where the terms without subscripts correspond to the three-dimensional shock. In an analogous manner, Eq. (4) along with the scalings for ξ, η, and τ may be used to relate the pressure distribution obtained in a two-dimensional experiment or a numerical solution of Eq. (3) to that for a three-dimensional focusing problem.

In conclusion, the focusing of a weak, three-dimensional shock wave at a cusp in a caustic has been studied. It has been shown that although the focusing is initially three-dimensional, the flow in the vicinity of the arête is essentially two-dimensional. The details of the flow are governed by Eq. (3); once these are obtained, the pressure distribution is given by Eq. (4). It has also been shown that the similarity parameter used in Ref. 7 was unnecessary and, as a result, Eq. (4) delineates the dependence of the resultant pressure levels on the initial shape and strength of the shock. For three-dimensional shock waves satisfying the conditions discussed here, the similitude also allows us to determine once and for all the pressure distribution simply by analyzing, either experimentally or numerically, a two-dimensional focusing problem.

References

[1]Whitham, G. B., "A New Approach to the Problems of Shock Dynamics. Part 1. Two-Dimensional Problems," *Journal of Fluid Mechanics*, Vol. 2, 1957, pp. 146-171.

[2]Whitham, G. B., "A New Approach to the Problems of Shock Dynamics. Part 2. Three-Dimensional Problems," *Journal of Fluid Mechanics*, Vol. 5, 1959, pp. 369-386.

[3]Guiraud, J. P., "Acoustique Géometrique, Bruit Ballistique des Avions Supersoniques et Focalisation," *Journal de Mécanique*, Vol. 4, 1965, pp. 215-267.

[4]Hayes, W. D., "Similarity Rules for Nonlinear Acoustic Propagation through a Caustic," *2nd Conference on Sonic Boom Research*, NASA Special Publication No. 180, 1968, pp. 165-171.

[5]Pechuzal, G. and Kevorkian, J., "Supersonic-Transonic Flow Generated by a Thin Airfoil in a Stratified Atmosphere," *SIAM Journal of Applied Mathematics*, Vol. 33, 1977, pp. 8-33.

[6]Sturtevant, B. and Kulkarny, V. A., "The Focusing of Weak Shock Waves," *Journal of Fluid Mechanics*, Vol. 73, 1976, pp. 651-671.

[7]Cramer, M. S. and Seebass, A. R., "Focusing of Weak Shock Waves at an Arête," *Journal of Fluid Mechanics*, Vol. 88, 1978, pp. 209-222.

[8]Cramer, M. S., "The Focusing of Weak Shock Waves at an Axisymmetric Arête," to appear in the *Journal of Fluid Mechanics*.

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Approach to Self-Preservation in Plane Turbulent Wakes

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Introduction

THE purpose of this Note is to examine the manner in which moderate Reynolds number, plane turbulent wakes behind wake generators of different shapes approach a unique self-preserving state, and to point out what appear to be some surprising features of the process.

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Experiments

Table 1 lists details of the several wake generators used in the experiments discussed here. Except for two cases,[1,5] all of the wakes were generated in an open-circuit suction-type wind tunnel with a contraction ratio of about 10, and a 30 cm square, 4.27 m long test section. The wind speed was constant to within about 1.5% and the freestream turbulence was about 0.15%.[2] All mean velocity measurements were made with a round pitot tube of 1 mm o.d. using a micromanometer capable of reading 0.05 mm alcohol; no corrections for finite turbulence levels were attempted for the pitot measurements.

Background

By self-preservation, we mean here that the mean velocity and the Reynolds shear stress distributions must be independent of the streamwise position when normalized by the same velocity and length scales. In the asymptotic limit of vanishing velocity defect, a two-dimensional self-preserving (linear) turbulent wake is characterized by constant values of two parameters defined in Ref. 7 as $W=(w_0/U)\sqrt{(x/\theta)}$ and $\Delta=\delta/\sqrt{x\theta}$. Here, w_0 is the maximum of the velocity defect w, δ is the half-wake thickness given by the distance from the centerplane to where the defect is half the maximum, x and y are, respectively, the distances from the trailing edge of the wake generator and from the wake centerplane, and θ is the momentum thickness defined by

$$\theta=\int_{-\infty}^{\infty}(w/w_0)(1-w/w_0)\,\mathrm{d}y \qquad (1)$$

If the asymptotic self-preserving state is unique, the parameters W and Δ must assume universal values, say W^* and Δ^*. As appropriate to small but finite defect wakes, the nature of correction terms to W^* and Δ^* cannot be assessed on the basis of linear theory alone, but it was argued in Ref. 8 that the correction terms are of $0(w_0/U)$. Thus, the behavior of the measured values of W and Δ against w_0/U gives us a gross indication of the manner in which the unique asymptotic state (if one exists) is approached.

