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Decay of scalar variance in isotropic turbulence

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Two consequences of a recent theory for the decay of scalar variance in isotropic turbulence are shown to be in essential agreement with measurements.

Using a modified Richardson law for pair dispersion, Nelkin and Kerr[1] have recently proposed a simple theory for the decay of scalar fluctuations in homogeneous and isotropic turbulent field. The theory has two explicit consequences: First, if one fits a power law to the decay of the scalar variance $\langle\theta^2\rangle$, the decay exponent m, obtained from the log-log plot of $\langle\theta^2\rangle$ against the streamwise distance x in a wind tunnel, relaxes only rather slowly to its asymptotic value of 1.2. A second conclusion is that the product $\langle\theta^2\rangle L_\theta^3$ remains a constant independent of x, where L_θ is the integral length scale of scalar fluctuations. This latter result has also been arrived at by Chatwin and Sullivan[2] in a different, but related, situation involving a turbulent cloud of passive contaminants.

In a brief comparison with the experiments of Ref. 3, Nelkin and Kerr[1] observed that m did not vary by more than a factor of about 2 over the (fairly wide) range of measurements, in qualitative agreement with the theory. These authors also noted that the Warhaft– Lumley measurements[3] showed no tendency toward constant $\langle\theta^2\rangle L_\theta^3$. Unfortunately, no comparisons were made in Ref. 1 with the equally relevant measurements of Ref. 4. The purpose of this note is to point out that, in fact, there is some unexpected degree of agreement between the Nelkin–Kerr theory and the measurements of Ref. 4.

In Ref. 4, experiments were made for three locations x_s of the heating screen at 20, 34, and 54 mesh sizes downstream of the turbulence-generating grid. Two heating screens, of mesh size M_θ about 0.44 and 0.88 times the mesh size M of the momentum grid, were used. Figure 1 is a collection of data from these experiments on $\langle\theta^2\rangle L_\theta^3$. While there is some ambiguity for the $M_\theta/M=0.88$ case, it is seen that the product $\langle\theta^2\rangle L_\theta^3$ is indeed constant beyond a certain x (where x is measured from the grid), consistent with the expectation from the theory. For the case $M_\theta/M=0.44$ on which we shall concentrate further, this occurs for $x/x_s \gtrsim 1.9$. The initial sharp decrease presumably corresponds to the region of rapid adjustment of the scalar fluctuations to the superimposed turbulent velocity field.

A possible interpretation of the observed constancy of $\langle\theta^2\rangle L_\theta^3$ can be given if we note that, during scalar decay, the so-called Corrsin invariant[5] exists. That is,

$$\langle\theta^2\rangle \int_0^\infty r^2 f_\theta(r)\,dr = \text{const}, \tag{1}$$

where $f_\theta(r)$ is the two-point correlation function of the scalar fluctuation θ, and r is the separation distance. If one assumes that $f_\theta(r) \sim \exp(-r/L_\theta)$, it follows from Eq. (1) that $\langle\theta^2\rangle L_\theta^3 \sim$ const, as observed.

Sreenivasan *et al.*[4] found that, over the fairly extensive range of measurements, all the scalar variance decay data could be fitted by the power law

$$\langle\theta^2\rangle = \alpha(x/x_s - 1)^{-n}, \tag{2}$$

where α and n are constants, with n ($\simeq 2.23$) showing no perceptible dependence on the location of the heating screen within the range of measurements. The Nelkin–Kerr theory suggests, on the other hand, that

$$\langle\theta^2\rangle = \beta[(x/x_s)^{4/15} - B]^{-9/2}, \tag{3}$$

where β is a constant and

$$B = 1 - (R_0/L_0)^{2/3}. \tag{4}$$

Here, R_0 is proportional (but not equal) to the integral scale of the scalar fluctuations and L_0 is an integral

TABLE I. Ratio of R_0 to a characteristic integral scale of scalar fluctuations ($M_\theta/M = 0.44$).

x_s/M	R_0/M_θ	L_θ^*/M_θ	R_0/L_θ^*
20	0.082	0.41	0.20
34	0.10	0.51	0.20
54	0.12	0.55	0.22

