Rayleigh–Taylor turbulent mixing of immiscible, miscible and stratified fluids

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We propose a simple empirical model to describe the Rayleigh–Taylor (RT) turbulent mixing of immiscible, miscible and stratified fluids. For immiscible fluids, the rate of momentum loss is an invariant of the flow, whereas the energy dissipation rate increases linearly with time. Turbulent diffusion, accounted for through temperature fluctuations, does not terminate mixing but slows it down significantly. A stratified density distribution can stabilize RT mixing. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2009027]

Rayleigh–Taylor instability (RTI) occurs whenever fluids of different densities are accelerated against the density gradient.[1,2] Extensive interfacial mixing of the fluids ensues with time.[3] It plays a key role, for instance, in preventing the formation of "hot spot" in inertial confinement fusion, providing proper conditions for the synthesis of heavy mass elements in supernovae and determining the drop size distribution in sprays.[4–13] A grip on the mixing process and its dependence on the density ratio, diffusion, stratification and other physical factors is the basic objective of studies of RTI. We propose a simple heuristic model to describe turbulent mixing in RTI of immiscible, miscible and stratified fluids. Compressibility is not considered.

For incompressible immiscible fluids, observations[2,3,14–19] suggest the following evolution of RTI under sustained acceleration **g** (gravity). Small perturbations at the fluid interface grow exponentially with time. In the nonlinear regime a coherent structure of bubbles and spikes appears (see Fig. 1). Light (heavy) fluid with density $\rho_{l(h)}$ penetrates the heavy (light) fluid in bubbles (spikes), which move steadily for fluids with finite density ratio. The spatial period λ of the structure in the plane normal to the direction of gravity is set by the fastest-growing mode or the initial perturbation.[3,13,14] Shear-driven instabilities produce small-scale vortices on the sides of evolving spikes.[3,14–16] For a broad-band initial perturbation, the period λ may grow.[20,21] Eventually, a mixing zone develops[3,16–19] and its width $\tilde{h}$ in the direction of gravity increases quadratically with time, $\tilde{h}\sim gt^2$, $g=|\mathbf{g}|$. It is commonly accepted that diffusion and stratification reduce the growth rate of the instability,[13,22–25] but their quantitative influences on RTI evolution are poorly understood. The mixing flow is sensitive to the horizontal boundaries of the fluid tank (computational domain) though much less so to the vertical, and retains the memory of the initial conditions.[3,26,27]

For a long time the nonlinear dynamics of RTI was interpreted as a single-scale problem, characterized by the spatial period λ of the structure of bubbles and spikes.[12,20,21] This idea was implemented by the models[12,20,21] in a heuristic equation balancing the forces of inertia, buoyancy and drag with three adjustable parameters dependent on the density ratio. To derive a similarity solution $\tilde{h}\sim gt^2$ for the mixing zone, the models required the horizontal scale λ to grow proportional to the vertical scale $\lambda\sim\tilde{h}$ (the so-called "bubble merge"). The free parameters of the models were adjusted to fit the evolution of these scales in observations.[26] Despite significant efforts, the university of the scaling $\tilde{h}/gt^2$ and λ/gt^2 as well as the mechanism of the mixing process have remained open issues.[3,26–28]

A recent analysis[29] of the conservation laws based on group theory has found that the evolution of RTI has a multiscale character: The spatial period of the coherent structure λ and the vertical scale $\tilde{h}$ contribute independently to the nonlinear dynamics, whereas the postulates of single-scale models[12,20,21] violate the conservation laws. The present work accounts for the conclusions of Ref. 29 and suggests a simple heuristic model describing the RT mixing of immiscible, miscible and stratified fluids. We show that the growth of the horizontal scale is not a necessary condition for the mixing to occur. The rate of momentum loss per unit mass is an invariant of the flow, whereas the energy dissipation rate increases linearly with time. Turbulent diffusion reduces the mixing growth rate significantly, while stratification can stabilize the mixing process.

