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Review Article

Laminarescent, Relaminarizing and Retransitional Flows

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With 16 Figures

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Summary

This report examines in detail all accelerated turbulent boundary layers and subcritical pipe or channel flows undergoing relaminarization and possible retransition, with a view to evolving a broad picture in regard to the status of experiments in these flows, the trustworthiness or shortcomings of the data, the sources of difficulties peculiar to these flows, etc. With the hindsight so acquired, a discussion is provided of the directions in which future work would most usefully supplement the existing data.

Notation

a:	pipe radius or channel half-height
c_f:	skin-friction coefficient
H:	shape factor, δ^*/θ
K:	acceleration parameter, $\nu(dU_\infty/dx)/U_\infty^2$
k:	Karman constant
P:	kinematic pressure
Re:	Reynolds number, $U_{av}a/\nu$
R_θ:	momentum thickness Reynolds number, $U_\infty\theta/\nu$
T:	temperature
U, V:	mean velocity in x and y directions respectively
U_*:	friction velocity, $\tau_w^{1/2}$
u, v, w:	fluctuating velocity components in the x, y and z directions respectively
x, y, z:	streamwise, normal and spanwise Cartesian coordinates
Δ_p:	$\nu(dP/dx)/U_*^3$
Δ_τ	$\nu(\partial\tau/\partial y)/U_*^3$
δ:	boundary layer thickness, $U(\delta)/U_\infty = 0.995$
δ^*:	displacement thickness, $\int_{-\infty}^{\infty}(1 - U/U_\infty)\,dy$
θ:	momentum thickness, $\int_{-\infty}^{\infty}(U/U_\infty)\,(1 - U/U_\infty)\,dy$
Λ:	pressure gradient parameter, $(dP/dx)\,\delta/\tau_w$
ν:	kinematic viscosity coefficient
τ:	kinematic Reynolds shear stress, $\overline{-uv}$

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Suffixes

∞: free-stream values
w: wall values
av: section average values.

Superscript

+: variables normalized by U_* and ν
′: rms values

Overbar denotes averaging. Other variables used locally are defined at appropriate places.

1. Introduction

1.1 Motivation and Basis

Relaminarization is a process by which an initially turbulent flow is rendered effectively laminar. Laminarescence signifies the earlier stages of relaminarization — loosely, a precursor to relaminarization — in which large departures occur from the 'laws' empirically known to hold in the initial fully turbulent state. A relaminarized flow, if allowed a further development on removal of the agency responsible for creating relaminarization in the first place, will — as experience with direct transition suggests — become turbulent again, or undergo retransition.

All these phenomena are of (more or less) common occurrence — certainly more so than has been generally believed. Evidence of relaminarization and laminarescence has been found when a turbulent flow is subjected to effects of acceleration, suction, blowing, magnetic fields, stratification, rotation, curvature, heating, etc. As observable phenomena, they deserve our attention. Quite apart from the attention the phenomenon inherently demands, a study of relaminarization[1] serves two rather important and useful purposes. Firstly, relaminarization contains seeds of the science of "turbulence control", although the relevant technology seems still too far ahead. More immediate is its usefulness in turbulence modeling. Evidently, if a model shows the correct behaviour at all stages of the relaminarization process (starting from the fully developed turbulent through laminarescence to truly laminar), it can be said with reasonable confidence that the model contains the right kind of physics: at present, there exists no such model. It is precisely in establishing this physics that studies of relaminarizing flows find a persuasive place. Certainly, a turbulence model cannot even be considered as being roughly right unless it demonstrates an ability to simulate the essential features such as the "switch-off" of turbulence energy production at an appropriate point. An examination of relaminarizing flows with a view even to recognizing such "landmarks" and characterizing them by appropriate limits of operation would in itself be very useful.

In its widest generality implied above, an assessment of all that has been said about relaminarizing flows is at present a hopeless task. As an intriguing phenomenon, it has attracted the attention of a lot of people; as a difficult area, this interest has not always been backed by useful outcome: systematic experiments are very often lacking, and a *general* theory based on solid foundations is — and will be for some time — impossible. Considerable order has however been brought into this fluid situation by Narasimha and Sreenivasan [26] who suggested that, notwithstanding the diversity of flow situations in which relaminarization occurs, only three types of basic physical mechanism can be identified which, either individually or in some combination, bring about relaminarization. The first mechanism is one in which turbulence energy is destroyed or absorbed by an external body force such as buoyancy; a critical Richardson number governs the phenomenon here. In the second mechanism, turbulent stresses are dissipated through the action of a molecular parameter such as viscosity, and is governed by a critical Reynolds number or an analogous nondimensional parameter. The third mechanism, typified by an initially turbulent boundary layer relaminarizing under the influence of severe streamwise acceleration, is less simple. Here, a two-layer model seems appropriate. In the outer layer, the decay or destruction of turbulence is not significant; the turbulence structure there is essentially distorted by the rapid acceleration superimposed on it. A new viscous-dominated inner layer develops, in which the turbulence inherited from initial conditions decays. The interaction between the two layers is dynamically weak (except insofar as they provide appropriate "matching" boundary conditions for each other). The major factor in this type of relaminarization is the domination of the pressure forces over slowly responding Reynolds stresses in the outer layer, accompanied by the generation of a new laminar subboundary layer, which itself is maintained stable by the acceleration. Clearly, the laminarescent state in all these three cases can be expected to assume somewhat different characteristics also.

We may now ask whether enough is known about relaminarizing flows to invite a detailed examination that is timely and useful. The answer is that only two classes of relaminarizing flows — the highly accelerated turbulent boundary layers and subcritical pipe or channel flows — have been studied in enough detail to invite a justified attempt at a critical analysis; that generally is the task set for this report. For a discussion of the available material in other relaminarizing flows, reference must be made to Narasimha and Sreenivasan [26].

1.2 Scope and Plan of the Present Report

Specifically, the purpose of this report is to examine critically all the relaminarizing flows in these two classes mentioned above, with a view to obtaining a rather broad appreciation for the status of the experiments in these flows, and for what the measurements mean, the reliability of the data and their shortcomings, the sources of difficulties characteristic of these flows, etc. To provide a framework for further discussion, some physical characteristics of the relaminarizing boundary layers and some related conceptual difficulties are discussed in Chapter 2. In an ideal experiment on relaminarization, we require that the flow be unambiguously turbulent initially, with all the initial conditions documented in sufficient detail; such a flow must be brought to a final laminar-like state. During this process of relaminarization, all the mean and fluctuating parameters should be measured in sufficient detail; momentum balance and other checks as appro-

[1] From here onwards, where no confusion arises, relaminarization will be used in a comprehensive sense to include laminarescence.

priate should be made to insure the internal consistency of the measurements. Careful evaluation on this basis of the many existing experimental studies reveals that most of them do not strictly satisfy all the requirements and are generally beset with various measurement problems; these shortcomings are discussed in Chapter 3 with a view to helping better design of such experiments in the future; areas needing special attention will be emphasized. Chapters 4 and 5 respectively contain a discussion of the laminarescent and retransitional boundary layers. In Chapter 6, we describe, mostly qualitatively, a point of view that can explain a large part of the available measurements in these flows; with this as a reference, we will also be able to highlight the existing gaps — both in measurements and our understanding. Relaminarization in pipe and channel flows is discussed in Chapter 7, while Chapter 8 contains a summary of conclusions that this study has led to, and of the outlook for the future. Finally, the Appendix contains a commentary of the merits and drawbacks of each individual set of data in these flows.

2. Relaminarizing Boundary Layers: General Concepts

The first observations of relaminarization by acceleration were made by Sternberg [46] in a supersonic turbulent boundary layer negotiating a Prandtl-Meyer expansion. This and later studies (e.g., Sergienko and Gretsov [38], Narasimha and Viswanath [27]) in supersonic flows are however less detailed than is desirable for our purposes here. On the other hand, relatively more detailed studies now exist in incompressible turbulent boundary layers undergoing relaminarization by acceleration, and we shall direct our discussion only to these flows.

A typical experimental set-up is as follows. An incompressible turbulent boundary layer developing at constant pressure (say, on one of the wind-tunnel walls or a separately laid out flat plate) up to a 'point' x_0 is subjected to a sustainted large acceleration beyond x_0. This can be accomplished, for example, by fixing a liner of desired shape on the opposite wall of the wind-tunnel; see Fig. 1. Experiments show that the boundary layer asymptotically tends to a laminar-like state. In the past, more than two dozen flows of this type have been studied. (A somewhat special case is the flow between two converging flat plates, more about which will be said later; see Section 4.2.) All the relevant features of these flows have been listed in Table 1; for convenience each flow is given a code by which it will be referred to in this report.

2.1 General Characteristics and Broad Classification

Relaminarization (unlike transition to turbulence) is a gradual process, but is accompanied by drastic changes in the structure of the boundary layer: the boundary layer thins down, mean velocity profile departs from the well-known law of the wall[2] and law of the wake, the shape factor decreases first and then increases, the skin-friction and heat transfer coefficients increase before showing

[2] In the initial stages of acceleration, a large part of the velocity profile in the similarity region moves below the universal log-law, but as as the acceleration becomes stronger, an 'overshoot' is observed (see, e.g., Patel and Head [31]).

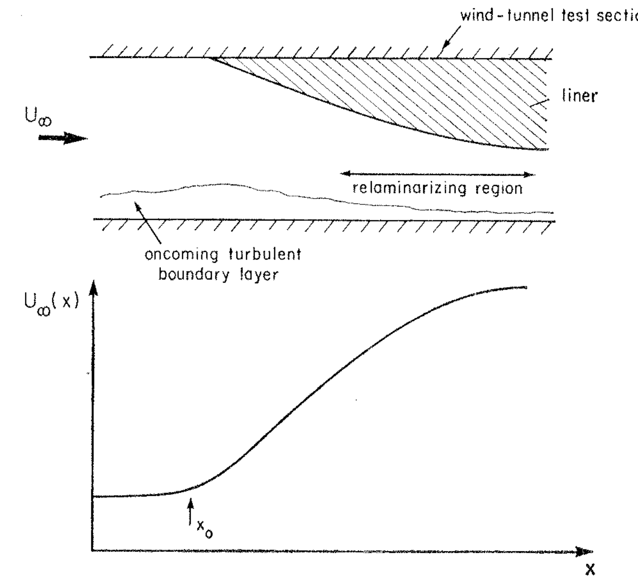


Fig. 1. Schematic of highly accelerated subsonic turbulent boundary layer undergoing relaminarization

a substantial decrease, the relative turbulence intensity goes down, etc. Under suitable conditions (e.g., Badri Narayanan and Ramjee [4]), the velocity profile that eventually results shows no significant departures from the laminar profile appropriate to the local conditions, but this need not always occur. Other accompanying gross changes are: a rapid decline in the rate of wall-layer bursting, which is now known to be a primary mechanism for the generation of turbulent energy (Kim et al. [16]); the spread of intermittency even to the wall region (Fiedler and Head [11]); elongation in the streamwise direction of the large eddy structure (Blackwelder and Kovasznay [7]); a fairly rapid decay of turbulent stresses near the wall consistent with the generation and growth of a new laminar layer there, etc. As will be shown in some detail in Section 3.5, one of the chief characteristics of this type of reversion is that, not too near the wall, the turbulent stresses do not decay in absolute magnitude; rather, they are 'diluted', as it were, by the increased local mean velocity.

It is clear that a *complete* prediction of the flow from first principles is not to be hoped for as yet, but the fact that the flow asymptotically reaches a laminar-like state gives us some justified hope for making rational approximations that will render parts at least of the flow amenable to quantitative analysis. To this end, it is useful to classify the flow broadly into various regions. Such a classification is attempted in Fig. 2. Since the pressure gradient in an incompressible flow cannot be abrupt, one expects a small initial region (region (a) in Fig. 2) in which the usual standard turbulent laws (e.g., the log-law) are valid without any modification. Acceleration effects become important beyond this region. There is a finite extent of the flow (region (b) in Fig. 2) in which substantial departures occur from the constant pressure turbulent behaviour but the flow is nevertheless

Table 1. *Summary of relaminarizing boundary layer data*

No.	Reference	Code	Manner of producing acceleration	Initial R_θ	Method of velocity measurement	Method of c_f measurement	Turbulence parameters measured	Momentum balance	Remarks
1	Back and Seban [1]	BS	Tunnel-wall liner	300	Pitot	wall-slope (linear)	—	Resonable	Sparse measurments, unknown initial conditions
2	Badri Narayanan and Ramjee [4]	BR1 BR2 BR3 BR4 BR5 BR6 BR7	Tunnel-wall liner (BR1–BR3) 40° Wedge (sink flow) (BR4–BR6) 9° Wedge (sink flow) (BR7)	1650 310 410 2050 1240 780 1700	Pitot and hot-wire	— Heat transfer gauge —	u'	— Reasonable —	Generally larger scatter than most other measurements. A priori estimates suggest that c_f measurements in BR2 and BR3 are reliable
3	Badri Narayanan et al. [5]	BRN	40° Wedge	1850	Pitot	—	u', v' and $\overline{uv}$	—	The only experiment with turbulent energy balance measurements
4	Blackwelder and Kovasznay [7]	BK	Two-dimensional contraction	2500	Hot-wire	wall-slope (linear)	u', v', w', $\overline{uv}$	Poor	The only experiment with w' measurement
5	Jones and Launder [13]	JL	Wedge (sink flow)	391, 338 475, 390, 340	Pitot	Stanton tube	u' and spectra	—	c_f also obtained by assuming self-similar sink flow
6	Launder [19]	L1 L2		320 1000	Pitot and Hot-wire	—	u', $\overline{uv}$	—	Data not clearly labeled and hence only of qualitative use
7	Launder and Stinchcombe [21]	LS	Wedge (sink flow)	200	Pitot and Hot-wire	—	u'	Poor*	—
8	Moretti and Kays [24]	MK	Variable height tunnel	1400 2800	—	—	—	—	Provide extensive heat-transfer measurements. Initial conditions and mean velocity distribution generally unknown. Among the many flows studied by the authors, the two that are reported in their paper are considered here
9	Okamoto and Misu [29]	OM1 OM2 OM3	Two-dimensional contraction	470 580 640	Pitot	wall-slope (linear)	—	Not good Reasonable	—
10	Patel and Head [31]	PH1 PH2	Centre body in a pipe	2100 5900	Pitot	Fence technique	—	Reasonable Poor	Some questions about uniqueness of initial conditions Boundary layer approximation violated ?
11	Schruab and Kline [37]	SK	Water channel with a flexible wall	590	Hot-wire	wall-slope (linear)	u' at one station	?	The only experiment to measure the wall burst rate. Working fluid is water
12	Simpson and Shackleton [40]	SS1 SS2	Two-dimensional contraction	1290 1650	Hot-wire	wall-slope (non-linear)	u'	Not good	Boundary layers developing under moderate adverse pressure gradient until acceleration (applies to rows 12 and 13)
13	Simpson and Wallace [41]	SW1 SW2	Flexible tunnel wall (sink flow)	1500 1700	Hot-wire	wall-slope (non-linear)	u'	Good Reasonable	
14	Sreenivasan [43]	S1 S2 S3	40° Wedge	675 940 1250	Pitot	—	—	—	S3 is an attempted repetition of BR1

* Poor according to Jones and Launder [13].

