost would be of the order of 30 crores per year, mostly because of the vast number of universities involved. There would be, in addition, a one-time expenditure on infrastructure, with a price tag in the vicinity of 100 crores.

These recommendations are indeed excellent, and closely resemble the thoughts I myself have held and expressed for a few years. I have left out parts of the recommendations of the workshop (such as raising the level of support for the National Board of Higher Mathematics by a factor of 8), and wish to add two comments. First, when this magnitude of money suddenly flows into mathematics, especially when only a small number of mathematicians among the many are outstanding, one has to take great care in maintaining quality. Some of the money would have to be spent on improving the level across the board (or at least in a few places), but some of it has to be spent only on people with promise[17]. Such people should be supported through generous fellowships that can be used flexibly for academic travel, purchase of computers, books, equipment and office facilities. There is again the question of egalitarianism vs elitism, but all successful countries have invented ways of negotiating this dilemma in a reasonably transparent and defensible manner. This is what India has to learn to do better. Secondly, I believe that there are no textbooks in the country that teach the student, in addition to problem-solving skills, how to create new mathematics. One can imagine that a concerted effort in this direction can be valuable: just think of the impact of books written by P. Halmos or by the Bourbaki regime. My feeling is that such books could be useful for many other countries as well.

Building a few strong applied mathematics groups

Building a sustained and high-class research department needs many elements, such as steady support, an atmosphere of free inquiry, meritocracy and openness to new ideas and paradigms. An oft-cited reason for the decline of a scientific group is the inbreeding that brings insularity. However, the truth is more complex. For some cohesiveness to emerge in a group, it is often desirable to build it from its own products. This ensures harmony and survival against disruptive interferences, while filling a department with prima donnas who do not respect each other guarantees its collapse. Building a group *and* keeping it on its toes requires a certain balance between components from within and without, and an enormous level of self-confidence and goodwill of a respected leader who puts the interests of his group above his own. This trait is rare in most places in the world, and India is no exception.

Let us now ask what one should do to build a few first-rate applied mathematics groups in India. I focus on applied mathematics because, as I said already, the subject is in transition in the country, and this is also the domain in which I can claim modest expertise. I should repeat that I have in mind people who know mathematics well and are willing to use that knowledge for solving hard and urgent problems of our times. They include problems of the environment, biotechnology, DNS sequencing, global change, spread of communicable diseases, communication, energy and security, financial markets, internet connectivity, the underlying common element among them being the multiscale behaviour and complexities of different sorts. In this connection, learning about Sobolev spaces and KAM surfaces may be far more important than special solutions of laminar boundary layer equations. I imagine that these applied mathematicians would be well versed in mathematics, including large-scale computing, and, even as they arm themselves with these tools, would be keen to solve problems of applied interest, usually in collaboration with those who know them directly. Such people are different from those who know one subject (say, elasticity and plasticity) well, and search for mathematical and computational tools that might suit their problems. I have come to the conclusion that the type of people I have in mind are far more essential for applied mathematics to flourish in India. They should be brought up as mathematicians foremost, perhaps in separate sections of applied mathematics.

On thinking about the infrastructure needed, I wish to emphasize that one cannot consider research and education separately, and this whole requires a few departments with rigorous undergraduate programmes, a few outstanding graduate programmes, and an ICTP-like centre to link people of different ideas and expertise. I will now add some details on all these fronts.

One ought to begin by choosing a few places in the country which already have a history of good undergraduate education, perhaps the ISIs and two or three universities, all of which should be loosely knit. A good curriculum will consist of many of the same courses taught to the pure mathematics students, without 'dumbing down'. Courses on the computational front will have to be added. This takes leadership on the parts of first-rate people, money not being the central issue. It is not clear how much of this change will come from within the mathematics community and how much of it will have to be forced from outside, but I imagine that it will require a combination of both. But I must emphasize that mathematics must be brought up in an environment which allows students to be open to a wide variety of applications across the board, including social sciences. This cannot be done by building specialized institutes of mathematics.

One would then start graduate programmes in a few selected places such as the TIFR Centre in Bangalore, IISc and two or three universities. The graduate and undergraduate programmes must be linked organically. It is difficult to start well-rounded graduate programmes instantly, but one could start by creating a nucleus, in each place, of a small group specializing in a few aspects of applied mathematics (e.g. biology, meteorology, oceanography, finance, communication, etc.). It takes two important ingredients: an unconditional acceptance by strong mathematics departments in the country, such as TIFR, that this is a valuable undertaking; and recruitment of at least a small number of outstanding people who will be given the freedom to do things.

