Some Studies of Non-Simple Pipe Flows

K.R. SREENIVASAN

SUMMARY A variety of phenomena occurs in pipe flows, especially if we stray away from straight circular pipes of uniform crosssection. This paper illustrates a few of the complexities arising from two relatively simple changes in geometry, namely, the sudden expansion and the coiling of a circular pipe.

1 INTRODUCTION

Pipe flows, far from being well-understood and dull, are very complex and highly interesting, and often show unexpected behaviours. Consider as an example a low speed, constant temperature, adiabatic flow in a long round pipe. The flow may be laminar or turbulent. Text books assert that, in a region sufficiently far away from the entrance, the static pressure varies linearly with the axial distance. Measurements, on the other hand, show that for air flow in a long, straight and thin tube (say, 6 mm diameter, 4000 diameters long), manometers located at equal intervals along the pipe length do not show equal readings; they increase with increasing downstream distance. (For the specific example chosen, and for a turbulent flow at a Reynolds number of the order of 10,000, the manometer reading over the last 100 diameters may be nearly twice as high as that, say, between 300 and 400 diameters.) Further, the wall shear stress is not simply proportional to the pressure drop.

This seemingly puzzling observation is not hard to understand, however. Without going into details (which can be worked out rather simply), we may note that, when the pipe is long and the axial pressure drop is substantial, the absolute pressure at the pipe entrance will have to be significantly higher than at the exit. In the present example, the pressure difference between the entrance and the exit will be of the order of one atmosphere. This gives rise to a substantial change in air density. With density a decreasing function of the axial distance, the flow will have to accelerate continuously, thus accounting for the observed behaviours. Thus, the classical notion of a linear pressure drop in a long pipe is exact (for gases) only in the limit of negligible pressure drop!

This is but one example of unsuspected behaviour. In the remainder of this paper, we shall discuss some intriguing phenomena arising from two simple changes in pipe geometry, namely, the sudden expansion and the coiling of a circular pipe. We shall not dwell at all on the complexities associated with flow of non-Newtonian fluids.

2 SUDDENLY EXPANDING PIPES

Consider a sudden expansion in a circular pipe shown in figure 1. Different phenomena occur in different ranges of parameter space, where the chief governing parameters are the Reynolds number Re, the axial distance x and the conditions upstream of the expansion. Here, all Reynolds numbers, unless specified otherwise, will be based on the diameter D of the downstream section and the bulk average velocity there. The origin for the downstream distance will be at the expansion itself. All results in this section refer to a diameter ratio of 2. Exceptions will be noted.

K R Sreenivasan is with the Mason Laboratory, Yale University, New Haven, Ct. 06520, U.S.A.
This paper was originally presented as one of nine keynote addresses at the Eighth Australasian Fluid Mechanics Conference held at the University of Newcastle, NSW 2308, 28 November - 2 December 1983. (Paper M1227 received 8 November 1983).

2.1 The Oscillatory Flow Regime

2.1a The Phenomenon: A hot wire located on the centreline of the pipe some distance downstream of the expansion will register, in a certain range of Re and for sufficiently smooth upstream conditions, oscillations of the type shown in figure 2, with amplitudes typically comparable to the average velocity in the downstream section. These oscillations are remarkable for their regularity and general repeatability (provided some care is taken, see below).

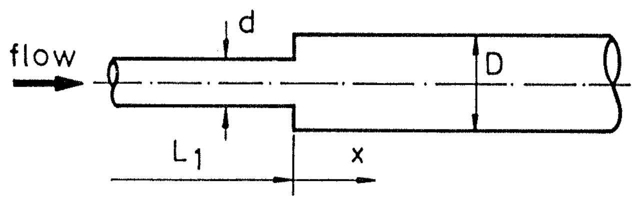

Figure 1 Schematic experimental configuration

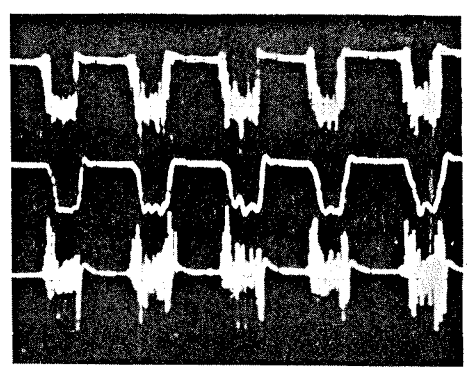

Figure 2 Oscillations seen by a hot-wire located on the pipe axis at x/D = 11. Re = 750, d = 0.635 cm, L_1 = 425 d. The uppermost trace is the unfiltered signal, the mid-trace is low-pass filtered below 10 Hz, the lowermost trace being high-pass filtered above 10 Hz. Most of the fluctuations seen in the last trace are below 500 Hz. Time scale: from left to right of figure, 5 sec.

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

These oscillations have several interesting properties. First, they appear when Re reaches a value of the order of 750, with the upper Reynolds number limit depending strongly on the degree of smoothness of the flow upstream of the expansion. If the entrance conditions to the upstream pipe are sufficiently smooth - say, in a qualitative way, smooth enough for the laminar-turbulent transition there to be delayed until an upst eam pipe Reynolds number of the order of 7500 is reached - the oscillatory phenomenon seen in figure 2 persists until an Re of around 1500. For less smooth conditions, the Reynolds number window shrinks, and the oscillations may diaappear altogether for certain conditions. In fact, if small levels of disturbance are artificially created just upstream of the expansion, or if the tube is squeezed hard asymmetrically at the expansion, the oscillations are disturbed rather strongly. They can even be controlled at will: for the conditions of figure 2, inserting a 0.24 mm diameter needle along a diameter through a hole carefully drilled just upstream of the expansion destroys the oscillations completely; removing the needle and resealing the hole restores them exactly. (The Reynolds number based on the maximum velocity in the upstream section and the needle diameter is approximately 50. The vortex shedding behind this needle, which we did indeed observe, perhaps creates enough asymmetry in the flow to prevent the oscillations from being formed. A slightly thinner needle, say, of 0.17 mm diameter, does not affect the oscillations, presumably because,its Reynolds number of 35 being lower than the critical value of about 40 (Kovaˍznay 1949), no vortex shedding appears.) Some further observations on this flow can be found in Sreenivasan & Strykowsky (1983a).

