Evolution of the centerline probability density function of temperature in a plane turbulent wake

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Measurements are presented of the evolution of the probability density function of the temperature fluctuation along the centerline of the wake of a heated circular cylinder in the range $4 \leq x/d \leq 300$.

I. INTRODUCTION

It has been shown[1,2] that the problem of turbulent mixing in chemically reacting flows can be more advantageously formulated in terms of the probability density function rather than in terms of the more familiar moments. In the limit of diffusion-controlled reactions, the probability density function of the reacting species can be obtained from those of a scalar admixture measured in the same class of shear flows.[3] Dopazo[4] formulated the problem of the evolution of the probability density function of temperature fluctuations and presented a simple closure scheme to obtain explicit results for an axisymmetric heated jet. His results compared favorably with the measurements of Venkataramani *et al.*[5] along the centerline of the jet. The reasonable degree of success attained with respect to a heated jet provides a motivation for similar measurements in other shear flows; a further motivation for these measurements is their general paucity in nonself-preserving regions of turbulent shear flows, where important fluid dynamical phenomena occur in turbulent mixing with and without chemical reactions. Clearly, the theoretical formulation is the simplest for the centerline evolution where the further complications due to intermittency are absent. Here, we present measurements of probability density functions of temperature fluctuations at several streamwise locations along a line in the centerplane of the wake of a heated circular cylinder. The range of streamwise distance covered is $4 \leqslant x/d \leqslant 300$, where d is the diameter of the cylinder.

II. DESCRIPTION OF EXPERIMENTS

A nichrome cylinder ($d \simeq 0.16$ cm) was mounted horizontally in a wind tunnel whose test section was 38 cm wide and 23 cm high; the cylinder was located well into the test section of the wind tunnel. The freestream velocity U_∞ was about 10.8 msec^{-1} ($U_\infty d/\nu \simeq 1160$). The cylinder was heated to about 200°C above ambient. Temperature measurements were made with a 0.6 μm diam platinum – 10% rhodium wire of about 0.8 mm length (wire resistance $\simeq 580\ \Omega$), operated "cold" at a constant current of 0.1 mA. The Corrsin – Obukhov scale η_θ (i.e., the smallest temperature scale of interest) at the last measurement station was estimated to be about 0.4 mm (and the corresponding frequency to be about 4.3 kHz). Although the wire cannot thus resolve the finest scales of the temperature field, it provides reasonable estimates for quantities not too sensitive to details in the upper end of the dissipation range. Detailed profiles of the mean temperature difference ΔT and the root-mean-square θ' of the temperature fluctuation θ were measured (*in situ*) at various x/d.

The temperature fluctuation θ was recorded on a Hewlett – Packard 3960 analog FM tape recorder, and was later digitized (with a 10 bit digitizer, including sign) and processed on a PDP 11/45 computer. The sampling frequency was 8000 Hz and the number of samples was 6×10^5; the real time record of 75 sec was found to be sufficient to reliably determine moments up to order 8 (available with the author upon request). Probability density functions were generated for different bins varying between 128 and 1024.

III. RESULTS AND DISCUSSION

The fluid-mechanical centerplane of the wake determined from the carefully established points of symmetry of the mean and root-mean-square temperature profiles was found to be shifted upward in the wind tunnel at a constant but small angle of about 1.3° to the geometric centerplane. The virtual origin x_0 for the wake growth was determined by plotting $(\Delta T)^{-2}$ and $1/\theta'^2$ against x/d, and extrapolating the asymptotic linear part of the curves to intersect the x axis (see Fig. 1). Both straight lines intersect the x axis at the same point to give $x_0 \simeq -75d$. This should be compared with $x_0 = -50d$ of Freymuth and Uberoi[6] and $x_0 = -40d$ of LaRue and Libby.[7] The discrepancies among these estimates in x_0 are not surprising in view of the nonuniqueness of the wake growth in the initial stages of its development.[8] For $x/d \geqslant 150$, the measured ΔT and θ' profiles were generally similar to those of Freymuth and Uberoi,[6] and LaRue and Libby[7]; for example, the centerline value of $\theta'/\Delta T$ was found to be 0.25 as against 0.21 of Ref. 6 and

