Observational Impact of Surrogacy on the Turbulent Energy Cascade

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The inertial-range dynamics of fully developed turbulence is often characterized in terms of structure functions $S_n(\Delta v_l) = \langle \Delta v_l^n \rangle \sim l^{\xi_n}$. The velocity increment $\Delta v_l = (\mathbf{v}(\mathbf{x}+\mathbf{l}) - \mathbf{v}(\mathbf{x})) \cdot \mathbf{l}/l$ is either taken as the component parallel (x) or perpendicular (y) to the mean flow direction. For scales $\eta \ll l \ll L$, where η is the dissipation scale and L the integral scale, the structure functions reveal power-laws with scaling exponents ξ_n. However, depending on the Reynolds number and the flow geometry, these scaling laws are either only approximate or come with a rather narrow scaling range.

A different way to extract scaling exponents is proposed by the Refined Similarity Hypothesis $\langle \Delta v_l^n \rangle \sim \langle \varepsilon_l^{n/3} \rangle l^{n/3}$, which relates structure functions to integral moments $\langle \varepsilon_l^n \rangle \sim l^{-\tau_n}$ of the energy dissipation and respective exponents via $\xi_n = n/3 - \tau_{n/3}$. The energy dissipation with viscosity ν is defined as

$$\varepsilon = \frac{\nu}{2} \sum_{i,j=1}^{3} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)^2 \tag{1}$$

and requires full knowledge about all three velocity components. Since in experimental data only one, at most two components of the velocity field are accessible, various surrogate forms are constructed:

$$\varepsilon_{\mathrm{sur1}}(x) = 15\nu(\partial_x v_x(x))^2 \, , \tag{2}$$

$$\varepsilon_{\mathrm{sur2}}(x) = \frac{15}{2}\nu(\partial_x v_y(x))^2 \, , \tag{3}$$

$$\varepsilon_{\mathrm{sur3}}(x) = \frac{15}{4}\nu \left[2(\partial_x v_x(x))^2 + (\partial_x v_y(x))^2 \right] \, . \tag{4}$$

Upon assuming isotropy, all three constructions have the same mean value as (1). Their coarse-grained amplitudes are determined via

$$\varepsilon_l(x) = \frac{1}{l} \int_{x-l/2}^{x+l/2} \varepsilon(x')dx' \ . \tag{5}$$

Fig. 1 illustrates the second-order moment based on (2) for an atmospheric boundary layer record [1]. Its Reynolds number based on the Taylor microscale $\lambda = \sqrt{\langle v_x^2 \rangle / \langle (\partial_x v_x)^2 \rangle}$ is $R_\lambda = 9000$, its estimated ratio between integral length and dissipation scale is $L/\eta = 5\times 10^4$ and it comes with a longitudinal as well as a transverse velocity component. The logarithmic local slope of $\langle \varepsilon_l^2 \rangle$ turns out to be constant only in the upper part of the inertial range, where it is equal to $\tau_2 = 0.20$. The same outcome holds for the other two surrogate forms (3) and (4). For a turbulent flow with such a large Reynolds number this result is to some degree surprising and for the moment leaves open the question as to why the scaling range does not extend more into the intermediate inertial scale range.

