An update on the intermittency exponent in turbulence

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The issue of experimental determination of the intermittency exponent μ, is revisited and it is shown that the "best" estimate for it is 0.25 ± 0.05. This "best" estimate is obtained from recent atmospheric data, and is based on several different techniques.

Ever since Kolmogorov[1] introduced the intermittency exponent, more than a dozen attempts have been made to obtain its numerical value from experiments. The resulting estimates have varied from as low a value as 0.18 to as high as 0.7. (See Ref. 2 for a summary of early measurements, Table IV of Ref. 3 for measurements until about 1975 and Refs. 4–10 for experiments made thereafter.) It is unsatisfactory that this exponent, which plays a moderately important role in the theoretical framework of turbulence, should be known with no better certainty. We present in this Brief Communication a brief critique of the various definitions and experimental techniques used for determining the intermittency exponent, and provide the best estimate for it from recent atmospheric data.

Some of this variability is clearly due to differences in the definitions of the intermittency exponent, and it is therefore essential to describe all of them briefly. We find it convenient to adopt different symbols to denote the different definitions of the intermittency exponent. First, there is the constant μ_1 in Kolmogorov's lognormal hypothesis[1] given by

$$\sigma^2=A+\mu_1 \log(L/r). \tag{1}$$

Here, σ^2 is the variance of the logarithm of ϵ_r, which is the energy dissipation averaged over an interval of size r, L is the "macro" or "integral" scale of turbulence, and the additive constant A is presumed to depend on the large scale of the flow.

On the basis of a generic self-similar cascade model—for relevant background, see Refs. 2 and 11; for a more modern interpretation in terms of multifractals, see Refs. 9, 10, and 12—we can write

$$\langle \epsilon_r^2 \rangle \sim r^{-\mu_2}. \tag{2}$$

From Novikov's[11] argument (discussed at some length in Ref. 2, Sec. 25), one can also write for homogeneous turbulence that

$$\langle \epsilon(x)\epsilon(x+r) \rangle \sim r^{-\mu_3}. \tag{3}$$

By Fourier transforming Eq. (3), one obtains for the spectral density of the energy dissipation $E_1^\epsilon(k)\sim k^{1-\mu_3}$, which we shall write as

$$E_1^\epsilon(k)\sim k^{1-\mu_4}. \tag{4}$$

Finally, use has also been made in the past of the definition that

$$\langle [\epsilon(x)-\langle\epsilon\rangle][\epsilon(x+r)-\langle\epsilon\rangle] \rangle \sim r^{-\mu_5}. \tag{5}$$

Equation (5) is equivalent to Eq. (3) only at very high Reynolds numbers.

It is safe to say that $\mu_2=\mu_3$, and that $\mu_3=\mu_4$ trivially. However, Eq. (4) may not yield satisfactory results because of crossover effects discussed by Nelkin;[13] these effects are important at all finite Reynolds numbers. Further, the scaling region apparent in one procedure may well be different from that in another. The quantity we shall call intermittency exponent is $\mu=\mu_2=\mu_3$. The constants μ_4 and μ_5 are good approximations to μ only if the Reynolds number is very high. The constant μ_1 is not related to the other μ's except through the hypothesis of some cascade model.

We are now in a position to comment on experiments in more specific terms. Most of the early estimates of the intermittency exponent were made by using Eq. (4). Nelkin[13] pointed out the difficulty in estimating μ via μ_4 when the scaling range is finite and, with this background and a model calculation, deduced a value of 0.25 from Kholmyansky's[14] spectral data. Antonia *et al.*[5] followed up with an immediate experiment in a fairly high Reynolds number jet ($R_\lambda=966$), and confirmed Nelkin's estimate. They obtained, by means of Eq. (3), the exponent to be around 0.2. This latter estimate was confirmed by Anselmet *et al.*,[6] who also used Eq. (3). It is interesting that Anselmet *et al.* found, from the same data at a moderate Reynolds number, that μ_5 was about 0.45, emphasizing the finite Reynolds number effect to which we have already alluded. The recent multifractal scaling work[9,10] has yielded a value of about 0.25 ± 0.05. These same experiments also showed that the measured value of μ_1 was about 0.23, in close agreement with 0.25. This may indicate that some kind of cascade process is not a bad model for turbulence.

Despite this seemingly conclusive evidence that the intermittency constant is about 0.25 ± 0.05, older estimates have often found favor in the literature. For example, Baker and Gibson,[7] and Gibson[8] have used a value of 0.5 for μ. Gibson (private communication) has indicated how the smaller value of 0.25 would imply, when used in conjunction with the lognormal model for the energy dissipation, an astronomical value for the energy-containing scale. It seemed that an effort at revisiting the issue, and settling it conclusively if possible, was worthwhile.

