ACCURACY OF MOMENTS OF VELOCITY AND SCALAR FLUCTUATIONS IN THE ATMOSPHERIC SURFACE LAYER

K. R. SREENIVASAN, A. J. CHAMBERS, and R. A. ANTONIA
Department of Mechanical Engineering, University of Newcastle, New South Wales, 2308, Australia

(Received in final form 11 October, 1977).

Abstract. A detailed accuracy analysis is presented for moments, up to order four, of both velocity (horizontal u and vertical w) and scalar (temperature θ and humidity q) fluctuations, as well as of the products uw, $w\theta$ and wq, in the atmospheric surface layer. The high-order moments and integral time scales required for this analysis are evaluated from data obtained at a height of about 5 m above the ocean surface under stability conditions corresponding to $z/L \simeq -0.05$. Measured moments and probability density functions of some of the individual fluctuations show departures from Gaussianity, but these are sufficiently small to enable good estimates to be obtained using Gaussian instead of measured moments. For the products, the assumption of joint Gaussianity for individual fluctuations provides a reasonable, though somewhat conservative, estimate for the integration times required. The concept of Reynolds number similarity implies that differences in integration time requirements for flows at different Reynolds numbers arise exclusively from differences in integral time scales. A first approximation to the integral time scales relevant to atmospheric flows is presented.

1. Introduction

The accurate measurement of momentum, heat and moisture fluxes is of paramount importance to the study of the atmospheric surface layer over ocean and land. Unfortunately, measurements published in the literature exhibit considerable scatter. While technical difficulties associated with these measurements may account for part of the scatter, there is also the possibility that this scatter is due to non-stationarity of the flow and/or inadequate length of record used to determine the fluxes. To establish the record length required to determine, within given limits, the average values of products, information about the (not usually available) high-order moments of products is required. For this reason, few error analyses are available in the literature. The only exception is Wyngaard (1973), who provided a useful estimate of averaging times of products uw and $w\theta$ (where u and w are the horizontal and vertical velocity fluctuations and θ is the temperature fluctuation), by considering measurements of the 1968 Kansas field experiment, and making a rough order of magnitude assumption on the integral time scales associated with these products. The difficulty, as stated by Stewart (1974), has been for some time that "the theory of the statistical behaviour of variables such as the product is not well understood", because of the highly non-Gaussian nature of the products. Fortunately, this assertion is not really valid because a large body of information (e.g., Antonia and Atkinson, 1973; Gupta and Kaplan, 1972; Lu and Willmarth, 1972) is now available for laboratory turbulent boundary layers, on the shapes of probability density functions and on high-order moments of products. These studies have also revealed that the assumption of joint Gaussianity of the individual

Boundary-Layer Meteorology **14** (1978) 341–359.

fluctuations is in reasonable agreement with measurements in the fully turbulent part of the boundary layer.

In this paper, this favourable situation is exploited to provide error statistics of high-order moments (up to order four) of products as well as individual fluctuations forming the product. The moments and integral time scales, for individual fluctuations as well as products, are evaluated for data obtained from an experimental investigation of the marine surface layer in Bass Strait (Antonia *et al.*, 1977). Similar estimates are also provided with the assumption of Gaussianity for the individual fluctuations. Present data on high-order moments are discussed in Section 4, and compared with other similar measurements in both laboratory and atmospheric boundary layers. Using the concept of Reynolds number similarity, it is argued that the results inferred from the present data (at least those not involving temperature fluctuations) are generally valid for surface layers of zero or near-zero values of $-z/L$. Consequently, in accuracy estimates of moments of products, use of laboratory values (obtained in neutral boundary layers) for the required high-order moments may be acceptable in atmospheric flows with small but non-zero $-z/L$; the only crucial factor is the integral time scale of products as well as power of products, relevant to the particular situation. A further simplification is possible because the ratio of integral time scales of higher powers of fluctuations to that of the first power is essentially the same as for laboratory flows. A first approximation to this ratio is explicitly given. For the integral time scales of the first powers themselves, it is argued that at least some of the present non-dimensional results would be valid for other neutral or near-neutral atmospheric surface layers. The accuracy of the present data on integral time scales is also assessed in the appendix.

2. Experimental Technique

Measurements of u, w, θ and q were recorded on Kingfish B, the ESSO-BHP natural gas platform which stands in Bass Strait (148° 9′E, 38° 36′S) about 80 km off the Gippsland coast of Victoria, Australia. The instruments for recording the above signals were mounted at a height z of about 5 m above the mean water level (on a vertical pipe), supported at the end of a horizontal boom fastened to one of the western platform legs. The horizontal velocity fluctuation u was obtained with a hot wire (5 μm diameter, ~0.8 mm length) operated by a DISA 55M01, constant-temperature anemometer. The vertical velocity fluctuation w was obtained using a Gill propeller. Temperature θ was measured with a cold wire (0.6 μm diameter platinum, ~0.8 mm length) operated by a constant-current anemometer. The value of the current was low enough (~0.1 mA) for the wire to be sensitive to temperature fluctuations only. Low-frequency temperature fluctuations were also obtained by a thermistor. The humidity fluctuation q was obtained using a Lyman-alpha humidiometer. Neither the hot-wire anemometer nor the Lyman-alpha humidiometer was linearized. Over the whole experiment, wind conditions were stationary and corresponded to a $z/L \approx -0.05$, where L is the Monin–Obukhov length. It is essentially this observed stationarity that enables a meaningful definition of integral scales and high-order moments.

Voltages proportional to u, w, q and θ fluctuations were recorded on a four-channel Hewlett–Packard 3960 FM tape recorder. The recording speed was 24 mm s^{-1} (-3 dB point of tape recorder 375 Hz). The tapes were played back and digitized at a sampling frequency of 20 Hz in the Faculty of Engineering Computing Centre at the University of Sydney. Prior to digitization, the signals were low-pass filtered with the -3 dB cut-off frequency set at 10 Hz. The digital records were processed both on a PDP 11/45 computer and on an ICL 1904A computer at the University of Newcastle. Further details of experimental conditions and techniques may be found in Antonia *et al.* (1977).

3. Accuracy of Measurements

All moments of u, w, θ and q were computed from the relation

$$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n p(x)\,\mathrm{d}x\,, \tag{1}$$

where $p(x)$ is the probability density function of x, normalized such that $\int_{-\infty}^{\infty} p(x)\,\mathrm{d}x = 1$. All probability density functions were generated for numbers of equal bins varying between 128 and 1024. For some test cases, moments computed according to Equation (1) were in excellent agreement with those computed directly from the time series according to the relation

$$\langle x^n \rangle = \frac{1}{T}\int_{0}^{T} x^n(t)\,\mathrm{d}t\,. \tag{1a}$$

For the products uw, $w\theta$ and wq, however, because of the sharp peaks in the probability density functions, greater accuracy can be expected if moments are computed from the time series. Consequently, all moments in the case of products were computed according to Equation (1a).

