

## TRANSITION INTERMITTENCY IN OPEN FLOWS, AND INTERMITTENCY ROUTES TO CHAOS

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The intermittent transition to turbulence in open flows (mainly pipe flows) is examined in the context of intermittency routes to chaos. Preliminary conclusions are that some quantitative connections can be discerned, but that they are incomplete. In a similar manner, connections with phase transition and other critical phenomena are also imperfect. Some measurements which we hope will be helpful in developing alternative models describing the essentials of the phenomenon are described. Some difficulties are highlighted.

### 1. Introduction

This paper is a part of an overall effort related to the exploration of quantitative connections between chaos in dissipative dynamical systems on the one hand, and transition and turbulence in the so-called open flow systems on the other. Open flows by definition possess a preferred direction, and there is a flux of mass across its boundaries. At least in some circumstances this elementary feature of open flows renders the nature of flow instability convective, as opposed to being absolute, which is the case observed in closed flow systems. This can have profound consequences on the origin of turbulence in open flow systems, which may in turn render our task quite difficult.

It has been known for over a hundred years now [1] that transition to turbulence in pipe flows occurs intermittently. For example, the velocity measured on the centerline at a fixed axial location in the pipe is typically as shown in fig. 1. It is this intermittent transition to turbulence that is our concern here. With increase in Reynolds number, the fraction of time that the flow is in the turbulent state increases, until eventually the flow is continuously turbulent. One observes qualitatively similar intermittency in the advanced stages of transition to turbulence in boundary layers (fig.

2), and channel (i.e., plane Poiseuille) flows, to which also we shall make a brief reference here.

Equally well known now is that many low-dimensional dynamical systems approach a chaotic state in an intermittent fashion, qualitatively similar to the intermittent transition to turbulence just

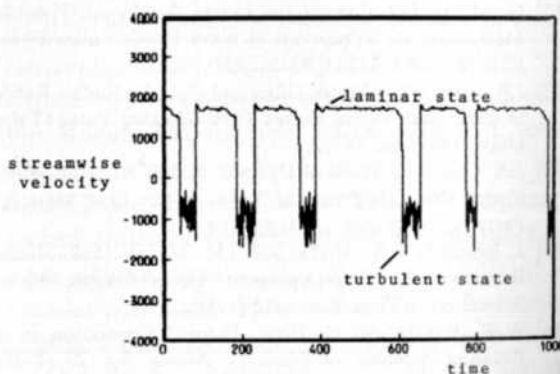


Fig. 1. The streamwise (or axial) velocity measured as a function of time on the centerline of a pipe flow. The measuring tool is a standard hot wire operated on a constant temperature mode. The signal oscillates apparently randomly between an essentially steady laminar state and a turbulent state. For a given axial position, a velocity trace obtained simultaneously at another radial position will show a coincident alternation between the two states, but the amplitude difference between the two states is a function of radial position. Both the ordinate and the abscissa are drawn to arbitrary scales.

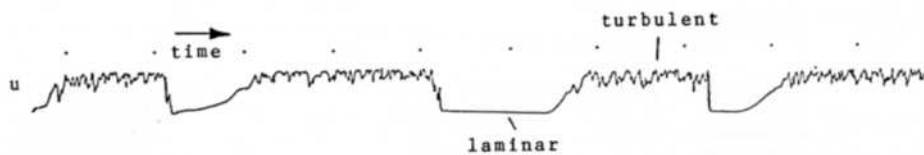


Fig. 2. Oscillograms of velocity fluctuations in the advanced stages of transition to turbulence in a constant-pressure boundary layer. Time interval between markers is 1/60 s. Source: ref. 17.

discussed. Fig. 3 is an example. The Lorenz equations [2], the Logistic map [3], and the RCL oscillator [4] are some of the other simple examples. Pomeau and Manneville [5] identified three generic intermittency routes which they called Type I, Type II and Type III—each differing from the other in terms of how the eigenvalues of the Floquet matrix, describing the return map linearized around a closed trajectory, cross the unit circle. Type I intermittency occurs when the linear stability of the limit cycle is lost by an eigenvalue of the Floquet matrix leaving the unit circle at  $+1$ , Type III when the crossing occurs at  $-1$ , and Type II when two complex conjugate eigenvalues simultaneously cross the unit circle. That these intermittent routes to chaos are relevant to fluid flow phenomena governed by partial differential equations has been demonstrated, for example, by Bergé et al. [6] and Dubois et al. [7] in the

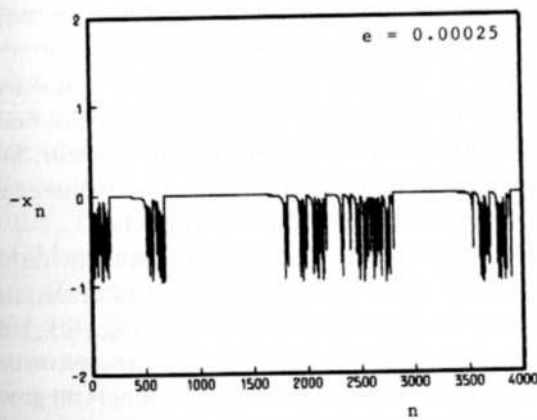


Fig. 3. Iterations of the one-dimensional map  $x_{n+1} = x_n(1 + e) + (1 - e)x_n^2 \pmod{1}$ ;  $e = 0.00025$  for this plot. The map is the result of a reduction [24] from the so-called Type II intermittency [5]. The relevant aspects of this type of intermittency route to chaos are mentioned in the text. The qualitative analogy with fig. 1 is quite obvious.

Rayleigh–Benard experiment, and by Pomeau et al. [8] in the Belousov–Zhabotinsky reaction.

It may be argued on the basis of these interesting findings that intermittent transition in open flows may belong to some kind of universality class: Even though as already mentioned open flows are different in several non-trivial ways from the highly confined flows (see also [9]), it looks reasonable to ask whether there are any connections between the intermittency routes to chaos mentioned above and the intermittency routes to turbulence in open flows. As we shall see, the process involved in the latter are more complex; it is to their partial characterization that this paper is devoted.

