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ON THE VARIATION OF THE TURBULENT PRANDTL NUMBER IN SHEAR FLOWS

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ABSTRACT

Although the rapid distortion formulation proposed by Townsend (1976) qualitatively describes the variation in the turbulent Prandtl number Pr_t among different shear flows, direct measurements of Pr_t indicate that the experimental variation of Pr_t is underestimated by his formulation.

In turbulent flow calculation methods, one often uses the turbulent Prandtl number Pr_t, which is the ratio of turbulent momentum diffusivity to turbulent thermal diffusivity. Strictly speaking, the concept of turbulent diffusivities assumes the applicability of gradient transport models. Although it is well-known that gradient transport models do not adequately represent the physical transport processes in turbulent shear flows, it has been demonstrated [1] that the flow symmetries, which force the location of zero stress to coincide with location of mean velocity or concentration gradient, render the concept operationally useful. It is in this spirit that we discuss the turbulent Prandtl number here.

In flows with variations in mean velocity and temperature confined primarily to the direction y, Pr_t is defined, in the usual notation, as

$$Pr_t = \frac{-\overline{uv}/(\partial U/\partial y)}{-\overline{v\theta}/(\partial T/\partial y)} \quad . \tag{1}$$

Few direct estimates of Pr_t using (1) have been made, but several estimates of Pr_t, which assume turbulent diffusivities compatible in an overall sense with measured mean velocity and temperature distributions, are available. These estimates indicate that Pr_t varies in different shear flows, being smaller in free shear flows (e.g. [2]) than in boundary layers or pipe flows. An explanation for this variation has been provided by Townsend [3], who used ideas of rapid distortion to show that the turbulent Prandtl number increases with increasing total strain. He also showed that the total strain in boundary layers is larger than in free shear flows, thus explaining in a qualitative manner the observed variations of Pr_t from one flow to another. To our knowledge, there has been no attempt to verify quantitatively Townsend's theoretical result, presumably because it is not clear how literally Townsend meant his own formulation. This note represents an attempt at such a verification using experimental data for Pr_t, deduced from (1) using measured fluxes. We consider in this note only those flows for which Pr_t as well as the total strain could be estimated with reasonable confidence.

Townsend hypothesised that the total strain is governed by a transport equation (see [3], p.214) whose solution would give the total strain. Townsend [3] evaluated an effective total strain α (Townsend's notation) for several standard flows. For two of the flows listed in Table I, the tabulated values of α are

TABLE I

Effective Total Strain, Stress-Intensity Ratio and Turbulent Prandtl Number in Different Shear Flows

Flow	α [3]	α_1	$\lvert\overline{uv}\rvert/\overline{q^2}$	Pr_t
Two-dimensional mixing layer [4]	$4^\dagger$	2.3	0.225	0.4
Two-dimensional jet [5]	6.1	4.3	0.165	0.65
Two-dimensional boundary layer [6]	10-15	5.2	0.150	0.95
Quasi-homogeneous shear flow with uniform mean temperature gradient [7]	---	5.6	0.140	1.1

†Not from [3], but our own estimate (see text).

those obtained by Townsend. Townsend obtained a value of 7.5 for the mixing layer but we estimate (using Townsend's method, see Section 6.8 of [3]) that a more appropriate value for the mixing layer of [3], which develops in the near region of a plane jet with a laminar boundary layer at the nozzle exit, and for which we present the Prandtl number data, is about 4.