Records of duration varying between 20 and 66 min were examined, and an average obtained over a number of runs varying between 9 and 15. Running averages of normalized moments of u and w are shown in Figures 1 and 2 for a typical run. Although the discussion in this paper is restricted to moments only up to order four, higher order moments are also presented here to provide a useful indication of the accuracy of the sixth- and eighth-order moments, which are used in error estimates of the third- and fourth-order moments, respectively. Figures 1 and 2 indicate that flatness, superflatness (i.e., $\langle (x - \langle x \rangle)^6 \rangle / \sigma_x^6$) and hyperflatness (i.e., $\langle (x - \langle x \rangle)^8 \rangle / \sigma_x^8$) factors converge to within about 10% of their final values in about half the duration of the total record used to obtain the present statistics. Here σ_x is the standard deviation of x defined by $\sigma_x = \langle (x - \langle x \rangle)^2 \rangle^{1/2}$. In the case of

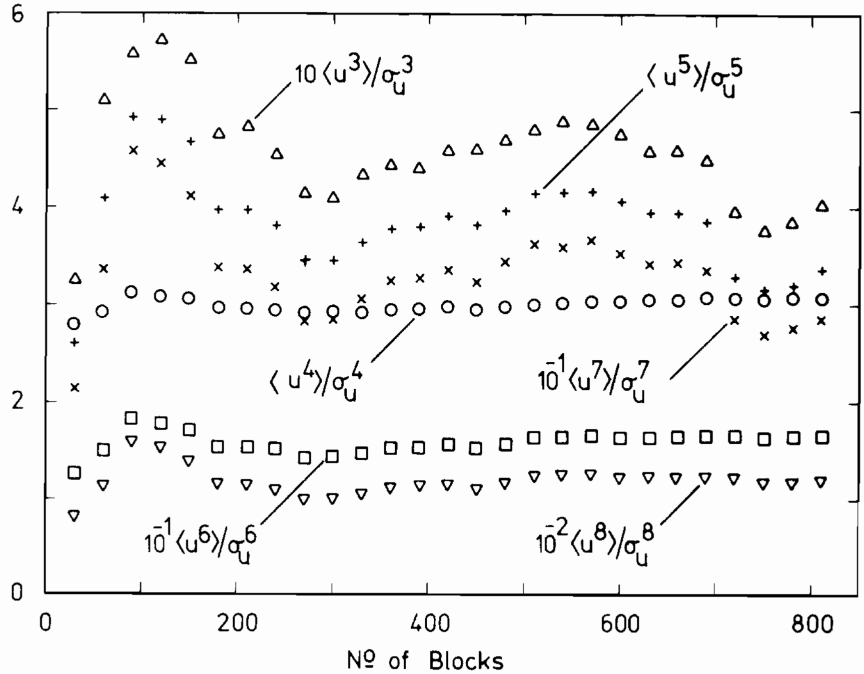

Fig. 1. Variation with record length of the normalised central moments of the horizontal velocity fluctuation. Each block corresponds to a duration of 4.267 s.

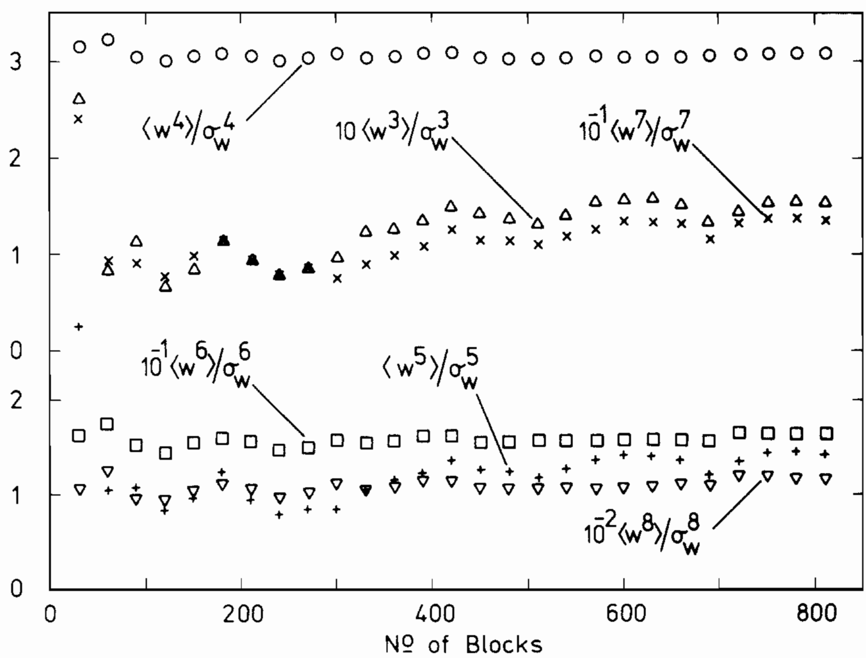

Fig. 2. Variation with record length of the normalized central moments of the vertical velocity fluctuation w.

odd-order moments, the convergence is poorer (especially for u) than for even-order moments, as the trend of the even-order running moments is overemphasized in odd-order moments. It is worth noting that all even-order moments, up to the eighth, of u and w are remarkably close to the appropriate Gaussian values. Moments of other quantities show a qualitatively similar behaviour. (This fact is extensively used in the error analysis in this section.) Odd-order moments of u show a somewhat larger departure from Gaussianity than those of w.

Running central moments of the product uw are given in Figure 3. Again, there is a considerable trend in odd-order moments. Greater reliance can therefore be placed only on even-order moments up to 4, both in the case of fluctuations and their products.

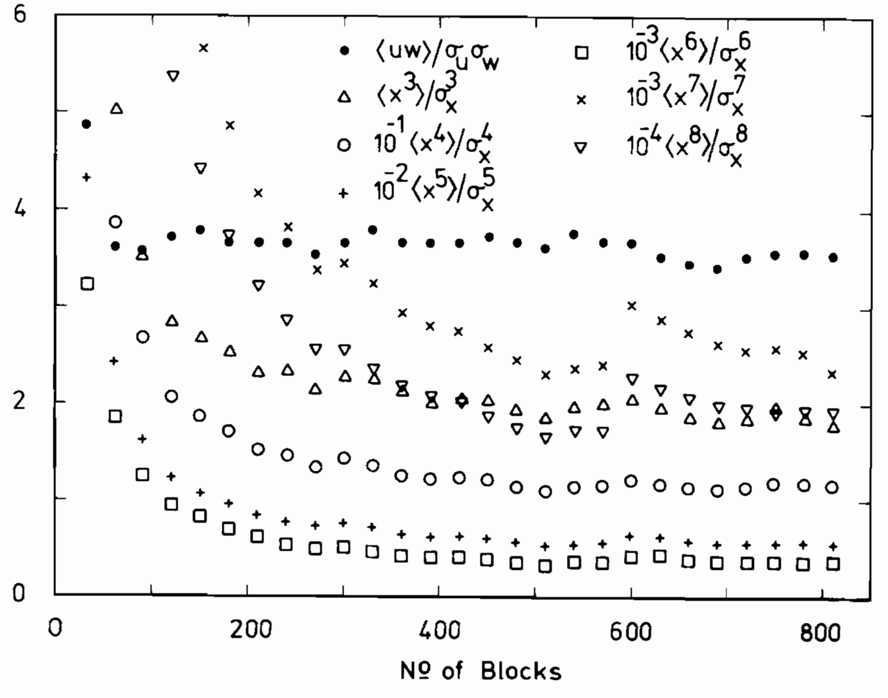

Fig. 3. Variation with record length of the normalized moments of $x = uw - \langle uw\rangle$.