## 2. The physical phenomenon

It is useful to recapitulate briefly the physical mechanism responsible for the temporal intermittency observed in fig. 1. Evidence from our own work—at least in pipes whose length is of the order of a few hundred diameters—as well as that of others (chiefly Wygnanski and Champagne, [10]), suggests that ‘disturbances’, whose developed state corresponds to the turbulent regions in the intermittent signal, arise locally (in radial, azimuthal as well as axial directions) in the entrance region of the pipe where the flow is laminar and steady, and is not fully developed (see fig. 4a). Once created in the boundary layer region, the disturbance quickly spreads over the entire cross-section of the pipe, and moves like an independent entity within the pipe; laminar regions are present both upstream and downstream of this entity, which now goes by the name ‘slug’. (They have

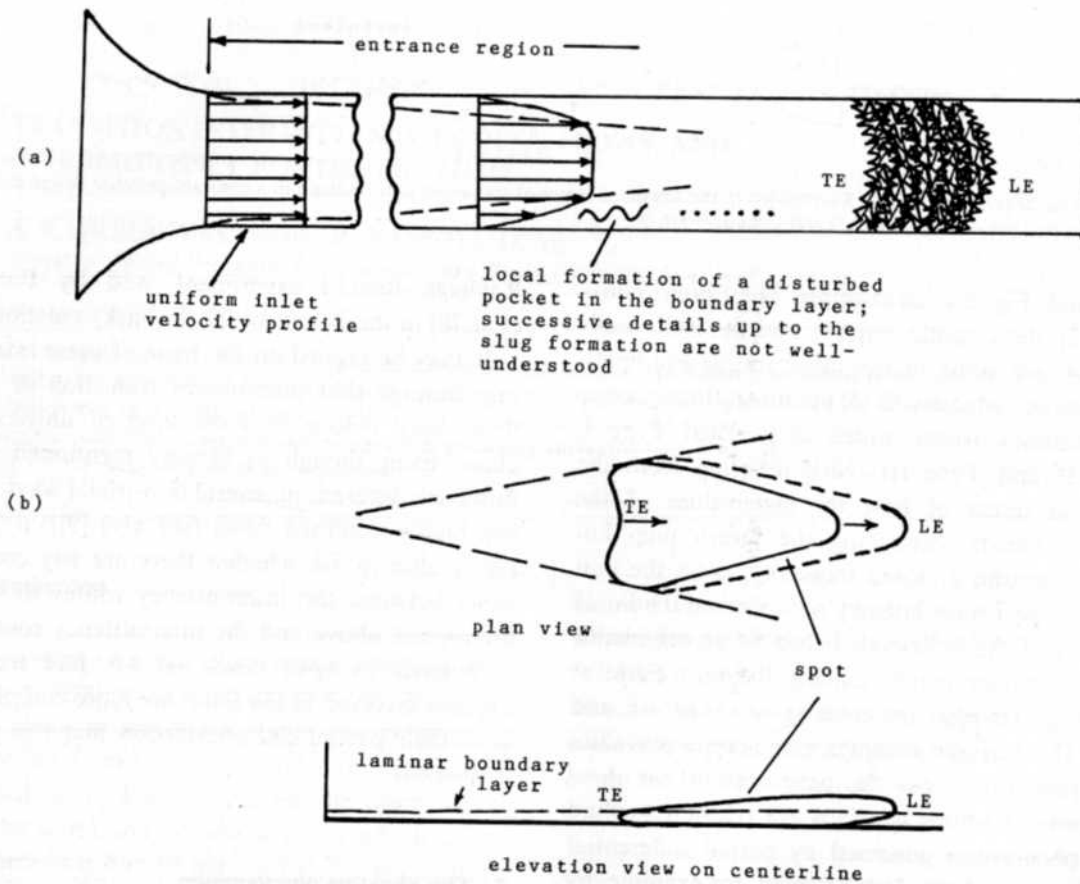


Fig. 4. A schematic of intermittent transition structures in (a) pipe flows, (b) boundary layers.

been called 'flashes of turbulence' by Reynolds [1] and 'plugs' by several workers, for example, Tritton [11].) A probe fixed at any point in the flow alternately sees the procession of slugs with laminar regions interspersed between them; the output signal consists of intermittent excursions from the laminar to the turbulent state, followed by the return (in the Eulerian frame of reference) in some stochastic manner to the former. The physical reasons for the return to the laminar state are the following. Usually in most pipe flows, it is the pressure difference between the inlet and the exit that is held constant. A given pressure difference can support a larger mass flow when the flow is laminar than when it is turbulent. As the slugs form and grow, the increased friction due to the turbulent flow in them produces a reduction in mass flux, thus inhibiting the instabilities at the

inlet. New slugs are most likely to be born only after the first slug completely passes out of the pipe. It is not hard to argue similarly that intermittent transition can occur also when the mass flux is fixed but momentum variations occur, but it is possible that the two types of intermittencies do not share the same detailed properties.

Corresponding to slugs in pipe flows, the transitional structure in boundary layers and in channel flows is the so-called turbulent spot (fig. 4b). Unlike the slugs which are constrained except in the axial direction, the boundary layer spots can grow in all directions. (There are some non-trivial differences between spots in channel flows and boundary layers, but these details are not relevant here.)

It turns out that the speed of propagation  $U_{lc}$  of the leading edge of the slug or a spot is different

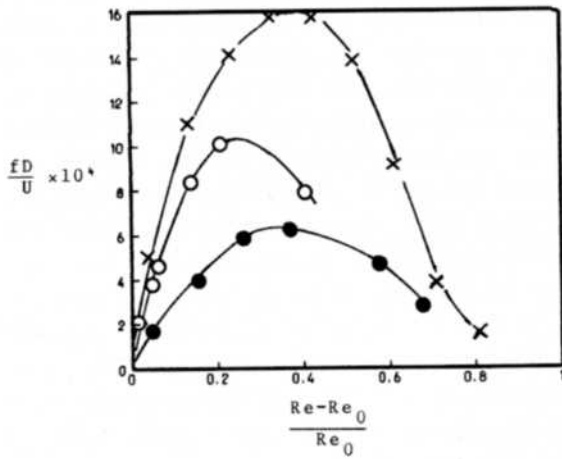


Fig. 5. Typical variation of the frequency of slugs as a function of  $Re$  for three streamwise locations. The topmost and lowermost curves are from Wagnanski and Champagne [10] corresponding to  $(Re_0, x/D)$  of  $(21 \times 10^3, 220)$  and  $(19 \times 10^3, 395)$ , respectively. The middle curve,  $Re_0 = 3900$  and  $x/D = 335$ , is from present measurements.  $U$  is the so-called bulk velocity (= flow rate/cross-sectional area of the pipe).  $Re = UD/\nu$ .