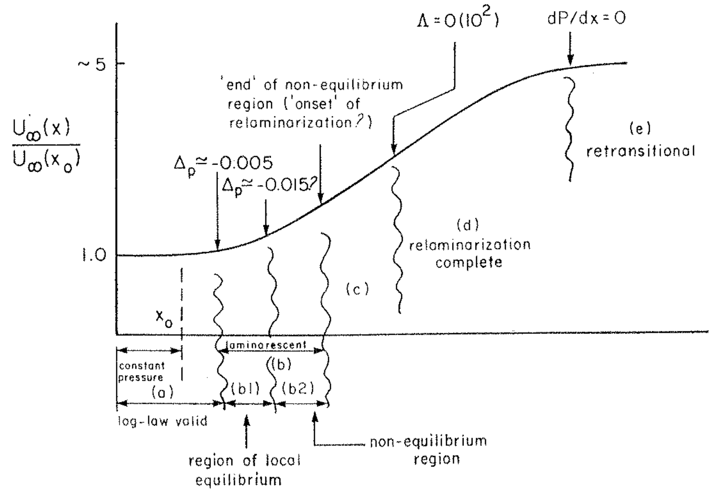


Fig. 2. Broad classification of the flow field into various regions

turbulent. Following Schraub and Kline [37], we shall denote this region *laminarescent*. As the pressure gradient effects become stronger, the flow ceases to be fully turbulent (e.g., region (c) and beyond in Fig. 2). Under sustained acceleration, a state is subsequently reached (region (d)) in which relaminarization can be said to be complete (in the sense to be made more precise in Section 2.4). If the acceleration relaxes, as it inevitably does in practice, the boundary layer eventually undergoes retransition to turbulence (region (e)), thus completing the cycle.

2.2 The 'Laminarescent' Region

Region (a) is straightforward, and the only point of debate here concerns the 'boundary' between regions (a) and (b). It appears logical to characterize this boundary by the parameter $\Delta_p (\equiv \nu(dp/dx)/U_*^3)$ which arises naturally when the equations of motion are written in wall-coordinates. Patel's [30] measurements suggest that this boundary occurs when Δ_p reaches roughly about -0.005. The 'laminarescent' region could then be considered to begin around here.

The use of the word 'laminarescent' does not necessarily imply that the development of the flow into a laminar state is inevitable, but does indicate a tendency towards it. In fact, we shall argue in Section 4.2 that by properly tailoring the pressure gradient, it is possible to maintain the flow in a laminarescent state without ever attaining complete relaminarization.

The laminarescent state of the boundary layer shares some characteristics with several other flows in which the turbulence structure has been affected by an external agency such as buoyancy, but to unearth these points of similarity and divergence will be another task, and shall not concern us here. In fact, we shall use the word specifically as restricted to relaminarizing boundary layers.

A further subdivision of the laminarescent region is useful. In the first (the region (b1) of Fig. 2), the boundary layer is in local equilibrium under the addi-

tional influence of the pressure gradient. This part is amenable to a local analysis, and new similarity laws can be formulated; this will be discussed in Section 6.1. As the pressure gradient becomes stronger, the turbulent boundary layer ceases to be in equilibrium, and local analysis can no longer be expected to hold. This non-equilibrium region is nevertheless turbulent and is characterized by large normal stress gradient near the wall. From the point of view of basic understanding, this as well as region (c) are the least understood at the present time; special efforts should be made in this direction.

While fundamental difficulties remain with regard to regions (b2) and (c), our ignorance here does not appear to be very crucial from a pragmatic point of view because, when the flow acceleration is steep, all details of the flow in the early stages of relaminarization get crowded. In particular, the extent of the laminarescent region will be small, especially when the flow Reynolds number is initially not very large. As we shall see later, this introduces an unusual simplicity in the calculation of relaminarizing boundary layers.

2.3 The 'Onset' of Relaminarization

Since relaminarization is not catastrophic, its onset is an illdefined concept (see e.g., Launder [19], Schraub and Kline [37], Narasimha and Sreenivasan [25]). For this reason, no satisfactory criterion for its occurrence has yet been given; all criteria proposed so far are uncertain at two levels, firstly in the way one recognizes the occurrence of relaminarization (the 'physical criterion'), and secondly in specifying the precise parameter which can be used as a convenient indicator of its occurrence (the 'parametric criterion'). For example, it is possible to recognize the 'end' of the non-equilibrium region (b2) with the 'onset' of relaminarization, but to give a convenient parametric criterion for it is quite another problem.

Many criteria (of both types) have been proposed in the past, but we shall confine ourselves to a brief consideration of some of them. Patel and Head [31] identified the occurrence of relaminarization with the departure of the inner layer flow from a version of the modified wall-law which explicitly incorporates, in an *empirical* way, the stress-gradient near the wall. They suggested that this occurred when the stress-gradient parameter $\Delta_\tau \left(\equiv \nu(\partial\tau/\partial y)/U_*^3\right)$ reaches a critical value of -0.009. Unfortunately, their proposal suffers from many internal inconsistencies (as discussed at some length by Narasimha and Sreenivasan [25]). We may also note that the Patel-Head version of the modified wall-law is an empirical substitute to the inner law (6.2) derived by rational methods in Section 6.1. At any rate, judging the occurrence of relaminarization by the breakdown of an inner law whose basis is local similarity — albeit different from the constant-pressure case — essentially ignores the existence of the non-equilibrium turbulent region (b2). Moretti and Kays [24] identified relaminarization to have occurred when heat transfer rates dropped below some standard turbulent predictions, and used the parametric definition that the acceleration parameter $K \left(\equiv \nu(dU_\infty/dx)/U_\infty^2\right)$ reach a critical value of 3.5×10^{-6}. Somewhat similar suggestions have been made by Launder [19], Back and Seban [1] and several others. The chief criticism against K is that it is a free-stream parameter which takes no cognizance of the boundary layer whose relaminarization it is supposed to mark!

Another parametric criterion often used is that the parameter Δ_p reach a critical value of about -0.025 (see, e.g., Patel [30], Badri Narayanan and Ramjee [4]). Some other criteria take the form Kc_f^{-n}, with n varying between 3/2 and 1/2 (e.g., Launder and Stinchcombe [21], Back et al. [2], Schraub and Kline [37]). Notice that $\Delta_p \equiv Kc_f^{-3/2}$. Again, all arguments leading to Kc_f^{-n} invariably depend on the similarity of the flow in the wall region, with one argument differing from another only in detailed approximations made in the use of wall similarity. Breakdown of similarity does not imply relaminarization!

One possible sign that the flow has ceased to be fully turbulent is the spread of intermittency all the way to the wall. Fiedler and Head [11] have indeed observed intermittency right up to the wall in the early stages of relaminarization (their physical criterion), and have speculated that a parametric criterion for it could be specified in terms of the occurrence of a minimum[3] in the variation of the shape factor, H. The minimum H criterion was also inferred by Patel and Head [31], but unfortunately there are serious difficulties associated with their inference (see Narasimha and Sreenivasan [25]). In general, the same favourable pressure gradient which makes the velocity profile fuller and hence H lower also decreases the boundary layer Reynolds number whose effect is to increase H. Thus, it is conceivable that H passes through a minimum without having much direct connection with relaminarization.

Another possible, but by no means obvious, physical definition of the 'onset' of relaminarization is the cessation of bursting (Schraub and Kline [37]). The difficulties associated with this definition are two-fold. Firstly, the bursting rate decreases monotonically even in region (b1) (Kline et al. [17]), and there is thus no way of distinguishing region (b1) from region (b2), or of identifying a point at which bursting ceases. This latter problem is not serious if we agree to require that the properly scaled bursting rate should fall to a specified small fraction of that expected if the boundary layer were in equilibrium under the local conditions. Although we cannot precisely specify this last quantity with our current understanding of turbulent boundary layers, this too may be of minor importance in practice if the specified fraction is small (say 5%). The second and more practical difficulty is related to the considerable complexity of burst measurement, which makes a simple substitute parametric criterion very essential. Kline et al. [17] noted that the bursting rate practically ceases when the parameter reaches about 3.7×10^{-6}. We have already noted the difficulties in using a critical K.

Whether the occurrence of minimum H, critical K or Kc_f^{-n} are compatible with one another is not obvious; indeed, no unambiguous relation can be expected among them. Nevertheless, these criteria are often useful as rough indicators of some drastic changes occurring in the accelerated turbulent boundary layer. It should be emphasized that the convenience associated with the use of any of these criteria should not mask our concern about their basic inadequacy as markers of the 'onset' of relaminarization.

Finally, a separate discussion is needed on the criterion proposed by Badri Narayanan and Ramjee [4]. These authors argued that the physical criterion for relaminarization should be sought in the decrease in the peak value of the relative turbulence intensity; all other observations and criteria, they argued, pertain only to the phenomenon of accelerated turbulent boundary layers without necessarily involving relaminarization. They suggested that the effect of the favourable pressure gradient was merely to reduce the Reynolds number which, on reaching a critical value, allows the decay of turbulent fluctuations to occur. On the basis of their experiments, they argued that $(u'/U_\infty)_{\max}$ starts decreasing around $R_\theta = 300 \pm 100$, and that this should serve as a parametric criterion for the onset of relaminarization.

[3] Fiedler and Head [11] also suggested that the minimum value would be about 1.28.

The fundamental presupposition of their proposal is that all (normalized) turbulent stresses decay more or less simultaneously. This however leads to an immediate difficulty in view of the later experiments (flow BK) which show that v'/U_∞ and w'/U_∞ need not decrease at the same point at which u'/U_∞ does. As regards their R_θ-criterion, one sees that while in the flow $BR2$ the $(u'/U_\infty)_{\max}$ starts decaying at $R_\theta \simeq 250$, in the flow BK this occurs around $R_\theta \simeq 2000$!

We may here make a somewhat more general remark about an often-held view that low Reynolds numbers ($R_\theta \simeq 300$—400) are prerequisite to the occurrence of relaminarization. This argument derives support from the fact that in several experiments on relaminarizing boundary layers, K reaches its suggested critical value ($\simeq 3.5 \times 10^{-6}$) whenever R_θ is in the range 300—400. This general impression is largely due to the fairly low Reynolds numbers attained in most academic wind tunnels (see Section 3.1). A close look at all the data suggests that this conclusion does not hold when the initial Reynolds numbers are even moderately high. For example, in the flow BK, K reaches 3.5×10^{-6} when R_θ is around 1600.

2.4 Completion of Relaminarization

In contrast to the onset, the completion of relaminarization can be defined without much ambiguity. Relaminarization can be defined to be complete (Narasimha and Sreenivasan [25]) when the effect of the Reynolds stresses on the mean flow development becomes negligible. This of course is an asymptotic attainment, but we will agree to identify a boundary layer as fully relaminarized if we can predict to within a 'few percent' all mean flow parameters including skin-friction and heat transfer, *without invoking any specific closure hypothesis*.

In the sense described above, relaminarization is complete in region (d) of Fig. 2. The extent of the intermediate region (c), and indeed whether at all it exists, depends largely on the way the downstream boundary of region (b2) is identified. Since turbulent stresses no longer affect the mean flow dynamics in region (d), it can in principle be analyzed exactly. Indeed, Narasimha and Sreenivasan [25] have done this through an asymptotic analysis, which will be briefly mentioned in Chapter 6.

3. Relaminarizing Boundary Layers: Measurements

3.1 Initial Conditions

One of the features of existing relaminarizing flows is that the initial Reynolds numbers are generally low. Fig. 3 shows the histogram of relaminarizing boundary ylaers plotted against the initial Reynolds number R_θ. It is seen that the largest fraction of experiments is in the vicinity of $R_\theta \sim 500$, and only one experiment with $R_\theta > 2500$!

The difficulty seems to be inherent in the usual manner in which the desired accelerations are produced, which is to attach a suitably shaped liner on one of the wind-tunnel walls in the test section; other variations of this are essentially equivalent for this discussion. Accommodating the liner reduces not only the useful development length for the boundary layer, but also the test-section speed due to the blockage effect. Thus, even in wind tunnels nominally designed to produce a high Reynolds number *constant-pressure* boundary layer, the two effects mentioned above cumulatively reduce the initial flow Reynolds number. Most academic

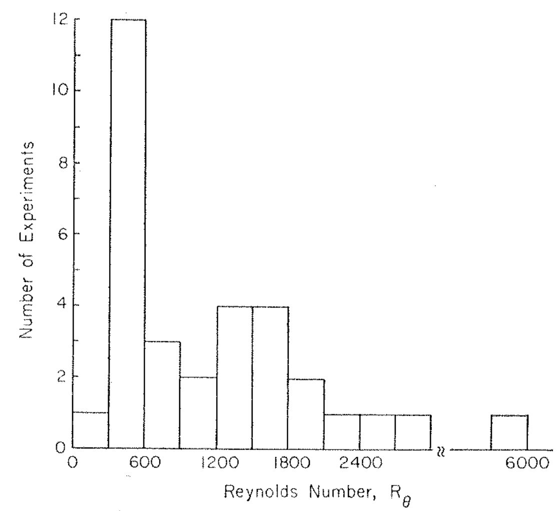


Fig. 3. Distribution of the number of experiments with respect to the initial flow Reynolds number, R_θ

wind tunnels are not big (typically, the characteristic test section dimension is about 30 cm), and are operated at speeds not too far from about 30 m sec^{-1}, so that the maximum attainable R_θ is itself not very large, even under design conditions, so that the concentration of experiments around an initial $R_\theta \approx 500$ (while being unfortunate) is not surprising.

Since the self-professed goal of the experiments here has been to demonstrate the occurrence of relaminarization, and to document quantitively the progression from a fully turbulent to a laminarized state, it is clearly essential to ensure that the initial boundary layer is fully-turbulent and can be described by known 'laws'. Since our understanding of anything but a constant-pressure boundary layer is deplorably small, initially we want to have a constant-pressure boundary layer with a sizeable log-region and a 'fully developed' wake component; in particular, it is undesirable to have perturbed boundary layers or those developing initially under an adverse pressure gradient. Only then can the initial functions needed for the calculation methods, when not measured, can be prescribed with

relatively high degree of confidence[4]. A criterion due to Coles [10] suggests that a minimum R_θ of about 5000 would be desirable for the initial constant-pressure flow. As a working rule of thumb, one then needs a wind-tunnel designed to produce *under normal circumstances* clean constant-pressure boundary layers with R_θ of about 10000 or more.

In most of the existing relaminarizing flows, one inescapable question arises: How much of the details observed for low-R_θ flows hold for high-R_θ flows? One hopes that essentially the same phenomenon occurs in both cases, but it is possible that many features presently associated with relaminarization take a different appearance in high R_θ flows. The inherent difficulty is that large and small scale features cannot easily be separated in low R_θ flows, and many details of the phenomenon are masked. In particular, persistent familiarity with low Reynolds number flows has often led to the mistaken notion that low Reynolds numbers are essential for relaminarization.