An estimate of the accuracy of the first-order moment may be obtained with the use of an expression given by Lumley and Panofsky (1964) or Tennekes and Lumley (1972), i.e.,

$$\varepsilon^2 = \frac{\langle x^2\rangle - \langle x\rangle^2}{\langle x\rangle^2}\frac{2\tau_1}{T_x}. \tag{2}$$

In Equation (2), ε^2 is the mean-square relative error, determined by integration over a duration T_x, of the mean value of a stationary random signal x whose true mean and variance are $\langle x \rangle$ and $\langle x^2 \rangle$, respectively, and whose integral time scale is τ_1. Expression (2) can be extended to estimate the mean-square error of any moment of orde1 n, by replacing x by x^n and τ_1 by the appropriate integral time scale τ_n associated with x^n, i.e.,

$$\varepsilon^2 = \left(\frac{F_{2n}}{F_n^2} - 1\right) 2 \frac{\tau_n}{T_x}, \tag{2a}$$

where $F_{2n} = \langle x^{2n} \rangle \langle x^2 \rangle^n$ and $F_n = \langle x^n \rangle / \langle x^2 \rangle^{n/2}$. In the case of the product xy (x and y may represent u, w, θ or q), it is convenient to discuss moments of $xy - \langle xy \rangle$, rather than moments of xy.

Values of the integral time scale τ_n were obtained from the autocorrelation curves of x^n or $(xy - \langle xy \rangle)^n$. Details of the method adopted of deriving autocorrelations, as well as of their accuracy, are discussed in the appendix. The magnitude of τ_1 normalized by the ratio z/U, as well as the ratio τ_n/τ_1 ($n \le 4$), are given in Table I for the quantities u, w, θ, q, $uw - \langle uw \rangle$, $w\theta - \langle w\theta \rangle$ and $wq - \langle wq \rangle$. Wyngaard

TABLE I

Integral time scales τ_1 and the ratio τ_n/τ_1 ($U \simeq 9.1$ m s^{-1}, $z = 5.5$ m). Numbers in parentheses are values obtained from sub-records of 400-s duration. Numbers in square brackets against u and q are evaluated from time series directly, according to Equation (A2), with $T_1 = 80$ s

		τ_n/τ_1		
Quantity	$\tau_1 U/z$	$n = 2$	$n = 3$	$n = 4$
u	3.9	0.70	0.74	0.50
	[3.8]	(0.60)	(0.72)	(0.43)
w	1.5	0.56	0.64	049
θ	4.9	0.73	0.70	0.51
q	4.1	0.69	0.73	0.53
	[4.7]			
$uw - \langle uw \rangle$	1.2	0.80	0.50	0.39
$w\theta - \langle w\theta \rangle$	1.2	0.73	0.67	0.44
$wq - \langle wq \rangle$	1.1	0.64	0.59	0.48

(1973) assumed that $\tau_2 \approx z/U$ to obtain estimates of integration times required to achieve some specified accuracy in second-order moments of w and θ. In this case, it follows from Equation (2a) that

$$T_{w^2, \theta^2} \simeq \frac{4z}{\varepsilon^2 U} \tag{3}$$

when the flatness factor of w or θ is approximately equal to 3. This last assumption appears to be reasonable (Section 4) for all the fluctuations considered here. The present value of T for w^2 is in reasonable agreement with that obtained from

Equation (3) since the scale $\tau_2 U/z$ is close to unity in the case of w^2 (Table I). For the second moments of u, θ and q, however, Equation (3) underestimates the integration time significantly as the corresponding integral scales are higher than z/U. For obtaining flatness factors within a specified error, similar estimates for the required integration time can be made using $\tau_4 U/z$ obtainable from Table I, provided reasonable estimates for the eighth-order moments are available. Sample data shown in Figures 1 and 2 provide at least partial proof that the eighth-order moments obtained in the present runs are reasonably accurate for this purpose. The resulting estimates of integration times are shown in Table II. The corresponding estimates using moments appropriate to a Gaussian variable (i.e., $F_4 = 3$ and $F_8 = 105$) are very nearly the same because, as mentioned above, all even-order

TABLE II

Integration times required to determine moments to accuracies of 10% and 20%. For products, numbers in parentheses are estimates based on Equation (10) using the measured value of r

	Time T, min			
	Present estimates		Wyngaard (1974)	
x	$\varepsilon = 0.1$	$\varepsilon = 0.2$	$\varepsilon = 0.1$	$\varepsilon = 0.2$
u^2	12.1	3.0	4.0	1.0
w^2	3.4	0.9	4.0	1.0
θ^2	18.1	4.5	—	—
q^2	10.8	2.7	—	—
u^4	53.1	13.3	21.5	5.4
w^4	17.6	4.4	21.5	5.4
θ^4	56.2	14.1	—	—
q^4	36.8	9.2	—	—
$(uw - \langle uw \rangle)^2$	20.4	5.1	—	—
	(20.2)	(5.1)		
$(w\theta - \langle w\theta \rangle)^2$	15.8	3.9	—	—
	(16.1)	(4.0)		
$(wq - \langle wq \rangle)^2$	10.3	2.57	—	—
	(14.1)	(3.5)		
$(uw - \langle uw \rangle)^3$	150	38	—	—
	(218)	(54.5)		
$(w\theta - \langle w\theta \rangle)^3$	389	97	—	—
	(310)	(77.8)		
$(wq - \langle wq \rangle)^3$	99	25	—	—
	(146)	(36.5)		
$(uw - \langle uw \rangle)^4$	136	34.0	—	—
	(218)	(54.5)		
$(w\theta - \langle w\theta \rangle)^4$	90.0	22.5	—	—
	(231)	(57.8)		
$(wq - \langle wq \rangle)^4$	85.6	21.4	—	—
	(300)	(75.3)		

moments up to the eighth are close to Gaussian values. From Table II, it appears that, for the integration times used in the present runs, the second- and fourth-order moments are accurate to within about ±10%. Unfortunately, to obtain odd-order moments to the same accuracy, very long integration times are necessary. For example, to obtain the skewness values to within ±20% accuracy, Equation (2) suggests that an integration time of the order of $2\frac{1}{2}$–3 hours is required for u and θ, and substantially longer record durations for w and q. The reason for this is the numerically small value of skewness; if the skewness is exactly zero, the integration time required is indeterminate. A less pessimistic and a more realistic error specification for near-zero odd-order moments should perhaps be in terms of an absolute error band, say ±0.1. From this point of view, integration times used in the present runs seem quite adequate for the skewness values too.

