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Possible Effects of Small-Scale Intermittency in Turbulent Reacting Flows

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Abstract. It is now well established that quantities such as energy dissipation, scalar dissipation and enstrophy possess huge fluctuations in turbulent flows, and that the fluctuations become increasingly stronger with increasing Reynolds number of the flow. The effects of this small-scale "intermittency" on various aspects of reacting flows have not been addressed fully. This paper draws brief attention to a few possible effects on reaction rates, flame extinction, flamelet approximation, conditional moment closure methods, and so forth, besides commenting on possible effects on the resolution requirements of direct numerical simulations of turbulence. We also discuss the likelihood that large-amplitude events in a given class of shear flows are characteristic of that class, and that, plausible estimates of such quantities cannot be made, in general, on the hypothesis that large and small scales are independent. Finally, we briefly describe some ideas from multifractals as a potentially useful tool for an economical handling of a few of the problems touched upon here.

Key words: turbulent reacting flows, intermittency, resolution in DNS, multifractal scaling

1. Introduction

The phenomenology of turbulent combustion has evolved from the broad understanding that one has acquired about laminar flames. This starting point suggests that the two quantities that play predominant roles are the fluctuations of the conserved (or passive) scalar and its dissipation rate [1–3]. The fluctuations of the two variables are fundamentally different in character. The scalar fluctuations are essentially independent of the Reynolds number but dependent on the large structure of the flow, which itself depends on details such as boundary and initial conditions. No universal theory for them is possible, although much of value can be said in the self-preserving state (if such a state exists at all in common flame geometries). In parts of turbulent flows that are not too close to boundaries, solid or otherwise, such variables possess probability density functions (PDFs) that are not far from Gaussian. Though never exactly Gaussian, the main point is that the first few moments of these variables contain much of the information about their statistical nature. On the other hand, variables such as energy dissipation and enstrophy—and scalar dissipation if the flows disperse an admixture—fluctuate wildly as a function of position and time, and their PDFs possess extremely long tails [4, 5]. Being dependent on the fluid temperature, which itself shows spatial variations, chemical reactions will also fluctuate in space. Such variables are called "intermittent".[1] A characteristic of intermittency is that extreme amplitudes are far more probable than one may estimate naively from Gaussian considerations. For instance, for energy dissipation, at a Taylor microscale Reynolds number of the order 10,000, events which are some 6 standard deviation away from the mean occur about 150,000 times more frequently than for Gaussian; for some 10 standard deviations, this ratio is an astronomically large value of about 10^{18} [10]. Another characteristic is the clustering of fluctuations of different amplitudes, about which little has been explored. Intermittent variables cannot be treated in the same way as those with relatively mild variations, such as the turbulent velocity or scalar. For the latter, a good estimate of the magnitude of the fluctuation is the standard deviation but it provides a poor indicator for intermittent variables. Different levels of fluctuations are characterized better by different moments of the variable; equivalently, to describe intermittent quantities with any degree of faithfulness, one needs the entire PDF itself.

In all outlooks on turbulent combustion—such as models based on fast-chemistry [11], flamelets [3] and conditional moment closures [12]—scalar dissipation plays an important role. The question to which we will pay some attention is whether intermittency or, equivalently, large and not so rare fluctuations are of much consequence to issues affecting chemical reactions and combustion in turbulent flows. So far, modelling of turbulent combustion takes no explicit account of intermittency.[2] Even though such models work reasonably well, at least in special circumstances, that the fluctuations indeed occur often and cause difficulties in the interpretation of results can be seen easily in direct numerical simulations, for example those of Overholt and Pope [13].

The purpose of this paper is to point out the general consequences of intermittency on aspects of reacting flows, though few new results will be derived. It is an appropriate topic for a paper celebrating Robert Antonia's 60th birthday because he has pursued the characterization of intermittency for a good part of his career in turbulence research. Given its special context, we shall not give many details and refer, instead, to earlier papers as appropriate. The perspective will be Eulerian, mostly because of the author's belief that it is better understood at present than Lagrangian intermittency [14]. In Section 2, we discuss the lognormal approximation commonly

[1] In the literature on reacting flows, enough care has not been invested on distinguishing this intermittency of small scales from the so-called "outer" intermittency of Corrsin and Kistler [6]. The effects of the latter are obviously important, and examples of how they can be incorporated into calculation methods can be found in Libby and Williams [7] and Kuznetsov et al. [8]. Here, we shall use intermittency to mean small-scale intermittency. Although we shall be concerned technically with the intermittency of both inertial and dissipative regions, the latter is of special interest in combustion. It is of historic interest to note that this dissipation intermittency was discovered more than 50 years ago by Batchelor and Townsend [9], but is yet to make significant inroads into the combustion literature.

[2] Through the use of *measured* conditional densities, some effects of intermittency are indirectly incorporated into calculation methods—see, for example, Klimenko and Bilger [12].