from the trailing edge speed  $U_{te}$ . Clearly, slugs (or spots) grow with distance if  $U_{le} > U_{te}$ . In the following we concentrate on the slugs. If the slugs are generated at some mean frequency, and if more than one slug resides in the pipe at the same time, the leading edge of a slug could catch up with the trailing edge of the preceding one, resulting in merger and a consequent reduction in their passage frequency with axial distance. These two factors could then provide a plausible mechanism for the streamwise dependence, at any given Reynolds number, of all measured temporal quantities. The two most important parameters in the problem are thus the Reynolds number  $Re$  and the (normalized) axial distance  $x/D$ , where  $D$  is the pipe diameter. The frequency of the slugs, for example, depends on both of these parameters, as shown in fig. 5. A characteristic value of this frequency (say, the peak value) varies inversely with  $x/D$  (fig. 6), and seems to be independent of the Reynolds number. (Strictly, one must plot on the abscissa the quantity  $x - x_0$ , where  $x_0$  is a virtual origin for the slugs; it is possible that the scatter in the plot partly originates from this

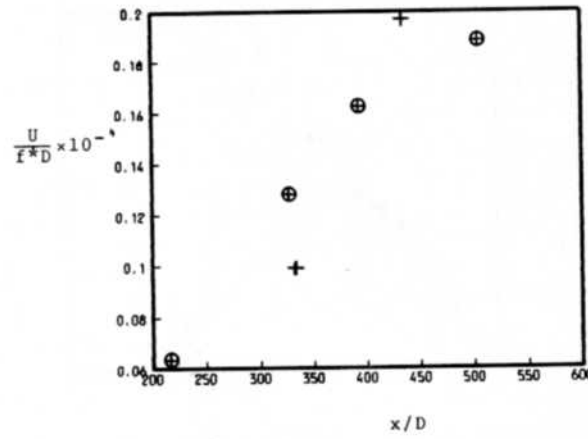


Fig. 6. Inverse of the peak slug frequency ( $f^*D/U$ ) as a function of the streamwise distance. Noting that all measurements were made at the exit of pipes of different lengths, the streamwise distance must really be understood to mean the pipe length. The circled points are from Wagnanski and Champagne [10].

source. However, since  $x$  is relatively large compared to  $x_0$ , this is believed to be of small consequence.)

Several points must be made explicit. Both in figs. 5 and 6 (and in the others to follow),  $x/D$  is really  $L/D$ , where  $L$  is the pipe length. That is, measurements were actually made at the exit of pipes of different lengths. Although we have retained the notation  $x/D$  above in conformity with previous practice, it is not clear to us that measurements made at different axial locations of the same long pipe will show the behaviors of figs. 5 and 6. A look at fig. 5 shows that the mean length of slugs ( $\sim U/f$ ) is greater than the pipe length, suggesting that more than one fully developed slug is unlikely to reside in a pipe at any given instant of time. Thus, the probability of merger is quite small. The reason for the observed reduction in the characteristic frequency with pipe length (fig. 6) must then be attributed largely to the reduction of the formation frequency of slugs with pipe length. This makes sense if we remember that longer pipes mean longer slugs which take longer to pass the entire pipe. We conclude that detailed and careful measurements at several stations in

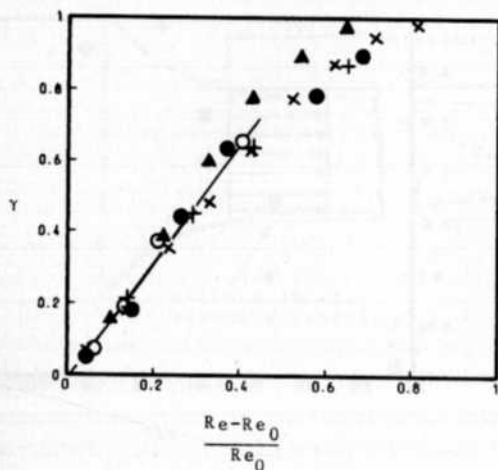


Fig. 7. Typical data on the intermittency factor  $\gamma$  as a function of  $(Re - Re_0)/Re_0$ .  $\blacktriangle$ :  $L/D = 505$  [10];  $+$ :  $L/D = 435$  (present). Other symbols as in fig. 5.

extremely long pipes (say, length to diameter ratio  $> 10^4$ ) is overdue.

As one varies the flow Reynolds number, the appearance of the intermittent state is quite abrupt. The intermittency factor  $\gamma$ , defined as the fraction of time the flow is turbulent, appears to vary approximately linearly with the Reynolds number. By a backward extrapolation to zero of the measured intermittency factor, one can define a unique value of the onset Reynolds number  $Re_0$ . Fig. 7 shows that  $\gamma$  is a unique linear function of  $Re - Re_0$  in a certain non-trivial neighborhood of  $Re_0$ ;  $x/D$  or  $L/D$  is thus an inconsequential parameter for this quantity.

A reasonable goal now is to describe in phase space the main features of these processes. Returning now to fig. 1, it appears plausible to think that the steady laminar state is essentially zero-dimensional—that is, a proper orthogonal decomposition of the temporal signal contains no time dependent function. (Unfortunately, estimates of statistical properties such as the entropy and dimension from the velocity signal obtained entirely in the laminar state, for example just before the onset of intermittency, is dominated by the high-dimensional, low-amplitude noise overriding the laminar motion. The noise here does not arise merely from instrumentation or other 'purely ran-

dom' fluctuations in the background; as mentioned elsewhere [9], the background 'noise' in most open flow systems is usually dominated by large-scale pressure fluctuations which are far from being structureless.) From this fixed point, the motion escapes to an attractor representing the turbulent state, and gets reinjected near the fixed point at apparently random intervals. Two relevant questions can be asked: 1) Can one quantitatively capture by a low-dimensional map the essential dynamics of this intermittent motion from the fixed point? 2) What are the characteristics of the chaotic attractor? Answers to these questions are attempted below.