Some of what I just mentioned has already been happening. What is not happening – and I regard this as the crucial element – is this: as students advance in their studies, whether undergraduate or graduate, they should be frequently exposed to people who are immersed in solving serious problems in global change, energy and environment, pharmaceuticals, biology and biotechnology, finance, cryptography and communication – again without diluting the problems to academic and sanitized level. The point is to expose the students to the best and the most rigorous of mathematics there is, and also to practical problems in their most naked manifestation, and let talented people do what they can through this dual exposure. It should be understood that many people will not contribute much, but some will certainly take up the challenge: they are the ones who need the support of the sort discussed earlier.

For applied mathematics to flourish, we want someone to say, 'Here is a problem of great consequence', and someone else to say, 'I know how to solve it and can do it'. In the ideal world, it is one and the same person. In general, they are different and it is not easy to bring about contact between these two sets of people. Here is where I think that an ICTP-like institution (say, on mathematical sciences) can be of immense value. I have made this case often to many scientific colleagues in the country. Such a centre cannot be seen in isolation but ought to be regarded as integral parts of broad undergraduate and graduate programmes to upgrade mathematics, on the one hand, and, on the other hand, to applications from industry, defence, health, environment and society. It works better as a package.

I am not fond of creating new administrative mechanisms but should make a few related remarks for the sake of completeness. The first principle to remember is that nothing will be gained by increasing administrative overhead needlessly. There is already a successful entity called the National Board for Higher Mathematics, set up under the umbrella of the Atomic Energy Commission, with funding coming from the Department of Atomic Energy. The goal of this Board is to foster the development of higher mathematics in the country by helping mathematics centres and supporting researchers at the doctoral and postdoctoral levels. By and large, it has focused on pure mathematics and has done its job with an eye on quality. It will probably be useful to set up extensions of the Board, with one component dedicated to applied mathematics and another to undergraduate education. The first act is simply an extension of the Board's current mandate, while the second is a qualitatively different add-on. With uncompromising commitment to quality as it now does, and a lot of goodwill, the Board can indeed succeed.

The faculty and the role of expatriate Indians

A major problem in applied mathematics at present is the absence of a sizeable pool of outstanding faculty. There are too few applied mathematicians obtaining Ph Ds in the country. In principle, this shortage could be compensated by recruiting Indian Ph Ds from the US and Europe. There are the usual problems of getting Indians to return from abroad to universities in India, but the additional difficulty is that few of them work in applied mathematics. I believe that this situation is due to the fact that few good Bachelor's and Master's programmes exist in applied mathematics in India (unlike B Math and B Stat of ISI, and B Sc of CMI, which cater to pure mathematics). This is why I earlier emphasized the need for viable undergraduate and Master's programmes.

In spite of this problem (or perhaps because of it), India must use its expatriates with imagination. The relation between them and the academics in India is somewhat ambiguous, and each group does not understand the motives of the other well enough. I should hasten to add that my own relationship in this regard is excellent, and I cannot – and do not – complain about the openness of the Indian scientific community towards me personally. However, I wish to go beyond and note that it is ironic that many expatriate Indians want to get involved in the country and many people in the country are willing for that to happen, yet a suitable system does not exist for this beneficial interaction to occur. The Chinese, for instance, seem to have done it better (though they have had their share of dramas[18]). It is clear that the expatriates have to approach the Indian issues with humility and understanding, and the local scientists have to deal with expatriates with tolerance and flexibility. I then foresee immensely beneficial interactions. Perhaps one has to set up some well advertised and prestigious short-term fellowships[19] for first-rate Indian expatriates. These fellowships should pay well and, at the same time, include specific responsibilities. One has to go with the other.

Summary

I have presented an incomplete and – even to me – an unsatisfactory picture of mathematics in India through its history to the present, making some suggestions for its enhancement. My final comments have focused on applied mathematics for understandable reasons. A central planning effort is not the best way to go, because many elements described here depend on local initiatives, but it is clear that there should be an agent to articulate a vision and promise support, nurturing those who are willing and able to do the needed work. There indeed are such people and agents who can rise to the challenge.

Let me close with a broad statement. Like all sweeping thoughts, it is bound to be true only loosely, so it has to be taken as such. Western Europe, where scientific thought has seen its zenith in recent centuries, had essentially lost all connection to Greek traditions following the collapse of the Roman Empire. It was rebuilt after several centuries in the Renaissance period. That the West has been able to reach great heights since then has to do much with economics and politics, but the open-mindedness and persistence to create a tradition in science has played an important role. This example shows that it is possible to recreate something of the lost tradition even after several centuries, but it may be long in coming; it comes only when the combination of motivation and the number of interested people exceeds certain threshold values. This is what one hopes will occur in India, especially in the face of challenges posed by the IT and business sectors.