3.2 Mean Velocity Distribution and Boundary Layer Thickness

In the initial stages of acceleration, the velocity profile in the wall-region becomes non-linear, the log-law disappears, the wake region diminishes and the velocity in (what would be expected to be) the buffer layer and lower log-region overshoots the standard logarithmic line. Unfortunately, the order in which these various changes occur, or indeed whether there is such an order, is not at all clear; the data are in fact often conflicting. At low Reynolds numbers, the *SK* and *BR*-data show that the disappearance of the wake region precedes the breakdown of the log-law. The *SK*-data further suggest that the measured velocity profile in the inner viscous region deviates from the $U^+ = y^+$ line even at $y^+ \simeq 2$ well upstream of where the log-law begins to get affected. But the *OM* flows — also at low Reynolds numbers — and the flow *BK* at moderately large Reynolds numbers, show a well-defined $U^+ = y^+$ line until the very end of the acceleration region, well after the wake and log-regions have disappeared altogether; its extent in y^+ units is even greater than that in the constant-pressure flows. The conclusion from the *BR*-data is uncertain because of the scatter but, closest to the wall, these as well as the *SS*-data occasionally show departures from the $U^+ = y^+$ line which are in the wrong direction! The *PH*-data, on the other hand, show that the appearance of significant non-linearity in the wall slightly precedes the disappearance of the wake and logarithmic regions.

It is precisely near the wall that the greatest care in measurement is required. Typically, during the late stages of relaminarization, about 90% of the velocity variation occurs in the lowest 10% of what is usually referred to as the boundary layer thickness. A significant part of the displacement and momentum thickness comes from this region, which is usually of the order of about 0.1 mm in many experiments. This is a factor to be carefully evaluated in assessing the accuracy of δ^* and θ in very thin boundary layer flows.

Towards the late stages of relaminarization, approach to the free stream velocity occurs very slowly in the outer 90% of the sheared region so that it is

[4] This is quite important since in no experiment can all initial functions required for all computational methods be anticipated and measured.

barely possible to define accurately a boundary layer thickness in the usual way. If the height of the test-section is not very large in comparison with the initial boundary layer thickness, or if there is a weak unsteadiness in the wind-tunnel, or if the free-stream turbulence is not small, it is easily possible for two different people to measure significantly differing boundary layer thicknesses; not much quantitative significance can thus be attached to δ values quoted in the literature. Quite often (e.g., Ramjee [34], Brinich and Neumann [8]), a large part of this weakly sheared region is missed in the measurements, which accounts for the unjustified claims that there is considerable negative entrainment or 'detrainment' in the accelerating region.

3.3 Skin-Friction Measurements

Generally, the skin-friction measurements (where made) are the least reliable among the mean flow parameters. The skin-friction initially rises as the turbulent boundary layer responds 'normally' to the acceleration and, with the exception of the sink flows, generally decreases in the relaminarizing region. In the sink flows, the local laminar c_f could be larger than that of the initial turbulent boundary layer, and so c_f may in fact increase with x. When there is a maximum in the c_f, it usually occurs downstream of where K and Δ_p reach their 'critical' values.

The usual methods of measurement of skin-friction in constant-pressure flows depend on the existence of similarity laws in known form, and hence do not hold in relaminarizing boundary layers. This accounts for the fact that less than half the experiments have attempted this measurement in relaminarizing boundary layers. The limit of accuracy quoted is often of the order of $\pm 15\%$, although larger errors seem quite likely. Skin-friction has been measured by three different methods in accelerating boundary layers, and each deserves some comments in turn.

(*a*) *Wall-slope method:* In this method, one measures the slope of the mean velocity distribution $(\partial U/\partial y)$ at the wall. Two versions exist. In the simplest case (Blackwelder and Kovasznay [7], Schraub and Kline [37], Okamoto and Misu [29]), one linearly extrapolates to the wall the closest few points measured near the wall. But as the linear region is usually small in extent (e.g., Schraub and Kline [37]), a more reasonable and sophisticated method (e.g., Simpson and Wallace [41]) is to expand the velocity profile near the wall to two terms as

$$U/U_\infty = \frac{c_f}{2}\left(\frac{yU_\infty}{\nu}\right) - \frac{K}{2}\left(\frac{yU_\infty}{\nu}\right)^2, \tag{3.1}$$

and determine c_f by the best fit. Typically, the first five data points cover the region $y^+ \lesssim 10$.

Since the velocity profile is a lot steeper in the accelerated region than in the constant-pressure case (note that c_f may drop in the accelerating region but not τ_w), the accuracy of the method depends on the number of experimental points that one can reliably obtain in the small region close to the wall; we have already remarked on the difficulties that seem to occur here. A single hot-wire has quite often (e.g., flows *BK*, *SW*, *SS*) been used to get as close to the wall as possible, but the generally large fluctuations and shear in the wall region contribute to

measurement uncertainties. Further uncertainties result because the hot-wire position is not known very accurately. This may partly account for the discrepancies among various experiments in trends of the measured mean velocity near the wall.

(*b*) *Fence technique:* A fence is essentially a square-edged step in the surface, whose height h and streamwise extent are very small in comparison with its width; other geometries are also possible. Usually, two static pressure holes are drilled immediately in front and to the rear of the step respectively. The step is entirely submerged in the viscous sub-layer of the turbulent boundary layer, and one relates the measured pressure difference across the fence to the wall stress. The technique has been used by Patel [30] and Patel and Head [31] in low speed flows, and by Nash-Webber [28] in compressible flows.

At very low fence Reynolds numbers R_f $(\equiv hU_*/\nu)$, the pressure difference δp across the fence is proportional (as in Stokes' flow) to the first power of a characteristic velocity U_m of the oncoming stream; the characteristic velocity U_m could be the mean velocity at some height βh $(\beta < 1)$. For large R_f, on the other hand, one has

$$\delta p \propto U_m{}^2.$$

In the intermediate range of R_f, it seems reasonable to put

$$\delta p = aU_m{}^b, \tag{3.2}$$

where a and b are constants depending, among other things, on the geometry of the fence and the fence Reynolds number R_f.

In constant-pressure flows, if h is small, one can write

$$U_m = \beta\left(\frac{hU_*{}^2}{\nu}\right), \tag{3.3}$$

so that (3.2) becomes

$$\delta p^0 = a\left(\beta\frac{hU_*{}^2}{\nu}\right)^b \tag{3.4}$$

where δp^0 is the pressure difference δp across the fence when $dp/dx = 0$. In the presence of a strong streamwise pressure gradient, the relationship between the characteristic velocity U_m and the wall stress τ_w is no longer as simple as in (3.3). Instead, we should be guided by (3.1), which can be written in the form

$$U/U_* = y^+ + \frac{1}{2}\Delta p(y^+)^2 + 0(y^3), \tag{3.5a}$$

from which it follows that

$$U_m = \beta\left(\frac{hU_*{}^2}{\nu}\right)[1 + (\beta/2)\,\Delta p\,h^+ + 0(h^2)]. \tag{3.5b}$$

Using (3.2), (3.4) and (3.5b), we get

$$\delta p \simeq \delta p^0[1 + (\beta/2)\,\Delta p\,y^+]^b. \tag{3.5c}$$

Patel [30] determined that $b \simeq 1.5$ and $\beta = 2/3$, in the range $4 < R_f < 8$.

Nash-Webber [28] tried to improve the situation by assuming that the $O(y^3)$ terms in (3.5a) are of the type $\Delta p^2(y^+)^3$. The appropriate term should involve $\left(\frac{\partial^2 \tau}{\partial y^2}\right)$, which cannot be represented easily by $\Delta p^2(y^+)^3$, and hence it is not clear (in terms of the present discussion) how rational this improvement is. Incidentally, Nash-Webber chose $b = 1.31$.

It appears from here that the fence technique is at best only plausible. The difficulty clearly is that the model used in determining the relation (3.5c) is a highly simplified and inadequate model for the rather complex flow field around the fence. In reality, the flow is unsteady with large amplitude fluctuations, and the ultimate defense of the technique can only rest on the actual performance. So far, it appears that it is at best comparable in accuracy to the measurements made by the wall-slope method.

(c) *Surface heat transfer gauge:* In this technique, the local heat transfer rate from a small heating element embedded into the wall is related to the skin-friction. The design and use of such heating elements requires some care, and the relevant latest technology can be found in Rubesin et al. [36].

If (a) the thermal boundary layer of the heat transfer gauge is thin enough to be submerged entirely within the linear region of the mean velocity profile, and (b) the streamwise extent of the heating element is not so small as to render the boundary layer approximation in the thermal layer (i.e., $\partial T/\partial x \ll \partial T/\partial y$) invalid, it has been shown (Liepmann and Skinner [22], Spence and Brown [42]) that the heat transfer rate q_w and the wall shear stress τ_w are related by

$$q_w = \alpha \tau_w^{1/3},$$

where, for a given fluid and heating element, α is the same constant in laminar as well as turbulent flows. In principle, this makes the heat transfer gauge an ideal instrument to use in relaminarizing flows. However, because the momentum boundary layers are unusually thin in the acceleration region, the requirement (a) is hard to satisfy unless a very small element and low overheat ratios are used. If the element is very small, however, (b) becomes difficult to satisfy. Spence and Brown [42] give plausible limits within which the size of the heating element must lie for these conditions to hold. For air, they are

$$7.8 < U_* l/\nu < 46 \tag{3.6}$$

where l is the effective 'length' of the heating element in the streamwise direction.

Badri Narayanan and Ramjee [4] used this technique in measuring skin-friction in relaminarizing flows. They used a wall-embedded hot-wire (15 μm dia., 0.635 cm span) for the heating element, and calibrated it in a two-dimensional channel. The effective streamwise dimension of this heating element can be expected to be a few times the actual 'length' (Brown [9])[5]. This effective length can in principle be obtained by the use of the calibration curve for the heat transfer gauge. A typical calibration curve from Ramjee [34] gives an effective

[5] This occurs because the temperature distribution across a heating element is not top-hat, but is spread out to either side.

'length' which is half the actual 'length', an impossibility! There is thus a possible error in the calibration curve of Ramjee. Assuming (as appears plausible; see Brown [9]) the effective dimension to be about four times the actual 'length' of the sensing element, we would have (for *BR*2)

$$U_* l\nu \lesssim 5,$$

which from (3.6) is a bit marginal on the requirement (b) above. Since (3.6) is only a rough criterion, and the requirement (a) is amply satisfied, we expect a priori that the c_f measurements in the *BR* experiments are likely to be reliable.

3.4 Two-Dimensionality

For those experiments in which all the mean parameters (including c_f) have been measured at fairly close intervals, it is possible (and useful) to make a two-dimensionality check by evaluating both sides of the momentum integral equation

$$\frac{d\theta}{dx} + \frac{1}{U_\infty}\frac{dU_\infty}{dx}(H + 2)\,\theta = \frac{c_f}{2}. \tag{3.7}$$

In relaminarizing flows where there is a rapid variation of the fluctuations in the streamwise direction, it may be thought necessary to add to RHS the term

$$\frac{d}{dx}\int\limits_0^Y (\overline{u^2} - \overline{v^2})\,dy,$$

where Y is some height outside the boundary layer. However, on evaluating this term for the BK data (see Fig. 4c), it was found to be small (at most 5% of c_f), so that it will be disregarded uniformly.

By integrating (3.7), we can write

$$\theta(x) = \theta(x_0) + \int\limits_{x_0}^{x}\left[\frac{c_f}{2} - \frac{1}{U_\infty}\frac{dU_\infty}{d\xi}(H + 2)\,\theta\right]d\xi, \tag{3.8}$$

so that the measured $\theta(x)$ can be compared with that implied by (3.8).

Equations (3.7) and (3.8) have both their advantages. Since both $d\theta/dx$ and dU_∞/dx on the left side of (3.7) are fairly large quantities with opposite signs, c_f evaluated as the difference of two large numbers cannot be expected to be very accurate; the integrated version (3.8) overcomes this disadvantage. However, since any measurement errors become cumulative in the $\theta(x)$ calculated according to (3.8), it does not give us the correct idea of where precisely the imbalance occurs. So, here, both (3.7) and (3.8) have been used in all cases as a test of two-dimensionality, and will be referred to as needed.

Figures 4 show typical cases illustrating good, marginal and poor momentum balance, and the nature of the two-dimensional momentum balance for each flow is shown qualitatively in Table 1. For flows in which the momentum balance is designated as poor, the imbalance is much larger than the combined uncertainty of the two sides of (3.7). The fact that there are very few experiments with good

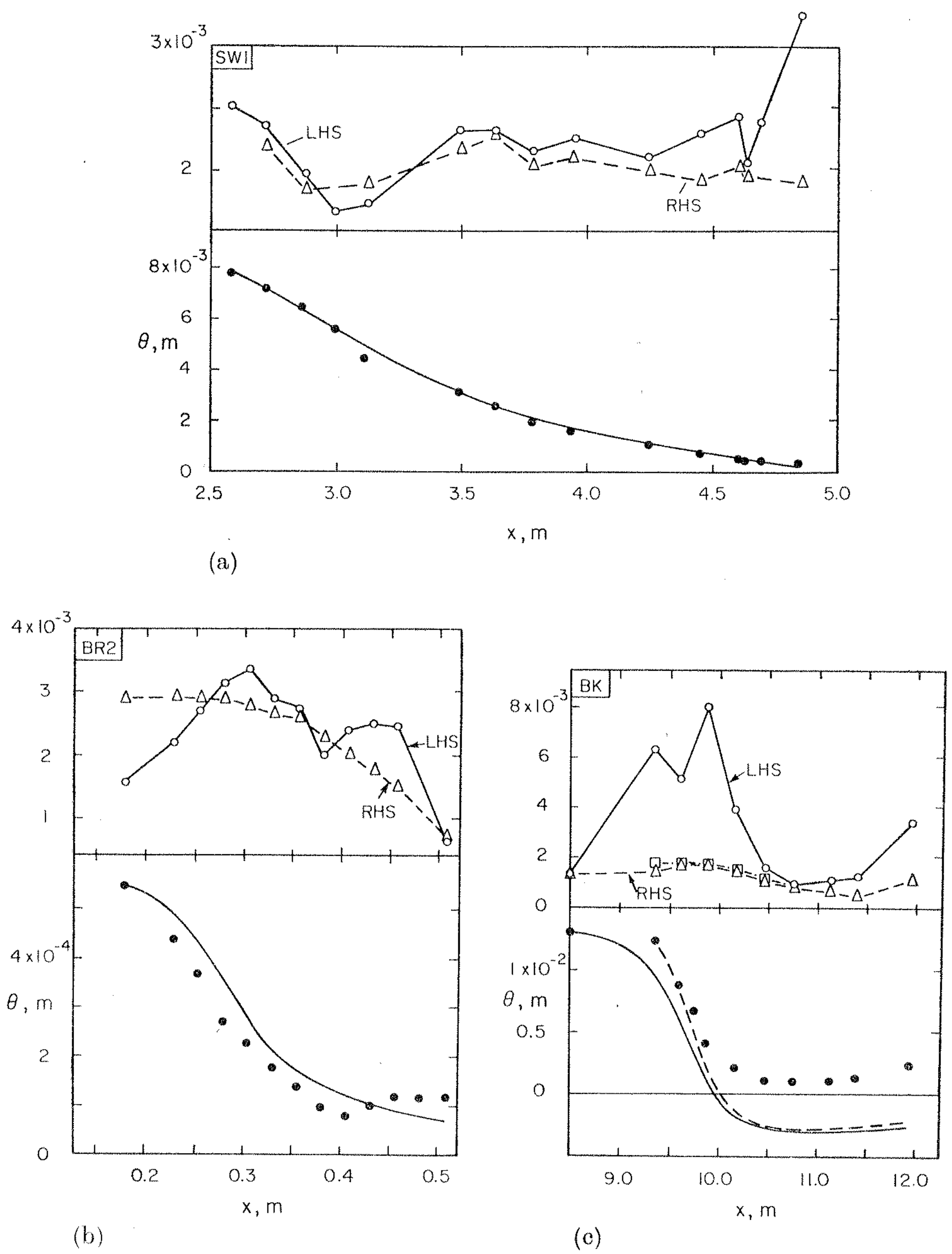


Fig. 4. Typical two-dimensional momentum balance in relaminarizing boundary layers: (a) Good, (b) Marginal, and (c) Poor. The upper part of each figure compares the left hand side (○) of Eq. (3.7) with the right hand side (△). The lower part compares $\theta(x)$ evaluated from Eq. (3.8) with the measured values (●). In Fig. 4c, □ indicates $c_f/2 + \frac{d}{dx}\int_0^Y (\overline{u^2} - \overline{v^2})\, dy$, and —— indicates $\theta(x)$ evaluated from (3.8) with the origin x_0 chosen at $9.36m$

momentum balance and the alarmingly high degree of imbalance in some flows compel us to seek explanations external to the possible experimental errors.