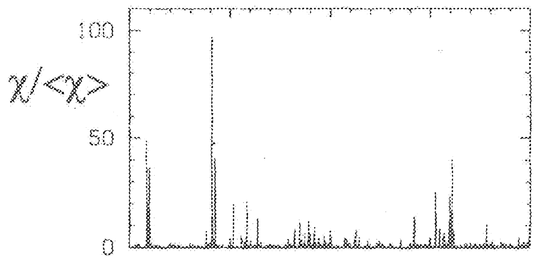

Figure 1. The streamwise component of the scalar dissipation obtained on the centerline of a heated cylinder; Taylor's hypothesis has been used to interpret the time trace as a spatial cut. The cylinder Reynolds number is about 12,000 and the measurements are made at about 90 cylinder diameters downstream on the wake centerplane. Thus, the horizontal distance in the figure approximately corresponds to a distance of 10 longitudinal integrals scales. The vertical axis is this surrogate scalar dissipation normalized by its mean value. Peaks of the order of a 100 times the mean value occur in this trace. Longer traces reveal several such peaks. The intermittent nature of scalar dissipation is evident.

used for the PDFs of intermittent variables. A few effects of small-scale intermittency are described in Section 3, while, in Section 4, we draw attention to the stringent resolution constraints imposed by intermittency on direct numerical simulations of turbulence. The nonuniversal effects in large-amplitude events in dissipation are discussed in Section 5. This feature has some important consequences for the correlation of events involving large amplitudes of fluctuations. In Section 6, we discuss the probabilistic and geometric interpretations of multifractals and illustrate how they can be useful for our purposes. The paper concludes with Section 7 containing a few summary remarks.

2. Lognormality and the Variance of Scalar Dissipation

The fluctuations in scalar dissipation possess characteristics that are not strongly dependent on the flow, and are thus "nearly universal". Putting some substance to this assertion has consumed much time and effort in the literature, see Sreenivasan and Antonia [5] for a summary; see, also, the caveat to be discussed in Section 5. Figure 1 shows a typical time trace of the scalar dissipation rate, χ, in a moderate-Reynolds-number turbulent wake. Even at the modest Reynolds number of the flow, the intermittent character of the scalar dissipation is evident. The fluctuations become even wilder as the Reynolds number increases. At very high Reynolds numbers—which may or may not be relevant to flames—the fluctuations can be expected to be essentially singular-like in some regions in the flow and of negligible magnitude in others.

Kolmogorov's [15] genius was to regard the logarithm of an intermittent variable as Gaussian.[3] This idea may appear natural to us with the hindsight of more than

[3] The original proposal was for energy dissipation, for which the notion of an energy cascade makes the proposal somewhat palatable physically; but the general idea is extendable to all intermittent phenomena.

40 years of living with it, but it was a significant departure from the thinking at the time. The technical shortcomings of this suggestion have been underscored for sometime (see [4] for a summary), and it appears clear that some consequences of lognormality do not agree with data; for example, the so-called scaling exponents for high-order quantities, to the extent that they can be trusted, show a qualitatively different trend from that for the lognormal distribution [4, 5]. Nevertheless, lognormality is a useful working approximation, where an analytically tractable and reasonably good approximation for the PDF is desired.[4] A basic quantity that appears in combustion theories is the variance of the logarithm of the scalar dissipation. This can be obtained from measurements as follows.

One of Kolmogorov's [15] assumptions (not germane to his so-called third hypothesis and is independent of it) was that the variance would assume the form

$$\sigma^2_{\ln \chi_r} = A + \mu_\theta \ln(L/r), \tag{1}$$

where L is the large scale of the flow—to be specific, the longitudinal integral scale—and r is a local averaging scale for χ; that is,

$$\chi_r = r^{-1} \int_x^{x+r} \chi \, dx. \tag{2}$$

In general, because it is thought that the coefficient μ_θ in front of the logarithmic term is a universal constant, there have been many efforts made to measure μ_θ (for a partial list, see [18–22]). Since the jet flow is a useful paradigm for turbulent non-premixed flames, we will comment on that flow first. We shall also assume that the additive constant A does not vary a great deal within a given class of flows, and that Equation (1) can be extended all the way to $r = \eta$, where η is the Kolmogorov scale, thus enabling us to evaluate the variance of $\ln \chi$ itself. It follows from Prasad et al. [23] that

$$\sigma^2_{\ln \chi} = 0.8 + 0.29 \ln R_d, \tag{3}$$

where R_d is the nozzle Reynolds number of the jet.

For boundary layer flows, the measurements of all three components of the scalar dissipation [24] has suggested that

$$\sigma^2 \approx 0.35 + 0.26 \ \ln R_L, \tag{4}$$

where R_L is the Reynolds number based on L and the root-mean-square velocity fluctuation, u.