### 3. The route to chaos

Fig. 8 shows a close-up of the vicinity of the velocity signal near the leading edge of a typical slug. Corresponding to the laminar as well as this interface regions, we have constructed by discretization a return map of  $u_{n+1}$  vs  $u_n$  (fig. 9). A close look in the vicinity of the fixed point shows that the map is much like that from which fig. 3 was constructed. Secondly, the slope of the return map near the fixed point is close to but greater than unity. This shows that the fixed point is unstable once the onset of intermittency occurs; the laminar and interface regions are thus merely a reflection of the duration spent in the narrow channel in the vicinity of the fixed point. There is some hope, then, that the dynamics of the leading edge interface can be described (approximately) by a one dimensional map of some kind, for example that used in fig. 3 (for small  $x_n$ ). This observation lends some emphasis to our original question of possible connections to the generic intermittency routes to chaos. We must right away note a simple fact: Pipe flows strictly belong to neither type of intermittency mentioned in section 1, an obvious reason being that, unlike in the Pomeau-Manneville formulation, intermittent transition in open flows (see especially fig. 2) occurs from a *steady state* and not from a limit cycle. (For a

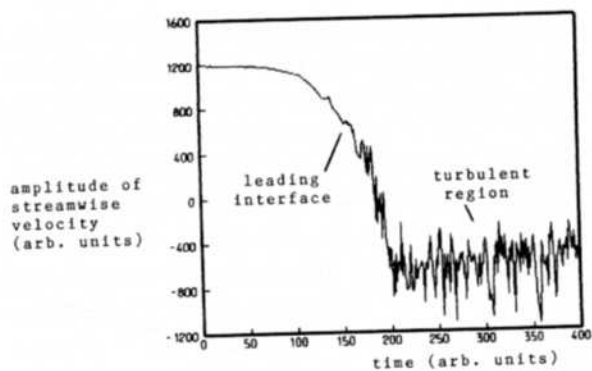


Fig. 8. A close-up of the velocity signal near the leading edge of a typical slug.

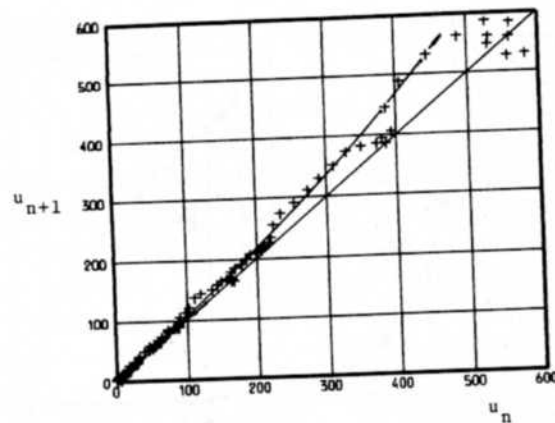


Fig. 9. The return map of  $u_{n+1}$  vs  $u_n$  for the interface region shown in fig. 8, obtained by the discretization of the signal. The origin is the fixed point representing the laminar state.

brief reexamination of this point see section 6.) This may be interpreted to mean that Poincaré sections of the Pomeau–Manneville intermittencies have a direct bearing on pipe flow transition, but it will unfold that this is not the entire story. As we shall show soon, this is related to the non-uniform manner in which the motion in phase space gets reinjected to the vicinity of the unstable fixed point. (We take the view that to label reinjection by ‘relaminarization’ – as is often done – is to miss the point altogether. While in the Eulerian frame of reference one sees an alternation between laminar and turbulent states, this does not imply relaminarization of fluid that was once turbulent. As must be clear from section 2, in the Lagrangian frame of reference, there is no relaminarization of fluid entrained by a slug: one is talking merely about the slug/no-slug situation.)

All three types of intermittencies mentioned in section 1 make definite predictions for certain statistical quantities of the intermittent signals, against which the outcomes of experiments can be tested. Apart from the nature of the return maps themselves, the important predictions concern the probability distribution for the duration of the laminar regions; from this distribution one can in particular calculate their mean duration as a function of the departure from the critical value of the control parameter,  $Re - Re_0$  here. At any rate, it is useful to measure these quantities in the hope that they will help us build alternative models.

Fig. 10 shows a plot of the average length  $L_l$  of the laminar regions as a function of  $Re - Re_0$ . The data for several experimental conditions all tend to show that  $L_l \sim (Re - Re_0)^{-1}$ . This behavior is common to both Type II and Type III intermittencies. The measured inverse cumulative distributions for the length of the laminar intervals (fig. 11) follows the expression

$$P(l > l_0) \sim [e / \exp(4el_0 - 1)]^{1/2}, \quad (1)$$

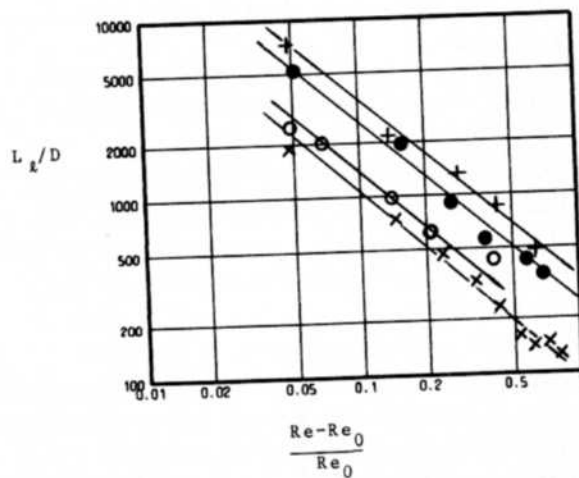


Fig. 10. The mean length of the laminar regions in the measured velocity signals, plotted as a function of the distance from the critical Reynolds number. Lines correspond to the  $-1$  power predicted for Type II and Type III intermittencies. +,  $L/D = 435$ ,  $Re_0 = 4480$ . Other symbols as in fig. 5.