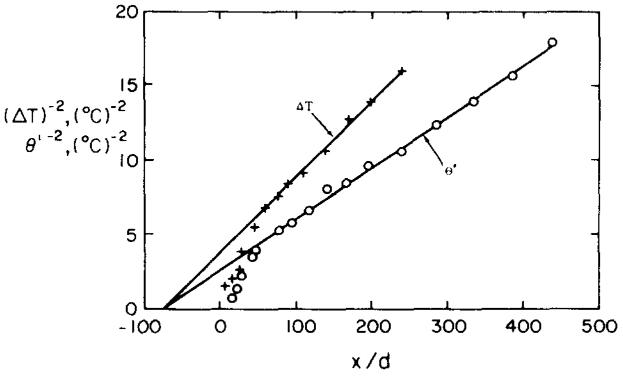

FIG. 1. The centerline mean and root-mean-square temperature variation.

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0.28 of Ref. 7. At $x/d = 300$, the measured half-wake thickness was about 4.2 cm, so that the aspect ratio at that station was about 9.

Figures 2(a) and (b) show how an initially positively skewed probability density function at $x/d = 4$ gradually evolves into a negatively skewed shape at $x/d = 300$, characteristic of its asymptotic state in a fully developed wake; also shown for comparison is the Gaussian with the same mean and standard deviation. A strong large eddy structure of the von Kármán vortex type present in the initial region of wake development (say, $x/d \lesssim 78$) pumps "fresh" low temperature fluid toward the centerline, thus giving rise to the observed peaks in the probability density function at some negative θ. The hot fluid, on the other hand, would have undergone sufficient mixing in its relatively longer history of motion from the cylinder, thus resulting in a spread-out probability density function toward positive θ. The two together explain the observed positive temperature skewness for $4 \lesssim x/d \lesssim 78$. Although the lowest attainable temperature is limited by the free-stream value, there is no evidence of an abrupt cutoff at the lower end of the probability density functions: clearly, the flow at the wake centerline is fully turbulent unlike, for example, the heated turbulent jet in a co-flowing stream which exhibits definite, although small, intermittency on its centerline (γ, the intermittency factor $\simeq 0.998$).[9] At about $x/d = 78$ the measured distribution is essentially symmetrical, and the departures from Gaussianity are small. The negative skewness that develops farther downstream into the asymptotic region suggests a restructuring of turbulent eddies in a way essentially different from that in the region $x/d \lesssim 78$.

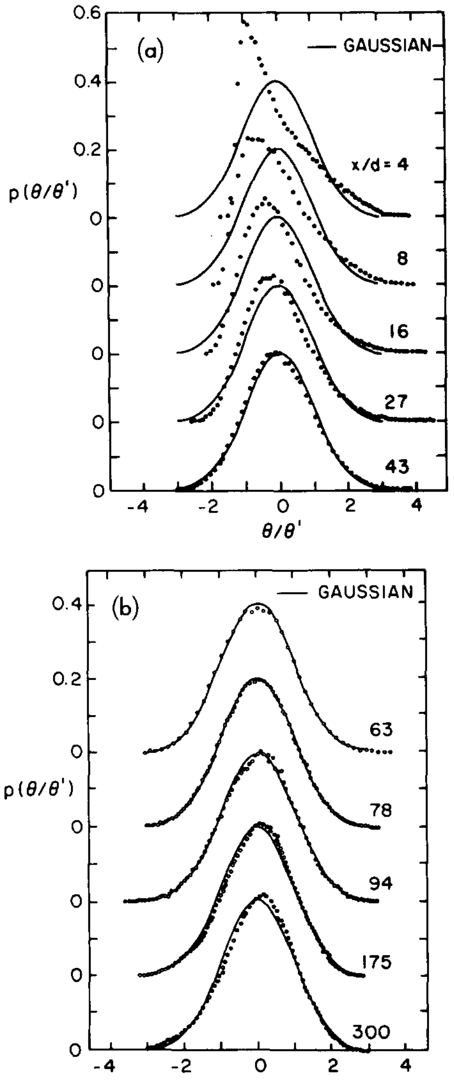

FIG. 2. Normalized probability density functions of temperature fluctuation θ along a line in the fluid-mechanical centerplane. (a) $x/d = 4, 8, 16, 27, 43$. (b) $x/d = 63, 78, 94, 175, 300$.