Even if our estimate for the variance of $\ln \chi$ is good to within some 10%, it does not follow that the variance of χ will be known to the same accuracy (because χ is an intermittent variable). Thus, it is useful to have an explicit form for the variance

[4]The log-Poisson model—see, for example, She & Waymire (1995)—has several advantages over the lognormal model (section 4). Although we will not pursue the log-Poisoon model here, it appears to be quite relevant to the most singular vortex structure (Vainshtein 2003). By inference, it may be speculated that the most singular events of scalar dissipation may similarly obey the log-Poisson model.

of χ itself. For the particular flow studied by Antonia and Sreenivasan [25], it was also possible to write the ratio

$$F_n \equiv \langle \chi_r^n \rangle / \langle \chi \rangle^n = \exp\left[\frac{1}{2} n(n-1)\sigma^2\right], \tag{5}$$

or,

$$\ln F_n = (n/2)(n-1)\ \ln(L/r)^{\mu_\theta}. \tag{6}$$

Again extending the relation all the way to $r = \eta$, and using the isotropic relation $R_L = R_\lambda^2/15$, where R_λ is the microscale Reynolds number based on u and the Taylor microscale λ, we have

$$F_2 \approx 0.7 R_\lambda^{0.52} \tag{7}$$

(since our interest here is $n = 2$). A summary of all existing measurements, summarized in Figure 8 of Sreenivasan and Antonia [5], shows that a better fit to the data, for $R_\lambda > 50$, is close to

$$F_2 \approx R_\lambda^{0.5}. \tag{8}$$

This is our recommendation for the R_λ-variation of the variance of the scalar dissipation. Indeed, our present understanding is that the moments F_n vary as power laws with R_λ, instead of logarithmically; measurements at many Reynolds numbers indeed show that the moments F_n vary with R_λ roughly as power laws, with the power law exponent increasing with n. The qualitative point to be made is that the PDFs of intermittent variables vary strongly with the Reynolds number, and this variation is not understood theoretically. Empirically, it is known that they can be approximated adequately by stretched exponentials or sums of two exponentials [26].

3. Some Elementary Examples of Intermittency Effects

To illustrate some qualitative effects of intermittency, consider Bilger's [11] beautiful formula for nonpremixed flames, which links the instantaneous chemical reaction rate per unit volume for a chosen species, w, to the chemistry and to the scalar dissipation rate χ, through the formula

$$w = \rho\beta(c)\chi. \tag{9}$$

Here, ρ is the density of the species and, in the fast reaction limit, the function β is a function merely of chemistry. The first trivial comment is that, since χ is intermittent, the product formation is likewise intermittent. The second comment is that the relation in Equation (9) may not always be valid even if the nominal situation is one of fast chemistry; this is so because, the fast chemistry limit, which is the basis of the above formula, may not apply locally in some parts of the flow. To see this, recall that a simple-minded measure of whether the reaction rate is fast or otherwise is the Damköhler number, D, which is the ratio of the mixing time-scale τ_m to the chemical reaction time-scale τ_r. We take the appropriate mixing time to be the Kolmogorov time-scale[5] an average value of which is given by $(\nu/\langle\epsilon\rangle)^{1/2}$. In practice, because the dissipation varies wildly, it is clear that one can define a local Kolmogorov time-scale, based on the instantaneous value of the energy dissipation, which, at the Reynolds numbers such as those studied by Karpetis and Barlow [27], can have thousand-fold variability within the flow. It is clear that the local value of the Damköhler number varies by a factor of about 30 for this reason alone. In reality, chemical-reaction scale is not a fixed number either, and that too varies for complex chemical reactions. Thus, while the reaction may be fast by an average measure, reactions can be fast in some regions of the flow and slow in others, at one and the same time. It would then be useful to know in what fraction of the flow the reaction is slow and in what fraction it can be regarded as fast. It is known that flame extinctions may occur when $D < D_{\text{crit}}$ (or when ϵ is large), but this criterion does not convey quantitative information if the local value of D is highly variable within the flow volume. Thus, in principle, Equation (9) cannot be taken to be valid in the entire space (because it relies on the fast-chemistry assumption); thus, perhaps belaboring the point, we can only note the negative result that

$$\langle w\rangle \neq \langle\rho\rangle\langle\beta(c)\rangle\langle\chi\rangle. \tag{10}$$

What changes the inequality to an approximate equality is the δ-function nature of the source term in the equation for one-step reaction, arising from the presumed discontinuity in the space of reacting species and the mixture fraction. Equivalently, a measure of the accuracy with which the inequality in Equation (10) can be replaced by an equality is the extent to which the above δ-function approximation can be regarded as valid.