References and end notes

1. Dampier, W. C., *A History of Science*, Cambridge University Press, Cambridge, 1929, p. 7; van Nooten, B., Binary numbers in Indian Antiquity. In *Computing Science in Ancient India* (eds Rao, T. R. N. and Kak, S.), 1993, pp. 21–39; Joseph, G. G., *The Crest of the Peacock: Non-European Roots of Mathematics*, Penguin Books, London, 1994; Jha, P., *Contributions of the Jainas to Astronomy and Mathematics*, The Mathematics Education (Siwan), 1984, vol. 18, pp. 98–107; Subbarayappa, B. V., India's contributions to the history of science. In *India's Contribution to World Thought and Culture* (eds Chandra, L. *et al.*), Vivekananda Rock Memorial Committee, Madras, 1970, pp. 47–66; Sen, S. N. and Bag, A. K., *The Sulbasutras*, Indian National Science Academy, New Delhi, 1983; and others. Because documentation from antiquity is difficult to come by, controversial claims have been advanced. A case in point is *Vedic Mathematics* authored by Sri Bharati Krsna Tirthaji Maharaja, and published posthumously by Motilal Banarsidas, Varanasi, 1965. A balanced critique of this book, along with other references, can be found in Dani, S. G., Vedic mathematics: Myth and reality, *Econ. Political Wkly*, 1993, **28**, 1577–1580. Other papers by this same author on this topic can be found on his website at TIFR, Mumbai. The tendency of ascribing greater insight to earlier workers can be justified is not uncommon, and one often carries present knowledge back to the past without justification. This tendency is even greater if one is emotionally attached to the people of earlier times. There is also the likelihood of reading too little in the accomplishments of the past. Even the work of specialized scholars cannot resolve all the issues to everyone's satisfaction. Establishing priorities in the work of the present day is itself sometimes controversial. So one can imagine the problems associated with the work of antiquity. Nevertheless, coming to terms with past accomplishments is central to building a healthy psyche of a people.
2. Sriram, M. S., Ramasubramanian, K. and Srinivas, M. D. (eds), *Five Hundred Years of Tantrasangraha: A Landmark in the History of Astronomy*, Indian Institute of Advanced Study, Shimla, 2002.
3. After I had completed writing this article, R. Ramakrishnan (ICTP, Trieste) drew my attention to the article 'Les Mathématiques en Inde' by M. Waldschmidt. It briefly covers the history of Indian mathematics from antiquity, including specific results from the Kerala School, goes over Ramanujan's work and provides a glimpse of the later mathematicians. This article brings us,