In the case of $uw-\langle uw\rangle$, $w\theta-\langle w\theta\rangle$ and $wq-\langle wq\rangle$, the integration times required are

$$\frac{UT_{uw-\langle uw\rangle}}{z}=\frac{2}{\varepsilon^2}\frac{\tau_1 U}{z}\left(\frac{\sigma^2_{uw-\langle uw\rangle}}{U_*^4}-1\right) \tag{4}$$

$$\frac{UT_{w\theta-\langle w\theta\rangle}}{z}=\frac{2}{\varepsilon^2}\frac{\tau_1 U}{z}\left(\frac{\sigma^2_{w\theta-\langle w\theta\rangle}}{U_*^2\theta_*^2}-1\right) \tag{5a}$$

and

$$\frac{UT_{wq-\langle wq\rangle}}{z}=\frac{2}{\varepsilon^2}\frac{\tau_1 U}{z}\left(\frac{\sigma^2_{wq-\langle wq\rangle}}{U_*^2Q_*^2}-1\right), \tag{5b}$$

where U_* $(=-\langle uw\rangle^{1/2}/U_{\text{ref}})$, θ_* $(=\langle w\theta\rangle/U_*)$ and Q_* $(=\langle wq\rangle/U_*)$ are the friction velocity, temperature and humidity, respectively; here, 5 m was used as reference height. Wyngaard assumed that $\tau_1\approx z/U$ and found that experimental results under nearly neutral conditions indicated a value of about 10 for the quantities within the circular brackets in Equations (4) and (5a) so that

$$\frac{UT_{uw-\langle uw\rangle}}{z}\approx\frac{UT_{w\theta-\langle w\theta\rangle}}{z}\approx\frac{20}{\varepsilon^2}. \tag{6}$$

No estimates were given for the corresponding quantities in Equation (5b). The present values of $\sigma_{uw-\langle uw\rangle}/U_*^2$, $\sigma_{w\theta-\langle w\theta\rangle}/(U_*\theta_*)$ and $\sigma_{wq-\langle wq\rangle}/(U_*Q_*)$ and appropriate integral time scales given in Table I lead to

$$\frac{UT_{uw-\langle uw\rangle}}{z}\approx\frac{30}{\varepsilon^2} \tag{7}$$

$$\frac{UT_{w\theta-\langle w\theta\rangle}}{z}\approx\frac{64}{\varepsilon^2} \tag{8a}$$

and

$$\frac{UT_{qw-\langle qw\rangle}}{z}\approx\frac{44}{\varepsilon^2}, \tag{8b}$$

corresponding to Equations (4), (5a) and (5b), respectively. Estimates given by Equations (7) and (8a) are larger than Wyngaard's estimate (6). The integration time required for $\langle w\theta\rangle$ appears to be the largest and is about twice as large as that for $\langle uw\rangle$.

In the case of higher-order powers of the products, estimates of T given in Table II were obtained from measured values of $\tau_n U/z$ (Table I), and the measured flatness and higher-order moments of products. For the third- and fourth-order moments of the products, the values of T given in Table II can be considered only as rough estimates, since the sixth- and eighth-order moments of products are more difficult to determine accurately than those of individual fluctuations (cf. Figures 1 and 3). It is worth noting that because of the substantially non-zero values of odd-order moments of the products, it is possible to obtain them to a better relative accuracy than those of the individual fluctuations, using records of reasonably long duration. For the present averaging times, Table II suggests that the error in estimating the third- and fourth-order moments is about 20% or less, and probably less than 10% in the case of second-order moments.

For products, an alternative plausible method of estimating integration times would be to use moments of the product xy, under the assumption that the joint probability density $p(x, y)$ of x and y is Gaussian*. For this case, the product xy has the probability density (see, e.g., Antonia and Atkinson, 1973; Lu and Willmarth, 1972)

$$p(xy)=\frac{\exp\,[\{r/(1-r^2)\}xy]}{\pi(1-r^2)^{1/2}}K_0\left(\frac{|xy|}{1-r^2}\right), \tag{9}$$

where r is the correlation coefficient $\langle xy\rangle/\sigma_x\sigma_y$ and K_0 is the zeroth-order modified Bessel function of the second kind. Antonia and Atkinson (1973) derived expressions for the skewness and flatness factor of xy. General expressions for the nth order moments of the products were given by Lu and Willmarth (1972) and Sreenivasan *et al.* (1977). It may be useful here to write explicitly the first eight moments of xy:

$$\begin{aligned}
&\langle xy\rangle = r\\
&\langle(xy-\langle xy\rangle)^2\rangle = 1+r^2\\
&\langle(xy-\langle xy\rangle)^3\rangle = 6r+2r^3\\
&\langle(xy-\langle xy\rangle)^4\rangle = 9+42r^2+9r^4\\
&\langle(xy-\langle xy\rangle)^5\rangle = 180r+320r^3+44r^5\\
&\langle(xy-\langle xy\rangle)^6\rangle = 225+2835r^2+2715r^4+625r^6\\
&\langle(xy-\langle xy\rangle)^7\rangle = 9450r+42210r^3+28014r^5+1854r^7\\
&\langle(xy-\langle xy\rangle)^8\rangle = 11025+270900r^2+630630r^4+263284r^6+16513r^8\,.
\end{aligned} \tag{10}$$

* In the present experiments, measured isoprobability density contours of $p(x, y)$, where x and y are u, w, θ or q, did not differ significantly from the Gaussian elliptic contours, so that this approximation should lead to reasonable error estimates, at least to a first approximation.

For the particular case when $r=0$, $\langle xy\rangle=0$ and $\langle (xy)^n\rangle=\langle x^n\rangle\langle y^n\rangle$; the hyperflatness $\langle xy\rangle^8/\langle (xy)^2\rangle^4=(105)^2$, i.e., the square of the hyperflatness factor of individual Gaussian components. For $r=\pm 0.25$, and ± 0.5 (which approximately cover the range of present measurements), the hyperflatness values are approximately 2.4×10^4 and 3.3×10^4, respectively. Although these are roughly of the same order of magnitude as the measured ones, in general, Gaussian high-order moments are larger than the measured values. Thus, estimates based on Equation (10), also shown in Table II, yield generally conservative values for the integration times.

It is worth emphasizing that when the accuracy of high-order moments of products is assessed by the use of formula (2a), it is important that the moments in (2a) correspond to those for a Gaussian joint probability density function, and not simply, as implied by McBean (1974), to those for a Gaussian probability density function of the individual variables. The reason is that for the products, integration times are larger and increase much faster with the order of the moment than for a Gaussian variable. Using typical values of 10 and 2×10^4 for flatness and hyperflatness, respectively, it is seen that as n increases from 1 to 2, and then from 2 to 4, the factor (F_{2n}/F_n^2-1) in Equation (2a) increases by about 10 and 20, respectively, while the corresponding increases would be about 2 and 5, respectively, for a Gaussian variable.