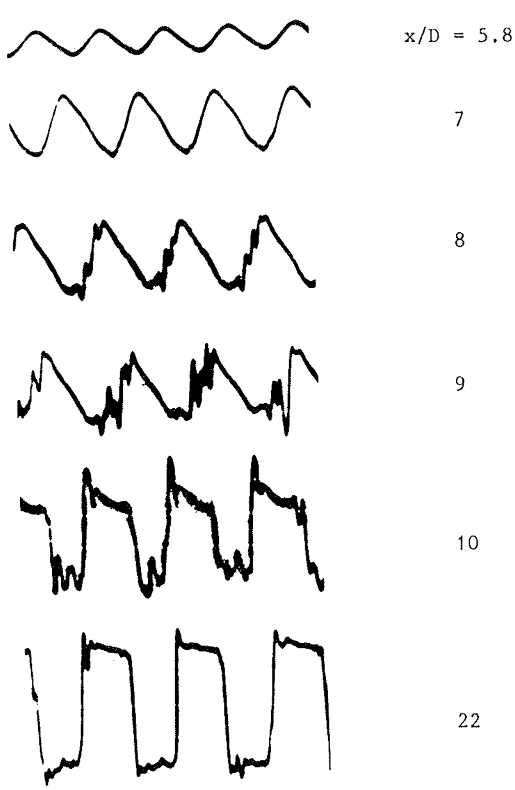

Figure 3 Development and growth of oscillations along pipe axis downstream of the expansion. Signals low-pass filtered to 10 Hz. Unfiltered signals at x/D of 5.8 and 7 are no different from the filtered ones; at other x/D, signals do develop an increasingly higher frequency content. Time scale: from left to right of figure, 4 sec.

2.1b In search of an explanation: How do these oscillations arise and what physical phenomenon do they represent? A partial answer can be seen from figure 3 which records the development and growth of oscillations along the pipe axis. It is immediately clear that they do not represent oscillations in mass flux (for, if they did, oscillations should have been seen with nearly equal intensity at all x/D) but, rather, may be representative of the instability developing downstream of the expansion.

For further clarification, we set up a simple flow visualization experiment in water. An initial problem was that any dye-introducing device placed upstream of the expansion would produce enough disturbance to destroy the flow oscillations. However, a dye streak introduced at the inlet to the upstream pipe itself (much as in the original experiments of Reynolds,1883) served our purpose quite well. The large area ratio of the contraction (≃150) damped out the disturbances produced by the dye-injecting needle to sufficiently small values so as not to be disruptive to the process that resulted in the oscillations in the first place.

We may summarize our flow visualization results as follows. The dye streak downstream of the expansion would remain straight and smooth for x/D of the order of 5, apparently unaffected by the expansion. Thereafter, depending on the precise value of the Reynolds number (as long as it exceeded a 'critical' value of about 750) it would develop rapidly growing oscillations (see figure 4a and compare it with figure 3), and would abruptly break down at some point; when this break-down occurred, the dye filled the entire pipe crosssection downstream, suggesting that the break-down and reattachment of the oncoming flow occur essentially simultaneously. Just as abruptly, however,

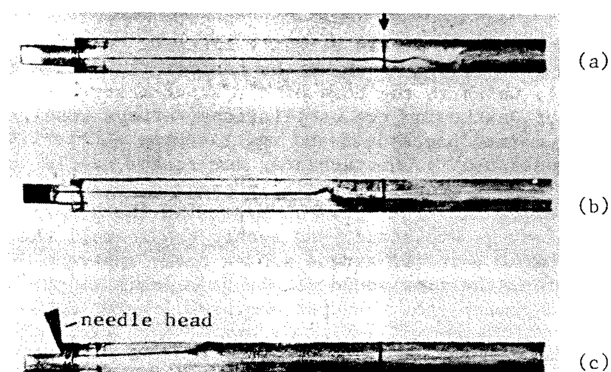

Figure 4 Flow visualization results for Re = 800. In (a), the break-down of the oncoming dye-streak occurs downstream of the mark indicated by the arrow, while in (b), this break-down occurs upstream of the mark. In (c), it is seen that the needle placed upstream of the expansion anchors the break-down point.

the reattachment would move back to a different point along the pipe, only to move forward to its original position later. This back and forth movement occurs essentially periodically, so that if one concentrated at a fixed observation station along the pipe axis (such as the mark in figures 4), one would alternately see an unruffled dye-streak or a situation in which the mixed-up dye filled the entire crosssection. An unruffled dye-streak at the observation station implies a velocity there that is characteristic of the jet-like oncoming flow from the upstream smaller pipe whereas, once the reattachment occurred, the flow would fill up the entire pipe, thus reducing the average velocity. This is essentially what makes a hot-wire see (as in figure 2) two different levels of velocity with periodic alternation between them. In fact, the upper level of oscillations in figure 2 corresponds roughly to the

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

centre-line velocity in the smaller pipe, while the lower level approximately to the average velocity that would result if the flow coming out of the smaller pipe filled the entire downstream pipe uniformly. Further, it may be seen that (cf. the uppermost trace of figure 2) the upper velocity level is essentially laminar-like, while the lower one is turbulent-looking, reflecting the fact that the lower level in the oscillations of figure 2 represents a turbulent situation downstream of reattachment.

Why does the reattachment point move back and forth so regularly? The answer lies probably in the complex interaction between the stability of the velocity field downstream of the expansion and the oscillatory pressure field further downstream. At this point, our knowledge of the process is meagre, but a possible (necessarily speculative) explanation follows.

The velocity distribution downstream of the expansion would be nearly parabolic in the core, but surrounded by a region of reverse flow. The resulting complex velocity distribution has several inflexion points, and is obviously prone to instabilities which are quite possibly excited in phase by the downstream pressure field, thus providing the mechanism for the regularity of the oscillations. These instabilities grow and eventually lead to the break-down of the flow at some point downstream. When this occurs, the turbulence that develops and the consequently increased pressure drop would shift the reattachment point upstream. One may surmise that this upstream shift of the reattachment point would restore the stability of the flow by altering the velocity distribution just enough, so that the reattachment point would now move downstream to its original position. This self-perpetuating act repeats itsef indefinitely.

Inserting a small needle slightly upstream of the expansion (see figure 4c where the head of the needle can be seen), which in the case of air experiments had the effect of destroying the oscillations, always resulted in a premature break-down and reattachment of the flow at around x/D of 4. Disturbances due to the needle upstream, or any other artificially created disturbance, would hasten the break-down by bypassing the normal oscillatory growth stage, and anchor the so well the reattachment point at around x/D =4 that, upstream of this point, the flow would simply be a laminar 'jet' of fluid coming from the upstream pipe — here, a hot-wire on the pipe axis would continuously record very nearly the peak velocity in the upstream pipe — whereas downstream of this point, it would simply record continuously the lower velocity corresponding approximately to that after reattachment. This is essentially why no oscillations were seen by the hot-wire.