As a second example, consider the notion of a flamelet. This pertains to the situation when the reaction regions are thin compared to regions over which scalar dissipation varies significantly. In this way, one can indeed effect closures by using stationary laminar flamelet calculations for a range of scalar dissipations rates, and weighting them by an appropriate PDF for the dissipation. This PDF may be a lognormal with its variance specified by Equation (3) or equivalent.[6] But if the scalar dissipation varies a great deal from one position to another, and the scales of these variations are much finer than the standard average measure of the Kolmogorov scale, it is not clear that the concept of flamelets would be suitable for large Reynolds numbers—certainly uniformly at all points within the reacting flow. Also, in transformed variables using mixture fraction gradients, the term that balances the reaction rates is a nonstandard diffusion term in which the scalar

[5]In the combustion literature, one nominally uses the large-eddy time scale L/u for τ_m but a better measure of the mixing time may be the smallest scalar scale. According to traditional thinking, this time scale is that of the Kolmogorov scale itself, for all Schmidt numbers equal to or greater than unity. It is easy to argue, though we shall not do so here, that even if we use the large-eddy time-scale to represent mixing, intermittency effects will be strong for small scales.

[6]In flamelet models so far, no allowance seems to have been made for the Reynolds number variation of the variance.

dissipation acts as an effective diffusion coefficient. Again, the consequences of an intermittent diffusion coefficient have not been explored; this would be fascinating to understand at some basic level.

As a third example, consider the effect of heat release on the dynamics of a reacting flow. In nonpremixed flames, if the density varies drastically as a result of heat release, one can imagine that details of the entrainment of the outside fluid will change, thus affecting the growth rate of the flow—but data do not seem to suggest large changes. It is reasonable to suppose that the major effects will be centered around the mean stoichiometric surface, but that is precisely where the specific volume changes little with respect to mixture fraction. Thus, Bilger [28] argues that this effect of heat release is relatively small for nonpremixed flames. However, this is not the case for premixed flames. For the flamelets to remain thin, presumably, the strain rate induced by the dilatation effects should be small compared to the turbulence strain rate which keeps the flamelets thin. That strain rate, given by $(\epsilon/\nu)^{1/2}$, is a strong function of position in a high Reynolds number turbulent flow, varying by a factor of the order 100 for typical cases. Thus, it is not possible to say, from any simple considerations, whether the heat release effects are small or large: there will always be regions where they are small and regions where they are large. If we know the fraction of the volume over which it is large or small, we may be able to obtain an average estimate of the effects. The main point is that estimates based on the average measure of the strain rates do not give the right answer.

As a final example, briefly consider conditional moment closures [12]. The second-order closure scheme already takes into account the effects of intermittency indirectly because it is based on the *observed* form of conditional expectations (which clearly incorporate intermittency appropriate to the chosen situation), but it is clear that low-order closure schemes do not take explicit account of large fluctuations (or regions dominated by small-scale turbulence). It is generally unclear that adopting increasingly high-order schemes will improve the accuracy, unless proper attention is paid to the variation of intermittency with the Reynolds number.

4. Effects of Dissipation Intermittency on Resolution Requirements of DNS

Dissipation intermittency poses stringent demands on the accuracy of direct numerical simulations of turbulence. Bilger [28] has remarked that there exists no scalar dissipation *measurements* with adequate resolution. This onus on experimentalists has been remarked upon by others as well [22]. Particular attention to this issue has been paid also by R.A. Antonia in several of his publications, see, for example, Antonia et al. [18].

We remark here that there exist no high-Reynolds-number scalar dissipation *simulations* of adequate resolution, either. (For a well-resolved calculation at low Reynolds numbers, see Yeung et al. [48]). It should be said right away that we

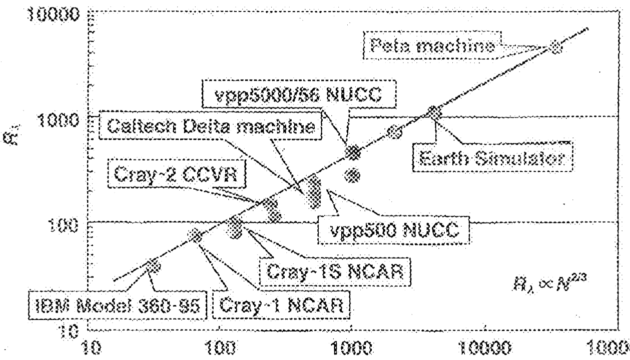

Figure 2. This figure shows that the Taylor microscale Reynolds number of direct numerical simulations have typically varied approximately as the 2/3-rds power of the number of grid points on the side of the periodic computational domain. This is consistent with the common practice of adopting the resolution nearly always to the average value of the Kolmogorov scale. This figure is adopted from an article published by Professor Y. Kaneda and his colleagues at the University of Nagoya, Japan, in a trade journal. Data from Yeung [31] are in excellent agreement with the trend of $N \propto R_\lambda^{2/3}$ fitted by Professor Kaneda and colleagues.

offer this remark in a constructive spirit, by no means to downplay the enormous contributions that direct numerical simulations have made for our understanding of turbulence. A proper resolution of the smallest scales may not alter our present knowledge of turbulence too much, but we cannot be certain until we verify it.[7]

For homogeneous turbulence in a periodic box, we know that

$$L/\langle\eta\rangle = R_L^{3/4} \propto R_\lambda^{3/2}. \tag{11}$$

Here, $\langle\eta\rangle = (\nu^3/\langle\epsilon\rangle)^{1/4}$ the usual (average) Kolmogorov scale, $\langle\epsilon\rangle$ the average rate of energy dissipation. (In previous sections, we simply used η to designate $\langle\eta\rangle$, but from here on we need to make the distinction between the average and instantaneous values).