Finally, for a given quantity x, Equation (2a) may be rewritten as

$$T_x U/z = m_x/\varepsilon^2 \tag{11a}$$

when F_n and F_{2n} in Equation (2a) are evaluated from measurement. In the case when F_n and F_{2n} in Equation (2a) are evaluated from the Gaussian probability density function for the fluctuations, and from Equation (9) for the products (using measured values of r), Equation (11a) may be replaced by

$$T_x U/z = g_x/\varepsilon^2\,. \tag{11b}$$

Then, the present error analysis can be summarized in terms of the two constants m_x and g_x listed in Table III.

TABLE III

Constants m_x and g_x in Equations (11a) and (11b)

x	m_x	g_x	x	m_x	g_x
u^2	12	12	$uw-\langle uw\rangle$	30	39
u^3	150	—	$(uw-\langle uw\rangle)^2$	20	20
u^4	53	53	$(uw-\langle uw\rangle)^3$	149	217
w^2	3	3	$(uw-\langle uw\rangle)^4$	134	217
w^3	—	—	$w\theta-\langle w\theta\rangle$	64	45
w^4	17	17	$(w\theta-\langle w\theta\rangle)^2$	16	16
θ^2	18	14	$(w\theta-\langle w\theta\rangle)^3$	386	308
θ^3	180	—	$(w\theta-\langle w\theta\rangle)^4$	90	229
θ^4	56	53	$wq-\langle wq\rangle$	44	12
q^2	11	11	$(wq-\langle wq\rangle)^2$	10	14
q^3	—	—	$(wq-\langle wq\rangle)^3$	98	145
q^4	37	47	$(wq-\langle wq\rangle)^4$	85	298

4. Discussion of Moments of Velocity and Scalar Fluctuations

Figure 4 shows the normalised probability density functions of u, w, θ and q for a typical run (duration 57.6 min) in the present experiment. Also shown are Gaussian functions with the same mean and variance. Mean values and standard deviations of skewness and flatness factors of the quantities u, w, θ and q obtained for a number of runs are shown in Table IV. Both u and θ are significantly skewed while q and w are remarkably symmetric about their mean values. The flatness factor of q shows the largest deviations from the Gaussian values of 3. The negative sign of S_u and the positive sign of S_θ are consistent with the notion that probability density functions of u and θ, at a height of 5 m above the sea surface under nearly neutral conditions, reflect the arrival of lower momentum fluid from the warm sea surface.

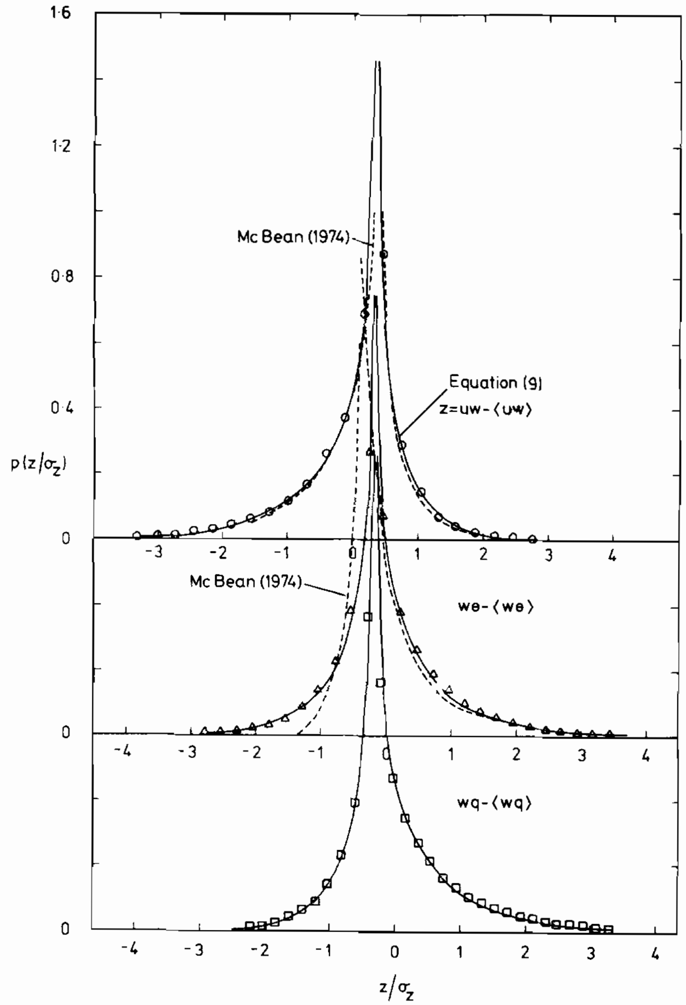

Fig. 4. Normalized probability density of fluctuations.

TABLE IV

Skewness and flatness data

Quantity	No. of runs	Mean	Standard deviation	Standard error of mean
S_u	16	−0.35	±0.05	0.01
F_u	16	2.94	0.23	0.06
S_w	22	0.02	0.10	0.02
F_w	22	3.16	0.13	0.03
S_θ	18	0.39	0.40	0.09
F_θ	18	3.05	0.39	0.09
S_q	16	−0.04	0.10	0.03
F_q	16	2.67	0.16	0.04

This idea is supported by the sign of S_w but not by that of S_q. The experimental correlation

$$z/L = -0.0137 \exp(4.396\, S_\theta) \tag{12}$$

obtained by Tillman (1972) for over-land measurements seems to be in fair agreement with the present data. For $z/L = -0.05$, the value of the stability parameter that prevailed over most of the present runs, correlation (12) yields $S_\theta \approx 0.3$, while the measured average value is about 0.4. Considering that this correlation was obtained from data with significant scatter, and the possible error in skewness measurements in general, the present value is in good agreement with comparable measurements over land.

Standard deviations of products uw, $w\theta$ and wq about their respective means, are given in Table V, normalized by U_*, θ_* and Q_*. Also included are values obtained from Equation (10) appropriate to the probability density function (9). Although the mean value of $\sigma_{uw-\langle uw\rangle}$ is significantly higher than the value of about 2.4

TABLE V

Regression lines of standard deviations of products on wind speed U_5 (m s^{-1}) and non-dimensional height z/L. Values in parentheses in the second column correspond to the probability density (9)