2.2 The Puff Region

Further downstream of the expansion, the smoothness or otherwise of the flow in the upstream pipe becomes irrelevant, and the Reynolds number and the downstream distance become the only relevant parameters. The downstream evolution of the flow for a fixed Reynolds number of 2200 is shown in figure 5. The flow is fully turbulent at x/D of 24, where the uppermost trace was obtained. With increasing distance, the signal is seen to build up in isolated regions while, at the same time, the general level of turbulence slowly diminishes elsewhere (see the middle two traces). Eventually, one has (as in the lowest trace of figure 5) nearly perfect laminar regions interspersed with characteristic signatures of structures known as puffs (Wygnanski & Champagne, 1973). Figure 6 presents the complementary information, namely, the flow evolution with increasing Reynolds number at a fixed x/D of 144. Below an Re of

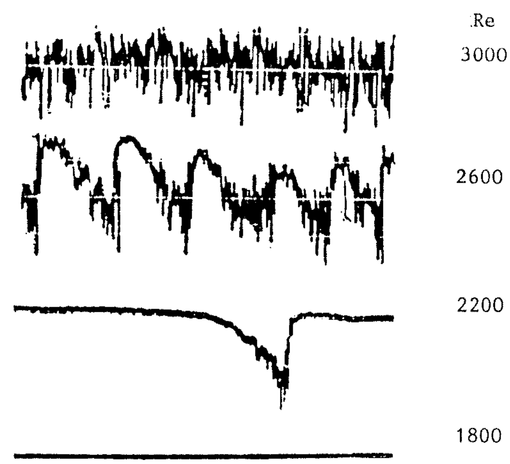

Figure 6 Oscillograms on pipe center-line. x/D = 144

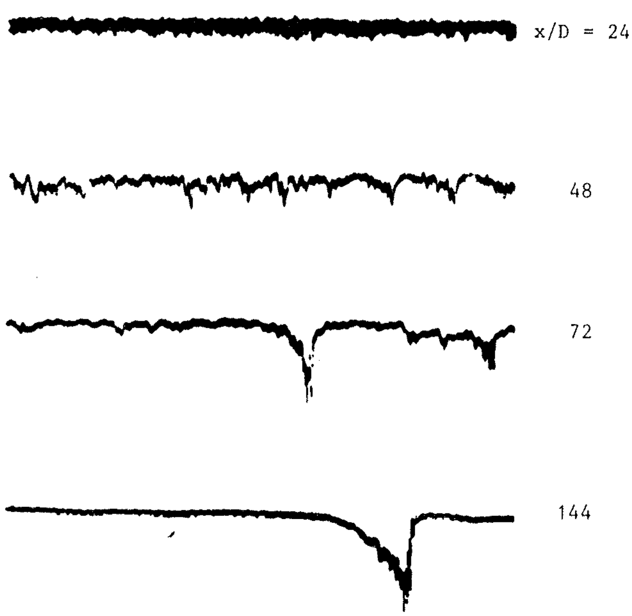

Figure 5 Oscillograms along the centre-line downstream of the expansion. Re = 2200.

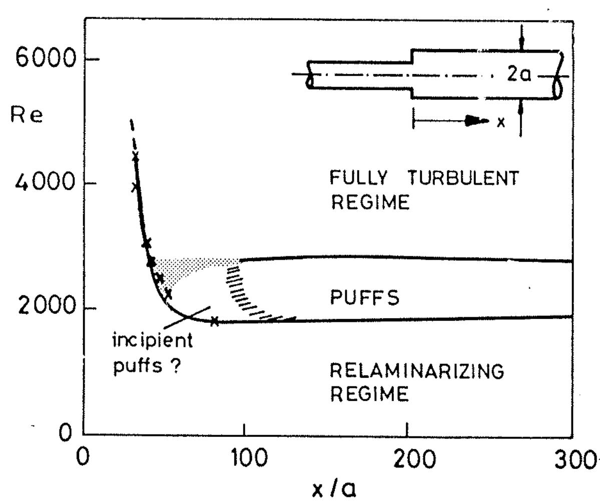

Figure 7 Boundaries between the turbulent, puff and relaminarizing regimes downstream of a sudden expansion.

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

about 2000, the flow is entirely laminar; considering that the expansion renders the flow downstream of it turbulent irrespective of whether the oncoming flow is turbulent or not, the above observation simply means that the flow is completely relaminarized for Re ≤2000 (see section 2.3). With increasing Reynolds numbers, puffs begin to appear more and more frequently, until eventually (for all practical purposes, beyond an Re of 2700) a fully turbulent flow results from the interaction and conglomeration of puffs.

By obtaining similar traces at different x/D, one can construct a map marking boundaries between the turbulent, puff and relaminarizing regimes (see figure 7). Similar maps have been constructed before for other cases by Wygnanski & Champagne (1973) and Champagne & Helland (1978). The map is self-explanatory in the region x/D ≧ 100. In the region marked 'incipient puffs?' one cannot see a distinct puff-like structure, but can recognise something similar (see the second trace from above in figure 5) which will evolve into puffs further downstream. Turbulence level downstream of the expansion seems always to decrease for a certain initial distance; whether it continues to decay or not depends on the Reynolds number. Crosses in the figure indicate the x/D positions where the minimum in the mean-square level of turbulence occurs for a given Re. The line joining the crosses thus demarcates the region of decaying turbulence to its left from that of increasing and stable levels of turbulence to its right.

It is known that puffs once formed may either merge with each other or split to form more than one (Wygnanski et al., 1975), depending on the Reynolds number. An equilibrium puff is one that does neither, and sustains itself indefinitely; it occurs around an Re of 2200. In structure, an equilibrium puff consists probably of several toroidal vortices (Rubin et al., 1980), and its occurrence follows a Poisson distribution rather well. Recently, Bandyopadhyay & Hussain (1983) seem to have identified the regeneration mechanism that allows the equilibrium puff to survive indefinitely in spite of the continually occurring turbulent energy dissipation. It appears that when the laminar flow from upstream of a puff enters it — figures 5 and 6 show that the puffs are relatively slow moving and have sharp upstream interface — rather well-organized vorticity is generated (much as in an axisymmetric jet) which breaks up into small-scale turbulence subsequently. In the incipient stage, one surmises that this same process of regeneration must gradually start to occur after being initiated via statistical fluctuations.