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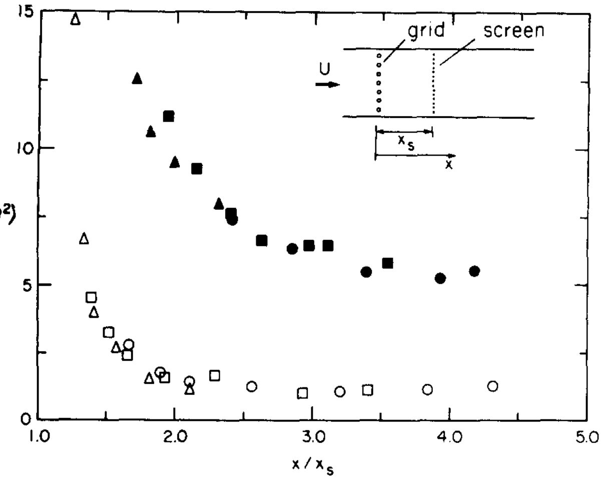

FIG. 1. Variation of $\langle\theta^2\rangle L_\theta^3$ from experiments of Ref. 4. Open symbols correspond to the case $M_\theta/M = 0.44$ and the solid symbols to $M_\theta/M = 0.88$. ○, $x_s/M = 20$; □, $x_s/M = 34$; △, $x_s/M = 54$. Inset shows the experimental configuration.

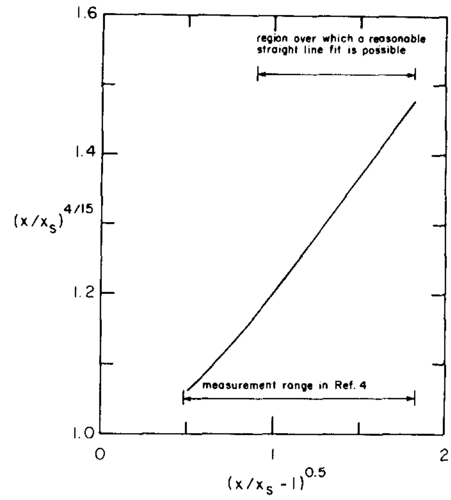

FIG. 2. A comparison of the experimentally determined decay law of Ref. 4 with the theoretical deductions of Ref. 1.

scale of turbulence. Consistency of Eqs. (2) and (3) requires that, over the measurement range, we should have

$$(x/x_s)^{4/15} \simeq A(x/x_s - 1)^{0.5} + B, \tag{5}$$

where A is another constant. Figure 2 shows that this relation again holds for $x/x_s \gtrsim 1.9$, with $B \simeq 0.875$. Nelkin and Kerr[1] also found $B \simeq 0.87$ for the Warhaft–Lumley[3] data. This apparent "universality" of the constant B may seem surprising in view of Eq. (4), which seems to predict a dependence of B on the ratio of the scalar scale to turbulence scale. This, however, will have to be examined more closely since R_0 is undefined in the sense that the scalar scale grows with distance from the heating screen. The correct interpretation of the observed "universality" of B is [from Eqs. (2) and (5)] that it implies a universality of the power law index n in Eq. (2). This is indeed consistent with the findings in Ref. 4.

Finally, we can tentatively assign a meaning to R_0. From Eq. (4), $B \simeq 0.875$ implies that $R_0/L_0 \simeq 0.044$. For the case $M_\theta/M = 0.44$, Table I shows the values of R_0 with L_0 chosen to be the transverse integral scale of turbulence at x_s. (We may note here that the integral scales in Ref. 4 were obtained by measuring the area up to the first zero under the measured two-point correlation function.) Also given in Table I is the integral scale L_θ^* of the scalar fluctuations at the point $(x/x_s \simeq 1.9)$ beyond which the Nelkin-Kerr theory seems to apply (see Fig. 1). It is clear from Table I that R_0 bears a constant ratio of about 0.2 to the characteristic integral scale L_θ^*.

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