Results

Figures 1 and 2 show, respectively, the variation of the parameters W and Δ with w_0/U; corresponding variation in x/θ ranges typically from about 10 to about 1000. (Where necessary, convergence corrections according to the suggestions of Refs. 2 and 9 have been applied to the data.) Although it is not surprising that different wakes approach self-preservation through different routes, the degree of variability and the non-monotonic behavior shown by W and Δ was unexpected. It should be emphasized that both δ/θ and w_0/U showed monotonic variations with x/θ for all of these wakes. Although all wakes seem to approach the asymptotic values W^* and Δ^* indicated on the figures (more about which will be said shortly), thus indicating an approach to *the* self-preservation state, there are substantial differences among them even when the defect ratio is as low as 5%: the large eddies seem to remember the manner of their generation even so far downstream! The wake behind a twin-plate generator appears to have the simplest behavior and attains self-preservation in the shortest distance (as was indeed found by Narasimha and Prabhu,[7] who first used it) probably because large eddies in the flow are rendered weak by the nature of the mean strain field that occurs there.

The relatively simple behavior of the wake parameters in a twin-plate generator wake led us to make detailed far-wake measurements with the sole purpose of determining the asymptotic values W^* and Δ^*. These were determined by extrapolating linearly to zero defect the parameters W and Δ obtained from measurements in the region of small but finite defect. The chief conclusion of these measurements, reported elsewhere,[8] can be stated as $W^*=1.63\pm0.02$, and

Table 1 Summary of wake data examined

Wake generator	Symbol	Source	UL/ν	Uθ/ν	C_D	Aspect ratio
Circular cylinder (L = 4.8)	○	Townsend[1]	835–8100	420–4500	1–1.1	–
		Prabhu[2]	4320	2400	1.1	64
		Bhutiani[3]	5720	3200	1.1	
		Prasanna Kumar[4]	5460	3060	1.1	
Square cylinder		Prasanna Kumar[4]				
(L = 2.1)	□		2240	1890	1.75	144
(L = 2.1)	◇		2240	2240	1.45	100
Flat plate (1.6, 600, .025, L = 2400)	+	Chevray and Kovasznay[5]	6.5×10^5	3160	.0016	310
(3.2, 152, L = 914)		Prasanna Kumar[4]	10^6	4000	.0013	96
Twin-plate (1.59, L = 24)	●	Prabhu[2]	2.8×10^4–4×10^4	1020–1500	.074	64
		Sreenivasan[6]	3.2×10^4	1180		

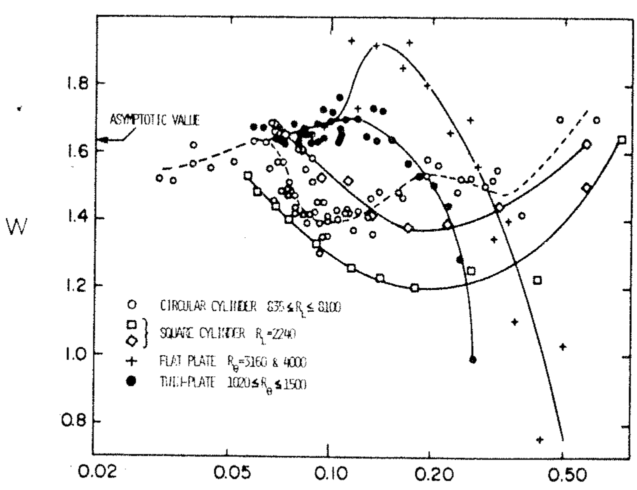

Fig. 1 Variation of the wake parameter W with the defect velocity ratio.