If the rapid distortion hypothesis is applicable, structural parameters such as the stress-intensity ratio $|\overline{uv}|/\overline{q^2}$, where $\overline{q^2}$ is the total turbulent intensity, depend only on the total strain. Townsend [3] assumed that the evolution of the turbulence structure in two-dimensional shear flows is essentially due to the rapid distortion of the superposed mean shear, and computed $|\overline{uv}|/\overline{q^2}$ as a function of the total strain. The values of total strain α_1, which correspond to these values of $|\overline{uv}|/\overline{q^2}$, are shown in Table I. These values, estimated from the upper part of Fig. 3.15 of [3] (with the eddy viscosity N = 0), are consistently smaller than α. Although the ratio α_1/α is not constant, the variation of α_1 in different flows is similar to that of α. The difference in magnitude between α and α_1 apparently reflects (small) differences in the stress-intensity ratio for the same flow. For internal consistency, we have used α_1, as the appropriate measure of total strain, to compare the experimental variation of Pr_t with Townsend's theory. The comparison, shown in Fig. 1, indicates that the overall change in Pr_t predicted by the

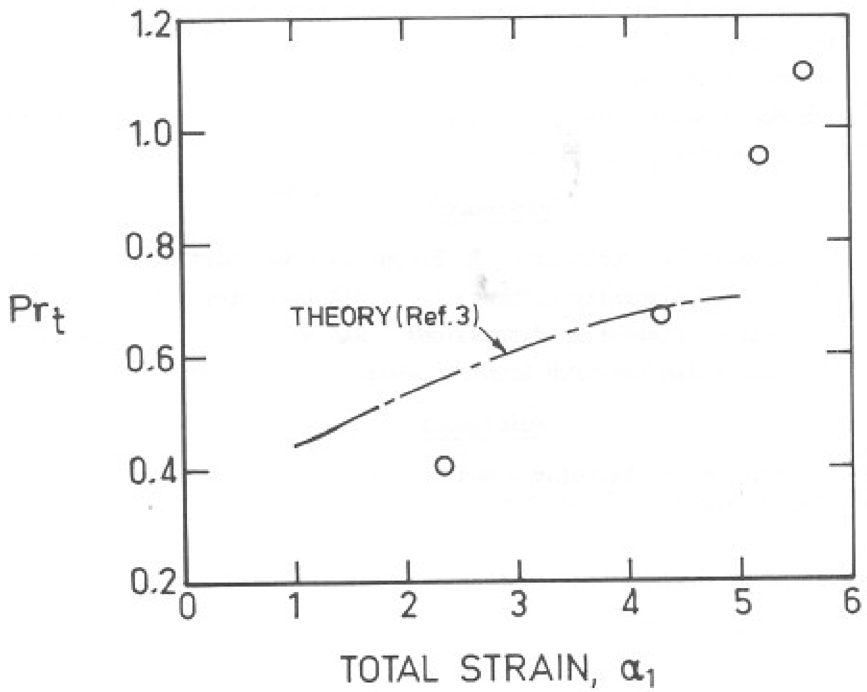

FIG. 1

Variation of Turbulent Prandtl Number as a Function of Total Strain
Theory [3]: —— - ——; Experiment: o (see Table I for further details)

theory underestimates the experimental variation, for the four flows considered in Table I. In particular, in the limit $\alpha_1 \to 1$, the theoretical value of Pr_t is about 0.4. Experimental values of Pr_t as small as 0.2 have been obtained [4] in a mixing layer.

It may be argued that the Pr_t-α_1 correlation shown by Fig. 1 may be fortuitous, firstly because rapid distortion is merely a limiting situation not strictly applicable to shear flows of the type considered here, and secondly because the actual values of Pr_t and total strain (no matter how one defines it) depend on the precise location in the flow. We should therefore re-emphasise the fact that rapid distortion arguments have been shown to be remarkably successful in describing the structural aspects of turbulent shear flows in a variety of contexts (e.g. [3] and [8]) and further that the values both of α_1 and Pr_t used in Fig. 1 are typical. For example, the Pr_t values correspond to a central "core region" of the flow where its variation with respect to y is not significant.

One further comment appears worth making. The data of Fig. 1 seem to suggest that Pr_t increases indefinitely and quite strongly with α_1; we suspect that this trend may partly be an artifact of the way we determined α_1. However, we know of no data that would refute such a variation. Further heat transfer experiments in homogeneous shear flows with larger values of total strain would be very useful from this point of view.

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