2.3 Relaminarizing Regime

For Re ≦ 2000, the measured mean velocity profiles acquire increasingly laminar-like shape with increasing downstream distance. One expects that for large x/D, the theoretical Poiseuille flow will be established asymptotically. At any station downstream of the expansion where the measured velocity distribution $u(x,r)$ — r being the radial distance from the centre-line — has not quite reached its asymptotic shape, one can write:

$$\begin{aligned} u(x,r) &= u_0(r) + \varepsilon u_1(x,r) + O(\varepsilon^2), \\ v(x,r) &= \varepsilon v_1(x,r) + O(\varepsilon^2), \\ p(x,r) &= p_0(x) + \varepsilon p_1(x,r) + O(\varepsilon^2), \end{aligned} \qquad (1)$$

where $u_0(r)$ and $p_0(x)$ are the asymptotic velocity and pressure distributions, and u_1, v_1 and p_1 are the departures of the axial and normal velocity components, respectively, and of the pressure, from the asymptotic distributions. (Note: $v_0 \equiv 0$.) We have retained only ε-order terms, which implies that (1) can be expected to hold only sufficiently far from the pipe expansion where departures from the asymptotic state are small. The parameter ε is the inverse of the characteristic Reynolds number based on the thickness of the inner laminar layer developing (in some asymptotic sense) from the expansion itself, and the average velocity in the pipe. It is these layers that grow and eventually merge to form the asymptotic shape of the velocity profile (Narasimha & Sreenivasan, 1979), the process being much like that in the entrance region of a straight pipe (Goldstein, 1938).

For the fluctuations too, we may write:

$$u' = \varepsilon u_1' + O(\varepsilon^2); \quad v' = \varepsilon v_1' + O(\varepsilon^2), \qquad (2)$$

the expectation being that in the asymptotic state the fluctuations are zero. We may now write the Reynolds shear stress τ as

$$\tau = -\overline{u'v'} = c_\tau(\tilde{u}\tilde{v}) = c_\tau\left[O(\varepsilon^2)\right], \qquad (3)$$

where c_τ is the correlation coefficient, the tilde denote root-mean-square values, and the last step in (3) follows from (2). Measurements show that during relaminarization of this type, not only do the fluctuations decay with distance but also become decorrelated (see, for example, Badrinarayanan, 1968); that is, c_τ tends to zero as $x/D \to \infty$ or $\varepsilon \to 0$. It is thus reasonable to take $c_\tau = o(1)$, so that, from (3), we may write

$$\tau = o(\varepsilon^2), \qquad (4)$$

or, that τ is higher order in smallness than ε^2. Using (1),(2) and (4) in the Reynolds averaged continuity and momentum equations, we obtain, to O(1):

$$\mu\left[\frac{d^2u_0}{dr^2} + \frac{1}{r}\frac{du_0}{dr}\right] = \frac{dp_0}{dx}, \quad u_0(a) = 0$$

whose solution, as expected, is the classical parabolic distribution. To order ε, we get:

$$\frac{1-\eta}{\eta}\frac{\partial}{\partial\xi}\left(\eta\frac{\partial u_1}{\partial\eta}\right) = 2\frac{\partial^2}{\partial\eta^2}\left(\eta\frac{\partial u_1}{\partial\eta}\right), \qquad (5)$$

where $\eta = r^2/a^2$, and $\xi = xa/\mathrm{Re}$; $\int_0^1 u_1 d\eta = 0$. Putting $\eta(\partial u_1/\partial\eta) = C\exp(-2\lambda\xi)\,\phi(\eta)$, we can write (5) as:

$$\phi'' + \lambda(1-\eta/\eta)\phi = 0, \qquad (6)$$

with $\phi(0) = 0$ and $\int^1 \phi\, d\eta = 0$. Our interest is in the first odd eigenfunction and the corresponding eigenvalue for (6).

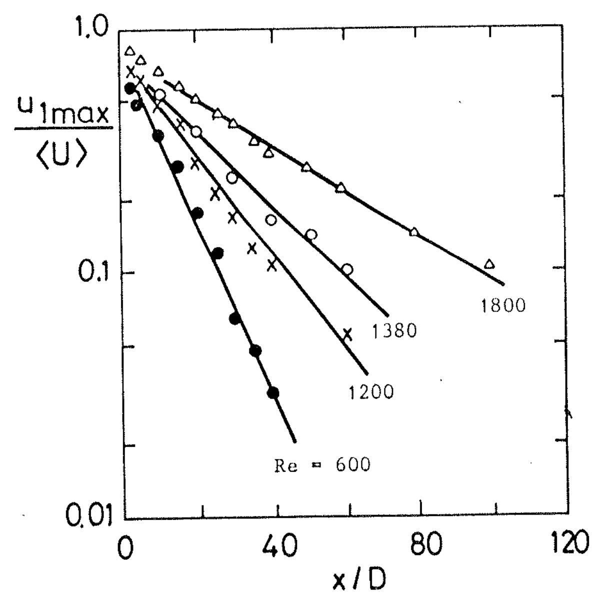

Figure 8 Exponential approach to the asymptotic state. ●,×,▲: sudden expansion data from Sibulkin (1962); D/d = 4.5. O, gradual expansion, Laufer (1962).

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

There are some nice consequences of this analysis. First, a characteristic value of u_1, say u_{1max} which is the centre-line value of u_1, should decay exponentially with x. Figure 8 shows this to be true to a good approximation. Interestingly, the exponential decay which, by virtue of having retained only two terms in (1), could a priori have been expected to hold only for large x/D, holds true quite close to the expansion, especially for low Reynolds numbers. Second, the rate at which u_{1max} decays is inversely proportional to the Reynolds number. (That is, if $u_{1max} \sim \exp(-mx/a)$, the product mRe should be a constant independent of Re.) Figure 9 shows that this is true not only for the sudden expansion case but also for gradual expansions and bifurcating pipes. Finally, figure 10 shows that the experimentally determined distribution of u_1/u_{1max} agrees quite closely with the approximate eigenfunction for (6). Again, the theory holds for x/D as low as 8.

3 FLOW IN HELICALLY COILED PIPES

Flow in curved pipes — which encompasses the topic under discussion— has been a subject of numerous investigations, but it appears that even some of the gross phenomena have not been understood. Our intention here is not to discuss curved pipe flows exhaustively — a recent survey by Berger et al.(1983) does this very well — but to point out a few interesting results.