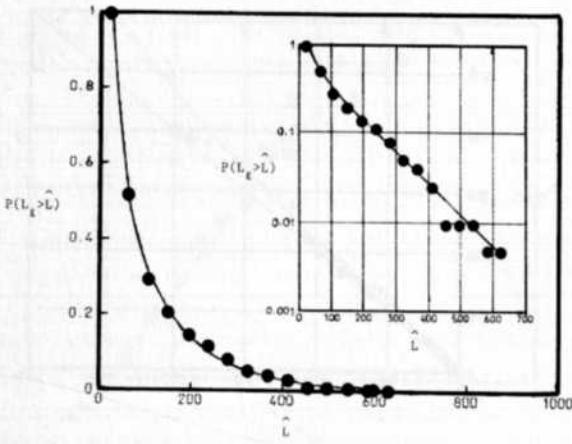


Fig. 11. The cumulative distribution of the laminar intervals.  $Re = 4725$ ,  $\gamma = 0.379$ ,  $e = 0.004$ . Note that, except for a small  $l$ , the behaviour of  $P$  is very nearly exponential.

which is a result known to hold for Type III intermittency. Here, the parameter  $e$  should be identified as being proportional to  $(Re - Re_0)/Re_0$ . The inset, which is an expanded log-linear plot shows that the fit is very good even towards the tail region.

In spite of these concurrences, one cannot identify the pipe flow with Type III intermittency for two reasons. Firstly, the hallmark of Type III intermittency is the subcritical period-doubling [7], with the primary effect of nonlinearity being a dramatic enhancement of the subharmonic component just before the flip to the chaotic state occurs. The system, instead of subsequently following the period doubling route to chaos, somehow decides to go the intermittency route. As already mentioned, the nonturbulent state is not a limit cycle. Secondly, and more importantly, in arriving at expression (1), the assumption has been made that whenever reinjection occurs from the chaotic attractor to the vicinity of the limit cycle, the distance from the fixed point of a Poincaré map where this reinjection occurs is uniformly distributed [12]. We have measured (see Appendix) the distribution of the reinjection distance from the unstable fixed point (in this case), and obtained the result that it is approximately an inverse power law (fig. 12) over a certain range. The

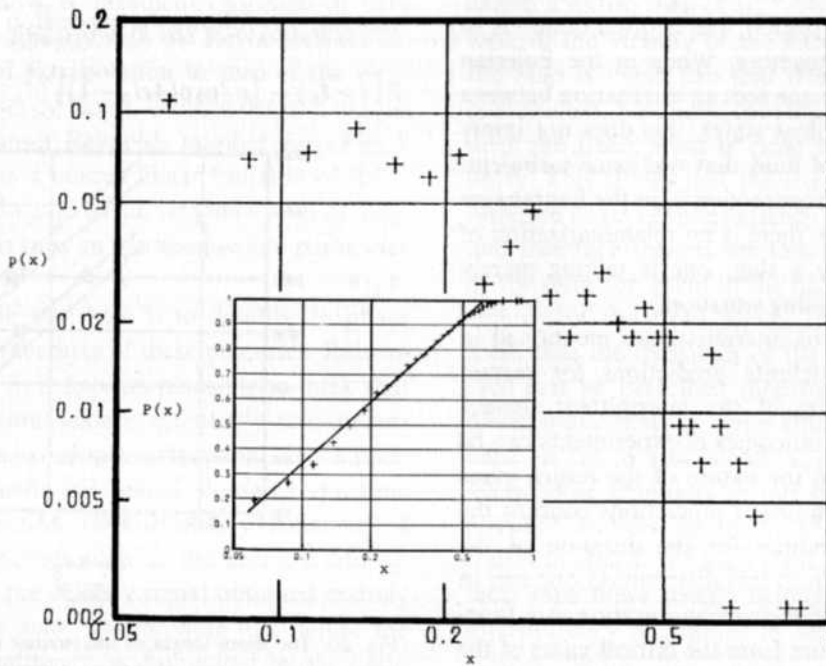


Fig. 12. The measured reinjection probability. For a discussion of how this was obtained, see Appendix. The inset which is a log-linear plot shows that the cumulative probability for the reinjection distance increases logarithmically with the reinjection distance from the fixed point (laminar state).

log-linear plot of the cumulative distribution shown in the inset is a less scattered comparison because of the averaging involved in the integrating process.

This nonlinearity associated with the reinjection probability adds an additional 'dimension' to the problem, and should be explicitly incorporated in any model of the problem. Using this empirically determined reinjection probability, it is easily shown that Type II intermittency also leads precisely to the expression (1) for the cumulative distribution of laminar lengths. This result, together with figs. 9 and 10, might be taken to indicate a closer connection with Type II. It is also worth recalling that the one-dimensional map from which fig. 3 was constructed was obtained after some simplification from Type II. Perhaps the connection is even closer if we realize that a suitably obtained Poincaré section of Type II intermittency is qualitatively similar to the measured velocity signals here (figs. 1 and 2).

Before closing this section, we note that, independent of the agreement between measurement in fig. 10 and the intermittency models, the almost exponential variation of the data (fig. 11) is pointing to some simple mechanism of slug generation (e.g., a Poisson process).

#### 4. The chaotic state

From traces of the type shown in fig. 1, we have constructed a composite velocity signal by stringing together all the turbulent patches; that is, by removing the laminar as well as the parts corresponding to the interface between the two states. For this composite signal, we have calculated the correlation dimension using the Grassberger-Procaccia [13] algorithm. Calculations show the scaling exponent of the correlation function is about 18. To the extent that one can trust calculations resulting in such large numbers and their interpretation, the dimension of the attractor is about 18. This relatively high dimension does not come as a surprise to us, because it is consistent

with our experience with most open flow systems [14]; for all Reynolds numbers except those very close to the onset of turbulence, low-dimensional attractors do not seem to exist. (The number of data points used in these calculations is not as large as is usually believed to be necessary for calculating dimensions of the order 18 reliably, but far fewer ( $\sim 3000$ ). We have however calculated the dimension from several independent patches of the composite signal each of which is about 3000 points long, and performed ensemble averaging over these segments. We have found on other occasions – to be described elsewhere – that this procedure gives stable numbers. In any case, the issue here is not whether the dimension is 18.1 or 18.2, but whether it is 2, 6 or 18. The safest conclusion to draw from here is that the dimension is *not* small, of the order of 5, say.)