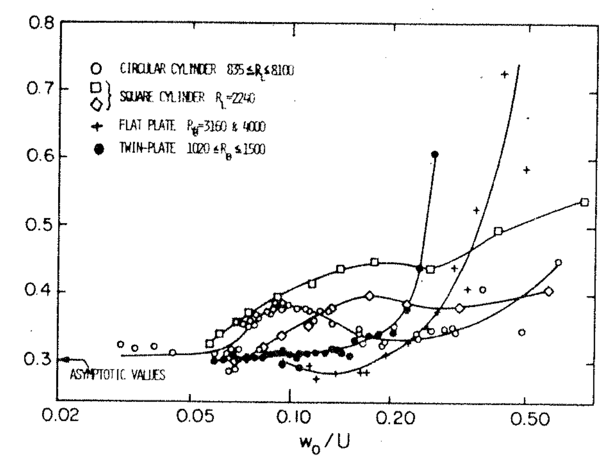

Fig. 2 Variation of the wake parameter Δ with the defect velocity ratio.

$\Delta^* = 0.3 \pm 0.005$. These are the asymptotic values marked in Figs. 1 and 2. One further relevant conclusion of Ref. 8 is that $I_1 = 2.06$ and $I_2 = 1.51$ for a self-preserving wake, where the integral parameters I_1 and I_2 are defined as

$$I_n = \int_{-\infty}^{\infty} (w/w_0)^n \mathrm{d}(y/\delta), \quad n = 1,2 \tag{2}$$

Now, it follows from Eqs. (1) and (2) that

$$\theta/\delta = (w_0/U)[I_1 - (w_0/U)I_2] \tag{3}$$

If the velocity profiles preserve their shapes when scaled on w_0 and δ, a plot of θ/δ vs w_0/U must be unique for each wake. If, further, this shape is the same for all wakes, data from all of them must collapse onto a common curve. Figure 3 shows that all of the data do indeed collapse together from almost immediately downstream of the wake generators, suggesting that all of the wakes—no matter how created—quickly assume essentially the same shape of *mean* velocity profile.

Also shown in Fig. 3 is Eq. (3) with I_1 and I_2 appropriate to the self-preserving wake. As this curve represents all of the data fairly well for $w_0/U \leq 0.15$, it is clear that the common mean velocity shape attained under this condition is indeed that of the asymptotic self-preserving wake itself. However, W and Δ have not attained a common value at this stage (see Figs. 1 and 2); this can only mean that the Reynolds stress profile does not preserve its shape on w_0 and δ—an observation found to be true from very limited measurements.

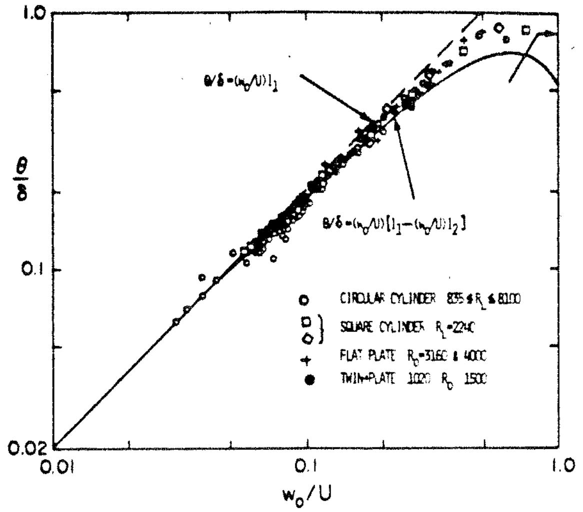

Fig. 3 A plot of θ/δ vs w_0/U. For $w_0/U \leq 0.05$, Eq. (4) and the linear approximation to it are indistinguishable.

Some relation can be expected to exist between the near-wake behavior and the boundary layers just before they leave the wake generators. The boundary layers near the trailing edge of both flat plates examined here were known to be turbulent; the boundary layers on the bluff bodies become unstable and turbulent soon after they separate. In view of this, it is quite surprising to find that θ/δ values for all wakes seem to approach, as $w_0/U \rightarrow 1$, the value ($\simeq 0.856$) appropriate to a flat-plate laminar boundary layer (with θ and δ both defined as done here for wakes); although the corresponding number for a turbulent boundary layer depends on the Reynolds number, it is at least an order of magnitude higher. As our measurements stopped short of extending all the way up to the wake generator only by a dozen or so momentum thicknesses, the last result should simply mean that, within a few momentum thicknesses after leaving the wake generator, the flow quickly readjusts as if the boundary layers leaving the wake generator were laminar, a curious behavior worth a closer examination!