(a) RTI of immiscible fluids. The dynamics of incompressible RT spike is governed by a balance per unit mass of the buoyant and dissipation forces[30] as

$$\frac{dh}{dt}=v,\quad \frac{dv}{dt}=\frac{\delta\rho}{\rho}g+F, \tag{1}$$

where v is the velocity, h is the position, $\delta\rho/\rho$ is the function of the density ratio with $\delta\rho=(\rho_h-\rho_l)/2$, $\rho=(\rho_h+\rho_l)/2$. With $\rho_h\leftrightarrow\rho_l$ and opposite signs for position, velocity, and dissipation force, system (1) describes the dynamics of the bubble. The spike (bubble) position is proportional to the vertical scale $h\sim\tilde{h}$ of the flow.[6,26–28] We consider fluids with similar densities, $\delta\rho/\rho\ll 1$ in (1), so the bubbles and spikes are

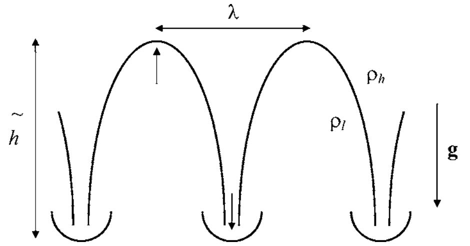

FIG. 1. Large-scale coherent structure of bubbles and spikes in the Rayleigh–Taylor instability: λ is the horizontal scale (spatial period), $\tilde{h}$ is the vertical scale, **g** is gravity and $\rho_{h(l)}$ is the density of the heavy (light) fluid. Arrows mark the direction of the fluid motion at the tip of the bubble (up) and spike (down). A roll-up of vortices results in a mushroom-type shape of the spike. For fluids with similar densities, $\rho_h \sim \rho_l$, the bubbles and spikes are nearly symmetric.

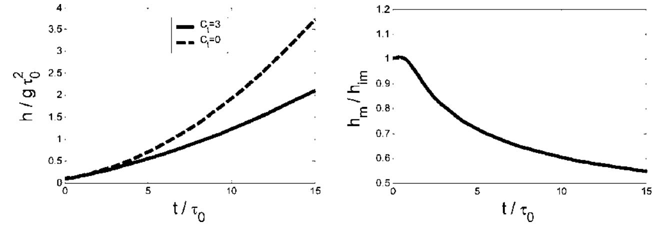

FIG. 2. The turbulent mixing of immiscible and miscible fluids with turbulent diffusion calculated through temperature fluctuations. In Eqs. (1) and (2), the characteristic length scale is $L=h$, the time scale is τ_0, the initial position, velocity and density ratio are $h_0=0.1g\tau_0^2$, $v_0=0$, and $(\delta\rho/\rho)_0=0.2$, and $C=3.67$; for the immiscible case $C_t=0$ and $h=h_{im}$ (dashing line, left plot), for the miscible case $h=h_m$ and $C_t=3$ (solid line, left plot); the ratio h_m/h_{im} decays with time (right plot).

nearly symmetric, and $h\approx\tilde{h}/2$. The dissipation force is the rate of momentum loss in the direction of gravity and $F=-\varepsilon/v$, where ε is the energy dissipation rate.[30] If there is no viscous time scale, dimensional grounds suggest[31,32] for the energy dissipation rate $\varepsilon=Cv^3/L$, where C is a constant, and L is the characteristic length scale of the flow.

The time dependence of asymptotic solutions for system (1) depends on whether the characteristic length scale of the flow is horizontal $(L\sim\lambda)$ or vertical $(L\sim\tilde{h}\sim h)$. If the relevant scale is the horizontal one, the solution is steady, $v \sim \sqrt{g\lambda}$ and $h\sim t\sqrt{g\lambda}$, as observed for nonlinear RTI.[2,14,15,29] In this regime the rate of momentum loss is $F=g\delta\rho/\rho$ and the energy dissipation rate is $\varepsilon=(g\delta\rho/\rho)^{3/2}(\lambda/C)^{1/2}$. If, on the other hand, the characteristic length scale is the vertical scale, $L\sim\tilde{h}\sim h$, then system (1) has the asymptotic solution with $v\sim gt$ and $h\sim gt^2$, and $F=(1-a)g\delta\rho/\rho$, where $a=(1+2C)^{-1}\sim 0.1$ according to observations.[3,26,27] The rate of momentum loss decreases compared to its value in the nonlinear regime, the flow length scale h grows, and the energy dissipation rate increases with time $\varepsilon=a(1-a)(g\delta\rho/\rho)^2t$.