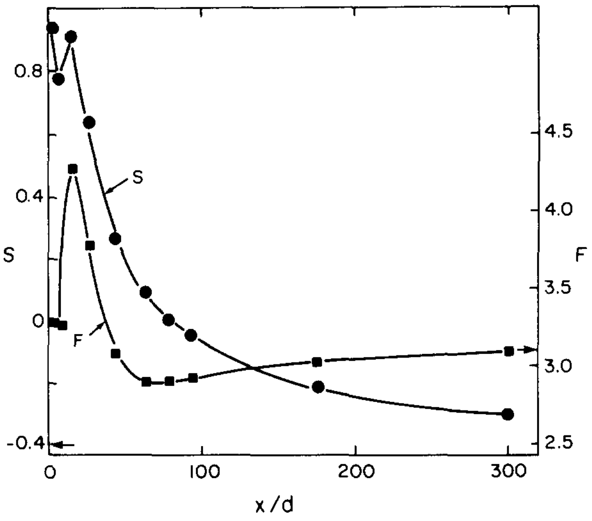

FIG. 3. Skewness and flatness factors as a function of x/d. Arrows on the ordinate are data from LaRue and Libby.[7]

Figure 3 shows a plot of the skewness S and flatness factor F evaluated from the measured probability density functions. Reasonably accurate measurements for

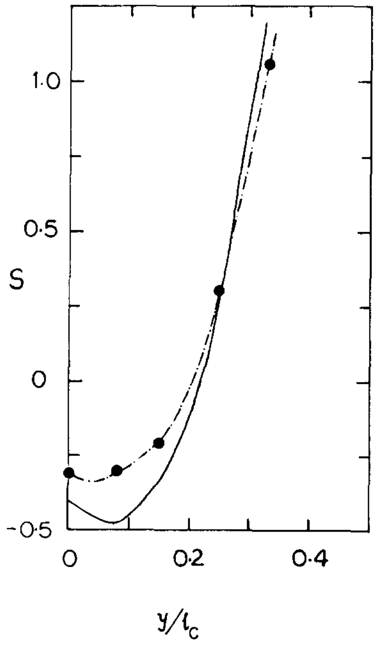

FIG. 4. A comparison of the present skewness data ($\bullet$, — • — • —) with those of Ref. 7 (——). $l_c \equiv [(x - x_0)d]^{1/2}$.

$x/d \lesssim 4$ were not possible, but the dip in the flatness factor very close to the cylinder suggests that the probability density functions undergo a strong change in character in this region. Another interesting feature is the nonmontonic approach of the probability density functions to the asymptotic state. The flatness factor reaches a minimum of 2.9 (at about the same x/d where the skewness is zero) before rising to about 3.1 at $x/d = 300$. Higher order moments up to order 8 (not presented here) also confirm this nonmonotonic approach to the asymptotic state. It is known[8] that mean flow parameters characterizing the self-preservation of the wake also show this nonmonotonicity.

Further evolution beyond $x/d = 300$ was found to be weak in the present experiments. This is partly reinforced by the centerline skewness and flatness data from LaRue and Libby[7] at $x/d = 400$, which are also included in Fig. 3. It is seen that their F is about the same as the present value at $x/d = 300$. The present off-axis flatness factors were also found to be generally close to the LaRue – Libby values. However, some differences were found between the two sets of skewness data (Fig. 4). Apart from the numerical differences especially close to the wake centerline, the present (somewhat scanty) data do not show a pronounced off-axis minimum. The total duration (75 sec) of the signal record used in the present skewness computations was sufficiently large for the slow convergence of the skewness to be a negligible factor in the observed discrepancy between the two sets of data.

One possible explanation of this discrepancy relates to the difference in the relative length of the sensors used in the two studies. The present sensor length was about $2\eta_\theta$ while it was about $0.5\eta_\theta$ in the LaRue–Libby measurements. The spatial averaging will thus have a relatively more pronounced effect in the present study. If the length scales comparable to η_θ contribute significantly to the temperature skewness, the spatial averaging that is likely to have occurred in the present measurements could explain a somewhat lower skewness than the LaRue – Libby value. In view of the general agreement in the flatness factor data, this explanation can be correct only if the length scales of the order of η_θ contribute relatively little to the flatness factor.

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