At lower Reynolds numbers (R_θ of the order of 300 or lower), preliminary measurements behind flat plates showed an even more complex behavior in which the parameters W and Δ showed discontinuous jumps when plotted against w_0/U. Presumably, some of this is associated with the Karman vortex patterns that have an important influence on flow development at these Reynolds numbers.

Acknowledgments

All of the wake parameters reported here have been obtained by a reanalysis of the raw data collected by my colleagues and me at the Indian Institute of Science, Bangalore, over a period of about four years. My thanks go to Drs. A. Prabhu, P. K. Bhutiani, and Mr. Prasanna Kumar, whose measurements I have used, and to Prof. R. Narasimha, whose constant interest in the matter actually spurred the analysis.

References

[1] Townsend, A. A., *The Structure of Turbulent Shear Flow*, University Press, Cambridge, U.K., 1956.

[2] Prabhu, A., "Non-equilibrium Wakes," Ph.D. Thesis, Dept. of Aero. Eng., Indian Institute of Science, Bangalore, 1971.

[3] Bhutiani, P. K., "Effect of Free Stream Turbulence on the Development of a Turbulent Wake," M.E. Dissertation, Dept. of Aero. Eng., Indian Institute of Science, Bangalore, 1972.

[4] Prasanna Kumar, "A Study of Wakes Behind Two-Dimensional Bodies," M.E. Dissertation, Dept. of Aero. Eng., Indian Institute of Science, Bangalore, 1972.

[5] Chevray, R. and Kovasznay, L.S.G., "Turbulence Measurements in the Wake of a Thin Flat Plate," *AIAA Journal*, Vol. 7, Aug. 1969, pp. 1641-1643.

[6] Sreenivasan, K. R., "Data on Constant-Pressure Two-Dimensional Turbulent Wakes," Indian Institute of Science, Bangalore, Aero. Eng. Rept. 74FM5, 1974.

[7] Narasimha, R. and Prabhu, A., "Equilibrium and Relaxation in Turbulent Wakes," *Journal of Fluid Mechanics*, Vol. 54, July 1972, pp. 1-17.

[8] Sreenivasan, K. R. and Narasimha, R., "Equilibrium Parameters for Two-Dimensional Turbulent Wakes,".

[9] Prabhu, A. and Narasimha, R., "Turbulent Non-Equilibrium Wakes," *Journal of Fluid Mechanics*, Vol. 54, July 1972, pp. 19-38.

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Transformation of the Equation Governing Disturbances of a Two-Dimensional Compressible Flow

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THIS Note concerns the propagation of small amplitude inviscid and unsteady disturbances on steady nonuniform mean flows. Problems of this type arise in a wide variety of fields, including aerodynamic noise, flutter, forced vibration, and buffeting of structures; for example, when a purely vortical disturbance is incident on an airfoil at *large* angles of attack. Indeed, the present discussion is restricted to the analysis of the distortion of vortical and entropic disturbances as they convect, with the mean flow, past a "bluff" object (flow separation is ignored) and to the generation of certain irrotational fields that permit the enforcement of boundary conditions on the body surface. A classical approach is to reconstruct the velocity field from the vorticity and then to obtain the pressure field from the former.[1] Goldstein[2] has developed a much simpler approach which requires the solution of a single inhomogeneous wave-like equation for the irrotational (roughly the acoustic) field. Because the mean flow is nonuniform, this equation has variable coefficients.

The purpose of this Note is to show that, when the behavior of the two-dimensional and compressible mean flow is approximated by the tangent gas relations,[3] the inhomogeneous wave equation can be transformed into a much simpler form involving only one variable coefficient. Further simplifications are possible when the mean flow is a small perturbation of a uniform stream.

We consider small amplitude disturbances superimposed on a steady, irrotational, compressible mean flow. Linearizing the equations of motion about the mean flow, and neglecting viscous effects, Goldstein[2] has shown that the perturbations are described by the following equations

$$u' = \nabla G + v \tag{1}$$

$$s' = b(X - IU_\infty t) \tag{2a}$$

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