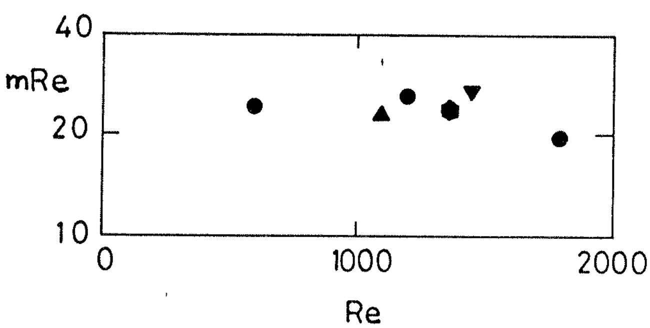

Figure 9 The product mRe in relaminarizing pipe flows. Sudden expansion: ●, Sibulkin, ▲, present. ⬢, gradual expansion, Laufer. ▼, branching pipe, Lynn & Sreenivasan (1982).

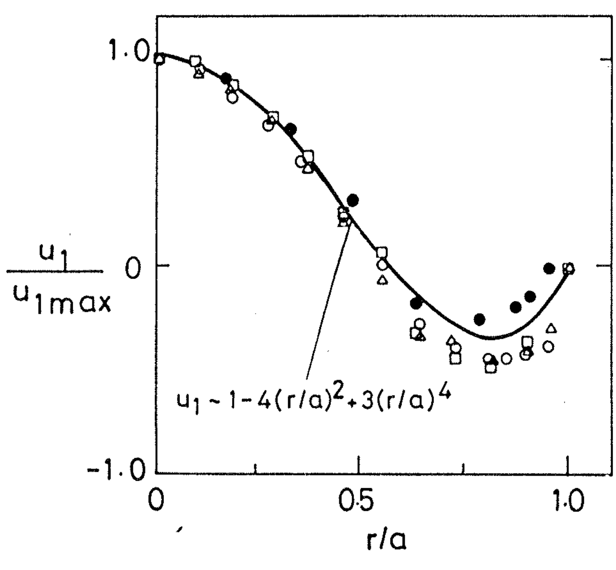

Figure 10 A comparison with theory of the departure of the measured distribution from the asymptotic parabolic profile. Sibulkin: O, x/D = 8; Δ,17; □,35. Present: ●, x/D = 17.5.

Consider a long straight section of a smooth pipe followed by a coiled section; following the coil is another long straight section (see figure 11). Several phenomena we want to discuss are related to the question of transitional Reynolds numbers in this set-up, and we shall mention this first.

Since we made no special attempt to keep the flow in upstream straight section unusually disturbance-free, the onset of transition occurred there at a Reynolds number of about 2300. Typically, this manifests itself in the form of puffs, and transition proceeds with increasing Reynolds numbers much as in figure 6. As determined from intermittency measurements, transition to turbulence is complete around an Re of 3200. This holds up to the entrance to the coil. Once inside the coil, the nature of transition depends, even at a fixed axial distance and for a given radius ratio (that is, the ratio of the radius of the pipe to the radius of curvature of the coil), strongly on the precise location in the pipe. It is not easy to determine, or even define convincingly, the onset of transition to turbulence (although a preliminary attempt has been made by Sreenivasan & Strykowsky, 1983b), but two limiting situations can be defined relatively unambiguously: the upper Reynolds number limit for the existence of a steady laminar flow ('steady laminar limit') and the lower Reynolds number limit at which the flow is turbulent everywhere in a given crosssection of the curved pipe ('turbulent limit'). Notice that in the special case of the straight pipe the steady laminar limit coincides with the onset of transition to turbulence; of course, the turbulent limit retains its meaning throughout of the completion of transition to turbulence.

Figure 12 shows both steady laminar and turbulent limits for the set-up shown in figure 11. (The data correspond to a pipe which was 173 diameters long upstream of the coil, had 20 $^1/_2$ turns in the coil and was 937 diameters long in the downstream straight section. The diameter 2a was 0.635 cm, and the radius ratio a/R was 0.058. The fluid was air. All transitional Reynolds number data were determined with a hot-wire.) One effect of the coil is to increase both the steady laminar and turbulent limits up to the end of about three turns or

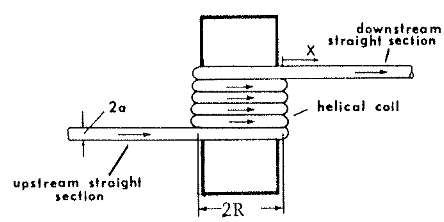

Figure 11 Schematic of the experimental set-up.

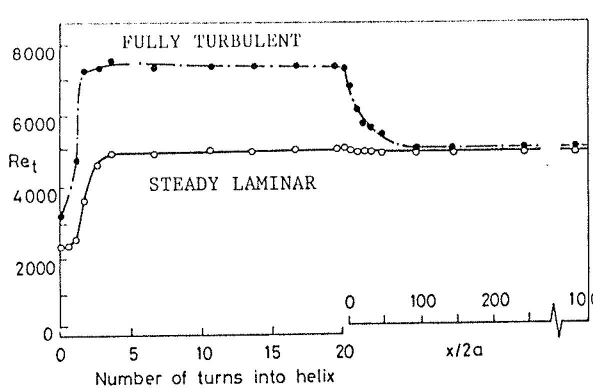

Figure 12 The steady laminar and turbulent Reynolds number limits for the set-up shown in figure 11.

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

so; thereafter, some asymptotic state seems to have been reached. In this asymptotic state, the flow remains laminar and steady for Reynolds numbers up to about 4800, and does not become fully turbulent until an Re of 7900 or so is reached; clearly, the gap between the two curves is larger inside the coil than that at the entrance to the coil. Perhaps surprizing is the behaviour downstream of the coil: while the turbulent limit drops as expected, the steady laminar limit does not, but stays approximately at the same elevated level as in the coiled section. In other words, the onset of turbulence has been permanently raised to an Re of 4800 in contrast to about 2300 in the upstream section!

Why does the flow remain steady and laminar for higher Reynolds numbers in the coiled section than it usually does in the upstream straight section? Can the asymptotic values of the two limiting Reynolds numbers be increased indefinitely? What makes the flow remain steady and laminar for Reynolds numbers as high as it does in the downstream straight section? Can that too be increased indefinitely? These are some of the obvious questions that come to our mind. In what follows, we shall attempt at least partial answers to these questions drawing largely from our continuing study of this flow.