more or less, up to the present, while paying special attention to recent Indo-French collaborations. It makes a worthy reading for anyone interested in Indian mathematics.
4. Divakaran, P. P., Natural and artificial in the language of the Malayalam text. In Workshop on the Generosity of Artificial Languages in an Asian Perspective, Amsterdam, 18–20 May 2006. In revised version, the article will appear in the *Journal of Indian Philosophy* under the title 'The First Textbook of calculus: Yuktibhasha'.
5. I have read at various places that Ibn Tariq and Al-Fazari (8th century), Al-Kindi (9th century), Al-Khwarizmi (9th century), Al-Qayarawani (9th century), Al-Uqlidisi (10th century), Ibn al-Samh (11th century), Al-Nasawi (11th century), Al-Beruni (11th century) and Ibn-Al-Saffar (11th century) were among the scholars who based their scientific treatises on Indian texts. Several of them made important and original contributions to geometry, algebra and trigonometry. Their contributions and connections to the Indian work need a separate discussion, which I am not equipped to provide here. Perhaps a comment of Al-Beruni, also sometimes spelled as Al-Biruni, could be provided here to exemplify the mood of the civilization at that time. He says, in his book *India* (translated by E. C. Sachau, with introduction by Q. Ahmad, National Book Trust of India, 1983): '… the Hindus believe that there is no country but theirs, no nation like theirs, no kings like theirs, no religion like theirs, no science like theirs… . They are by nature niggardly in communicating that which they know, and they take the greatest possible care to withhold it from men of another caste among their own people, still much more, of course, from any foreigner…' If they traveled and mixed with other nations, they would soon change their mind, for their ancestors were not as narrow-minded as the present generation….' The country has undergone great upheavals over the millennium since this description applied, and their effects are still to be assessed adequately.
6. The first translation of Euclid into Sanskrit is credited to Al-Beruni (AD 973–1048). Translations by Indians seem to have occurred later, perhaps from Persian and Arabic translations.
7. Hardy, G. H., *A Mathematician's Apology*, Cambridge University Press, Cambridge, 1940.
8. Kanigel, R., *The Man who Knew Infinity: A Life of the Genius of Ramanujan*, Charles Scribner's Sons, New York, 1991; Berndt, B. C. and Rankin, R. A. (eds), *Ramanujan: Essays and Surveys*, Hindustan Book Agency, New Delhi, 2003 (the original publication was produced by the American Mathematical Society in 2001).
9. Narasimha, R., The chequered histories of epistemology and science. In *Different Types of History* (ed. Ray, B.), PHISPC Monograph Series on History of Philosophy, Science and Culture in India, 2007. Two other articles by the same author make interesting reading and are included here for that reason: Axiomatism and computational positivism. In *Science in India* (ed. Narlikar, J. V.), PHISPC Monograph Series on History of Philosophy, Science and Culture in India, 2007, chapter 2, pp. 85–113; The *Yukti* of classical Indian astronomical science. In *Foundations of Sciences* (ed. Sreekantan, B. V.), PHISPC Monograph Series on History of Philosophy, Science and Culture in India, 2007.
10. Narasimhan, R., *The Coming of Age of Mathematics in India*, Miscellanea Mathematica, Springer-Verlag, Berlin, 1991, pp. 235–258. For another highly readable account of this topic, see, Raghunathan, M. S., Artless innocents and ivory-tower sophisticates: Some personalities on the Indian mathematical scene. *Curr. Sci.*, 2003, **85**, 526–536.
11. I should perhaps explain myself at the risk of being regarded as naïve. A casual study of the ancient Indian mathematics suggests that what may have been lacking in it was the modern sense of proof. One could then fall into the trap of false security if one is not aware of it, or be fooled into thinking a particular result to be general. Building a solid tradition on that basis becomes difficult. The notion of how to build continually on the basis of the past seems to be a great hallmark of Western science. On the other hand, much of mathematics is the creature of intuition, without which it will reduce itself to the banal.
12. Weil, A., *The Apprenticeship of a Mathematician – Autobiography of André Weil*, Birkhauser, 1992; see also Weil, A., *Collected Works*, Springer-Verlag, New York, 1980, pp. 129–131.
13. M. S. Raghunathan has aptly made this point on more than one occasion; see, for example, his speech at the Symposium on the Importance of Science and Technology in our Society, 'Revitalising science', Seminar 546, March 2005. But it is clear that the number of such mathematicians is still too small in India. Apparently, even in France where the tradition of mathematics is strong, it is getting more and more difficult to interest good students in mathematics, so India is not alone in this quandary.
14. *Current Science* routinely publishes anguished items on the failure of the university system in India. A succinct analysis is contained in a pair of Guest Editorials by S. E. Hasnain, 'Crisis in the University: Increasing the Funding Gap between Universities and National Laboratories/Scientific Research Institutions', *Curr. Sci*., 2006, **91**, 1435–1436; 1591–1592. The first editorial outlines problems, while the second outlines some possible solutions.
15. The advocacy for creating a base of a substantial number of mathematicians should not be regarded as supporting mathematics research of dubious quality. There is indeed a need to identify quality research that is not trivial or simply driven by fashion. This is a non-trivial task for which no easy recipes exist. Historically, great schools have emerged even in difficult economic and political circumstances. For example, theoretical physics and mathematics thrived at an extraordinarily level in Germany between the First and Second World Wars. Hungary and Russia had excellent mathematics school even in the worst of times.
16. Workshop report on 'Perspectives and Future Prospects in Higher Mathematics', National Institute of Advanced Studies, Bangalore, No. R4-06. For a summary under the same title, see Bhogle, S., *Curr. Sci.*, 2007, **92**, 13–16.
17. Already, there exist fellowships such as the Swarnajayanthi. Though I know some of its recipients personally, I do not have a concrete idea of the impact of these fellowships. It is almost certainly too early to tell.
18. The influx of expatriate Chinese scientists at high salaries is seen as a mixed blessing by the Chinese communities of scholars. On the one hand, this has improved the working conditions and salaries of most Chinese academic. On the other hand, some chosen ones are perceived to enjoy too much say in scientific policy and the distribution of resources. A dramatic case of conflict is described in Nasar, S. and Gruber, D., Manifold destiny. *The New Yorker*, 28 August 2006, pp. 44–57. It must be pointed out, however, that the account described in the article has been contested by the people in question.
19. R. Narasimha has informed me that such fellowships have recently been created on the basis of the recommendations of the Scientific Advisory Committee to the Prime Minister. They should carry a level of prestige (for instance, they should be named after a great Indian scientist) and assign specific, though broadly defined, responsibilities to the Fellows.

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