A comparison of the energy dissipation rate and the rate of momentum loss suggests two distinct mechanisms for how a transition may occur from the nonlinear to turbulent regime of RTI. In the former case, the energy dissipation rate ε is comparable in the nonlinear and turbulent regimes so that $\varepsilon\sim(g\delta\rho/\rho)^{3/2}\lambda^{1/2}\sim(g\delta\rho/\rho)^2t$. Hence, the horizontal scale λ grows quadratically with time, $\lambda\sim t^2g\delta\rho/\rho$. This scenario was used by merger models.[20,21] In the latter case, the rate of momentum loss $F=-\varepsilon/v$ has the value $F=g\delta\rho/\rho$ in the nonlinear regime with $L\sim\lambda$, and is reduced to the value $F=(1-a)g\delta\rho/\rho$ in the turbulent regime with $L\sim\tilde{h}\sim h$. Therefore, the turbulent mixing can develop if the vertical h scale dominates the flow, and is regarded as the integral scale for energy dissipation. The dissipation occurs in small-scale structures[6,14,15] produced by shear at the fluid interface.

RT turbulent mixing has two invariants: the rate of momentum gain $g\delta\rho/\rho$ and the rate of momentum loss $F=-\varepsilon/v=(1-a)g\delta\rho/\rho$ in the direction of gravity. The energy dissipation rate grows linearly with time $\varepsilon\sim v^3/L \sim(g\delta\rho/\rho)^2t$ and is not an invariant. In Kolmogorov turbulence[31] the invariance of energy dissipation rate ε is compatible[31,32] with the existence of the inertial range, direct cascade and $k^{-5/3}$ velocity spectrum: The energy injected at large scales $\varepsilon\sim v^2(v/L)$ is transferred without loss through the inertial range and dissipated at small scales $\varepsilon\sim(vL)(v/L)^2$. Whether these fundamental concepts are applicable to an accelerating flow is unclear in the absence of rigorous analysis or trustworthy guidance from experiments and simulations. We see, however, that for describing RT turbulent mixing the rate of momentum loss is a better indicator than the energy dissipation rate.

(b) RTI of miscible fluids. The transport of scalars (such as temperature or molecular diffusion) decreases the buoyant force and changes the mixing properties. In experiments and simulations of miscible fluids[16,17,19,22,27] the value of h/gt^2 shows significant scatter and sensitivity to the initial conditions, which existing models[12,20,21] do not explain. To study the influence of scalar transport on the mixing process, we assume $\rho\sim T$ and $\delta\rho/\rho\sim\delta T/T$, where T is the temperature and $\delta T/T$ is the temperature contrast between the cold, heavy fluid and the warm, light fluid, and account for the turbulent diffusion through the temperature fluctuations. According to Refs. 31 and 32, for nearly the homogeneous case the rate of temperature change is φ/T, where the function $\varphi=\chi(\nabla T)^2$ and χ is conductivity. Dimensional grounds suggest[31,32] $\varphi \sim(vL)(\delta T/L)^2\sim(v/L)(\delta T)^2$, and for small $\delta\rho/\rho\ll 1$ the decrease in the density contrast is $d(\delta\rho/\rho)/dt\sim-(v/L) \times(\delta T/T)^2\sim-(v/L)(\delta\rho/\rho)^2$. The system of governing equations has the form

$$\frac{dh}{dt}=v,\quad \frac{dv}{dt}=\Theta g-C\frac{v^2}{L},\quad \frac{d\Theta}{dt}=-C_t\frac{v}{L}\Theta^2, \tag{2}$$

where $C_t>0$ is a constant and $\Theta=\delta\rho/\rho$. For turbulent case with $L\sim h$ in Eq. (2), the density ratio $\Theta\to 0$ as $t\to\infty$ and $h\sim\exp(1/\Theta)$, and we have $h\sim gt^2/\ln(gt^2/h_0)$ to the lowest order, where h_0 is the initial position. Therefore, the turbulent diffusion cannot terminate the mixing development but reduces its growth rate significantly, see Fig. 2. The dependence $h\sim gt^2/\ln(gt^2/h_0)$ implies that the values of h/gt^2 measured at large but distinct moments of time are different constants, and the memory of the initial conditions is retained.