3.1 Stabilization Effects and Relaminarization

Within the coil, the flow near the inside wall sees a convex curvature whose effect has long been known to be stabilizing. However, the concave curvature associated with the outside wall is known to be destabilizing, and so, the explanation for the net stabilization effect observed in the present circumstances is a bit subtle. The clue lies in the behaviour of the mean velocity distribution. Essentially because of the centrifugal forces, the peak of the velocity in the plane of the helix moves to the outside; typically for a radius ratio of about 0.058, the peak occurs at a distance from the outer wall of a tenth of the pipe diameter. Over the bulk of the profile from the inside to the peak, the sense of the mean flow vorticity is the same as the 'angular velocity' in the pipe, so that, by Rayleigh's criterion — for a statement of the criterion most appropriate in the present context, see Coles (1965) — the flow is stable. There is however a small region near the outside wall where the mean vorticity and the 'angular velocity' vectors are oppositely aligned. But this region is quite thin for fairly large curvatures, and the governing instability there is of the boundary layer type. This 'boundary layer' too will be stable unless the Reynolds number based on its thickness is above the appropriate critical value; then and only then will the onset of instability and possible transition to turbulence occur. This explanation, in spirit due essentially to Lighthill (1970), cannot be complete because of the three-dimensionality of the velocity field but appears very reasonable.

One consequence of these stabilization effects is that, in a certain Reynolds number range (for the conditions of figure 12, $2300 \leq Re \leq 4800$), a turbulent flow entering the coil can be expected to become laminar at some point in the coil. That does indeed happen, as can be seen from the oscillograms of figure 13. The flow, which begins its journey in a fully turbulent state at the inlet to the coil, has become completely laminar by about two turns in the coil. In fact, near the inside surface, the flow has lost most of its turbulent characteristics only half a turn into the coil!

3.2 Radius Ratio and Other Effects

We set up several coiled pipe flows in order to determine the effect of radius ratio on the asymptotic values of the steady laminar and turbulent limits discussed with respect to figure 12. Since the parameter govern-

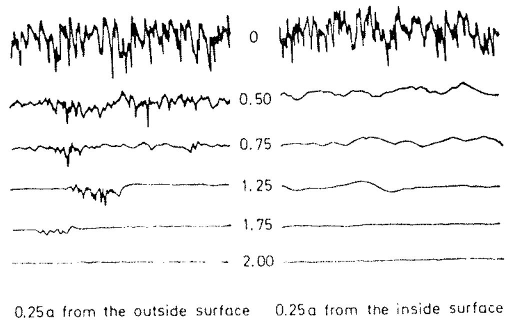

Figure 13 Typical oscillograms of hot-wire traces during relaminarization. Re = 3450. a/R = 0.058. The numbers marked in the middle of the figure correspond to the number of turns into the coil.

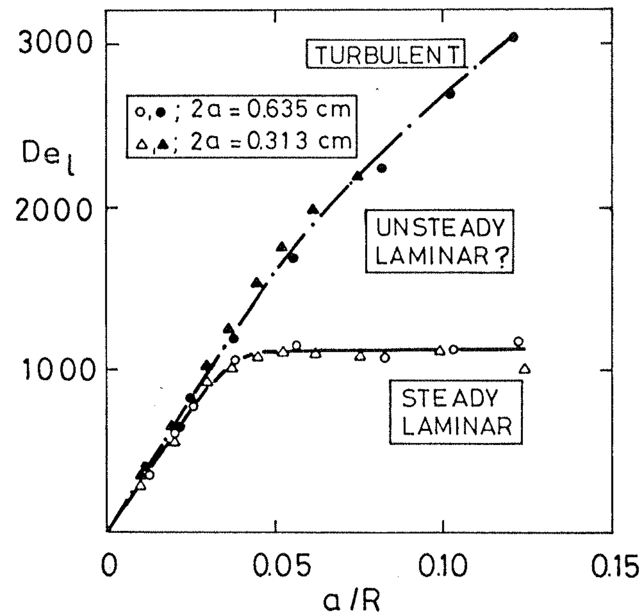

Figure 14 Asymptotic values of the limiting Dean numbers in the coiled section, measured at the end of 20 turns for all radius ratios.

ing the dynamic similarity in curved pipes is the so-called Dean number (White, 1929),

$$De = Re(a/R)^{0.5}, \qquad (7)$$

the data obtained were plotted (see figure 14) as the limiting Dean numbers against the radius ratio a/R. It is seen that the limiting Dean numbers (whose meaning is the same as that of the asymptotic limiting Reynolds numbers of figure 12) increase with increasing tightness of the coil until an a/R of 0.04. Thereafter, we note that steady laminar flow cannot be found for Dean numbers above about 1100, however large the radius ratio. (This constancy in the steady laminar limit for the Dean number actually implies a decrease in the corresponding Reynolds number for increasingly larger a/R; see equation (7).) On the other hand, the turbulent limit appears to increase monotonically (in terms of both the Dean and Reynolds numbers) with the radius ratio.

If we replot the data of figure 14 in terms of Reynolds numbers instead of Dean numbers (Sreenivasan & Strykowski, 1983b), it can be seen that the lower curve of figure 14 shows a peak for a/R of 0.039 and Re of 5400. This simply means that the most stable conditions

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

obtain for a radius ratio of 0.039 and the highest Reynolds number for which a steady laminar flow is possible in the asymptotic state in the coiled pipes is 5400. This appears to be so independent of how large the critical Reynolds number is upstream of the coil (or, how smooth the inlet is to the upstream straight pipe); our data at this point are not extensive but seem enough to hold this view. We therefore conclude that, if we can maintain the flow upstream of the coil steady and laminar for Re $\geq$ 5400, it actually loses its stability upon entering the coil – a notion that seems to have been ruled out in the long history of curved pipe flows! Curvature in this case is not always stabilizing!

3.3 Unsteady Laminar Flow in Tightly Coiled Pipes?

For convenience, we shall call coiled pipes with a/R higher than about 0.04 (corresponding to the flat lower curve in figure 14) as tightly coiled pipes. We shall now qualitatively examine the nature of the flow with increasing Dean number for a typical tightly coiled pipe. Figure 15 shows several oscillograms, all obtained at the end of 10 turns of a coiled pipe with radius ratio 0.1. It is useful to note that the traces look much the same over most of the crosssection of the pipe, except perhaps in the vicinity of the outside wall in the plane of the coil. It is clear that, while the flow loses its steady characteristics in the neighbourhood of a De of 1100, it is still laminar-looking up to a much higher Dean number (see the second, third and the fourth traces in figure 15). We therefore make a hypothesis that the stable laminar state yields to another laminar state, with transition to turbulence

If an intermediate unsteady laminar state does indeed occur, it is clear that theoretical analyses of the laminar motion must somehow incorporate this at large Dean numbers. This failing may well be the chief reason why, Van Dyke's (1978) extension to De $\rightarrow \infty$ of Dean's(1927) analysis of laminar motion in curved pipes, while being technically sound, yeilds results with qualitatively incorrect dependence of friction factor on the Dean number. We may also note that Van Dyke's friction factors are lower than the experimentally measured ones, as should indeed be expected if the present discussion is correct.