One possible explanation lies in the mean streamline curvature produced by the deflection of the flow by the liner that produces the acceleration. This effect can be expected to spread only minimally to the boundary layer on the opposite flat wall if the boundary layer is a small fraction of the total test-section height. Unfortunately, this is not true in general: the ratio of the boundary layer thickness to tunnel height is initially of the order of 0.15 (sometimes even higher), and the liner occupies a significant part of the test-section height. Although the curvature effect is negligible near the wall, it increases directly with distance away from the wall. For example, at $x = 9.8$ m and $y/\delta = 0.8$ in the fow BK, the streamline curvature is of the order of 0.002 cm^{-1}. This can induce a normal pressure gradient of about 20% of the streamwise pressure gradient, which in turn can produce a secondary motion that is clearly not accounted for by the momentum integral equations (3.7) and (3.8). Thus, an imbalance will inevitably be observed unless the boundary layers are thin compared with the wind-tunnel height.

Curvature affects the momentum balance in another indirect way in a test-section of finite span. Launder (private communication) suggests that the normal pressure gradient (associated with the streamwise curvature discussed above) is greater in the side-wall boundary layers than that required by the slow-moving fluid there to describe the same streamwise path as described by fluid elements in the main part of the flow. Consequently (referring now to Fig. 5), the flow has a tendency to go down the side walls leading to a thickening of θ; later in the acceleration region, however, the pressure on the flat plate is higher than on the curved wall, thus leading to a flow up the side walls and to a too small θ.

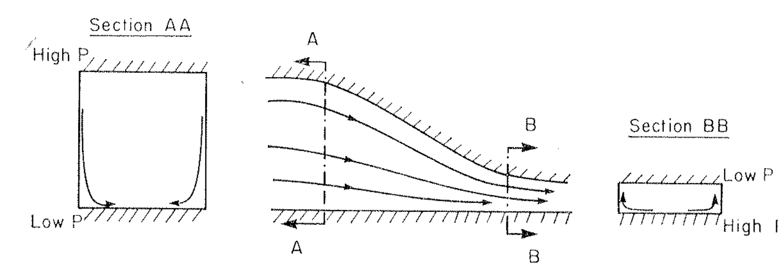


Fig. 5: The effect of finite-size test sections in inducing two-dimensional momentum imbalances

3.5 *Turbulence Stresses*

Most experimenters have measured only u fluctuation (because it is the simplest), and our understanding even of these measurements is not complete. There is, for example a substantial difference in the rates of decay of $\overline{u^2}$ near the wall between flow at low Reynolds numbers (flows BR2 and BR3, for example), and those at moderately high Reynolds numbers (flow BK). A definite need for more systematic measurements exists.

In most flows studied so far, the excessive thinning of the relaminarizing boundary layers renders the near-wall region inaccessible to crosswires of the usual size. This is unfortunate because important phenomena occur fairly close to the wall, and v', w' and $\overline{uv}$ measurements are quite helpful in elucidating the details. It is clear then that thicker boundary layers or laser-Doppler velocimeter measurements would help.

Even in the outer layer where these difficulties are absent, v' and w' measurements have been reported only in one flow, BK. Unfortunately, this flow has a poor momentum balance (see Section 3.4), but one suspects that this imbalance may not affect the turbulence structure drastically.

In spite of these difficulties, some general patterns can be discussed. Again, it is convenient to refer to the broad flow classification of Section 2.1. In region (a), the turbulent boundary layer responds normally to acceleration by increased values of turbulence intensities, and this increase extends also to a part of region (b) until the reduction in bursting rate (and thus in turbulence energy production), coupled with sustained energy dissipation, reduces the turbulence intensities near the wall. Figure 6 shows for the flow BR2 the behaviour of streamwise turbulence intensity along a mean streamline in the inner layer. Notice that the initial rise is followed by a significant drop in the region $0.3 \text{ m} \lesssim x \lesssim 0.4 \text{ m}$ (roughly corresponding to region (d) of Fig. 2). Measurements of v' and w' and $-\overline{uv}$ are not available close to wall, the closest ever made (in flow BK) being about 1/10th of the boundary layer thickness midway through the acceleration. If we take a rough guidance from flow BK, we come to the conclusion that v fluctuations show the most rapid decay. This is as it should be, in view of the direct connection between v fluctuation and turbulent bursting which essentially vanishes during relaminarization (Schraub and Kline [37]). This same result also implies that $-\overline{uv}$ must decay to small values in the inner layer, but direct measurements are lacking. Finally, the quick rise in $\overline{u^2}$ observed in Fig. 6 is an indication of retransition of the laminarized boundary layer to turbulence (see Chapter 5 and Section 6.4).

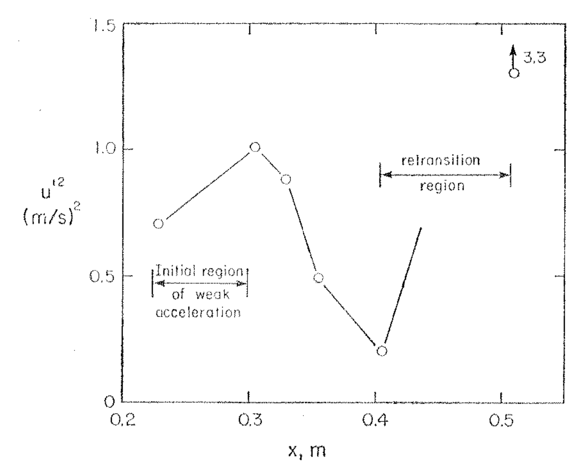


Fig. 6. The decay of streamwise turbulent intensity along a streamline in the inner region of a relaminarizing boundary layer. Initially, the intensity increases as the boundary layer responds to the acceleration; as relaminarization sets in, it shows a significant decay. The final sharp rise is due to retransition to turbulence

It should be noted that all *relative* turbulence intensities decay everywhere in the boundary layer (excepting region (a) and part of (b)) during the relaminarization process. But the decrease in absolute intensities generally occurs only near the wall. The exception to this last statement is u fluctuation which shows some decay everywhere. Fig. 7 shows how $\overline{(v/U_\infty)^2}$ decays everywhere (Fig. 7a) while there is a small reduction in $\overline{v^2}$ only fairly close to wall (Fig. 7b). Elsewhere, v^2 either remains constant or even shows a slight increase. This is true of $\overline{w^2}$ too.

In particular, the Reynolds shear stress in the outer layer remains essentially constant along a given mean streamline (Fig. 8). Again, because the mean velocity continually increases with x, the *relative* shear stress shows a monotonic and rapid decline with x.

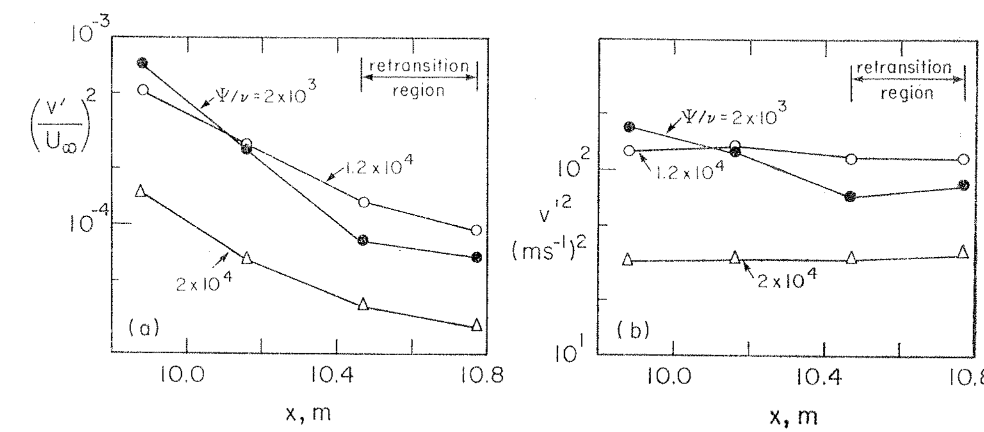


Fig. 7. The variation of turbulence intensity in the normal direction along three streamlines. Fig. 7a shows that the relative intensity decreases everywhere, while Fig. 7b shows that there is no reduction in absolute intensities except quite close to the wall. Data from [7]

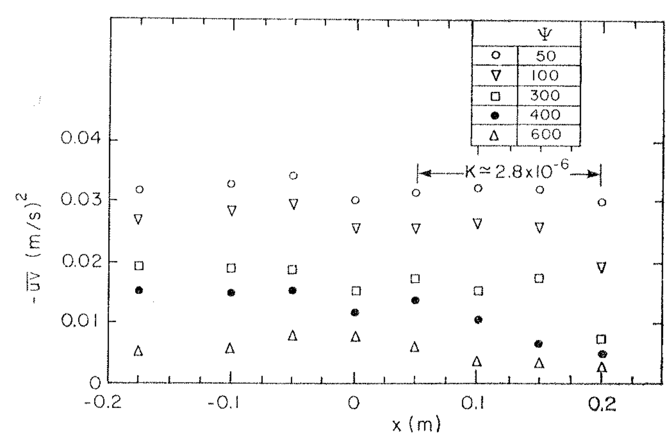


Fig. 8. The 'freezing' of Reynolds shear stress along outer streamlines during relaminarization. Data from [5]

4. Laminarescent Boundary Layers

4.1 General

An examination of the laminarescent flows is very useful and provides challenges for developing computational turbulence models. Flow prediction in region (d) where relaminarization is complete is relatively simple compared with that in laminarescent region (b). This may at first seem ironical, but is obvious if we realize that the former is a limiting case: for a model to predict reasonably the fully relaminarized region (d), it only needs to be able to switch off turbulence energy production at the appropriate point and merely recognize that turbulent fluctuations beyond that point are irrelevant to mean flow dynamics; the results can then be expected to be relatively insensitive to the details of the turbulence model unless it is doing gross violation to the physics. On the other hand, for a model to predict laminarescent flows, it is essential, among other things, to maintain the correct amount of production and dissipation everywhere. Thus, a great many details which are irrelevant in the fully relaminarized flow become relevant here.

When the flow accelerations are sharp, the laminarescent region becomes narrow in extent and is not generally amenable to a detailed examination. It was suggested in Section 2.2 that, if the acceleration (suitably expressed) can be maintained, as in sink flows, at the appropriate level over a considerably large distance, the laminarescent state can be studied more conveniently. Implicit in this statement is the assumption that the history of acceleration is immaterial. This is clearly so in region (b1) of Fig. 2 where the acceleration, in spite of being strong enough to produce significant departures from constant-pressure laws, varies weakly enough for the flow to be in local equilibrium. It is essentially in the study of this equilibrium region that sink flows are most useful. In the later stages of laminarescence (region (b2) of Fig. 2) where no local description can be expected to hold and history effects become relevant, this convenience does not exist.

4.2 Sink Flows

Sink flows are flows developing between two convergent planes. Both self-similar laminar and turbulent sink flows are possible. Indeed, sink flows form the only class of turbulent flows with varying free-stream velocity in which complete similarity may be attained (because both turbulence and viscous length scales 'grow' at the same rate). In these flows, the Reynolds number (no matter how defined) is invariant with x, and the skin-friction coefficient remains constant; the acceleration parameter K is a constant at all stations and completely describes the self-similar sink flow — whether laminar or turbulent. If the acceleration is very large, we expect that, asymptotically, an initally turbulent boundary layer becomes effectively laminar. If the acceleration is very weak, the turbulent boundary layer retains a structure that differs in no significant way from that of a constant-pressure flow. For some intermediate K, we expect the laminarescent state to occur.

To make matters clear, let us consider the two extreme cases first. From the

known solution for a laminar sink flow, the variation of the integral parameters can be written (see, e.g., Rosenhead [35]) as:

$$KR_\theta^2 = 0.14$$

$$c_f K^{-1/2} = 2.31$$

and

$$H = 2.07. \quad (4.1)$$

For the other extreme limit, we shall consider (for simplicity) the mixing length solution of Launder and Jones [20] in which the mixing length l was assumed (van Driest [49]) to be given by

$$l = ky[1 - \exp(-y^+/A^+)]. \quad (4.2)$$

Here, A^+ may be interpreted as a dimensionless sublayer thickness and is a constant (= 26) according to [49].

Figs. 9a, b display these two solutions in terms of the two integral parameters R_θ and H as functions of K. Referring to Fig. 9a, all laminar self-similar flows plot

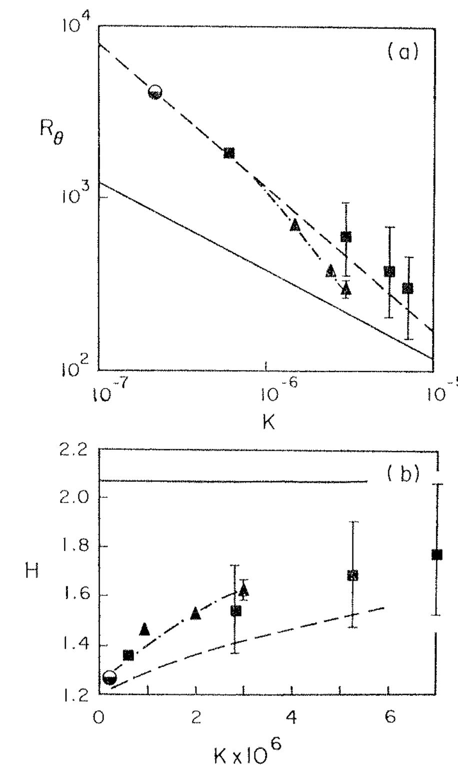


Fig. 9. The variation with K of integral parameters (a) R_θ and (b) H in laminarescent flows. ———, exact laminar solution [35]; — —, fully turbulent calculations [20]; —·—·—, mean through data for $K \lesssim 2.5 \times 10^{-6}$. For larger K, no selfpreserving flow appears realizable; the integral parameters then continuously vary even when K is held constant. ◒ from [12]; ■ from [4]; △ from [13]. Adopted from [13]

on the lower curve. On the other hand, turbulent self-similar flows will plot on the upper curve only as long as their turbulence structure is not too different from that of a constant-pressure turbulent boundary layer. When substantial departures from the latter occur, deviations from the upper curve can be observed. Similar considerations hold also for Fig. 9b. If this intermediate state is self-similar, the integral parameters of the flow can be represented by a single point on each of the Figs. 9a, b, and the parameters K (or Δ_p, since c_f is a constant) and R_θ (or H) define the flow completely. Anticipating from Chapter 6 that this is true also of the equilibrium laminarescent region (b1), we conclude that the equilibrium laminarescent region with given local values of K and R_θ has the same structure as that of a self-similar sink flow with the same K and R_θ. Not much can however be said if no self-similarity is attained in the sink flow.