(c) RTI in stratified fluids. This situation occurs in inertial confinement fusion (ICF) with embedded interface or in direct-drive experiments with picked pulses,[23–25] in stellar

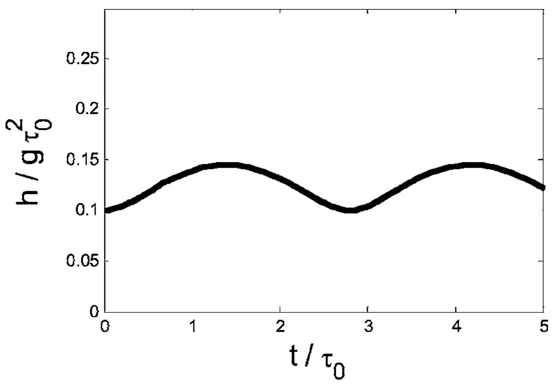

FIG. 3. The spike propagation in a stratified density distribution: in Eq. (3) $C=3.67$, the length scale is $L=h$, the time scale is τ_0, and the initial position, velocity and density ratio are $h_0=0.1g\tau_0^2$, $\nu_0=0$ and $(\delta\rho/\rho)_0=0.2$.

non-Boussinesq convection[9,10] (plumes of dense fluid propagating in a light stratified fluid), and other phenomena.[13] The influence of stratification on the nonlinear evolution of RTI is not well understood. Theories[13,33,34] have described the growth of linear RTI for various density profiles. Simulations of multilayer ICF capsules reported that, in a stratified medium, the width of the mixing zone approaches a constant value and the accelerated mixing does not develop.[24] Simulations of stellar convection[9,10] found similar results. To estimate the influence of stratification on the mixing development, we neglect compressibility, as in Refs. 13 and 33, and consider the propagation of a dense incompressible spike in a light fluid with stratified density profile. The increase in the light fluid density results in a decrease in the buoyant force and $d\Theta/dt=(d\Theta/d\rho_l)(d\rho_l/dh)(dh/dt)$. As an example, we assume $\rho_l=\rho_0(h/h_0)^G$, $\rho_0=\rho_l|_{h=h_0}$ and derive

$$\frac{dh}{dt}=\nu, \quad \frac{d\nu}{dt}=\Theta g - C\frac{\nu^2}{L},$$
$$\frac{d}{dt}\frac{1}{(1+\Theta)}=\frac{(1-\Theta_0)}{2(1+\Theta_0)}\frac{d}{dt}\left(\frac{h}{h_0}\right)^G, \tag{3}$$

where $\Theta_0=\Theta|_{h=h_0}$. If the characteristic length of the flow is the horizontal scale, $L\sim\lambda$, the solution for system (2) has the form $(h/h_0)^G=1+2(\Theta_0-\Theta)/(1-\Theta_0)(1+\Theta)$. Asymptotically, $\nu=\Theta=0$ and $h=h^*$ with $(h^*/h_0)^G=(1+\Theta_0)/(1-\Theta_0)$. This equilibrium is accompanied by oscillations, $\nu,\Theta,h-h^*\sim\exp(iwt)$, which are gravity waves with frequency $w^2=Gg/2h^*$. If the characteristic length is the vertical scale, $L\sim h$, the asymptotic dynamics is qualitatively similar to that for $L\sim\lambda$ (see Fig. 3). The equilibrium point is a circle in the phase space of dynamical system (3), and without dissipation, the oscillation amplitude is finite. In practice, these oscillations are damped, and stratification may result in the appearance of a layered structure. Compressibility, neglected in (3), may result in generation of acoustic waves and may destroy the spike.[10] The nonlinear dynamics of RTI in compressible stratified fluids is a fundamental and hard problem,[13,33,34] and its complete consideration is beyond the scope of the present work. According to our simple model, in a stratified density profile, incompressible spikes propagate to a certain distance, until the density difference is zero, and the mixing development is terminated.