We need the computational box to have at least one integral scale L in it, and, indeed, one usually works with box sizes whose size is of the order of an integral scale. The highest resolution aimed for is roughly $\langle\eta\rangle$. Thus the linear dimension of the box

$$N = L/\langle\eta\rangle \propto R_\lambda^{3/2}. \tag{12}$$

This is satisfied by the empirical data from all past computations (see Figure 2).

But, is the resolution based on $\langle\eta\rangle$ adequate? Perhaps it is so for some purposes, but not for all. As we have already seen, scales much smaller than $\langle\eta\rangle$ do indeed exist in high-Reynolds-number flows, which means that the resolution required is

[7]Perhaps needlessly, we note that there are differences of principle between the effects of poor resolution in experiments and in simulations. In the former, the flow develops on its own, and whatever one can measure with adequate resolution can be said to be measured correctly; in simulations, if one does not resolve scales properly, it is not obvious that the properties of the resolved scales are necessarily correct.

not $\langle\eta\rangle$, but the smallest value that η can assume—the ratio $\eta_{\min}/\langle\eta\rangle$ becoming smaller with increasing Reynolds number. That is, if we wish to resolve all possible scales in the flow, the resolution required is not $\langle\eta\rangle$, but

$$\eta_{\min} = (\nu^3/\epsilon_{\max})^{1/4}, \tag{13}$$

where $\epsilon_{\max}$ is the largest value of the dissipation rate in the intermittent distribution in space.

This ratio is not easy to estimate, though it seems reasonable to suppose that it should exist. Sreenivasan and Meneveau [29] provide plausible estimates for the ratio $\epsilon_{\max}/\langle\epsilon\rangle$ via the measured multifractal exponents of ϵ (see Section 6).[8] The formula is

$$\eta_{\min}/\langle\eta\rangle = R_\lambda^{-(3/2)(1-\alpha_{\min})/(3+\alpha_{\min})} \approx R_\lambda^{-1/2}, \tag{14}$$

where α_{min} is the exponent corresponding to ϵ_{max}, the strongest spike of the energy dissipation.[9] We do not presently have infallible estimates for $\alpha_{\min}$, because the strongest singularities are quite hard to measure faithfully, but a plausible estimate is $\alpha_{\min}$ is not far from zero. This estimate has been used in the second step of Equation (14). If $\eta_{\min}$ is the resolution desired, we will have, from Equations (12) and (14),

$$L/\eta_{\min} = (L/\langle\eta\rangle)(\langle\eta\rangle/\eta_{\min}) = R_\lambda^{3/2} \times R_\lambda^{1/2} = R_\lambda^2. \tag{15}$$

If we fix the number of grid points in the box and increase the resolution as noted, we can only allow for smaller value of L (noting that we should at least have one L in the box). Thus, we can attain only a lower Reynolds number. We then have the relation

$$N \propto R_\lambda^2, \tag{16}$$

or

$$R_\lambda \propto N^{1/2}, \tag{17}$$

instead of the traditional $N^{2/3}$ of Figure 2. This suggests that the prevailing expectation of the highest computable R_λ, namely Equation (12), is more optimistic than is indeed the case; the optimism is related to the fact that we have not paid attention to sub-Kolmogorov scales while setting goals for grid resolution. For instance, had the monumental and record-breaking calculations on the Earth Simulator [30], using a box with N = 4096, aimed for the resolution which we have advocated above, the R_λ attained would have been 300–400 instead of the 1200 now attained with the resolution of $\langle\eta\rangle$.

[8] Since sub-Kolmogorov scales were not measured in these experiments, the present estimates can only be approximate, but the main qualitative point will remain unaffected.

[9] The quantity $\epsilon_{\max}$ does not exist for a strictly lognormal distribution. One, however, imagines that it is finite in a given flow. An advantage of the log-Poisson distribution over lognormal is that a finite value of $\epsilon_{\max}$ exists for the former.

These considerations are especially stringent for passive scalars, for which these estimates are not dissimilar in spirit (though more involved in detail), especially when the Schmidt numbers are larger than unity.

5. Non-Universality of Large-Amplitude Fluctuations of Intermittent Quantities

We shall illustrate the basic idea of this section by considering energy dissipation as the example, but similar notions will hold for scalar dissipation. We shall not mention a number of technical details, for which reference should be made to the Ph.D. thesis of Dhruva [32]; some relevant information for the passive scalar dissipation in standard shear flows can be found in Kailasnath et al. [33]. However, the details omitted should not affect the understanding of the basic point. Consider differences in velocity between two neighboring points which are a distance r apart. Consider their square, namely Δu_r^2. The behavior of the average of this quantity for $r \ll L$ is of primary interest in turbulence theory [34, 35, 4]. For $r = \eta$, it is enough to state that Δu_r^2 is the same (modulo the kinematic viscosity) as one of the components of the energy dissipation (and hence, in some rough sense, can be regarded as representative of the full energy dissipation). Our interest here is in understanding the behavior of the variance of Δu_r conditioned on the large scale velocity. If we replace u by the scalar concentration, and replace the large scale velocity by the mixture fraction, the analogous interest would be in the behavior of scalar dissipation conditioned on the mixture fraction variable. This is a quantity of direct interest in modelling of reacting flows.