Several sink flows have been studied in the past (see Table 2), and are discussed in Chapter 6 and Appendix. A few general observations can however be made here by examining the integral parameters R_θ and H. Data for a selected few experiments have been plotted on Figs. 9a, b (with others omitted in interests of clarity). As was to be expected from a discussion in Section 2.1, no significant departures from the constant-pressure turbulent boundary layer behaviour occur when the K values are small ($\simeq 10^{-7}$). As K increases, such departures do set in. For a given K, we recall that there is a unique state of self-preserving flow whose Reynolds number lies between the laminar and turbulent curves of Fig. 9a. Such self-preserving sink flows can however be produced within reasonable length of wind-tunnel only if one can match (by some trial and error, as Jones and Launder [13] do) the initial flow Reynolds number with that expected in the self-preserving final state. This matching appears possible only for K values lower than about 2.5×10^{-6}. Larger K implies lower R_θ (see Fig. 9a). Recalling Preston's [33] argument that there is a minimum R_θ below which no decent turbulent boundary layer can exist, we conclude that there is an upper limit for K values possible for a self-preserving sink flow; this upper limit on K appears to be $\sim 2.5 \times 10^{-6}$. For flows with larger K, the initial Reynolds numbers are invariably higher than the matching sink-flow values, and therefore a continuous adjustment from the fully developed turbulent to fully laminar states occurs (see especially the BR flow with $K = 7 \times 10^{-6}$) even if K is held a constant. In this regard, such flows are no different from other relaminarizing flows with varying K.

Table 2. *List of sink flows*

Source	K	Initial R_θ (approx.)	Comments
Herring and Norbury [12]	2×10^{-7}	3750	Fully-turbulent flow (mild acceleration)
Launder and Stinchcombe [21]	7×10^{-7}	1000	Poor momentum balance
	1.3×10^{-6}	400	
	3×10^{-6}	200	
Badri Narayanan and Ramjee [4]	2.9×10^{-6}	2050	Not self-preserving
	5.2×10^{-6}	1240	
	7×10^{-6}	780	
	7×10^{-7}	1730	Self-preserving
Julien, Kays and Moffat [14]; Loyd, Moffat and Kays [23]	6×10^{-7}	1600	Not self-preserving
	8×10^{-7}	1200	
	1.4×10^{-6}	920	
	2×10^{-6}	1000	
	2.5×10^{-6}	560	
Jones and Launder [13]	1.5×10^{-6}	710	
	2.5×10^{-6}	390, 340	Self-preserving
	3×10^{-6}	470, 390, 340	
Simpson and Wallace [41]	2.2×10^{-6}	1530	Not self-preserving
	3.2×10^{-6}	1700	

Among all the laminarescent sink flows listed in Table 2, only three flows of Jones and Launder [13] and one of Badri Narayanan and Ramjee [4] are reasonably self-preserving. (Possibly for reasons already mentioned, the Jones-Launder flows with $K = 3 \times 10^{-6}$ were not quite self-preserving.) The Jones-Launder experiments are the most detailed to-date, and should form a good basis for developing turbulent models for computing laminarescent flows.

5. Retransitional Boundary Layers

Unfortunately, only a few experiments (e.g., flows L and BK) have paid attention to this problem, and these are clearly sketchy. From flow visualization and an observation of oscilloscope traces near the wall, Launder [19] concluded that retransition to turbulence occurs in essentially the same way as in natural transition, (i.e., by the formation and growth of turbulent spots). The newly created turbulence can be clearly distinguished from the low-frequency background turbulence: Blackwelder and Kovasznay [7] in fact traced contours of constant intermittency in restransitional boundary layers. Others (e.g., BR and PH) inferred retransition to have occurred when some mean parameter such as the shape factor or skin-friction showed a drastic change in character. It has often been suggested that the occurrence of a maximum in the shape factor marks the 'retransition point', but this can only be a rough guide. Further work on relaminarizing boundary layers must pay attention to this poorly studied region.

A much better appreciation for the nature of retransition and the 'point' of its occurrence can be obtained with a little background of the ideas described in Section 6.3. We therefore relegate further discussion of this topic to Section 6.4.

6. The Physics of Relaminarizing Boundary Layers

A motivation for our analysis here is helping the development of turbulence modeling for the computation of highly accelerated boundary layers. Currently available turbulence models can handle flows with weak accelerations, but to enable them also to handle large accelerations, it is necessary for us to understand the physics of relaminarizing boundary layers. It is thus appropriate at this point to ask what aspects of the data we have sifted through so far can be explained from general arguments that do not invoke detailed turbulence modeling; such aspects of the flow, if any, will serve as reliable anchor points for further development.

It is convenient to return to the flow classification of Fig. 2. Region (a) need not concern us here, because the constant pressure laws are valid here without any modification. We shall begin with a brief account of a rational analysis of region (b1).

6.1 The Equilibrium Laminarescent Region

In that initial part of the laminarescent region where the pressure-gradient *variations* are not large, it can be expected that a local similarity analysis is possible — i.e., the flow at a given x depends only on parameters at that x and not on the upstream history of the flow. The relevant local parameters are ν, δ and U_* (as in the constant-pressure boundary layers) and $\alpha \equiv \varrho^{-1}(dP/dx)$, which appears as the additional factor. Three independent length scales can be formed out of these four parameters: ν/U_*, U_*^2/α and δ. If the Reynolds number is large and the pressure gradient moderate enough to justify the assumption that

$$\nu/U_* \ll U_*^2/\alpha \ll \delta, \tag{6.1}$$

the necessary similarity theory (based on dimensional analysis and the method of matched asymptotic expansions) has been given by Kader and Yaglom [15] for the mean velocity distribution in a boundary layer developing in an adverse pressure-gradient. Briefly, Kader and Yaglom recover, by invoking the Reynolds number similarity, the usual log-law as the matching region between the inner viscous layer and an intermediate pressure-gradient layer characterized by the length scale U_*^2/α. The matching region between the pressure-gradient and outer regions (the latter of which, incidentally, obeys a modified defect law) comes out to be a half-power law. Kader and Yaglom [15] found that this predicted half-power dependence of the mean velocity on the distance from the wall did in fact apply over a significant fraction of the boundary layer. In the context of the laminarescent region where (6.1) can be expected to have some validity, essentially the same arguments suggest that over a middle portion of the boundary layer the mean velocity distribution will be given by

$$U(y) = k_1(\alpha y)^{1/2} + k_2, \tag{6.2}$$

where k_1 and k_2 are constants. Clearly, the extent of this half-power layer in relation to that of the log-layer depends on the magnitude of the pressure gradient, Reynolds number, etc.

As we noted in Section 4.2, the Jones-Launder data are probably the best in this class, and it is therefore useful to examine whether these experiments bear out (6.2). Fig. 10 shows that this is indeed so, and a reasonable half-power region exists.

Notice that these locally similar boundary layers are determined (in so far as the analysis is valid) essentially by R_θ and Δ_p. This is to be compared with the equilibrium sink flow boundary layers also determined by R_θ and K (or Δ_p, noting that c_f is a constant in those flows). This is the basis for our claim that equilibrium sink flows with a given R_θ and K differ in no essential respects from the equilibrium laminarescent boundarys whose local K and R_θ are the same.

The existence of the half-power law in the equilibrium region suggests that the boundary between regions (b1) and (b2) of Fig. 2 can be determined in principle by the breakdown of the half-power law. In practice, however, this requires a more complete analysis of all the equilibrium laminarescent flows with a veiw to determing the constants k_1 and k_2 in (6.2). This analysis is currently under way, and will be reported separately.

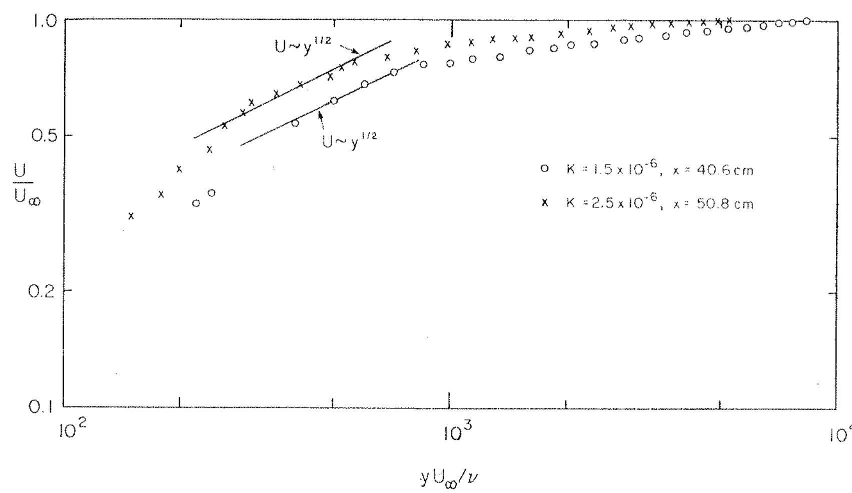


Fig. 10. The half-power law in equilibrium laminarescent flows. Data from [13]

6.2 The Island of Ignorance

For definiteness, the downstream 'boundary' of region (b2) will be identified with the 'onset' of relaminarization; although conceptually appropriate, this is admittedly an impotent definition in the absence of a simple parametric criterion. It is doubtful that any of the existing parametric criteria (see Section 2.2) are strictly relevant in this regard, even though several of them *may* serve as rough indicators. This is likely to remain a region of ignorance unless our understanding of region (b2) improves; at present, there is no clue to this end.

Region (c) of Fig. 2 is one in which, figuratively, relaminarization 'matures', from its 'onset' at the upstream 'boundary' to its completion at the downstream 'boundary'. Once relaminarization is complete, the flow becomes amenable to a detailed analysis. So, it appears quite reasonable to denote regions (b2) and (c) together as an island of ignorance in which there are no results of broad-based validity.

On a pragmatic level, however, suitable interpolations between the earlier (i.e., up to the end of region (b1)) and later (i.e., beyond (d)) stages of relaminarization give meaningful estimates of flow parameters in this island.

6.3 The Quasi-Laminar Region

By our definition, relaminarization is complete when the mean flow can be predicted without any recourse to an explicit turbulence model. This stipulation does not necessarily call for any significant decay in absolute magnitude of the

Reynolds stresses in a major part of the flow, but for the acceleration to reach such magnitudes that the turbulent stresses are effectively overwhelmed by this dominating single force; in spite of being comparable in absolute magnitude to their initial values, the turbulent stresses become relatively unimportant in determining the mean flow dynamics. Random fluctuations inherited from previous history might still remain in a large part of the flow and may be important for determining, for example, quantities such as the pressure fluctuations, but these are no longer relevant to mean flow dynamics. This region has thus been called the 'quasi-laminar' region by Narasimha and Sreenivasan [25].

A convenient (and potentially very valuable) way of analyzing this region is through an asymptotic analysis (the 'quasi-laminar' theory) in which the large parameter Λ is the ratio of the pressure gradient to the characteristic Reynolds stress gradient, defined by $(dP/dx)\,\delta/\tau$. For large values of Λ, Narasimha and Sreenivasan [25] have shown how one can split the mean flow field into a inner laminar subboundary layer and an inviscid but rotational outer layer. The development of these two layers can be evaluated independently by rather simple techniques, and a uniformly valid solution constructed.

It is not *a priori obvious* what values Λ must take in order that the pressure-gradient forces dominate the flow dynamics, but Narasimha and Sreenivasan [25] have shown that all flow parameters, including c_f and heat transfer coefficient, can be predicted quite well when Λ reaches[6] about 50.

In most of the flows listed in Table 1, the acceleration is generally so sharp that it takes very little physical distance for Λ to attain values of the order 50; the initial phases of relaminarization get crowded and it is often not possible to distinguish clearly all the regions marked in Fig. 1. It is this fortunate fact that renders the quasi-laminar theory valid over a much wider region of the relaminarizing boundary layer than could have been anticipated initially.

We shall here not recall the details of the theory except to note qualitatively some of its essential features. Narasimha and Sreenivasan [25] assumed that the narrow extent of the initial region renders (without much error) the inner-outer division of the flow valid almost from the point of commencement x_0 of the pressure gradient, and then explored its consequences. Their detailed calculations have shown that the outer parameters (such as R_θ and H) can be predicted quite well all the way from x_0, while this is not true of the inner parameters (such as c_f and heat transfer coefficient). The failure of the quasi-laminar inner solution near x_0 is clearly due to the fact that the inner layer is still turbulence-dominated and Λ has not attained large values. The unexpected success of the outer solution almost from x_0 (through the island of ignorance) needs an explanation. Firstly, the integral parameters R_θ and H change only slowly near x_0 and, *for short distances*, are not terribly affected even if the details of the calculations are not quite correct. Secondly, the inner layer contributions to these integral parameters are so small where they are incorrect (i.e., near x_0) that reasonable

[6] Specifically, in quoting this number, the Reynolds stress τ in the definition of Λ has been replaced by that in the fully-developed region before acceleration (the justification being that the shear stress is frozen during acceleration, as has been observed and can be shown by analysis). If the boundary layer is initially self-preserving, τ can be replaced by the wall shear stress there.

estimates of the outer parameters can be obtained from the quasi-laminar theory all the way from x_0.

We have already remarked that fully-turbulent calculations must be valid in region (a) near x_0. For c_f and heat transfer coefficient (which the quasi-laminar theory does not predict well until $\Lambda \gtrsim 50$), we are justified, on a pragmatic basis, to ask whether these fully-turbulent calculations have extended acceptance (to within reasonable accuracy) beyond region (a). Experiments show that such is the case (chiefly because these wall parameters also change smoothly near x_0) but, in general, these calculations cannot be extended right up to the beginning of region (d). Thus, there exists a gap in c_f prediction where neither the fully-turbulent nor the quasi-laminar calculations are valid (see Fig. 11). Clearly, the extent

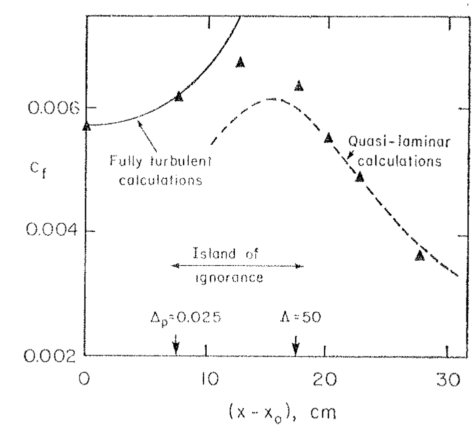


Fig. 11. The variation of skin friction coefficient with streamwise distance in a relaminarizing boundary layer. In the early upstream part, c_f can be predicted using fully turbulent calculations, while in the later stages the predictions from the quasi-laminar theory are valid. In the in-between region of ignorance neither calculation is valid

of this gap depends on the degree of sophistication employed in the fully-turbulent calculations but, with our current capability, we can tentatively identify this gap to coincide with the regions (b2) and (c) of Fig. 2. That essentially is the 'island of ignorance' mentioned in Section 6.2.