3.4 The Downstream Straight Section

We may now return briefly to the flow in the straight section downstream of the coil. We recall from figure 12 that the Reynolds number corresponding to the steady laminar limit stays essentially at the same elevated level as in the coiled section. We have found this to be true (Sreenivasan & Strykowsky 1983b) for coils with several other radius ratios too. This seemed surprizing at first, but is natural upon recollecting that the critical Reynolds number for a pipe flow (i.e., the Poiseuille flow), as determined theoretically from linear disturbance theory, is strictly infinite. In practice, the flow undergoes transition at finite and variable Reynolds numbers depending on the level of disturbances. Because of the continual dissipation of turbulence and other disturbances in the thin boundary layer-like regions, it is possible that the coiled section acts like a very successful filter that removes the most critical disturbances, or at least diminishes their amplitude, alters their frequency content, or both, in such a way that the remainder of the disturbances does not become critical until after a fairly high Reynolds number (depending on the radius ratio of the coil, number of turns, etc.) is attained. The picture is made more complicated by the fact that there is a strong swirl at the inlet to the downstream straight section, and the boundary layers that get established in the developing region may in fact set the upper limit to the transitional Reynolds number there. These and other questions cannot be settled

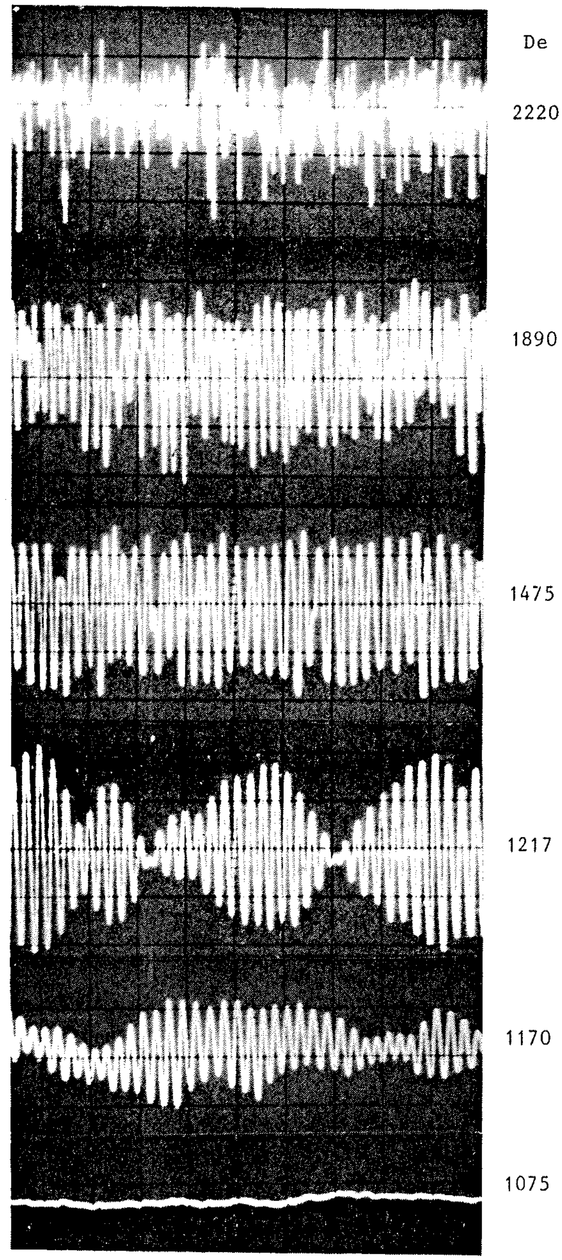

Figure 15 Oscillograms of hot-wire signals, a/R = 0.1. The gain for the top four traces is the same, but it increases by 2 each time for the following two traces. The time scale is the same for all traces.

without careful and quantitative studies, but it may be relevant to point out that an artificial disturbance, created immediately after the flow exits from the coil by inserting a fine needle across the pipe diameter (recall the arrangement in section 2.1), does not affect the transitional Reynolds number in the downstream straight section. On the other hand, the same needle, when inserted further downstream (say, x/D of 100), results in a precipitous drop in the steady laminar limit to around 2200.

Finally, we may note that the gap between the loss of steady laminar motion and the completion of transition to turbulence is relatively quite small (of the order of 0.5% of the steady laminar limit) in the downstream straight section. This catastrophic transition, not uncommon in pipes with relatively smooth inlets – the coil seems to serve much the same purpose indirectly – is quite different from that characteristic of the

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984

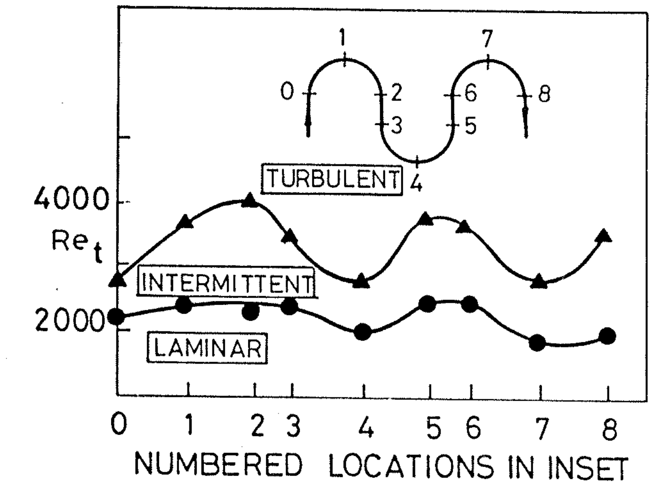

Figure 16 Transitional Reynolds numbers in a 'heat-exchanger-type' pipe configuration shown in the inset.

process in the upstream straight section with no specially smooth inlet, and is marked by the appearance of the so-called slugs (in contrast to puffs upstream) which are regions of turbulence filling the entire pipe section comparable in length to the pipe length itself, characterized by relatively sharp laminar-turbulent interfaces at both the front and back ends. (For a discussion of slugs, see, for example, Pantulu(1962), Lindgren (1969), Wygnanski & Champagne (1973), etc.)