Parameter	Mean ± std. dev.	Linear regression	(± std. dev.)	Correl.	Std. deviation Intercept	Std. deviation Slope
$\sigma_{uw-\langle uw\rangle}/U_*^2$	3.49 ± 0.36	$3.13 - 6.24z/L$	(0.39)	0.18	±0.53	±10.03
	(4.25)	$5.44 - 0.22U_5$	(0.37)	0.38	±1.50	±0.17
$\sigma_{w\theta-\langle w\theta\rangle}/U_*\theta_*$	5.15 ± 1.06	$5.546 + 6.85z/L$	(1.15)	0.15	±0.86	±13.86
	(4.58)	$4.543 + 0.07U_5$	(1.16)	0.07	±2.59	±0.30
$\sigma_{wq-\langle wq\rangle}/U_*Q_*$	3.19 ± 0.66	$4.779 - 27.66z/L$	(0.72)	0.34	±1.49	±28.94
	(2.80)	$5.276 - 0.23U_5$	(0.73)	0.17	±4.49	±0.50

obtained by McBean (1974) for $z/L \approx -0.05$, it is in good agreement with the range of values for the Kansas experiment reported by Wyngaard (1973). The present mean value $\sigma_{w\theta-\langle w\theta \rangle}$ is significantly higher than either Wyngaard's or McBean's value. These differences in the rms levels are perhaps caused by large-scale fluctuations which contribute little to the stress or scalar fluxes. McBean (1974) suggested that the ratios $\sigma_{uw-\langle uw \rangle}/U_*^2$ and $\sigma_{w\theta-\langle w\theta \rangle}/(U_*\theta_*)$ might be considered as measures of efficiency for momentum and heat transfer processes, respectively. On the basis of this criterion, the results of Table V would indicate that the heat transfer process may be more efficient than either humidity or momentum transfer processes. Linear regression of σ on z/L and the wind velocity at the 5-m height are also given in Table V. The statistical significance of the regression lines is poor (because of the low correlation coefficient and small range of z/L or U_5) but the trend of the variations of σ vs z/L is in agreement with the results of McBean and Wyngaard. In particular, the efficiency of both momentum and moisture transfers would be impaired, whilst the heat transfer efficiency improves, as instability increases.

A comparison between the measured probability density functions of products uw, $w\theta$ and wq (centered about their means), and the probability density function (9), shown in Figure 5, suggests that, over a significant range, the assumption of joint Gaussianity is good for the pairs of fluctuations (u, w), (w, θ) and (w, q). Also shown in the figure are the measurements (renormalized here to unity area in the present variables) of McBean (1974) over land at a height of 2 m ($z/L \approx -0.06$). The agreement in the case of uw is good, emphasizing the similarity of momentum transfer over land and water. In the case of $w\theta$, if the two distributions are considered typical over water and land, the short negative tail in McBean's data indicates that, over land, during the 'events' ($w>0$, $\theta<0$) and/or ($w<0$, $\theta>0$), small amplitudes are more probable and large amplitudes less probable than for comparable events over the ocean. On the other hand, heat transport over land and water, associated with events ($w>0$, $\theta>0$) and/or ($w<0$, $\theta<0$), seems to take place in an essentially similar manner. Considering that u and θ are negatively correlated, these events can be identified respectively with the outward ejection of low momentum fluid from warmer water and a sweep towards the sea surface of high momentum air parcels. Note also that σ_θ/θ_* was significantly higher than observed over land at comparable value of z/L.

Mean values of skewness and flatness factors of the product $xy - \langle xy \rangle$ are given in Table VI. Also given are the correlation coefficient r, and the appropriate Gaussian values for the skewness S and flatness factor F. The agreement between the measured and Gaussian values of S and F is good in the case of uw, $w\theta$ and $u\theta$, and somewhat poor for the products that include the quantity q.

According to Priestley (1959), $z/L \simeq -0.05$ is on the border between free and forced convection regimes. Hence, it is important to consider whether the present data on moments are valid for more general stability conditions. We note here that the transition between forced and free convection regimes is gradual, and different 'critical' values of z/L can be obtained when different flow parameters are

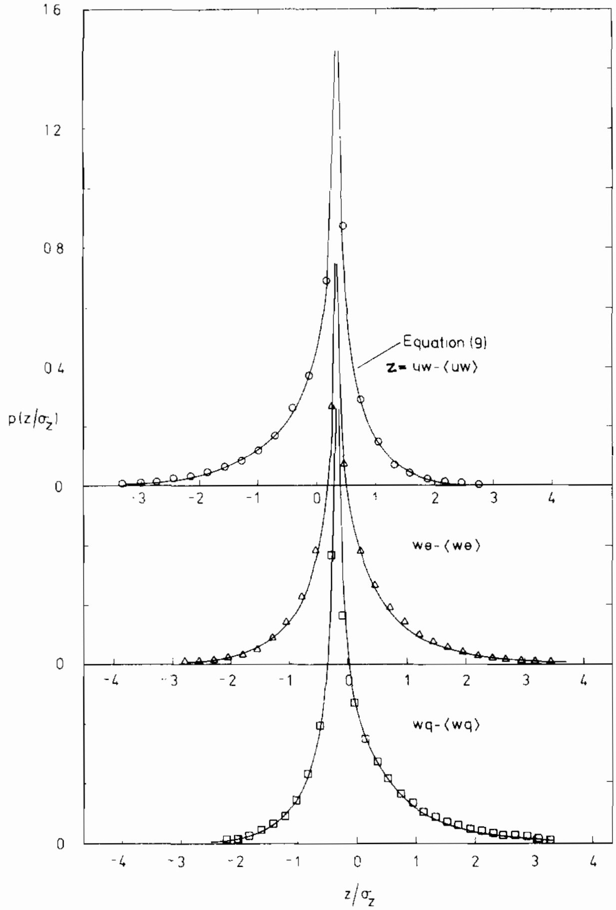

Fig. 5. Normalized probability density of products of fluctuations.

considered. For example, data of Wyngaard *et al.* (1971) indicate that this transition occurs at $-z/L \simeq 0.4$ when the behaviour of σ_w/U_* is used as the criterion, while a critical value of $z/L \simeq -0.04$ can be chosen when the behaviour of σ_θ/θ_* is considered. For the normalized moments considered here, the appropriate critical values are not known *a priori*. Although Pries (1970) provides data on the skewness of u and w for different values of z/L, the trends of these quantities with z/L are unfortunately substantially different at heights of 15–16 m and 90–91 m.

There are some indications that the present data pertaining to u and w (and possibly q) are not very different from those appropriate to neutral conditions of atmospheric stability. Antonia *et al.* (1977) have already noted that σ_u/U_*, σ_w/U_* and σ_q/Q_* obtained from present measurements are generally consistent with those obtained in nearly neutral surface layers. Further, the present values of about -1.4 and 10.8 for the skewness and flatness factor of $uw - \langle uw \rangle$ are in good agreement with the values of about -1.3 and 10, respectively, obtained by

TABLE VI

Mean and standard deviation of skewness and flatness factor of $xy - \langle xy \rangle$

Quantity	Flatness factor F or skewness S	No. of runs	Mean	Standard deviation	Standard error of mean	Correlation coefficient r	Gaussian flatness or skewness
$uw - \langle uw \rangle$	F	15	10.76	1.87	0.48	−0.25	10.33
$w\theta - \langle w\theta \rangle$	F	12	10.03	1.92	0.55	+0.23	10.14
$wq - \langle wq \rangle$	F	9	7.73	0.81	0.27	+0.42	12.06
$uq - \langle uq \rangle$	F	10	8.50	1.46	0.46	−0.62	13.81
$u\theta - \langle u\theta \rangle$	F	11	12.02	3.48	1.05	−0.39	11.75
$q\theta - \langle q\theta \rangle$	F	10	8.99	1.84	0.58	+0.65	14.01
$uw - \langle uw \rangle$	S	15	−1.42	0.42	0.11	−0.25	−1.40
$w\theta - \langle w\theta \rangle$	S	12	+1.08	0.24	0.07	+0.23	+1.30
$wq - \langle wq \rangle$	S	9	+1.24	0.49	0.16	+0.42	+2.09
$uq - \langle uq \rangle$	S	10	−1.96	0.30	0.10	−0.62	−2.58
$u\theta - \langle u\theta \rangle$	S	11	−2.02	0.66	0.20	−0.39	−1.99
$q\theta - \langle q\theta \rangle$	S	10	+1.89	0.38	0.12	+0.65	+2.62