Figure 3 shows the conditional average $\langle \Delta u_r^2 \mid u = u_0 \rangle$, where u is the large-scale velocity; the data were obtained in a high-Reynolds-number atmospheric turbulence about 35 m above the ground. The precise definition of the large scale velocity does not make a difference to the conclusions to be drawn, as shown by Dhruva [32], and one may therefore regard the velocity at the midpoint of the interval r as a suitable measure of u_0. Each curve in the figure corresponds to a particular value of r, all values of which lie in the inertial range. In Kolmogorov's phenomenology, the inertial-range scales are regarded as independent of the large scale—that is, the conditional averages of Figure 3 should be independent of u_0. In contrast, measurements show that the dependence is very strong for large numerical magnitudes of the large-scale velocity. One cannot argue that the dependence exists because the Reynolds number is low ($R_\lambda \approx 10,000$ here). Thus, the dependence of the conditional averages on the large-scale velocity must be regarded as a reality. More information on this facet can be found in [36, 37, 32].

This observation has several consequences. One of them is that the scaling properties, discussed at some length in Monin and Yaglom [35] and Frisch [4], are "contaminated" by this behavior. How to remove this contamination is the subject of Sreenivasan and Dhruva [10], and does not concern us here directly. What concerns us is the fact that large amplitude events are not necessarily universal, and depend

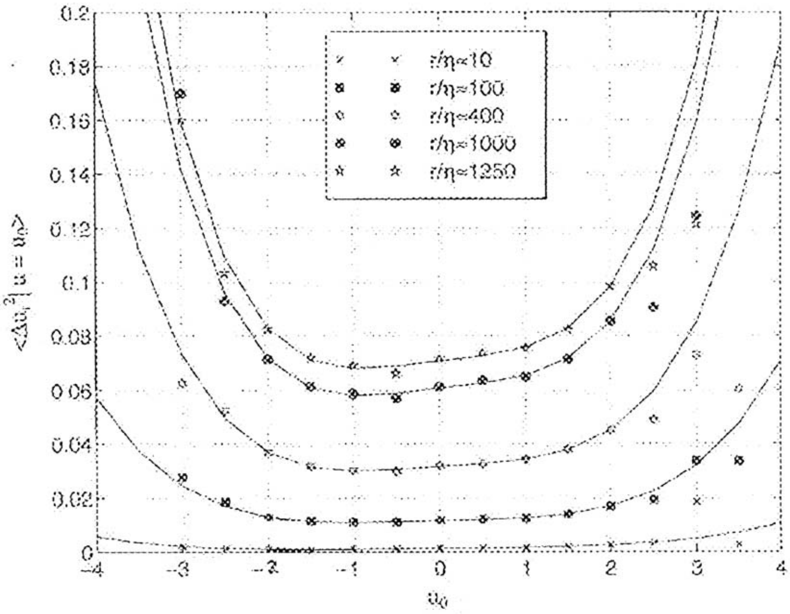

Figure 3. The variance of the velocity increments for five separation scales r, all in the inertial range, conditioned on the large-scale velocity u_0. The latter is plotted as a multiple of its variance. If the inertial-range velocity increments are independent of the large scale, these conditional variances should not vary with the large-scale velocity.

on the large-scale, or on the nature of "forcing"—as is known in the jargon. In particular, the independence of the large-scale and small-scale quantities, which has been the linchpin of Kolmogorov's phenomenology, cannot be used without a second thought. (Unpublished experiments and simulations in our group have shown that the dependence shown in Figure 4 does not exist for homogeneous and isotropic turbulence, which seems to suggest that the small scales in that flow are not affected by large-scale forcing. Perhaps, then, the independence of small-scale from the large scale is to be restricted to the case of homogeneous and isotropic turbulence. In other flows, except when the large-scale velocity is weak, there is a coupling between the large-scale and small-scale quantities. This is, in fact, the thinking in the modern literature on turbulence.)

Once we allow for the possibility that in shear flows (and flames) the large scale will have an influence on large-amplitude fluctuations of the small scale, the standard inferences drawn from Kolmogorov's phenomenology will need modifications. For instance, consider the (conditional) covariance of a reacting species and the scalar dissipation fluctuations; this quantity is of interest in conditional closure methods [12]. Simple notions of the independence of the large and small scales suggest that this covariance must be negligible, but strong correlations of large amplitude events suggests that it does not necessarily vanish even at very high Reynolds numbers. This would vanish only in homogeneous and isotropic turbulence, which suggests that it is a different paradigm of turbulence from the sheared turbulent flows. A minor amazement is that in each shear flow, successful efforts can be made to retrieve the part that is equivalent to homogeneous and isotropic turbulence [10, 38].