We conclude that pipe flow transition exhibits partial similarities with known intermittency routes to chaos – especially with Type II – but it does not strictly belong to any of them, at least because of the preferential nonlinearity in the reinjection mechanism. Although the dynamics appears low-dimensional on the interface region, it is clearly not so elsewhere. For this reason, it is helpful to examine the problem from another point of view.

#### 5. Analogy with phase transitions

As we already mentioned in section 2, the change of state from a laminar to a turbulent one occurs in pipe flows essentially discontinuously at an onset Reynolds number  $Re_0$ , and at any instant at a spatial location it is easy to say to which of the two states the fluid flow belongs. Above this onset Reynolds number the laminar and turbulent phases can be thought of as coexisting, with the fraction of time the flow is turbulent increasing monotonically with the Reynolds number; in the intermittent regime all the mean flow properties (such as the pressure drop in the pipe) change continuously from the laminar values to the fully turbulent values. Following the lead of Dhawan and



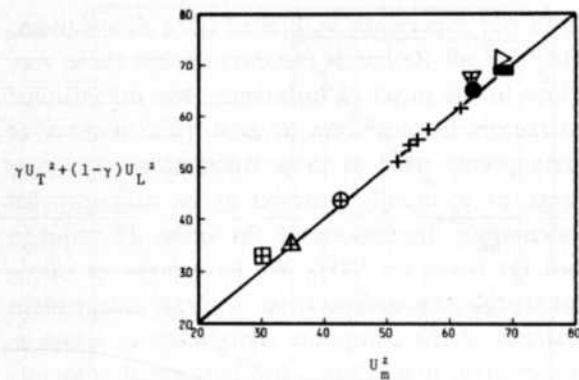


Fig. 13. The measured time-mean square of the streamwise velocity compared with the sum of  $\gamma$  times the turbulent value (for that Reynolds number) and  $(1-\gamma)$  times the laminar value.  $\square$ ,  $Re = 4214$ ,  $\gamma = 0.87$ ;  $\blacktriangle$ , 4006, 0.72;  $\circ$ , 4110, 0.54;  $\nabla$ , 4774, 0.32;  $\bullet$ , 4882, 0.43;  $\blacktriangleright$ , 4451, 0.05;  $\blacksquare$ , 4559, 0.16. For calibration purposes, completely laminar values (+) have also been plotted. We suspect that this agreement will not hold so well if the critical Reynolds number is large.

Narasimha [15] in boundary layers, we show in fig. 13 that, at any given Reynolds number during the intermittent transition, one can express to a good approximation some measured time average flow properties (such as the dynamic head on the pipe axis) as a linear combination of the laminar and fully turbulent properties appropriate to that Reynolds number. Noting that the intermittency factor itself appears to vary linearly with  $Re - Re_0$  (see fig. 6), it is clear that flow properties in the vicinity of  $Re_0$  can be expressed as linear combination of the laminar and turbulent ones, with  $\gamma$  replaced by  $(1 - Re/Re_0)$ .

The above description tempts us to explore possible connections with phase transitions. Since all phase transitions can be described in similar terms, the crucial step is to identify an order parameter, which is such that it takes on different values in coexisting phases, and jumps discontinuously in the course of the phase transition; the magnitude of the jump is zero at the critical point. As an example, the order parameter in the gas-liquid phase transition is the difference between the actual density and the density at the critical point.

While many details are not clear and the analogy has not yet been pushed to its logical conclusion, one can identify an order parameter with the (normalized) difference speed  $\Delta U$  between  $U_{le}$ , the leading edge speed, and  $U_{te}$ , the trailing edge speed, of the slug or the spot. Fig. 14 shows that in all flows in which  $\Delta U$  has been measured to-date, the relationship

$$\Delta U \approx \alpha (Re - Re_c)^{1/2} \quad (2)$$

holds quite well in a nontrivial neighbourhood of  $Re_c$ , where  $Re_c$  is a 'critical' Reynolds number akin to the critical temperature in the gas-liquid phase transition. It is surprising that this should be so, considering that the four flows studied in fig. 14 are quite different in detail; they range, on one extreme, from spots which grow in all directions to slugs on the other extreme which are constrained in all but the axial direction.\* We also find it very interesting that the 'critical exponent' must take on the classical value of 0.5.

For the boundary layer,  $Re_c = 200$  according to fig. 14. This suggests that attempts to create sustained spots below  $Re_c$  must necessarily fail because, interpreted literally, fig. 14 suggests that the trailing edge should then travel faster than the leading edge. If this does occur we would have on our hand a case of relaminarization but, in reality, spot-like structures below  $Re_c$  will break up and decay. To our knowledge, detailed tests relating to this issue have not been made. In the literature on spots, we have found no documentation of spots

\*For all cases but that involving boundary layer spots, a linear fit between  $\Delta U$  and  $(Re - Re_c)$  is not unthinkable, but the fit (2) is a bit better when  $Re - Re_c$  is not too large. Also, we believe that the departure from (2) in fig. 14d, for example, is largely due to the fact that the flows were generally set up in pipes which were not long enough for the fully developed parabolic state to emerge. This means that the leading edge speed of the slug, which is essentially equal to the largest speed anywhere in the flow field, cannot be as high as it would be if one had a parabolic distribution of velocity ahead of the slug. Data from Alavyoon et al. [31] in plane Poiseuille flow became available too late for inclusion here, but they follow the equation  $\Delta U^2 = 0.727 \times 10^{-4} (Re - 800)$ ; the fit appears as unambiguous as for the boundary layer data of fig. 14(a).

