*Journal of Low Temperature Physics, Vol. 138, Nos. 3/4, February 2005 (© 2005)*
DOI: 10.1007/s10909-005-2253-8

# Grid Generated Turbulence in Helium II

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*We present experimental data on decaying turbulence generated by towing a grid of crossed tines through a stationary sample of He II. The present data were obtained using a grid of substantially improved design from that used in previous investigations of grid generated superfluid turbulence. An important preliminary observation is that both the magnitude and temperature dependence of the effective kinematic viscosity of the turbulence—deduced from measurements of the attenuation of second sound as a function of time—are not substantially altered by details of the grid used to generate the turbulent flow.*

*PACS numbers:47.37+q, 47.27.Gs, 67.40.Vs*

## 1. Introduction

Superfluid turbulence has long been an area of interest, although particularly focused toward flows generated by the application of a heat current in helium II; i.e., thermal counterflow [1]. The classical analogue of the counterflow turbulence is thermal convection and has been an object of considerable interest recently[2]. Our object of interest here is the strong similarity that appears to exist between grid turbulence in classical fluids and in helium II, a quantum fluid [3–8]. Generally, while the study of turbulence in superfluids has some direct practical relevance, for instance to the important use of helium II as a coolant for superconducting devices, it is also possible that such studies can shed new light on problems in classical turbulence. Our perspective is the latter.

Previous studies of grid-generated turbulence in helium II [3–7] were performed using a grid of rather *unconventional* design: the "grid" in fact, simply consisted of four parallel rectangular tines crossed by a single tine at 45 degrees, to which a centered pull rod was attached [3,4] (see inset to figure 1). While this grid possessed the canonical 65% open fraction, its shape differed radically from that typically used in (classical) grid turbulence research. The motivation for the original design was drawn from the smallness of the helium II channel (1 cm on a side) together with the need for structural rigidity and integrity of the grid, which is pulled repeatedly at high speeds.

It is known, however, that the grid design has a major difference on the nature of turbulence generated in air [9]. Ideally, one should have many monoplaner tines crossing at right angles to each other so that at least, say, 10-20 meshes (the mesh size may be defined as the tine-to-tine distance in the direction of either set of orthogonal tines including half of each tine thickness in that direction) can be accommodated across the channel.

Briefly stated, our goal is to use a standard grid and repeat the measurements of superfluid turbulence behind the towed grid and to extract the same level of information as in the past [3–7].

## 2. Apparatus

Except for the variation of grid design, all measurements discussed here have made use of the same apparatus and general procedures, details of which are given below.

The turbulence is generated by towing a grid through a stationary sample of He II contained within a square copper channel of 1 $cm^2$ cross-section. The grid is attached to a central stainless steel pulling rod of diameter 0.24 cm which exits the cryostat via a pair of tight sliding seals. Outside the cryostat, the rod is attached to a linear servo motor that positions the grid with 0.01 cm accuracy and provides the towing velocity, $v_g$, up to roughly 2.5 m/s.

The original measurements were taken using a 65% open brass monoplanar grid of rectangular tines, 1.5 mm thick, with a mesh size, M of 0.167 cm[3,4], corresponding, as noted above, to three full meshes across the channel. The present measurements reported here were taken using a grid formed from 28 rectangular tines of width 0.012 cm—14 in each of two orthogonal directions—forming 13 full meshes across the channel of approximate dimension 0.064 cm. In contrast with the old grid, attachment was made at the outside corners of the grid and then to the centered rod several centimeters above the plane of the grid.

For the results to be presented, we have restricted attention to a relatively high mesh Reynolds number $Re_M = v_g M\rho/\mu = 1.5 \times 10^5$, where $\mu$ is the dynamic viscosity of the normal fluid and $\rho$ the total density. The high $Re_M$ allows a larger inertial scaling regime.

In order to probe the quantized vortex line density resulting from pulling the grid, we excite and detect second sound using vibrating superleaks mounted flush on opposing walls of the channel. The channel acts as a second sound resonator and typically a high harmonic of the fundamental frequency is used corresponding to $25 - 40$ kHz.

The length of quantized vortex line per unit volume $L$ is obtained from second sound measurements through the relation

$$L = \frac{16\Delta_0}{B\kappa}\left(\frac{A_0}{A} - 1\right), \tag{1}$$

where $A$ and $A_0$ are the amplitudes of the second sound standing wave resonance with and without vortices present, respectively, $\Delta_0$ is the full width at half maximum of the second sound resonance peak in the absence of vortices, B is the mutual friction constant and $\kappa$ is the quantum of circulation.

It has been shown[4] that the following formula applies more generally and includes the case of large attenuation:

$$L = \frac{8u_2}{\pi B\kappa d}\ln\left[\frac{1 + p^2P + \sqrt{2p^2P + p^4P^2}}{1 + P + \sqrt{2P + P^2}}\right], \tag{2}$$

where $p = A_0/A$ and $P = 1 - \cos(2\pi d\Delta_0/u_2)$, with $u_2$ the second sound velocity and $d$ is the width of the channel.

The information on line density we obtain is averaged over the measurement volume of order $\mathrm{d}^3 \cong 1\ \mathrm{cm}^3$. An example of decaying line density obtained with the present grid is shown in Fig. 1 for the temperature of 1.5 K, covering the inertial range of time that will be considered for further data analysis. Similar data were obtained for other temperatures as well up to 2.0 K.