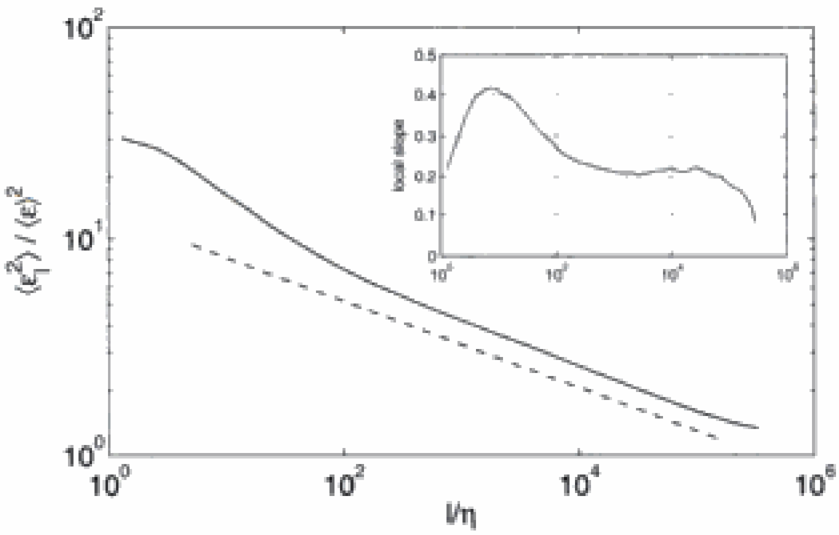

Fig. 1. Second-order integral moment $\langle \varepsilon_l^2 \rangle$ based on the surrogate energy dissipation (2) for an atmospheric boundary layer with $R_\lambda = 9000$. The dashed straight line has a logarithmic slope $\tau_2 = 0.2$. Inset shows the logarithmic local slope.

The second-order integral moment is closely related to the two-point correlation function:

$$\langle \varepsilon_l^2 \rangle = \frac{1}{l^2} \int_l dx_1 \int_l dx_2 \, \langle \varepsilon_{\rm sur}(x_1) \varepsilon_{\rm sur}(x_2) \rangle \ . \tag{6}$$

Fig. 2 compares the two-point correlator obtained from the surrogate forms (2), (3) and (4). All three variants reveal a rigorous power-law scaling behavior within the extended inertial range $15\eta \le d \le 0.3L$ and the corresponding scaling exponents are within $\tau_2 = 0.20 \pm 0.01$, showing little differences. Only for small two-point distances $d \to \eta$ the two-point correlators begin to differ. Whereas the variants based on (2) and (3) practically remain identical, the two-point correlation based on (4) is weaker for $d \leq 10\eta$; see inset of Fig. 2.

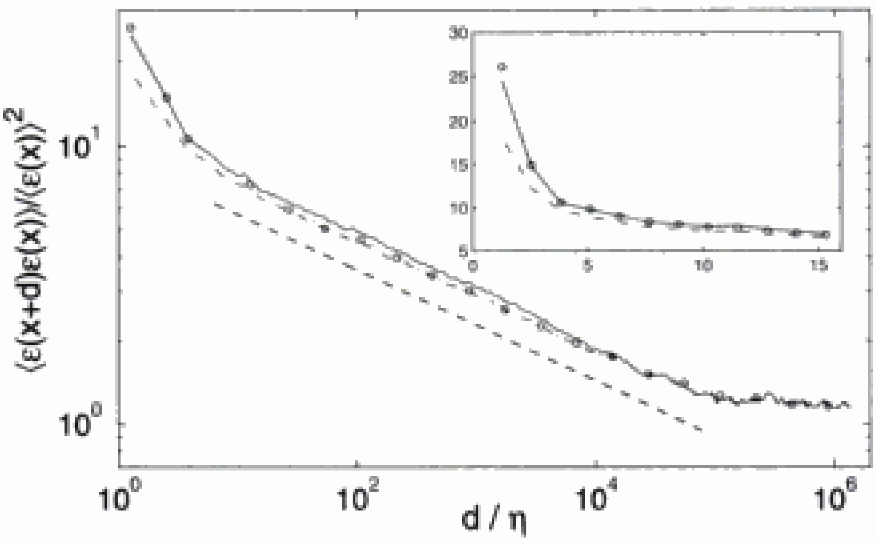

Fig. 2. Normalized two-point correlation function of the surrogate energy dissipation (2) (full line), (3) (dotted line with circles), and (4) (dot-dashed line). The dashed straight line has a logarithmic slope $\tau_2 = 0.2$. Inset magnifies the behavior for short separation distances.

When compared to the true energy dissipation, the expression (4) appears to be closer to (1) than the other two variants (2) and (3). This allows to speculate that if one adds more terms from the full list of (1), the extra-strong two-point correlations at small separation distances $d \leq 15\eta$ reduce further, perhaps even vanish once the surrogate field has converged to the true field.