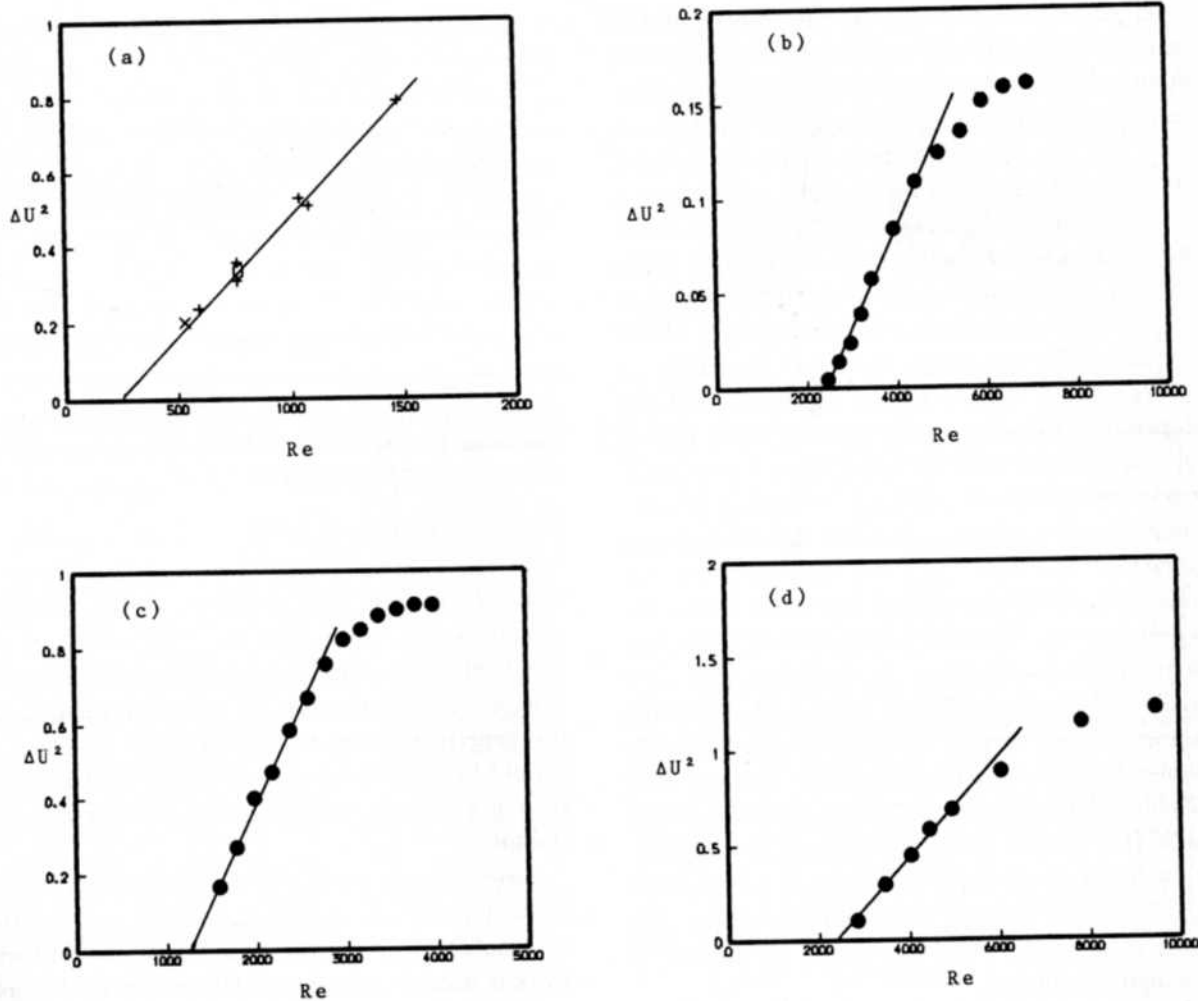


Fig. 14. The difference between the propagation speeds of the leading and trailing edges of the slug as a function of the flow Reynolds number. (a) Spots in a two-dimensional boundary layer:  $Re$  is based on the freestream speed  $U_\infty$  and the displacement thickness. The normalizing speed for  $\Delta U$  is  $U_\infty$ .  $\times$ , Cantwell et al. [24];  $+$ , Wygnanski [18];  $\circ$ , Zilberman (see Wygnanski [18]). Since Reynolds number (no matter how defined) increases with streamwise distance in boundary layers those used here are the Reynolds numbers at which spots were created. —,  $3.65 \times 10^{-4}$  ( $Re = 200$ ). (b) Spots on an axisymmetric body. When the spots grow to sufficient sizes, they wrap around the body. Data from Rao [25]. —,  $5.6 \times 10^{-5}$  ( $Re = 2500$ ); Reynolds numbers are based on the boundary layer thickness. (c) Transitional structure in a rectangular pipe, aspect ratio 4.0; Reynolds numbers are based on the hydraulic radius. Data from Sherlin [26]. —,  $5.0 \times 10^{-4}$  ( $Re = 1240$ ); (d) Slugs in circular pipes. Data from one experimental run, present. —,  $2.56 \times 10^{-4}$  ( $Re = 2350$ ). Similar data have been obtained by Lindgren [27], Pantulu [28], Coles [29] and Wygnanski & Champagne [10].

generated below  $Re_c$ , the lowest such Reynolds number being around 210 due to Elder [16]. Although Elder did not make specific claims that spot generation attempts below  $Re_c$  were unsuccessful, the absence of any documentation contrary to our conclusion must be deemed to be significant. Similarly for pipes attempts to gener-

ate slugs below a Reynolds number of about 2400 are known to be unsuccessful.

We draw attention to two minor matters. First, the constant  $\alpha$  in each of the four flows is of the same order of magnitude when proper account is taken of the differences in the definitions of the Reynolds number and different normalizing speeds

used in  $\Delta U$ . Second, the rate of spread of the spanwise extent of the spots (the 'width') in constant pressure boundary layers is only a weakly increasing function of the Reynolds number. (It goes up by about 20% in a Reynolds number differing by a factor of about 3 in the experiments of Schubauer and Klebanoff [17], and by about half as much in a similar Reynolds number range in Wygnanski's [18] experiments.) Not enough data exist on the Reynolds number dependence of the growth of the spot height normal to the plane.

We should remark on the likelihood that the expression (2) may signify nothing more than a characteristic shared by propagation fronts in diverse circumstances, where a power-law usually describes the relation between the propagation speed of the front and the distance from the critical value of the control parameter. Some examples are the speed of propagation of the turbulence front produced by an oscillating grid in a tank of still water [32], the speed with which the upper (lower) surface vortex propagates into the lower (upper) vortex in a short aspect ratio ( $= 1.25$ ) Taylor-Couette apparatus housing only two vortex rolls [19, 20], the propagation speed of solidification fronts in dendritic growths [21] the speed of the so-called 'directed lattice animals' in percolation theory [22], etc. Even this is an interesting enough conclusion.