Wyngaard and Izumi (1973) in a neutral surface layer over land. Gupta and Kaplan* (1972) obtain values of about −1.2 and 11.2 in an isothermal laboratory boundary layer at two Reynolds numbers, while Danh (1976) obtained about −1.8 and 11, respectively, also in a laboratory boundary layer. We know that Reynolds number similarity implies that the laboratory values must be equal to the atmospheric values under near-neutral conditions. The good agreement between the present data and the laboratory isothermal data is then a reasonable indication that the statistics of uw are not very sensitive to small departures of $-z/L$ from zero. This conclusion is supported by the data of McBean (1974) who obtained values of −1.3 and 10.2 as averages for all unstable cases he considered; these values changed only to −1.5 and 11.7 when some neutral and some stable cases were also included.

On the other hand, the position relating to $w\theta$ is less conclusive. The present values of 1.1 and 10 for the skewness and flatness factor of $w\theta - \langle w\theta \rangle$ are significantly lower than McBean's average values of 2.3 and 16.1, respectively, over land under unstable conditions. In view of our earlier remarks on the possible differences between heat transfer over land and ocean, it seems difficult to draw definite conclusions, based on this evidence, about the effect of stability. We do not know of any other comparable statistics of $w\theta$ over the ocean. It is worth noting however that Danh (1976) obtained 1.6 and 12, respectively, for the skewness and flatness factor of $w\theta - \langle w\theta \rangle$ in a (slightly heated) neutral laboratory boundary layer.

Corresponding data for wq do not seem to be available in the literature; we tentatively expect (remembering that σ_q/Q_* is, unlike σ_θ/θ_*, consistent with other

* Both Gupta and Kaplan (1972) and Wyngaard and Izumi (1973) evaluated non-central moments. They have here been converted to central moments.

measurements reported at small values of $-z/L$) that the statistics of wq will not be as sensitive to z/L as those of $w\theta$.

In conclusion, it appears that the statistics of quantities not involving θ are in general not very sensitive to the precise value of $-z/L$ if it is small. The general applicability of Reynolds number similarity allows us the use of moments obtained in neutral laboratory boundary layers in Equation (2a) for accuracy estimates in atmospheric surface layers in which $-z/L$ is small but not necessarily zero. However, the integral time scales will have to be evaluated in each case. It is worth emphasizing that the concept of integral scales in atmospheric flows is not as well defined as in laboratory flows because the distinction between 'trends' in mean flow and the lowest frequency of interest for turbulence measurements is not clear. Generally, in the literature, a high-pass filter is set at some arbitrary value; for instance, McBean (1974) uses a value of 0.003 Hz. Consequently, the present value of τ_1 must be considered only as a reasonable estimate.

However, the ratio τ_n/τ_1 will in general not be sensitive to different methods of computing the integral scales, as Table I shows. The considerable reduction in the ratio τ_n/τ_1 $(n > 1)$ is of some practical importance in the assessment of the accuracy of high-order moments. The only theoretical treatment of this problem seems to be that mentioned in Lumley (1970). Lumley quotes Alekseev's calculations for a Gaussian process, assuming an exponential form for the autocorrelation function. The results are $\tau_2/\tau_1 \approx 0.88$ and $\tau_3/\tau_1 \approx 0.69$. Sreenivasan *et al.* (1977) examined all the available laboratory data on τ_n/τ_1 and concluded that this ratio behaves in an essentially similar manner for all laboratory flows. It would be useful to ascertain if this result also holds for atmospheric flows. In Figure 6, a comparison is made between the present values of τ_n/τ_1 $(n \leq 4)$ for u and θ with the corresponding laboratory data obtained on the centre-line of a slightly heated axisymmetric

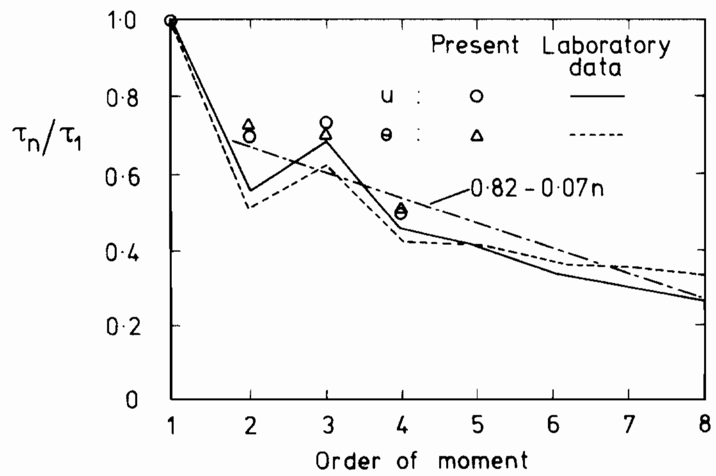

Fig. 6. Comparison of the ratio τ_n/τ_1 between laboratory flows and atmospheric measurements.

turbulent jet. The reasonable agreement with the laboratory data up to τ_4 suggests that, to a first approximation, laboratory data can be used to extrapolate for τ_n/τ_1, $n \geqslant 4$. Although τ_n/τ_1 does not necessarily decrease monotonically, a useful first approximation would be

$$\tau_n/\tau_1 = 0.82\text{–}0.07n.$$

With only a knowledge of τ_1, this result can be used with Equation (2a) to enable error estimates to be made.

5. Conclusions

To a first approximation, data on high-order moments required for accuracy estimates can be obtained using Gaussian values for the individual fluctuations, and the values appropriate to the joint Gaussianity assumption for the products. Error estimates based on actually measured moments can be obtained using Equation (11a) and the constants m_x listed in Table III, which correspond to the surface layer over ocean for $z/L \simeq -0.05$. Normalised moments of fluctuations, and at least of the product uw, do not differ significantly from those corresponding to neutral conditions. The concept of Reynolds number similarity will then suggest that a useful approximation to moments can also be obtained from laboratory measurements in neutral boundary layers. The integral time scales required for this purpose will however have to be determined in each case. To a first approximation, the present non-dimensional estimates $\tau_1 U/z$ and τ_n/τ_1, obtained from auto-correlation measurements of powers of fluctuations and products, can be used in atmospheric boundary layers. In particular, the ratio τ_n/τ_1, which appears to be nearly the same for all flows, can be crudely approximated by the relation

$$\tau_n/\tau_1 = 0.82 - 0.07n,$$

for $n > 1$.