The theory also provides a framework for understanding the behaviour of fluctuating quantities. Again, inner and outer layers can be considered separately. In the outer layer, one can show that the distortion of turbulence vortex lines due to turbulent motion is much smaller than that produced by the mean rate of strain (i.e., the acceleration), and the viscous effects are anyway small. Thus, the so-called rapid-distortion theory (e.g., Batchelor and Proudman [6]) can be applied to calculate the change in turbulence intensities. If necessary, these calculations can be corrected for departures from isotropy using the theory of Sreenivasan and Narasimha [45]. Figs. 12a, b, and c show that these combined calculations are quite successful for u but somewhat less satisfactory for v and w.

Further, one can show in the outer layer that the turbulent shear stress equation (see, e.g., Townsend [48]) reduces under the additional assumption that the pressure-rate-of-strain terms are negligible (Sreenivasan and Narasimha [44]), to

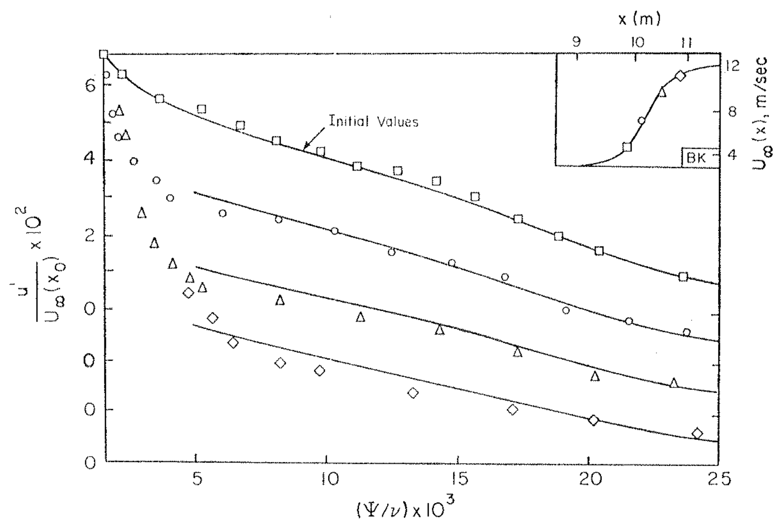
$$\frac{D(\overline{uv})}{Dt} = 0$$

or

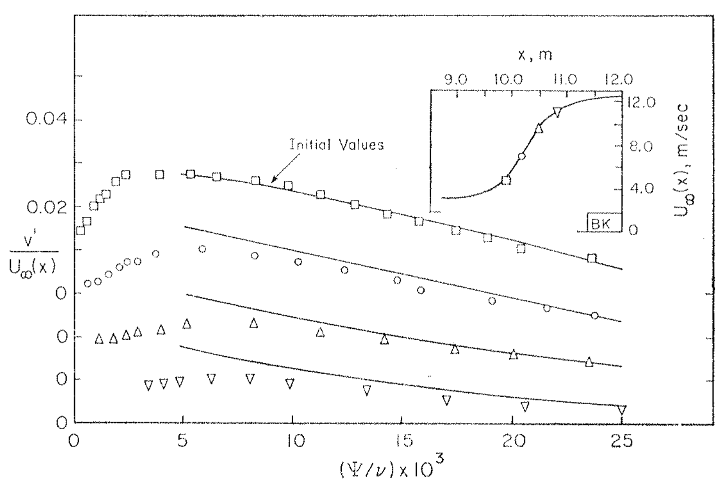
$$\overline{uv} = \text{constant}$$

along a given streamline, as observed (Fig. 8).

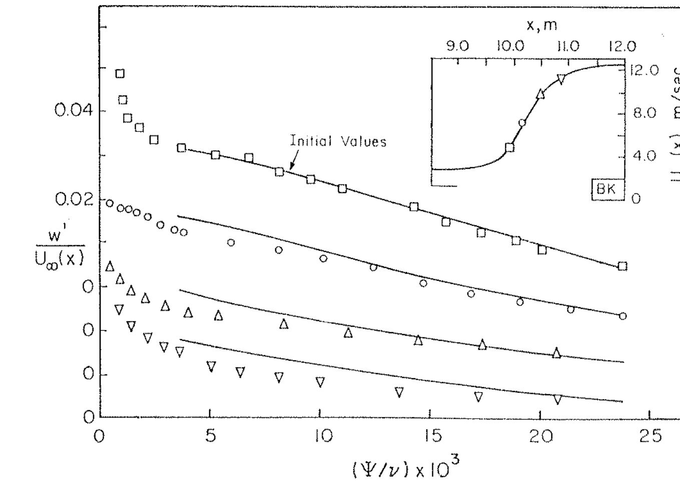
The fact that c_f in region (d) can be predicted well by the quasilaminar theory suggests that the inner layer must become effectively viscous-dominated by then, and that the Reynolds shear stress must have effectively decayed to zero. Un-



(a)



(b)



(c)

Fig. 12. Comparison of measured turbulence intensity distributions in the outer layer. (a) u, (b) v, and (c) w. Inset in each case shows the free-stream velocity variation and the location of measurement stations. Data from [7]

fortunately, as we remarked in Section 3.5, no direct measurements of Reynolds shear stress are available close to the wall. The only measurements made near the wall are of u, and these fluctuations persist even in region (d). In the absence of Reynolds shear stress, these existing (uncorrelated) fluctuations can be treated as perturbations superposed on a laminar layer, and can be shown to decay according to a characteristic power law. Again, this prediction is consistent with observations (Narasimha and Sreenivasan [25]).

6.4 *Retransition*

The same physical reasoning can be used to shed some light on retransition. Now, the inner layer, which would have reached an effectively laminar state in region (d), develops under somewhat unusual conditions because the turbulence in the outer layer does not necessarily decay (see Section 3.5). Thus, the inner layer sees a highly disturbed 'free-stream'. The inner layer remains essentially laminar in spite of this disturbed state of the 'outside' flow only due to the strong stabilizing influence of the favourable pressure gradient that the inner layer sees in the 'free-stream'. Soon after this favourable pressure gradient is released, we expect the inner layer to become unstable, and undergo transition to turbulence.

Narasimha and Sreenivasan [25] have analyzed the retransitional boundary layer data and have interpreted them in terms of the instability of the inner layer. Fig. 13 shows a summary of their calculations. The inner layer (displacement thickness) Reynolds number increases with distance as shown for the three flows PH1, BR2 and L. Also shown in the figure is the critical Reynolds number for the inner layer as estimated from the correlation given by Stuart in Rosenhead ([35], p. 543), in terms of the profile shape factor (for the inner layer, of course), which

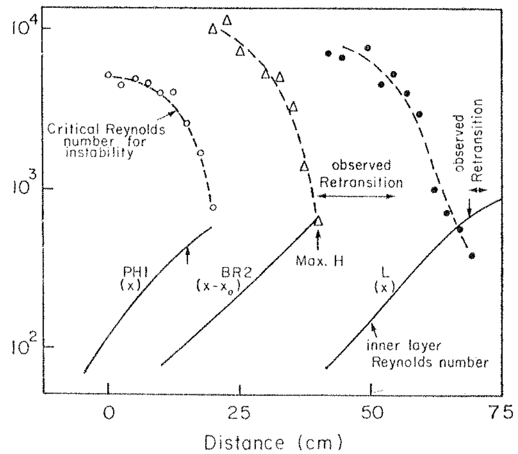


Fig. 13. Determination of the instability points for the inner layer. Dashed curves represent the critical (displacement thickness) Reynolds number for instability, obtained from the correlation of Stuart (in [35]) with the profile shape factor; data are based on the inner layer shape factor obtained from quasi-laminar calculations [25]. Solid curves represent the quasi-laminar solution [25] for the inner layer Reynolds number based on inner layer displacement thickness and the matching velocity between the inner and outer layers. The intersection of these curves gives the instability points. Arrow marks location of maximum H. Adopted from [25]

itself can be obtained by, for example, the Thwaites [47] method. The experimentally observed retransition region as well the location of maximum H are both shown. Although retransition point has not been precisely located in the experiments, it appears that retransition follows *immediately* after the onset of instability, which itself occurs soon after the pressure gradient is released. The highly disturbed 'free-stream' provides a short-circuit for the retransition process of the inner layer, and only the last stages of natural or direct transition, such as the formation and growth of spots, will be observed. All experimental evidence is in support of this conclusion; Blackwelder and Kovasznay [7] in fact observed the formation of new turbulent spots in the wall layer ($8 \lesssim y^+ \lesssim 30$) when the acceleration had decreased sufficiently.

7. Relaminarizing Pipe and Channel Flows

7.1 General

Experimental investigations of relaminarizing pipe and channel flows have been reported by Laufer [18], Sibulkin [39] and Badri Narayanan [3]; see Table 3. Some detailed measurements are now also being made by Champagne[7].

The general experimental arrangement in these flows involves a gradual enlargement of a pipe or channel from one diameter or width to another, as illustrated in Fig. 14: the angle of devergence is kept sufficiently small to ensure

[7] Private communication.

Table 3. *Relaminarizing pipe and channel flows*

Source	Device	Re_1	Re_2
Laufer [18]	Pipe with 1° half-angle divergence	1750	700
Sibulkin [39]	(a) Pipe with 3° half-angle divergence	1350	300
		2700	600
	(b) Pipe with sudden expansion	4040	900
		5400	1200
Badri Narayanan [3]	Channel with 3° half-angle divergence	1875	625
		2700	865
		2940	980
		3750	1250
Champagne	Pipe with 1° half-angle divergence	3000	600
		5100	1020
		9650	1930
		12750	2550

that no flow separation occurs[8]. In this case, the Reynolds number (based on the section-average flow velocity U_{av} and pipe radius or channel half-height a) goes down from say Re_1 upstream of the divergence to Re_2 downstream. If Re_2 is sufficiently small (we shall discuss this in Section 7.5), it may be expected that an approaching turbulent flow becomes laminar eventually.

Relaminarizing flows of this class bear certain points of resemblance to and show significant points of departure from relaminarizing turbulent boundary layers. It appears best, however, to relegate a discussion of this point until later (see Setion 7.5).

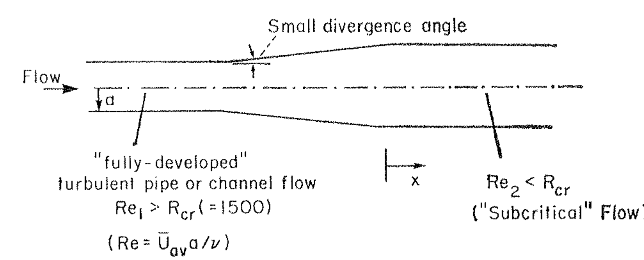


Fig. 14. Schematic of experimental apparatus for producing relaminarization in pipe or channel flow

7.2 Initial Conditions

As mentioned in Section 3.1, it is necessary to start with fully turbulent conditions upstream of the divergence section. Patel and Head [32] have drawn attention to the fact that the definition of fully-developed turbulent pipe flow is ambiguous for Reynolds numbers $Re_1 \lesssim 5000$. If we stipulate that in a fully-

[8] It is worth noting that Sibulkin [39] made measurements also in a pipe with an abrupt expansion and found no essential difference from those with a gradual section, at least beyond a 'few' diameters downstream of the expansion. There is thus a case for using sudden expansions (see Section 8).

developed pipe flow the c_f — Re relations should obey the well-known Blasius skin-friction law and that the laminar-turbulent intermittency disappear, then $\mathrm{Re}_1 \gtrsim 1500$ seems an adequate criterion. However, in the range $1500 \lesssim \mathrm{Re}_1 \lesssim 5000$, Patel and Head [32] do not find a substantial log-law with universal constants. It thus appears that pipe flows with $\mathrm{Re}_1 < 5000$ may not be fully developed at least in one sense. An ideal test case in this type of flow must therefore have an upstream Reynolds number $\mathrm{Re}_1 \gtrsim 5000$. This enables a confident prescription of the various initial functions needed as input for computations in turbulence modeling.

Pipe flows with Reynolds numbers $\lesssim 5000$ have a place that corresponds to the laminarescent state in the case of a relaminarizing boundary layer: Deviations from the constant-pressure laws occur but the flow in nevertheless fully turbulent. Unfortunately, apart from the Patel-Head data, there exists no other detailed and systematic measurements devoted to understanding the phenomenon of laminarescence here. This is clearly an area requiring further attention.

Perhaps somewhat surprisingly, the corresponding case of the channel flow appears less uncertain (Patel and Head [32]), and an upstream Reynolds number $\mathrm{Re}_1 \gtrsim 1500$ seems adequate for the oncoming turbulent channel flow to be fully developed from all the three points of view mentioned earlier. The reason for this difference between pipe and channel flows is not clear.

7.3 Mean Parameters

Notwithstanding the difference between pipe and channel flows in identifying the fully developed turbulent state, there appear no basic differences (either quantitative or qualitative) between them during relaminarization. They are thus referred to as a single class of flows here.

We may note in passing that no measurements have been made in the divergence section. Clearly, such measurements will be of some general interest for turbulence modeling.

Since the flow is developing even in the downstream constant-area section, the skin-friction coefficient cannot in principle be determined from the (nonlinear) pressure drop, but should either be measured directly or inferred from the slope at the wall of the measured velocity profiles. No direct measurements have so far been made, and the velocity measurements near the wall have not been detailed enough to infer c_f accurately. Nevertheless, within the possible accuracy of about $\pm 15\%$, Badri Narayanan [3] found that the c_f inferred from the slope at the wall of the mean volicity profile agreed fairly well with those evaluated from the pressure drop (which itself was not too drastically non-linear). Judging from these measurements, it appears that the skin-friction coefficient reaches laminar values sooner than, for example, the velocity distribution in the middle region reaches the characteristic laminar values. This latter happens relatively slowly at a rate that depends on the downstream Reynolds number.

This relatively rapid adjustment to laminar conditions near the wall as well as the observed growth with x of this inner region suggest that, during relaminarization, the viscous sublayer effectively gets converted to a new laminar boundary layer near the wall, rather like the entry region of the pipe, except that the core in the relaminarizing flow is sheared and carries the residue of an originally turbulent flow. In fact, Narasimha and Sreenivasan [26] have shown that the inner region grows like $x^{1/2}$, exactly as a laminar boundary layer under normal circumstances. When these inner layers from all around (for a pipe) or both sides (for a channel) merge, we have a fully laminar velocity profile.