Our results agree with available observations. For instance, for $L=h$, system (1) justifies the physical meaning of the model,[28] which did not use merger arguments and obtained good qualitative agreement with experiments[26,27] for the growth of vertical scale h/gt^2. The solution for system (2) indicates that RT turbulent mixing has no "universal" law $h\sim gt^2$ for miscible and immiscible fluids, in contrast to expectations[16,17,19,26,27] (see Fig. 2). The turbulent diffusion may explain the significant scatter of values h/gt^2 and sensitivity to the initial conditions observed in Refs. 16, 17, 19, 26, and 27. For stratified fluids, our model confirms the observations[9,10,23–25] quantitatively. The lack of data prevents us from making comparisons with RT observations of miscible and stratified fluids.

The experiments and simulations on RT turbulent mixing[26,27] focus mostly on identifying coefficients in the dependencies h/gt^2 and λ/gt^2, and on adjusting free parameters in the empirical models with various dependencies on the density ratio.[12,20,21,26–28] The foregoing results indicate that other observations are required to grasp the essentials of the mixing process. For instance, the merger mechanism[20,21] of the turbulent mixing with $\lambda\sim h$ presumes a "spherical" character of bubbles (spikes) evolution. This assumption has no reasonable grounds because RTI is essentially an anisotropic phenomenon: The flow dynamics in the direction of gravity differs from that in the normal plane.[2,3,14–19,29] The growth of horizontal scales in observations[26,27] may indicate a sensitivity of the flow to external boundaries of the fluid tank (domain) rather than solely to the "merging" character of the mixing. An experimental study of RTI with systematic variation of the aspect ratio of the boundary conditions may clarify this issue. On the other hand, if indeed small-scale structures are essential for the mixing development, then any process that advects vorticity from the interface into the bulk may decrease the dissipation of energy at the fluid interface and decelerate the mixing. Such situation may occur, for instance, in ablative RTI[4] or in premixed combustion in gravity field.[7] In contrast, the growth of the large scales is insensitive to the vorticity distribution. Monitoring the vorticity fractions in the bulk and at the interface may elucidate the role of small structures in RT turbulent mixing.

For isotropic Kolmogorov turbulence, the energy dissipation rate is a statistical invariant,[31,32] and the rate of momentum loss is not a diagnostic parameter. For RT turbulent mixing of incompressible fluids, the flow invariant is the rate of momentum loss in the direction of gravity, whereas the energy dissipation rate is time dependent. These values describe the structure of the interface and the energy transports. To date, none of them was monitored in RTI observations.

To summarize, the present phenomenological model describes RT turbulent mixing of immiscible, miscible, and stratified fluids. The model does not presume a single-scale character of the interface dynamics and distinguishes between the evolution of horizontal and vertical scales. For fluids with constant densities, the results obtained indicate two distinct mechanisms for the mixing development. The first is the traditional "merge" associated with the growth of horizontal scales. The second is associated with the production of small-scale structures and with the growth of the vertical scale, which plays the role of the integral scale for energy dissipation. In RT turbulent mixing, the rate of momentum loss is the flow invariant, whereas the energy dissipation rate is not, and the fundamental scaling properties of the accelerated flow differ from those of the classical Kolmogorov turbulence. The model considers the influence of turbulent diffusion and stratification on RT mixing. We show that turbulent diffusion calculated through the temperature fluctuations does not stop mixing, but decreases the value of h/gt^2 significantly, makes it time dependent and sensitive to the initial conditions. In a stratified density profile, incompressible bubbles and spikes propagate to a certain distance and the mixing process is terminated. Our model is based on dimensional grounds, and the results obtained can serve for future rigorous analysis and systematic experimental studies of accelerated mixing flows.

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