We systematically examined the various methods of determining the intermittency exponent using some recent data obtained in the atmosphere. Measurements were made in the atmospheric surface layer about 6 m above a long stretch of a wheat field canopy, and at a height of 2 m

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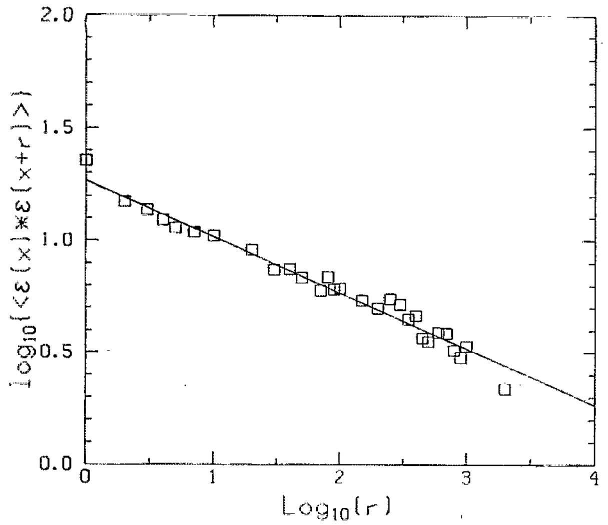

FIG. 1. The log–log plot of $\langle \epsilon(x)\epsilon(x+r)\rangle$ vs r (assuming Taylor's hypothesis). In this and the following two figures, r is expressed in terms of sampling units$=1/6000$ sec and the wind velocity was 6 m/sec. The best estimate of $\mu=0.25$. This so-called best estimate was obtained by trying various values of the exponent μ, and deciding upon the value of μ which yielded the best flat line in plots of (ordinate/$r^{-\mu}$) in Eq. (3).

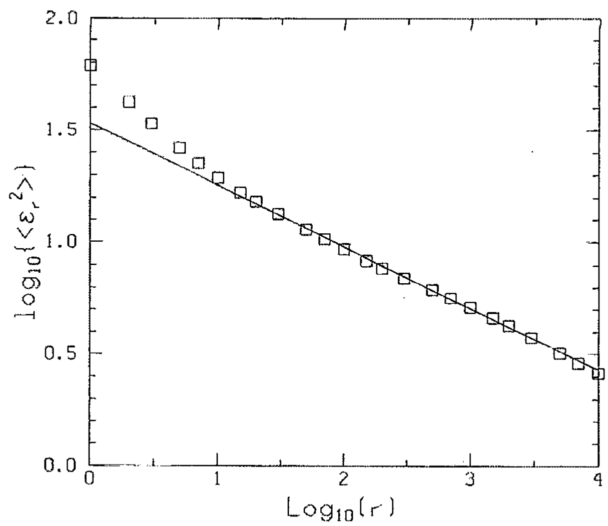

FIG. 2. The log–log plot of $\langle \epsilon_r^2\rangle$ vs r for the same set of data used in Fig. 1. The best slope is 0.27.

above the roof of a four-story building. The mean wind velocity varied between 4 and 6 m/sec. Velocity fluctuations were measured using a standard hot wire (5 μm in diameter, 0.6 mm long) operated on the constant temperature mode on a DISA 55M01 anemometer. The anemometer voltage was digitized on a 12-bit digitizer at a sampling frequency of 6000 Hz. This sampling frequency was found to be adequate for capturing most of the small-scale fluctuations. The linearization of the signal yielded a time trace of the streamwise velocity fluctuation $u(t)$. The dissipation field was approximated by $(du/dt)^2$. This assumes that the space derivative could be approximated by the time derivative according to Taylor's frozen flow hypothesis, and that one component of dissipation is an adequate representation of the total dissipation statistically. It is believed that these approximations were not critical to the determination of the intermittency exponent. The internal Reynolds number based on the root-mean-square velocity and the Taylor microscale varied between 1500 and 2000.

The following characteristics of the data reassured us about their quality. The power spectral density showed a sizeable scaling range with a slope of approximately $-5/3$ (actually slightly steeper). The third-order structure function varied linearly with the separation distance[15] for more than a decade of variation in the separation distance.

The most unambiguous methods of determining μ are from the Eqs. (2) and (3); to our knowledge, all previous measurements that used these methods have given a value close to 0.25. Figures 1 and 2 from the atmospheric data show that both Eqs. (3) and (2) yield essentially the same answer, namely, μ is about 0.25. (Note, however, that the scaling ranges in the two figures are somewhat different.)

While the experience of Anselmet *et al.*[6] would suggest that there is little point in using Eq. (5), we obtain more or less the same exponent irrespective of whether we use Eqs. (4) or (5). See Fig. 3. We tentatively conclude that the discrepancy in the two methods of measurement in the experiments of Anselmet *et al.* is their lower Reynolds number.