6. A Brief Note on Multifractals

6.1. Probabilistic interpretation

In the discussion above, we have said or implied that one needs to be able to take suitable weighted averages of the flow properties of interest, such as the chemical reaction rates, to take account of intermittency effects. Thus, the incorporation of intermittency is tantamount to finding the best way to average over quantities that fluctuate rapidly and wildly. No recourse to Central Limit theorem would be possible.

The standard practice would be to find the PDF of the appropriate intermittent variable and take averages. The shortcoming of this procedure is that the PDFs of intermittent variables are strongly dependent on the Reynolds number, and, for their use to be advantageous, one needs both the functional forms of the PDFs and their Reynolds number dependencies. Neither of them is known theoretically. In the multifractal framework, there are reasons to suppose that one can define quantities analogous to the PDF that are independent of the Reynolds number. This would be an advantage. In spirit, it follows Obukhov's [39] scenario, in which one first averages over flow regions, where the intermittent variable is within some narrow limits ("pure ensemble"), and then suitably weight them in the next step of averaging. It is also conceivable that intermittency demands more profound changes in outlook, but we shall not consider this option.

Recall that an intermittent variable in the limit of infinite Reynolds number is either infinitely large or zero. (For such variables, the notion of probability density becomes meaningless.) In practice, the Reynolds numbers are finite and there are cut-offs due to diffusion. Even so, as the Reynolds number increases, time traces or spatial cuts of scalar dissipation—just to cite an example—will become increasingly spiky. The best way to deal with such awkward signals is to locally smooth them over a nonoverlapping interval r, say. We can then study the properties of the smoothed variable as a function of the smoothing scale r. By extrapolating r to the smallest scale of interest, we can then say something about the unsmoothed variable itself. In this sense, smoothing is a convenient artifact. It is also physically a reasonable procedure because a spike at just one instant, or at one point in space, cannot have a large effect: it has to sustain itself for a finite period of time, or over a finite volume of space, for it to be effective in physical processes, such as flame extinction. Thus, we will deal with variables such as χ_r introduced earlier in Section 2. Considering, for simplicity, the distribution of χ_r on a line of unit length, it is clear that the number of nonoverlapping intervals is $1/r$.

As we stated already, Kolmogorov took the logarithm of an intermittent variable such as χ_r to obtain a Gaussian-looking variable. As a point of departure, consider the quantity

$$\alpha = \log(r\chi_r)/\log r. \tag{18}$$

In words, the so-called Hölder exponent α is the logarithm of the "stuff" inside the interval r (called the "measure") divided by the logarithm of the size of the interval. For most processes, α is positive and finite, and it is clear that it is a function of spatial position in the flow. (Note that both χ_r and r are assumed to have been normalized suitably in the definition of α here, but the normalizations are omitted here for slight convenience of writing.)

Now obtain the frequency distribution of α, i.e., the number $N_r(\alpha)$, the number of r-sized intervals having the Hölder exponent α. Divide $\log N_r(\alpha)$ by $\log(1/r)$, and call this ratio $f(\alpha)$. We have not identified this quantity in any way with r because, as $r \to 0$, $f(\alpha)$, which is a continuous function of α, tends to a well-defined limit [40–42]. The $f(\alpha)$ curve is called the multifractal spectrum for reasons that we shall see momentarily.

6.2. Geometric interpretation

Returning to the definition of α, let us rewrite Equation (18) as

$$\chi_r = r^{\alpha-1}, \tag{19}$$

where we draw attention to the fact that α is a function of spatial position. For all $\alpha < 1$, this formula shows that $\chi_r \to \infty$ as $r \to 0$; that is, χ_r will be singular. In practice, r never assumes a precisely zero value because it is smoothed out slightly, and one can only have "singular-like" behavior. It is also clear that the smaller the values of α the stronger the singular-like behavior (or more "spiky" the distribution of χ_r). Wherever $\alpha = 1$, χ_r is constant locally, and $\alpha > 1$ yields a smooth behavior, the degree of smoothness depending on how much larger a value than unity α assumes. Thus, one can imagine that, over the entire space of the flow, a variety of local behaviors of χ_r—whether singular-like, how strongly singular-like, whether relatively constant, whether smooth, and, if so, how smooth, and so forth—can be represented by a continuum of values of α within a range.