4 SOME OTHER EXAMPLES

In addition to the two non-simple pipe flows discussed in the previous sections, we have also examined in varying degrees of detail:

(a) pipe flow with a right angle bend,
(b) pipe flows which bifurcate into two equal or unequal branches, and
(c) typical heat exchanger pipes in which the flow reverses direction every half a turn; see inset to figure 16.

In engineering practice, these and other configurations are widely used. Several important gross parameters have been measured for a long time, and a number of working engineering correlations relating to their performance have been in existence also for a long time. But a more detailed look at any of these configurations reveals many interesting and unexplored facets. (See, for example, Tunstall & Harvey (1969) for a very curious phenomenon associated with sharp bends in fully developed pipe flows.) Perhaps, we are saying nothing but attesting to the obvious reality of the fascinating science of fluids!

We close our discussion of non-simple pipe flows with some data on the transitional Reynolds numbers measured in an example of (c) above. Again, we have plotted in figure 16 the two limiting Reynolds numbers (recall our discussion with respect to figure 12) as a function of position. It is seen that the two limiting Reynolds numbers first increase for the first half a turn or so, just as they do in the coiled pipes, but thereafter, follow the geometry of the pipe in some rough sense. The important point to note is that the onset and completion of transition occur around the same values common in straight pipes with disturbed inlet conditions. Clearly, this fact is important in heat exchangers and the like where it is no surprise to find this configuration, rather than a helically coiled pipe, in common use.

5. ACKNOWLEDGEMENTS

It is a pleasure to acknowledge contributions of Professor B.-T. Chu, Mr. T.B. Lynn, Professor M.V. Morkovin, Professor R. Narasimha, Mr. P.J. Strykowsky and Professor P.P. Wegener. I owe them all, especially Mr. Strykowsky, a debt of gratitude. Some of these studies were an outgrowth of a project on Turbulence Control sponsored by the Air Force Office of Scientific Research.

6. REFERENCES

BADRINARAYANAN, M.A. (1968) An Experimental Study of Reverse Transition in Two-Dimensional Channel flow, J. Fluid Mech., 31, 609-623.

BANDYOPADHYAY, P & HUSSAIN, A.K.M.F. (1983) The Organised Motion in 'Puffs' in Transitional Pipe Flow, Third Int. Symp. on Flow Vis. Univ. Michigan, Ann Arbor, Sept 1983.

BERGER, S.A., TALBOT, L. & YAO, L.-S. (1983) Flow in Curved Pipes. Ann. Rev. Fluid Mech. 15 , 461-512.

CHAMPAGNE, F.H. & HELLAND, K.N. (1978) Pipe Flow Relaminarization, Bull. Amer. Phy. Soc. 23 , Nov. 20.

COLES, D. (1965) Transition in a Circular Couette Flow, J. Fluid Mech. 21 , 385-425.

DEAN, W.R. (1927) Note on the Motion of Fluid in a Curved Pipe, Phil. Mag. J. Sci. 4 , 208-223.

GOLDSTEIN, S. (Editor) (1938) Modern Developments in Fluid Dynamics. Vol 1. Oxford: Clarendon Press.

KOVASZNAY, L.S.G. (1949) Hot-Wire Investigations of the Wake Behind Cylinders at Low Reynolds Numbers, Proc. Roy. Soc. Ser. A, 198 , 174-185.

LAUFER; J. (1962) Decay of Non-Isotropic Turbulent Field, In 'Miszellaneende Angewandte Mechanik, Festschrift Walter Tollmien,' Akademie-Verlag, Berlin.

LIGHTHILL, M.J. (1970) Turbulence. In 'Osborne Reynolds and Engineering Science Today' (D.M.McDowell & J.D. Jackson, Eds.), 83-146, Manchester Univ. Press.

LINDGREN, E.R. (1969) Propagation Velocity in Turbulent Slugs and Streaks in Transition Pipe Flow, Phys. Fluids, 12 , 418-425.

LYNN, T.B. & SREENIVASAN, K.R. (1982) The Flow in a Branching Pipe. Bull. Amer. Phys. Soc. 27, Paper EA3.

NARASIMHA, R. & SREENIVASAN, K.R. (1979) Relaminarization of Fluid Flows, Adv. Appl. Mech. 19, 221-309.

PANTULU, P.V. (1962) M.Sc. Thesis, Aero.Engg.Dept., Indian Inst. Sci., Bangalore.

REYNOLDS, O. (1883) An Experimental Investigation of the Circumstances which Determine Whether the Motion of Water Shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channel, Phil. Trans. Roy. Soc. 174, 935-982.

RUBIN, Y., WYGNANSKI, I. & HARITONIDIS, J.H. (1980) Further Observations on Transition in a Pipe. In 'Laminar-Turbulent Transition' (Eppler, R. & Fasel, H., Eds) Springer-Verlag, Berlin.

SIBULKIN, M. (1962) Transition from Turbulent to Laminar Pipe Flow, Phys. Fluids. 5, 280-284.

SREENIVASAN, K.R. & STRYKOWSKY; P.J. (1983a) An Instability Associated With a Sudden Expansion in a Pipe Flow, Preprint.

DISCUSSION ON THIS PAPER WILL BE ACCEPTED FOR PUBLICATION UNTIL 14 MARCH 1984.

SREENIVASAN, K.R. & STRYKOWSKY, P.J. (1983b) Stabilization Effects in Flow Through Helically Coiled Pipes, Experiments in Fluids, 1, 31-36.

TUNSTALL, M.J. & HARVEY, J.K. (1969) On the Effect of a Sharp Bend in a Fully Turbulent Flow, J. Fluid Mech. 34, 585-609.

VAN DYKE, M. (1978) Extended Stokes Series: Laminar Flow Through a Loosely Coiled Pipe, J. Fluid Mech. 86, 129-145.

WHITE, C.M. (1929) Streamline Flow Through Curved Pipes, Proc. Roy. Soc. Ser. A 123, 645-663.

WYGNANSKY, I. & CHAMPAGNE, F.H. (1973) On Transition in a Pipe, Part I: The Origin of Puffs and Slugs and the Flow in a Turbulent Slug, J. Fluid. Mech. 59, 281-335.

WYGNANSKY, I., SOKOLOV, M. & FRIEDMAN, D. (1975) On Transition in a Pipe, Part II: The Equilibrium Puff. J. Fluid Mech. 69, 283-304.

DISCUSSION ON THIS PAPER WILL BE ACCEPTED
FOR PUBLICATION UNTIL 14 MARCH 1984