7.4 Turbulence Fluctuations

In relaminarizing pipe and channel flows, all turbulent stresses decay in absolute magnitude. The rate of decay appears to be more rapid near the wall and in the middle region of the pipe or channel rather than at intermediate positions (Laufer [18]).

A chief characteristic of this decay is that the maximum intensities as well as those averaged over the cross-section decay exponentially with x (Badri Narayanan [3]). Turbulent intensities at a given y near the wall and close to the pipe or channel centre-line also decay exponentially. The exponential decay does not seem to be characteristic of fluctuations in the intermediate positions (Laufer [18]). Clearly, some more measurements are necessary to determine whether or not the exponential decay is a general result.

The normal velocity fluctuations decay faster than the streamwise fluctuations — a result consistent with the fact that most turbulence energy production occurs in the streamwise component whereas energy dissipation is roughly isotropic. No measurements of $\overline{w^2}$ have been reported, but there is no reason to expect any difference in behaviour from that of $\overline{v^2}$.

In contrast to the normal stresses, the Reynolds shear stress decays linearly with x (Badri Narayanan [3]). An important result is that the correlation coefficient $-\overline{uv}/u'v'$ drops with x, again the rate of decay depending on the Reynolds number. Clearly, then, a decorrelation mechanism is at work, and this forms an essential factor in the process of relaminarization here.

7.5 Onset and Completion of Relaminarization

(*a*) *Onset:* Unlike relaminarizing boundary layers, a definite criterion for the onset of relaminarization can be given in pipe and channel flows. Since the relaminarizing pipe or channel flow is affected by viscosity everywhere, it is only natural to seek a critical Reynolds number Re_{cr} at which the flow just ceases to maintain itself fully turbulent: for $\mathrm{Re} < \mathrm{Re}_{cr}$, the only possible asymptotic state of the flow is laminar.

The starting point is the observation (Badri Narayanan [3]) that

$$\overline{u^2}_{\max} \sim \exp(-k_0 x/2a),$$

where k_0 is a constant which decreases with the downstream Reynolds number Re_2. We may now define the critical Reynolds Re_{cr} as that Re_2 at which $k_0 = 0$; that is, that downstream Reynolds number at which the turbulence intensity just maintains itself.

Narasimha and Sreenivasan [26] have shown that all the available data suggest an unambiguous value of 1500 for Re_{cr}; the result is the same for both pipe and channel flows. (For the channel flow, this same conclusion had earlier been reached by Badri Narayanan [4].) They also showed that the constant k_0 depends on the Reynolds number as:

$$k_0 \sim (\mathrm{Re}_{cr} - \mathrm{Re}_2)^3 .$$

As a final remark, we may note that the definition of Re_{cr} is unambiguous (and same for both pipe and channel flows) in contrast to the definition of a critical Reynolds number for a fully developed turbulent pipe flow (Section 7.2).

(*b*) *Completion:* Following our earlier definition for boundary layers, relaminarization can be defined to be complete when the mean velocity profile can be predicted without any hypothesis on the Reynolds shear stress. In this case, this requires that the Reynolds shear stress decay to negligibly low values, although normal stresses may continue to exist. The rate at which Reynolds shear stress decays depends on the difference $\mathrm{Re}_{cr} - \mathrm{Re}_2$ and, in this sense, the completion of relaminazation is also Reynolds number dependent.

It is useful to contrast this with the highly accelerated turbulent boundary layer. The time scales involved in the process of relaminarization in the latter case are relatively short, and much of what is important happens before turbulent fluctuations in the outer layer adjust to the changed mean field; in particular, the Reynolds shear stress remains frozen along streamlines and no substantial decorrelation mechanism is in evidence. The processes in the inner layer are somewhat similar to the pipe and channel case, however. The u' fluctuations do decay and, although the Reynolds shear stress measurements have not been made very close to wall, there are strong indications that a decorrelation mechanism must be at work here too. The similarity perhaps ends here. The rate of bursting in the boundary layer — the phenomenon now believed to be responsible for turbulence energy production — decays exponentially on the pressure gradient parameter Λ (Narasimha and Sreenivasan [25]), leading to the speculation that the Reynolds number effects, if any, may be indirect even in the inner layer.

Returning now to the pipe and channel flows, Narasimha and Sreenivasan [26] have determined the mean velocity distribution by considering, in the region in which relaminarization is complete, the mean velocity to be a perturbation of the final asymptotic state. If one characterizes the departure from laminar conditions by the departure ΔU between the measured centre-line velocity and that expected in a laminar flow, the theory shows that

$$\Delta U \sim e^{-\alpha(x/2a)} \qquad (7.1)$$

where α is a constant for a given Reynolds number. Fig. 15 shows that this is indeed true beyond a certain x, the region of applicability of (7.1) moving closer to the exit of the diffuser plane as Re_2 decreases. In fact when $\mathrm{Re}_2 = 300$, (7.1) seems to hold for $x/2a \gtrsim 5$. Again, somewhat similar to the boundary layer situation, this asymptotic theory seems to hold far upstream of where it is expected to hold!

The detailed mean velocity distribution has also been computed for one case by Narasimha and Sreenivasan [26], and has been shown to be in good agreement with measurement, but only substantially downstream of where (7.1) starts being applicable.

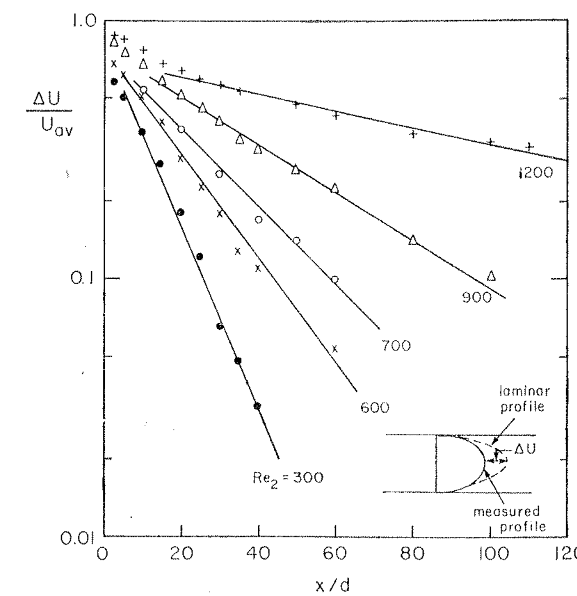


Fig. 15. Departure of measured mean velocity at the centre-line from the Poiseuille value in relaminarizing pipe flows. ●, ×, △ and + from [39] and ○ from [18]

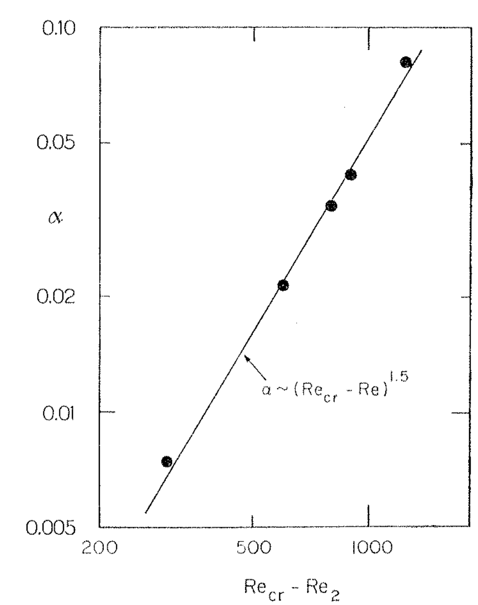


Fig. 16. Dependence on Reynolds number of the rate of approach of the centre-line mean velocity to the Poiseuille value

Fig. 16 shows that α increases as Re_2 decreases, and an empirical correlation (Fig. 15) shows that

$$\alpha \sim (\mathrm{Re}_{cr} - \mathrm{Re}_2)^{1.5}.$$

8. Conclusions and Recommendations

Among the many flow situations in which relaminarization is known to occur, only accelerating turbulent boundary layers and diverging pipe or channel flows have been studied in any detail. Experiments in other types of relaminarizing flows, even if occasionally excellent from several points of view, have mostly been isolated instances, and cannot be invoked for guidance in turbulence modeling; to reach this stage of completion, it is clearly necessary that more detailed and systematic studies be made. It is not enough to add 'one more parameter' to the list of measured quantities; measurements should be made also of the parameters already recorded in previous investigations. Specific comments, as restricted to the present class of relaminarizing flows, follow:

8.1 Relaminarizing Boundary Layers

In spite of the large number of experiments made on accelerating turbulent boundary layers, a close examination of the data reveals many loopholes in them. For example, enough variation in the range of parameters (such as the initial Reynolds number) does not exist, or many sets of data are incomplete in the sense that a crucial parameter like the skin-friction has not been measured. More disappointingly, even among these experiments in which all mean parameters have been measured, many show large momentum imbalances. None of these experiments can be recommended *without reservation* as ideal test cases for use in turbulence modeling and further computations chiefly because they suffer from at least one uncertainty whose influence on the subsequent flow development cannot be asserted conclusively.

These comments, however, present an imcomplete and thus unduly pessimistic picture. Although there is strictly no ideal test case, our discussion in Chapter 3 shows that *a few sets of data* do exist that are moderatlely good in the sense that:

(i) the measurement techniques seem plausible in principle and accuracy,

(ii) the flows satisfy the two-dimensional momentum integral equation (within reasonable limits), and illustrate most of the essential features of the relaminarization process,

(iii) checks on calculations and measurement accuracies (the latter inferred from cross checks) seem largely to bear out the authors' claims, and

(iv) the general reproducibility of the essential features (as judged by the behaviour of mean parameters, for instance) among these different experiments is realistically good.

A further saving grace is that each experiment suffers from a different sort of drawback, and has at least one strength. Thus, while we cannot put implicit faith in all details of any one single experiment, a selective use of the most correct features from these moderately good experiments, if judiciously done, serves as a useful (though decidedly inferior) substitute for a single ideal experiment. Therefore, computations on all or several of these moderately good experiments serves a reasonable purpose in developing turbulence modeling for high acceleration situations. In particular, we can conclude that a potential calculation method must concentrate on predicting:

(i) c_f measurements in BR2, BR3 and SW1, and

(ii) mean velocity data in SW1.

(iii) As the initial Reynolds numbers in the BR and SW flows are both low, a reasonably good addition (not alternative!) to (ii) is the flow PH1 (initial $R_\theta \approx 2000$).

Incidentally, the experimental configuration of PH1 is different from that in SW1. It is thus desirable to compute the mean velocity data in another flow with a different apparatus; OM3 can be considered a plausible choice.

As regards turbulence fluctuations, SW1 offers the best possible test case for u' if the history effects of the upstream deceleation region can be *shown* to be negligible (as they probably are, by virtue of the gradualness of the deceleration).

We may narrow down the choice somewhat further by identifying that the flow SW1 has the least number of uncertainties. Briefly, the mean parameters are measured fairly extensively in this flow, the quoted accuracies in experiments stand reasonable scrutiny, the flow satisfies the two-dimensional momentum integral equation quite well, the initial Reynolds number is at least moderately high, the streamwise fluctuations are measured in detail, etc. The possible shortcomings of this flow are:

(a) lack of v', w' and $\overline{uv}$ measurements;

(b) the boundary layer was developing under mild adverse pressure gradient before acceleration set in, so that its initial state is somewhat different from that of a constant-pressure boundary layer at the same Reynolds number. As the final state in the relaminarizing flows is not very likely to be sensitive to small changes in the initial condition, this may not be a serious problem.

(c) Although moderately high, it is desirable to have a higher R_θ initially.

For these reasons, the need for a new experiment in this area cannot be overemphasized. Clearly, a considerably higher degree of accuracy must be sought in this experiment; the time for mere 'demonstration' of the phenomenon is surely past. Further, the planning of such an experiment should take into account all the problems mentioned thus far. Some of the requirements and suggestions are summarized below.

(*i*) *High initial Reynolds number:* The initial R_θ should preferably be greater than 5000. We have already noted that the initial Reynolds number of an accelerated boundary layer will generally be much smaller than that of the constant-pressure flow for which the wind-tunnel is likely to have been designed originally. As a thumb rule, the future experiment should be planned only in wind-tunnels designed originally for producing constant-pressure boundary layers with R_θ in the vicinity of 10,000.

(*ii*) *Big wind-tunnels:* This last requirement almost immediately rules out many small research tunnels currently in existence in many academic institutions.

Bigger wind tunnels are also desirable to obtain fairly small curvature effects mentioned in Section 3.4. Since even small streamline curvature is known to produce severe effects on turbulence phenomena, as small a δ/R as possible must be obtained. Many existing experiments are unsatisfactory in this regard.

A further advantage in using bigger tunnels is that it is then relatively easy to create thicker boundary layers. Consequently, the near-wall region becomes more accessible to turbulence measurements not attempted until now.

(*iii*) *Two-dimensionality:* The flow must be ascertained to be two-dimensional everywhere, e.g., by measuring flow parameters at several spanwise locations. A momentum balance check, both in the differential and integral forms, must be made.

(*iv*) *Reliable skin-friction measurements:* At present, heat transfer gauge appears to be the most reliable method obtaining c_f. Due consideration (see Section 3.3) must be given to the size of the gauge and its overheat. It should be calibrated in a flow with known skin-friction, and the effective length determined. As a rough working rule, the Spence-Brown inequality (3.6) must be satisfied.

(*v*) *Duplication of techniques:* In the relaminarization regime, instances have been noted in which the mean velocity profiles measured with a Pitot-tube are significantly different from those measured with a hot-wire (e.g., Launder and Stinchcombe [29], raw data of BR1). It is quite likely that the quality of hot-wire measurements in these experiments was not very good, but there may also be a genuine difficulty because of the spreading intermittency. (Later experiments use either the Pitot-tube or the hot-wire and not both; see Table 1.) It is useful to measure mean velocity by both methods.

(*vi*) *The island of ignorance:* Finally, the region requiring most attention is what we have designated as the island of ignorance. Unfortunately, this region is barely distinct in many flows because of low flow Reynolds numbers. Serious attempts must be made to make detailed measurements especially in this region.

(*vii*) *Retransition:* While not being basic to the understanding of relaminarization itself, some attention must surely be paid to retransition of the relaminarized flow to turbulence. In particular, intermittency measurements in this region would help.

8.2 Laminarescent Boundary Layers

We have seen how sink flows listed in Table 2 are most helpful in elucidating the various details of the initial stages (region (b1)) of the relaminarizing boundary layer. It is, in this respect, hard to improve on the flows created by Jones and Launder [13], and we believe that these experiments can be used profitably in developing models capable of handling medium-range accelerations.

Finally, we may note that a part of our discussion is oriented towards delineating the differences in physical content in the different flow regimes (Section 2.1). We believe that it is advantageous to keep this picture in mind also in developing turbulence models capable of uniformly handling flows with mild as well as strong accelerations.

8.3 Subcritical Pipe and Channel Flows

Our earlier discussion (Section 7.2) shows that a pipe flow is fully developed only if initially $\mathrm{Re}_1 \gtrsim 5000$. In contrast to pipes, a fully developed channel flow appears possible for $\mathrm{Re}_1 \gtrsim 1500$. Since there are no discernible differences in relaminarization between pipe and channel flows, this is one reason why channel flows should perhaps be preferred to pipe flows.