In numerical simulations the full velocity field is accessible. Therefore they are particularly suitable for studying the difference between the true and surrogate energy dissipation. For the analysis here a small data set from a shear turbulence simulation [2] was available. Although the statistics are not very high the result is convincing enough to stress the surrogacy issue. Fig. 3 compares the two–point correlation of the dissipation obtained from the full field (1) with the one obtained from the surrogate field (2). Although the Taylor scale Reynolds number is only $R_\lambda = 99$ one can identify an approximate power law scaling range and both, the surrogate and true dissipation are identical in this range. Only for very small distances the two curves differ, where the correlator calculated from the surrogate field is showing the same extra-strong correlations as in experimental data. Note, that the strong increase of the correlation functions of the numerical data for the largest distances is an artifact of the periodic boundary conditions used in the simulation. This finding indicates the importance of the subtle surrogacy issue when interpreting data. The surrogacy of the energy dissipation alters the small scale behavior

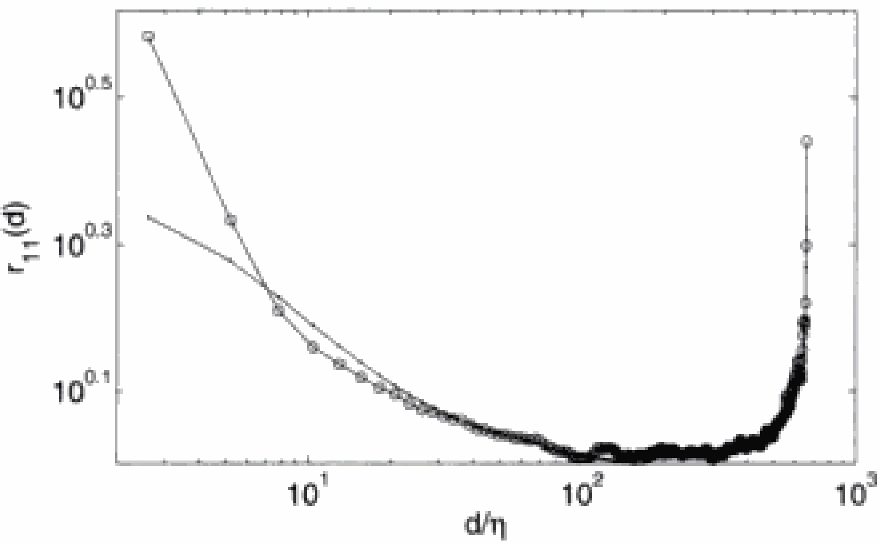

Fig. 3. Two-point correlator obtained from a numerical simulation. For the line with dots the true energy dissipation field (1) has been used and for the line with the open circles the one–component surrogate (2) has been used.

of the two-point correlation; fortunately, this leaves the rigorous scaling over the major part of the inertial range untouched.

In comparison with two-point correlations, the poor scaling of the integral moments appears in a new light. The extra-strong small-distance behavior of the two-point correlation can be roughly modeled with an additional δ-function at $d \approx \eta$, i.e. $\langle \varepsilon(x+d)\varepsilon(x) \rangle \approx a(\eta/d)^{\tau_2} + b\delta(d-\eta)$. Insertion into (6) then leads to $\langle \varepsilon_l^2 \rangle \approx a'(\eta/l)^{\tau_2} + b'(\eta/l)$, resulting in a slowly decreasing correction to the scaling term. This explains qualitatively the observed scale-dependence of the second-order integral moment: only in the upper part of the inertial range does the scaling term with exponent τ_2 dominate, whereas for the lower part strong deviations set in due to the small-distance behavior of the two-point correlation function, caused by the surrogacy effect.

More details on the observational impact of the surrogacy effect can be found in Ref. [3]. More follow-up discussions on two-point statistics of the turbulent energy cascade are given in Refs. [4, 5].

References

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