## 6. Discussion and conclusions

The behaviors described so far are not strictly applicable for large  $Re - Re_0$ . For example, as the intermittency factor approaches unity, increasingly larger departures occur between expression (1) and the measured probability distribution of laminar regions; similarly (2) is violated for large values of  $Re - Re_c$ . This in itself is no serious detraction, since all 'universality theories' aim to explain only the region immediately after the onset of intermittency. We want to emphasize one further point. For certain combinations of experimental conditions which are poorly understood,

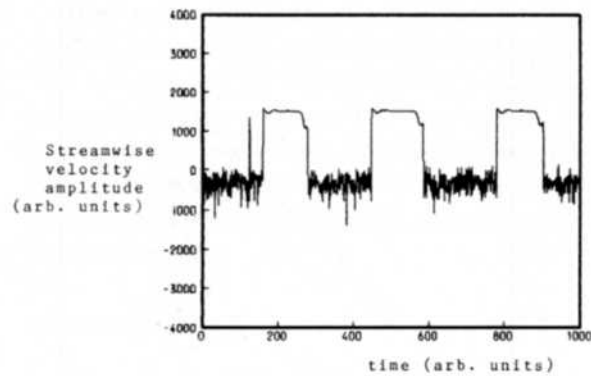


Fig. 15. A velocity signal measured on the pipe centerline for conditions different from those of fig. 1. Unfortunately, the differences are not documentable in detail.

the alternation between the two states occurs regularly (fig. 15); the distribution of the laminar intervals in this case obviously peaks sharply around some value. This last fact serves as a reminder of the complexity of the process involved. Further, even restricting to what one might call the generic features of this transition process, it should be clear from section 4 that the dynamics does not entirely reside on a low-dimensional attractor.

Nevertheless, several common features exist between pipe flow transition and purely mathematical models like one-dimensional maps; further work is needed to be completely certain of this, as well as about possible analogies to physical processes like phase transitions. In any case, a more realistic model than the existing ones need to be invented to duplicate the observed facts in detail. We think that a suitable modification of the Rössler equations [23] may serve this end to some extent.

One of our contentions has been that transition in the examples studied here occurs intermittently between a *steady state* and a chaotic one. In particular, the laminar regions do not correspond to any periodic states, as is especially clear from fig. 2. It may, however, be of interest to recall that in the experiments of Schubauer and Klebanoff [17] the first-born spots are generally accompanied by an undulating (*not steady*) laminar state, but

once a newly born spot sweeps by the fluid, it produces a 'calming effect' that subsequently eliminates the undulations in the laminar state.

Finally, we must remark that the standing of the conclusions of this paper is only preliminary unless substantiated by measurements in extremely long pipes (length to diameter ratios well in excess of  $10^4$ ) in which the mass flux (instead of the usual pressure drop) is held constant. We believe that such an experimental effort is worthwhile. Pipe flows are fascinating also because they provide counter examples to the commonly observed bifurcations, as well to many beliefs usually held, in dynamical systems. For example, 'noise-free' pipe flows are strictly stable at all Reynolds numbers, which clearly requires the presence of *sustained* noise for initiating transition; it is therefore not clear to what extent the intermittency statistics reflect the statistical properties of the noise itself. In contrast to transition to turbulence in convection problems (for example), much less appears to be known about the type of 'metastable' transition observed in pipe flows. The purpose of this paper is more than adequately served if it brings these problems to the attention of a wider audience than that customarily involved in them.

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#### Appendix

Transition from the turbulent to laminar state is very sharp as can be seen in fig. 1. (It is in fact

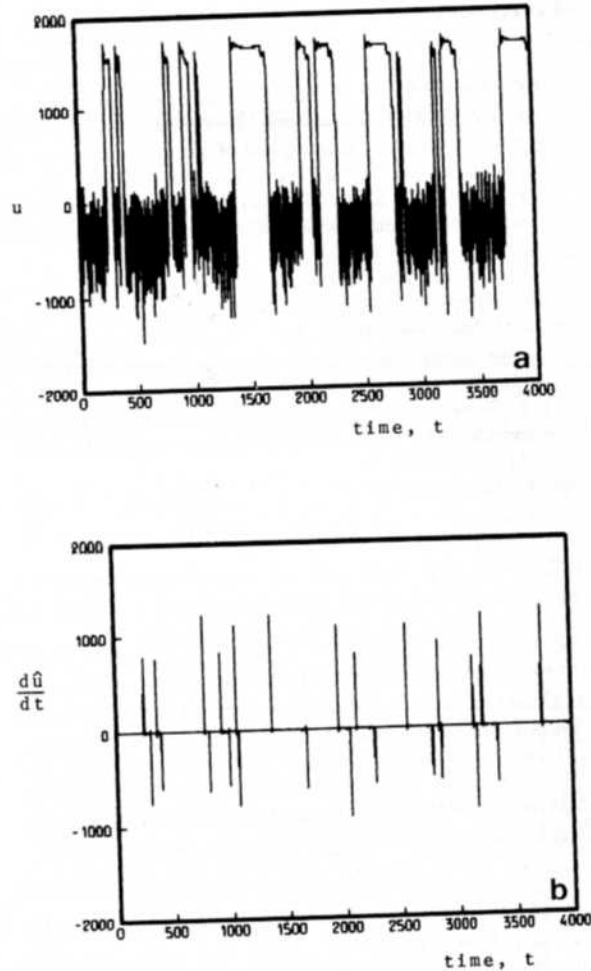


Fig. 16. The upper trace (a) is a velocity trace which, when modified as described in the text and differentiated, yields the lower trace (b).

sharper than the transition from the laminar to turbulent one.) It thus seems reasonable to associate reinjection with sharp velocity gradients. Hence a numerical differentiation was performed on the time trace, after substituting the turbulent state by a constant, say, 500 on the ordinate of fig. 16a. This modified signal  $u$ , when differentiated, looks as in fig. 16b. The reinjection point is then identified as the distance from the reference laminar state where the largest velocity gradient occurs. (Other plausible definitions yield much the same result.) Fig. 12 was obtained after rescaling the distance.

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