Downstream of the expansion, the Reynolds number must fall below Re_{cr} which, according to the best estimates available, is 1500. Notice that for channels Re_{cr} is the same as the minimum Reynolds for maintaining fully developed turbulence but *not* for pipe flows! The need for further study in this area cannot therefore be overemphasized.

We have seen that the rate of approach to laminar state depends on the difference $\mathrm{Re}_{cr} - \mathrm{Re}_2$. Thus, if relaminarization is to be complete within reasonable distances, it is clear that Re_2 cannot be just barely less than 1500, but will have to be substantially so.

If the initial Reynolds number is high (say, $\mathrm{Re}_1 = 5000$), and the downstream Reynolds number is low (say, $\mathrm{Re}_2 = 500$), a gradual expansion section (say, 3^0 half-angle) also becomes inconveniently long (about $400a$!). Following the example of Sibulkin (1962) who found no major effect due to a sudden expansion — evidently, flow separation does not seem to be very critical — one may without much hesitation use a sudden — rather than gradual — expansion.

Our Reynolds number criteria suggest that there exists no ideal experiment in pipes. The existing pipe and channel flows suffer also from other short comings: no accurate skin-friction measurements, no detailed data on turbulent fluctuations, and no clear documentation of initial conditions.

It is clear that there is a need for better measurements. All the considerations outlined above should be weighed before designing a new experiment.

In spite of these limitations, a lot can be learned from the general trends we noted in Section 7.5. Clearly, turbulence models must try to reproduce at least these general trands correctly. If, on the other hand, one were to attempt a detailed prediction of a single flow, both in terms of systematic study and measurement accuracy (as judged from the scatter in measurements, etc.), Laufer's [18] flow is perhaps the best choice, although in fact the other experiments (Sibulkin [39], Badri Narayanan [3]) are more detailed in many respects. The only problem would concern the initial conditions which are not measured in Laufer's flow upstream of the divergence, but the relevant data from other experiments under comparable conditions are in fact available (e.g., Patel and Head [32]).

Appendix: Commentary on the Data

A. Relaminarizing, Laminarescent and Retransitional Boundary Layers

Table 1 lists the sources of experimental data on relaminarizing and laminarescent boundary layers, and includes notes on the parameters measured as well as (where relevant) the method of measurement, the degree of momentum imbalance, initial conditions, etc. A close examination of this Table, along with the explanatory notes of Chapter 2, reveals why most flows have some problem or the other. One general and important criterion for selection of a flow as being trustworthy from the point of view of turbulence modeling is the possibility of an independent assessment through momentum balance of the internal consistency of mean flow measurements. This necessarily calls for skin-friction measurements; while those other flows without c_f measurements have had a very useful role to play in elucidating the phenomenon of relaminarization, there is no independent way of ascertaining their reliability. This emphasis on momentum balance seems quite justified in view of the large imbalances observed in many relaminarizing flows. If the cause for such an imbalance lies largely in the manner in which the flows are produced, it seems necessary to exercise great caution in future (especially because all relaminarizing flows tend to be produced in essentially the same way).

Detailed tabulation of all relevant data for flows recommended here as being reasonably good is available upon request from the author.

(*i*) *Back and Seban* [1]: Measurements are rather sparse and the initial Reynolds number is low. Initial conditions are not given, and skin-friction was obtained by the linear wall-slope method.

(*ii*) *Badri Narayanan and Ramjee* [4]: In BR1, the velocity profiles measured with a hot-wire differed from those that were measured with a Pitot-tube, and so an attempt was made to repeat these measurements. The repeat measurements (Sreenivasan [43]) differed in several respects from BR1. Most BR experiments have larger scatter than many of the other experiments considered here, but probably the most reliable c_f measurements have been made in BR2 and BR3 (using heat transfer gauge). The momentum balance in these two flows is reasonable, and for these reasons these two flows are among those that can be considered as possible candidates for computations. The major drawback is however their rather low initial Reynolds numbers.

(*iii*) *Badri Narayanan, Rajagopalan and Narasimha* [5]: No detailed measurements of mean parameters are available, but this is the only relaminarizing boundary layer in which turbulence energy balance measurements have been made. In spite of some obvious problems (e.g., the turbulence diffusion does not integrate to zero across the boundary layer), the measurements are interesting because they show that turbulence production in the outer layer is not negligible (and is larger than dissipation) even in the region of high acceleration. This flow is not ideally suited for computational purposes.

(*iv*) *Blackwelder and Kovasznay* [7]: This is perhaps the most detailed study made to-date of a relaminarizing boundary layer. Among the several noteworthy features of this flow are that the initial Reynolds number is not too low, and that a gradual progression of changes in the flow (including some features of retransition to turbulence) have been traced methodically; it is the only flow in which v' and w' fluctuations have been measured. It is therefore very unfortunate that the flow satisfies the momentum integral equation so poorly (Fig. 4c) that is cannot be considered a good test case. Possible effects of this imbalance on the fluctuating quantities cannot be established. As regards the c_f measurements which were made by the linear wall-slope method, there was indeed a substantial linear region in the velocity profiles measured with a single hot-wire; c_f measurement may thus not be unreliable.

(*v*) *Launder* [19]: The flow L1 cannot unfortunately be used for any quantitative assessment because of the uncertainty regarding the correspondence of these data to the streamwise stations and hence also the pressure gradient (the various stations as obtained from Launder's Fig. 12 and the number of mean velocity profiles presented in his Fig. 7 are not the same as the corresponding data in his Appendix II). The flow L2 provides valuable information about retransition to turbulence. Even here, there is a difficulty in the utilization of the data for quantitative analysis: there is no mention of the streamwise station, and the free-stream velocity, to which the data belong. Further, the mean energy balance presented in his Fig. 26 does not correspond exactly to L1 or L2. With regard to the shear stress measurements on streamlines presented in his Fig. 27, Launder does not specify the precise streamwise location. These difficulties however should not detract us from noting the valuable insight that this study has provided into the mechanics of relaminarization.

(*vi*) *Launder and Stinchcombe* [21]: These experiments (except possibly the one with the highest K, also the lowest Reynolds number) can be considered laminarescent in the sense discussed in Section 2.2. Being preliminary in nature, however, they are beset with several problems among which is the lack of two-dimensionality.

(*vii*) *Jones and Launder* [13]: Jones and Launder made measurements in several sink flow boundary layers, the nominal values of K being 1.5×10^{-6}, 2.5×10^{-6} and 3×10^{-6}. The last two consist of more than one experiment. All but the flows with the largest K are self-preserving, as revealed by mean and fluctuating flow measurement. Much of these authors' success in establishing roughly similar boundary layers in relatively short distances owes itself to the choice of initial R_θ close to the values corresponding to self-similar sink-flow boundary layers with the chosen K.

For a self-similar sink flow, K, H and R_θ are constants, and the two-dimensional momentum integral equation (3.7) reduces to

$$K R_\theta (H + 1) = c_f/2 \tag{A.1}$$

This relation was used by Jones and Launder to obtain c_f. The accuracy of these values depends on the accuracy to which the flows were actually maintained self-similar: for all but the flows with the highest K (where R_θ variations of the order of 20% were found), this was indeed a good approximation. Jones and Launder also used a Stanton tube for skin-friction measurements and found them to agree to within about 5% of those given by (A1). Because Jones and Launder convey the impression of having taken extra precautions to establish a two-dimensional flow, this agreement between the Stanton tube and momentum-integral estimates of c_f can be interpreted to imply the reasonableness of Stanton tube measurements when the tube is operated within the viscous sublayer. This conclusion is very significant because the JL flows show substantial deviations from the constant-pressure laws. In his survey on wall-jets, Launder (private communication) suggests that Stanton tube method of c_f measurements is in fact preferable to the wall-slope method or the surface fence technique.

It should be noted that the initial conditions in these flows have not been documented in full. Presumably, since these flows are self-preserving, the details of the initial conditions may not be too critical for computations. A chief source of difficulty is however that the published data do not include the acceleration history in a conveniently usable form.

(*viii*) *Moretti and Kays* [24]: These experiments put heavy emphasis on surface heat transfer measurements, and contain hardly any data about other quantities. For six of the many runs, the initial Reynolds numbers are specified or calculable, so that the other quantities required for initiating the computations can be estimated.

(*ix*) *Okamoto and Misu* [29]: Among the three experiments reported by these authors, the overall momentum balance is better in OM3 than in the other two; none of them is, however, excellent in this regard.

Mean velocity profiles were measured with a flattened Pitot-tube, and no corrections were made for the measurements near the wall of the mean velocity measurements; some uncertainty may be expected here. Further inconsistencies include discrepancies of the order of 15% between the θ values recalculated using the mean velocity data and those given by the authors. No turbulence measurements have been reported in usable form.

(*x*) *Patel and Head* [31]: In these experiments, a center body was used in a pipe to create the desired acceleration for the boundary layer on the pipe wall. First, a comment is necessary on the initial conditions. All the measurements in the PH experiments were made at a fixed station in the pipe while the pressure gradient itself was translated along the axis (by moving the centre body in the pipe) so that the origin and state of the initial turbulent boundary layer are, strictly speaking, different for each measured mean velocity profile. It is not clear how the effect of this uncertainty, more significant in PH1 (because of the smaller entry length) than in PH2, can be assessed on the data recorded.

The Patel/Head experiments are among the high Reynolds flows studied and deserve consideration at least for this reason. PH2 has the highest initial R_θ ($\simeq 5000$) among all the experiments made to-date, but unfortunately it cannot be used as a good test case. The chief problem is that PH2 is that it is not a plane boundary layer (very large two-dimensional momentum imbalances). The reason perhaps is that the flow was generated in a pipe whose radius was only twice that of the 'boundary layer' thickness. Even when treated as an axisymmetric flow, it shows considerable momentum imbalance. This may very well be because the velocity gradients everywhere in the flow are comparable in both streamwise and radial directions, so that the boundary layer approximations is itself questionable; it should however be remembered that the inner layer study of Patel and Head (which was their chief concern) is not necessarily invalidated by these considerations.

The fact that this flow is likely to be elliptic is no reason not to attempt calculations on, but it was felt that boundary layer flows should first be tried before attempting such calculations. Furthermore, if indeed the observed momentum imbalance is due to the flow ellipticity, no checks on the self-consistency of measurements is possible. For this, as well as reasons of uncertainty in the initial conditions, this flow is not considered as an ideal test case.

Although PH1 was conducted in the same experimental set-up, the degree of axisymmetry is negligible because of the smaller boundary layer thickness. The overall momentum balance is reasonable (except perhaps initially), and a slightly lower c_f (about 10 to 15%) would improve the situation even more. This degree of uncertainty of c_f measurements is not unexpected considering the difficulties with the fence technique (see Section 3.3). The initial Reynolds number is moderately high. Thus, in spite of some ambiguity associated with initial conditions, it appears worthwhile including this flow for computational purposes.

(*xi*) *Schraub and Kline* [37]: Because the authors present data at only three streamwise positions, it is hard to assess the momentum balance accurately in this flow. This is the only relaminarizing flow to make sublayer burst rate measurements. Schraub and Kline concluded that the bursting rate ceases when K_{crit} is reached, and therefore correlated the onset of relaminarization with K_{crit}. Even assuming that the cessation of bursting rate is a good criterion (probably more rational than most other criteria) for the onset of relaminarization, the burst rate decreases exponentially (see especially Narasimha and Sreenivasan [25]), showing no obvious break-points or plateau. Clearly, it is necessary to obtain more data on bursting at high Reynolds numbers.

(*xii*) *Simpson and Wallace* [41] *and Simpson and Shackleton* [40]: The two sets are clubbed together because they use essentially the same instrumentation and flow devices. We first note that each of the boundary layers in these experiments was developing under mild adverse pressure gradient before it was subjected to an acceleration. For example, in SS2 the free-stream velocity at upstream 'infinity' is about 4/3 that at the start of the acceleration; the initial conditions such as the mean velocity profile are characteristic of flows in midly adverse pressure gradient. Although there are no solid grounds to suspect that the possible 'history effects' would have a crucial influence on the subsequent flow development, this should be kept in mind while using the data. In particular, any calculation method meant for relaminarizing flows should take care not to specify the initial mean and *rms* velocity profiles according to 'standard' turbulent laws. For a method capable of uniformly handling decelerated, accelerated and relaminarizaing flows, this is in principle no shortcoming but, unfortunately, the authors do not give any data in the decelerated region.

The authors provide momentum balance checks for SW1 and a part of SW2. Our own estimates agree well with them for SW1, from which we conclude that SW1 is a good candidate for computations and turbulence modeling. Over a part of SW2, and in SS1 and SS2, the momentum balance is generally unsatisfactory even when $\pm 20\%$ uncertainty is allowed in c_f measurements.

(*xiii*) *Sreenivasan* [43]: Again, no skin-friction measurements are made here. As these experiments were made with considerable hindsight obtained from the BR experiments, we believe that all the mean parameters measured here are reliable. No independent checks are however possible. Further, the initial conditions are not specified in sufficient detail here. For these reasons, these flows cannot be recommended for computational purposes.

B. *Relaminarizing Pipe and Channel Flows*

Data on these flows are not as extensive as on relaminarizing boundary layers, and all the sources are listed in Table 3. Among these, Champagne's experiments are still under progress, and we shall not comment on them. Comments on other experiments follow:

(*i*) *Laufer* [18]: This is a single Reynolds number experiment. Mean velocity distribution measurements have been made at 10, 20, 30, 40, 50 and 60 diameters downstream of the expansion. At all but the last station, both *rms* velocity and spectral measurements of u fluctuation have been made. A very interesting result concerns the apparent similarity of the spectral density at all wave numbers, when the centre-line mean velocity and the integral scale of turbulence are used for normalization.

(*ii*) *Sibulkin* [30]: Essentially the same measurements as Laufer's were made in this remarkable paper. Although several Reynolds numbers have been examined, the data are not as extensive as in Laufer's paper. Sibulkin, too, found a tendency to spectral similarity, but the data were not as conclusive as in Laufer's. One rather important finding of Sibulkin is that the difference in the flow behaviour between gradual and sudden expansions is negligible from almost immediately downstream of the expansion.

(*iii*) *Badri Narayanan* [3]: Measurements have been made in a relaminarizing channel flow at four Reynolds numbers (see Table 3). The streamwise and normal (u and v) velocity fluctuation as well as the Reynolds shear stress $\overline{uv}$ have been measured. Like Laufer and Sibulkin, Badri Narayanan also found approximate similarity in the u' spectrum.

This is the only flow of this class in which the Reynolds shear stress as well as (approximate) energy balance measurements have been made. These latter measurements suggest that, at the flow Reynolds number $Re_2 = 865$, production was somewhat smaller than dissipation and that advection was small compared with either of them.

The instrumentation employed in these measurements (e.g., the average squarer) had severe shortcomings. Although there is no reason to doubt the general trend of the results (e.g., where they overlap with the pipe measurements, they are in qualitative agreement), there is an urgent need for repetition of these measurements under better experimental conditions.

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