We now use an alternative method for the determination of the intermittency exponent. This method uses the so-called multiplier distributions[4,11,12] appropriate to the inertial range. Briefly, the multipliers M_a are defined as the ratio of the energy flux (or the energy dissipation rate) contained in an eddy of size r in the inertial range to that in a box of size ar, where a is an integer that equals the number of subeddies into which a parent eddy is presumed to break up. The multipliers M_a are random variables that possess a well-defined probability density function, $p(M_a)$. In Ref. 12, it was shown that, for any given a, $p(M_a)$ is

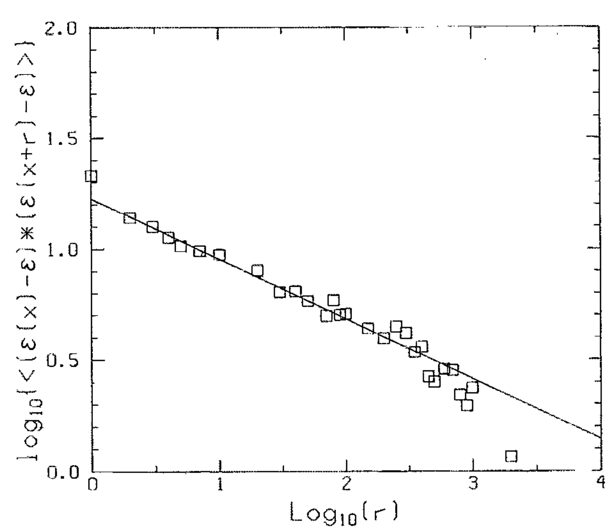

FIG. 3. $\langle[\epsilon(x)-\langle\epsilon\rangle][\epsilon(x+r)-\langle\epsilon\rangle]\rangle$ vs r for the same set of data used in Fig. 1. The best slope is 0.27.

independent of r for all r in the inertial range. Chhabra and Sreenivasan[12] obtained the distributions of these multipliers for several values of a, and published results for $a=2$, 3, and 5 (Fig. 2 of Ref. 12). The fact that $p(M_a)$ is independent of r for any given base a can be used to advantage in obtaining converged values of high-order moments that one would otherwise not be able to measure reliably. Although the probability density $p(M_a)$ depends on a, the various scaling exponents given by the multifractal spectrum are independent of a. In particular, the intermittency exponent is independent of the precise value of a chosen.

It is easy to compute μ_2 from multiplier distributions. Let us start with an eddy of size unity, and normalize the total energy dissipation in that eddy to be unity. Let such an eddy break down into a pieces, and let the energy dissipation contained in each piece (as a fraction of the total contained in the parent eddy) be picked randomly from the appropriate distribution $p(M_a)$. Let us examine the subeddies that have resulted after n steps in this cascade. By the definition of the multipliers, the amount of energy dissipation E_r contained in a box of size $r=a^{-n}$ is given by

$$E_r=\prod_{i=1}^{i=n} M_a(i). \tag{6}$$

Here, by the respective definitions of the two quantities, $E_r=r\,\epsilon_r$ (in one dimension). The product in Eq. (6) over the index i (which varies between 1 and the current step n of the cascade) occurs because the total energy dissipation E_r contained in a subeddy of size $r=a^{-n}$ is the product of the multipliers from one step to another starting with the scale unity. These multipliers will be chosen from the appropriate probability density function $p(M_a)$.

It follows that

$$\langle (E_r)^2\rangle=\left\langle \left[\prod M_a(i)\right]^2\right\rangle=\prod \langle M_a^2\rangle, \tag{7}$$

where the products are taken over all values between 1 and n, and the angle brackets denote probability averages over the appropriate multiplier distribution. The last step follows because the multipliers are statistically independent of the step of the cascade. Noting that $E_r=r\,\epsilon_r=a^{-n}\epsilon_r$ and that $\Pi\langle M_a^2\rangle=(\langle M_a^2\rangle)^n$, we can show by a few steps involving only algebra that

$$\mu=\log_a\langle M_a^2\rangle+2. \tag{8}$$

From the probability distribution $p(M_a)$ measured in Ref. 12, we obtain that $\mu=0.23\pm0.05$ for all three values of a (2, 3, and 5). This is consistent with the present estimates obtained by other methods. It should be noted that similar calculations made in Ref. 4 yielded values between 0.18 and 0.22. Van Atta and Yeh[4] also note that these values are consistent with Kholmyanski's result of 0.23.

Using the triangular approximation to $p(M_a)$ recommended in Ref. 12 for $a=2$, we can analytically show that $\langle M_a^2\rangle=7/24$, which gives $\mu=0.22$. For the p model of Ref. 16, also for $a=2$, $\langle M_a^2\rangle=0.29$, which gives $\mu=0.21$.

All these considerations lead us to conclude that the "best estimate" for the intermittency exponent is $\mu=0.25\pm0.05$.

Incidently, all our estimates[3,17] for the intermittency exponent in the case of the scalar dissipation are in the vicinity of 0.35.

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