Now, consider the set of all values α lying within a narrow band centered around a specified value, and ask the question: what is the fractal dimension of that iso-α set? By the definition of the fractal dimension, the number of intervals of size r which possess the particular value of α (in practice within a narrow band centered around it), namely $N_r(\alpha)$, will have the behavior

$$N_r(\alpha) = r^{-f}, \tag{20}$$

where f, which is a function of α, is the fractal dimension of the iso-α set. Taking logarithms on both sides yields the previous definition of $f(\alpha)$. Thus, for each α, the index $f(\alpha)$ represents the fractal dimension of the subsets of intervals of size r having the particular value of α [43]. As $r \to 0$, there is an increasing multitude of subsets, each characterized by its own value of the Hölder exponent α and the fractal dimension $f(\alpha)$. This is the reason for the name multifractals or, more formally, multifractal measures.

Meneveau et al. [21] developed a formalism for joint multifractal measures to describe joint statistics of two or more intermittent distributions. The idea is intuitively very similar to that described above, and the steps between single multifractal measures and joint multifractal measures are essentially no different from those between the PDF of a single variable and joint PDF of several variables. Joint multifractal measures will be invoked briefly in the next section without much explanation, but we expect that the reader will see the essential point.

6.3. An example of how multifractals can be used for averaging purposes

The exercise follows the pattern laid out in Sreenivasan and Meneveau [29]. Suppose that we are interested in knowing the fraction of the flow in which the scalar dissipation has singular-like features. We shall illustrate the idea by asking the equivalent question in one-dimensional cuts (either time traces or spatial cuts), but the situation is the same for the volume. We are interested in the fraction of unit interval corresponding to all $\alpha < 1$. This is given by

$$\int_{\alpha<1} (\eta/L)^{1-f(\alpha)}\,\mathrm{d}\alpha. \tag{21}$$

To see this, merely recall that the total number of η-sized intervals in L is L/η, and the number of sub-intervals corresponding to a fixed α is simply $(\eta/L)^{-f(\alpha)}$. This if one knows the $f(\alpha)$ curve, which, according to our discussion earlier, is independent of the Reynolds number, it is trivial to compute the integral for any Reynolds number. One can similarly ask: what fraction of energy dissipation is contained in the singular-like regions? Again, it is easy to see, from Equations (19) and (20), that this fraction will be given by

$$\int_{\alpha<1} (\eta/L)^{1-f(\alpha)} \times (\eta/L)^{\alpha-1}\,\mathrm{d}\alpha, \tag{22}$$

and that this integral too can be computed trivially if one knows $f(\alpha)$. These issues have been considered by Sreenivasan and Meneveau [29]—in particular, see their Figure 6. There is no theory for obtaining the functional form of $f(\alpha)$. For the energy and scalar dissipations, the empirically determined forms can be found in Sreenivasan [44].

Suppose, as another example, one is interested in the volume occupied by the singular part of the covariance of the reacting species and the scalar dissipation. The required fraction is given

$$\int_{\alpha<1,\alpha'<1} (\eta/L)^{-f(\alpha,\alpha')}\,\mathrm{d}\alpha\mathrm{d}\alpha', \tag{23}$$

where α' plays the same role for the reacting species as α does for χ. The joint multifractal spectrum $f(\alpha, \alpha')$ can be measured without much difficulty [21]. Similarly, if one needs to obtain the fraction of chemical-reaction rates produced in singular-like reaction regions (a quantity of interest for knowing whether the combustion is more in the form of a distributed reaction or of flamelets), it is easy to write it as

$$\int (\xi/L)^{1-f(\alpha')} \times (\xi/L)^{\alpha'-1}\, \mathrm{d}\alpha', \tag{24}$$

where ξ is the smallest length scale for the reaction rate. This type of analysis can be extended to all quantities of interest.

7. Concluding Remarks

In this paper, we have drawn attention to some issues concerning the intermittent behavior of various quantities in reacting flows, the scalar dissipation being the immediate paradigm. It seems plausible that, because of strongly nonlinear effects associated with reaction dynamics, extreme events (or regions dominated by small scales) should be of great consequence in reacting flows. We have considered a few possible effects on reaction rates such as flame extinction, flamelet approximation and conditional moment closure methods, and have commented on possible effects on the resolution requirements of direct numerical simulations of turbulence. A further factor is that the extreme events in the scalar field are nonuniversal in the presence of shear (present in all practical flames). The importance of extreme events is obviously tied to the importance that intermittency itself will have on reacting flow dynamics. Finally, we have discussed some aspects on how multifractals can be put to use for addressing some broad range of questions.

Since there is no formal theory for getting multifractal spectrum,[10] the question should be asked as to what extra value multifractals possess over the standard PDF methods. We reiterate that the main advantage is that the multifractal spectrum is independent of the Reynolds number, aside from staying close to the physical feel for intermittent variables. Still, the multifractal spectra have to be either measured or approximated in some general way. At present, there is a large body of intuitive and pragmatic understanding of multifractals that allows plausible approximations to be made. Often, very simple and analytically tractable approximations can be considered. One modest example for energy dissipation has been discussed in Sreenivasan & Stolovitzky [47]. Our firm belief is that most possibilities remain to be exploited.

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[10]The model problem of passive scalars, due to Kraichnan [45], is an exception: see Falkovich et al. [46].

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