## 3. Analysis

The power law decay of the superfluid line density shown in Fig. 1 is indicative of the classical behavior, if certain assumptions are made, as has been extensively discussed in previous works[7,8,10,11].

Briefly, these assumptions are the following. First, it is assumed [11] that the total energy dissipation per unit mass in the turbulent fluid, $\epsilon$ is given by

$$\epsilon = \nu'\kappa^2L^2, \tag{3}$$

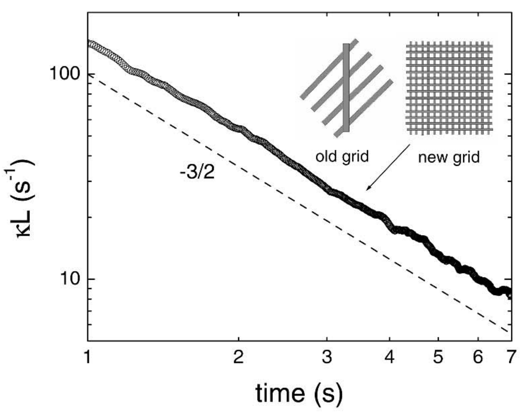


Fig. 1. The log-log plot of the decaying vortex line density versus time obtained with the new grid. The data correspond to $Re_M = 1.5 \times 10^5$ and a temperature 1.5 K. The grid passed the measurement volume at t=0 s, and only that part of the *inertial* range of the decay used for determining the effective kinematic viscosity of the turbulence (see text) is shown here. This procedure assumes a power law decay of the line density with exponent -3/2, illustrated for clarity by the displaced dashed line. Upper right inset illustrates schematically both the old and new grids. The latter was used to generate the data shown here.

where $\nu'$ is a proportionality constant with units of kinematic viscosity that need not be related to a property of the fluid, as it would in the analogous relation for classical fluids, where $\kappa^2 L^2$ is replaced by the mean squared vorticity of the flow and $\nu' = \nu$, the actual kinematic viscosity.

A second assumption is that a Kolmogorov-like energy spectrum applies to the coupled normal and superfluid components at length scales large compared with the average inter-vortex line spacing, so that

$$E(k) = C\epsilon^{2/3}k^{-5/3} \tag{4}$$

where $E(k)dk$ is the average turbulent energy per unit mass in the range of wavenumbers $dk$ and $C$ is the Kolmogorov constant. The value $C = 1.5$ used is close to accepted classical values[12].

From these two assumptions, we can calculate through successive integration and differentiation the expected decay rate for the line density per unit volume $L$ as

$$\kappa L = [(3C)^{3/2}d/\nu'^{1/2}]t^{-3/2}. \tag{5}$$

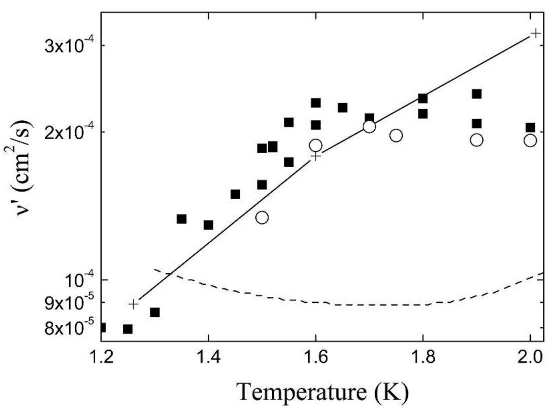


Fig. 2. The effective kinematic viscosity of the superfluid turbulence $\nu'$ deduced from vorticity decay data at various temperatures, and assuming a value of the Kolmogorov constant consistent with classical experiments. The dashed line represents $\nu = \eta/\rho$, where $\eta$ is the normal fluid viscosity and $\rho$ is the total density of helium II. The squares represent the kinematic viscosity deduced from decay data obtained using the old grid, while the open circles represent the present data obtained using a more conventional grid. The pluses connected by the solid line represent a model calculation for the effective kinematic viscosity[8].

The only unknown here is $\nu'$ which we get by fitting the above equation to the decay data in a so-called power-law part of the decay (see Fig. 1), where we expect the form of the energy spectrum given above to be applicable. Values of $\nu'$ so deduced are shown as a function of temperature in Fig. 2, for both types of grid. Also included in Fig. 2 are calculated values [8] for $\nu'$, as well as a line representing $\nu = \eta/\rho$, where $\eta$ is the normal fluid viscosity and $\rho$ is the total density of helium II. The reasonably close correspondence between $\nu$ and $\nu'$ may result simply from a numerical coincidence between the quantum of circulation $\kappa$ and $\nu$; indeed, it has been shown [8] that the factor $\nu'$ appearing in the energy dissipation relation is proportional to $\kappa$.

## 4. Conclusions

As can be appreciated from Fig. 2, the effective viscosities obtained with the new grid have the same qualitative dependence on temperature as those obtained previously. Likewise, the magnitudes are comparable, given the scatter in the data, and, although there appears to be a slight, systematic lowering of the effective viscosity, this could also be due to details of the analysis. We stress that these results are only preliminary and that a further and more comprehensive analysis is ongoing and will be published elsewhere. However, the main conclusion is reassuring at the least: that the empirically deduced effective kinematic viscosity for quantum turbulence appears essentially insensitive to details of the grid used to generate the turbulence flow, here demonstrated for radically different grids, and that previous results are not somehow artifacts of using a non-standard grid.

## 5. ACKNOWLEDGEMENTS

This research was funded by NSF grant # DMR-9529609.

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