Appendix: Accuracy of Integral Time Scales

In this paper, for the purpose of computing integral time scales, auto-correlation functions $\rho(t)$ were generally obtained from inverse Fourier transforms of spectral densities computed from an ensemble of sub-records of 50-s duration; in a few test cases, sub-records of up to 400 s were used. Although no explicit high-pass filtering was employed, the procedure will amount to an effective loss of information at the low-frequency end. This is not crucial to the calculation of auto-correlation functions provided $T_0 \ll T_s$, where T_s is the duration of the sub-record and T_0 is a measure of the lag time such that ρ is small for $t > T_0$. A characteristic measure of T_0 can be taken to be the smallest lag time for which $\rho(T_0) = 0$. This is justified for all signals considered here, because the negative magnitudes of their auto-correlation are not very large. For the present data, some sample auto-correlation functions evaluated directly from the time series showed that T_0 was in the range 15–20 s ($\approx$9–12 z/U) for u, q and θ, and about 3 s ($\approx$2 z/U) for w: thus the condition $T_0 \ll T_s$ is satisfied quite well for w and only marginally for u, θ and q. Obviously, for lag times t comparable to T_0, auto-correlation functions of u, q and θ cannot be trusted, except when sub-records of 400 s were used. Also, because of the so-called circular effect (Bendat and Piersol, 1971), it is possible that the values of auto-correlation functions are distorted even for $t < T_0$. This distortion must however be small if $\rho(t > T_0) \ll 1$; in the present case, $|\rho(t > T_0)|$ was always less than about 0.1. Some of these problems are discussed more explicitly in Sreenivasan *et al.* (1977).

These uncertainties are somewhat compounded by the fact that the integral scale computed from the relation

$$\rho(t) = \frac{1}{\overline{x^2} T} \int_0^T x(t')x(t'+t)\,\mathrm{d}t', \qquad T \to \infty, \tag{A1}$$

must strictly be zero for a random variable x with zero mean (e.g., Comte-Bellot and Corrsin, 1971). Clearly, for all signals whose autocorrelation changes sign only once, an upper bound to the integral time scale τ_1 can be obtained by modifying the definition of τ_1 to include the area under $\rho(t)$ only up to $t = T_0$, provided that calculations reproduce $\rho(t < T_0)$ faithfully. Even in cases where T_0/T_s is not very small, these modified estimates for the integral time scales should be reasonable. This can be seen from Table I, where the present values of τ_1 are compared, in the case of u and q, with those evaluated according to

$$\tau_1 = \int_0^{T_1} \rho(t)\,\mathrm{d}t, \tag{A2}$$

with T_1 set at 80 s and $\rho(t)$ evaluated according to Equation (A1).

The ratio τ_n/τ_1 $(n > 1)$ should however be quite reliable, because of the nearly identical manner in which τ_n (for all n) is affected by these problems. This is demonstrated in Table I by comparing the ratio τ_n/τ_1 for u obtained by using sub-records of 400-s duration, with present values obtained from subrecords of 50-s duration.

Acknowledgements

We acknowledge the generous cooperation of Dr I. S. F. Jones and the Australian Navy Research Laboratory. We are also indebted to Dr C. A. Friehe for his cooperation in the Bass Strait experiment, and Dr D. Britz for his help with computations.

This work represents part of a program of research supported by the Australian Research Grants Committee.

References

Antonia, R. A. and Atkinson, J. D.: 1973, 'High-order Moments of Reynolds Shear Stress Fluctuations in a Turbulent Boundary Layer', *J. Fluid Mech.* **58**, 581–593.

Antonia, R. A., Chambers, A. J., Rajagopalan, S., Sreenivasan, K. R. and Friehe, C. A.: 1977, 'Measurements of Turbulent Fluxes in Bass Strait', Report No. 7, Department of Mechanical Engineering, University of Newcastle. To appear in *J. Phys. Oceanography*.

Bendat, J. S. and Piersol, A. G.: 1971, *Random Data: Analysis and Measurement Procedures*, John Wiley, Interscience, New York, p. 313.

Comte-Bellot, G. and Corrsin, S.: 1971, 'Simple Eulerian Time Correlation of Full- and Narrow-band Velocity Signals in Grid-Generated "Isotropic" Turbulence" *J. Fluid Mech.* **48**, 273–337.

Danh, H. Q.: 1976, 'Turbulent Transfer Processes in Non-Isothermal Shear Flows', PhD Thesis, University of Sydney.

Gupta, A. K. and Kaplan, R. E.: 1972, 'Statistical Characteristics of Reynolds Stress in a Turbulent Boundary Layer', *Phys. Fluids* **15**, 981–985.

Lu, S. S. and Willmarth, W. W.: 1972, 'The Structure of the Reynolds Stress in a Turbulent Boundary Layer', Tech. Report 021490-2-T, Department of Aerospace Engineering, Gas Dynamics Laboratories, The University of Michigan.

Lumley, J. L.: 1970, *Stochastic Tools in Turbulence*, Academic Press, New York and London, p. 72.

Lumley, J. L. and Panofsky, H. A.: 1964, *The Structure of Atmospheric Turbulence*, Interscience-Wiley, New York, p. 37.

McBean, G. A.: 1974, 'The Turbulent Transfer Mechanisms: A Time Domain Analysis', *Quart. J. Roy. Meteorol. Soc.* **100**, 53–66.

Pries, T. H.: 1970, 'Wind Distribution Asymmetry in the Lowest 90 Metres and its Effect on Diffusion', *J. Appl. Meteorol.* **9**, 588–591.

Priestley, C. H. B.: 1959, *Turbulent Transfer in the Lower Atmosphere*, University of Chicago Press, Chicago and London, p. 45.

Sreenivasan, K. R., Britz, D., and Antonia, R. A.: 1977, 'On the Accuracy of Determining High-order Moments of Turbulent Fluctuations', Report No. 10, Department of Mechanical Engineering, University of Newcastle.

Stewart, R. W.: 1974, 'The Air-Sea Momentum Exchange', *Boundary-Layer Meteorol.* **6**, 151–167.

Tennekes, H. and Lumley, J. L.: 1972: *A First Course in Turbulence*, MIT Press, Cambridge, p. 212.

Tillman, J. E.: 1972, 'The Indirect Determination of Stability, Heat and Momentum Fluxes in the Atmospheric Boundary Layer from Simple Scalar Variables During Dry Unstable Conditions', *J. Appl. Meteorol.* **11**, 783–792.

Wyngaard, J. C.: 1973, 'On Surface-Layer Turbulence', in D. A. Hangen (ed.), *Workshop on Micrometeorology*, American Meteorological Society, 101–149.

Wyngaard, J. C. and Izumi, Y.: 1973, 'Comments on Statistical Characteristics of Reynolds Stress in a Turbulent Boundary Layer', *Phys. Fluids* **16**, 455–456.

Wyngaard, J. C., Coté, O. R. and Izumi, Y.: 1971, 'Local Free Convection, Similarity, and the Budgets of Shear Stress and Heat Flux', *J. Atmos. Sci.* **28**, 1171–1182.