Rayleigh Taylor instability with localized perturbations

R.J.R. Williams, AWE Aldermaston
Turbulent Mixing and Beyond
ICTP, Trieste

Thanks to Guy Dimonte, Antoine Llor and David Youngs, discussions with whom prompted some of these ideas.

Is the initial spectrum enough?

Effect of localized surface perturbations on RT instability

Accelerated interface linear modes:

Stable interface – $a \propto \exp i(k.x - \omega t)$ – wave-like modes

Unstable interface – $a \propto \exp ik.x \pm \lambda t$ – no translation component for exponentially growing mode

$\Rightarrow$ for unstable interfaces, relative *phase* of modes is important.

Here look at RT growth (Dimonte has discussed related issues for RM, e.g. at the recent IWPCTM).

RT growth as independent modes

Similar to Ramaprabhu et al., linear modes grow as

$$A \propto a_0 \exp\left[(\mathrm{At}\, gk)^{1/2}\, t\right]. \tag{1}$$

Width dominated by the modes just turning through nonlinearity: for spectrum,

$$w \simeq \frac{2\pi}{k_{\max}} \simeq a_0(k_{\max}) \exp\left[(\mathrm{At}\, gk)^{1/2}\, t\right]. \tag{2}$$

Eliminating $k_{\max} \Rightarrow$ implicit equation for $w(t)$:

$$w \simeq 2\pi \left[\ln \frac{a_0(2\pi/w)}{w}\right]^2 \mathrm{At}\, gt^2. \tag{3}$$

Self-similar spectra have $a_0(2\pi/w) \propto w \Rightarrow$ scaling factor α is constant.

RT growth as independent modes

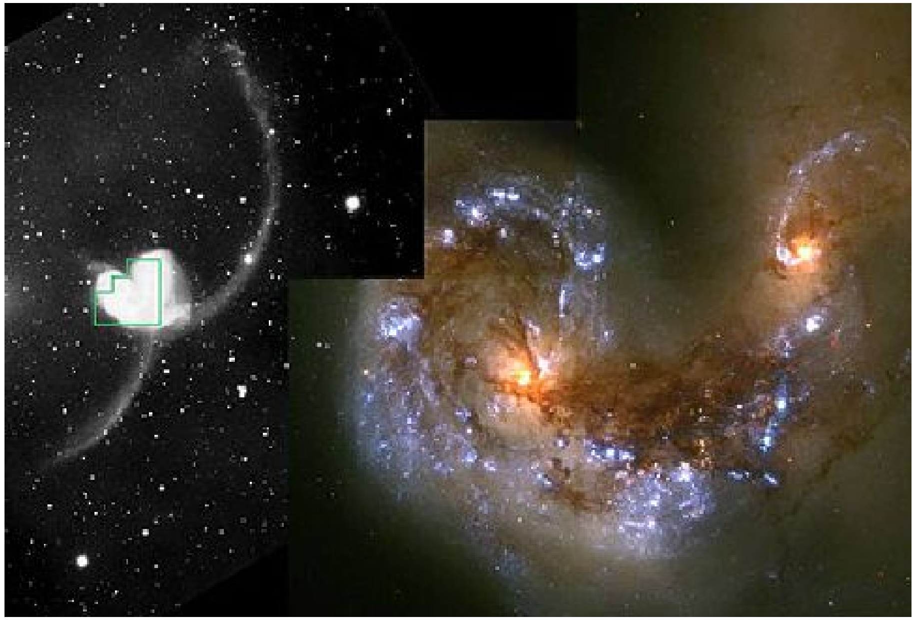
More generally, $w/\mathrm{At}\,gt^2$ is (weakly) dependent on a_0/w.
May be looked at as relating to the *initial* degree of nonlinearity of the modes which are currently going nonlinear.
Considering the local nature of unstable RT growth, initial nonlinearity may be better related to a filtered r.m.s. of the *real-space* value $\nabla_\perp a_0$, rather than the k-space equivalent ka_0.
Implications for the implementation of engineering models applied to real surfaces should be obvious.

Analogy to cosmological modelling

- Analysis of surface instabilities (and specification of initial conditions) generally based on Gaussian random fields.
- Origin in communications and wave prediction in 1940s/1950s (Kinsman 1965 for a brief history).
- Similar approach in cosmological modelling. After radiation decouples, gravitationally-unstable modes grow to form clusters, galaxies, galaxy clusters. Only at the nonlinear stages is there significant translation.
- Early-stage growth can be treated by Fourier mode superposition, but intermediate mode coupling physics is spatially local; nonlinear physics occurs within discrete localised structures.

In RT, mode-coupling effects will be different – *but still local* – and growth is in 2D not 3D.

Structure mergers



Merging in both cases. Gravitational collapse tends to leads to isolated galaxies surrounded by 'saturated' voids.

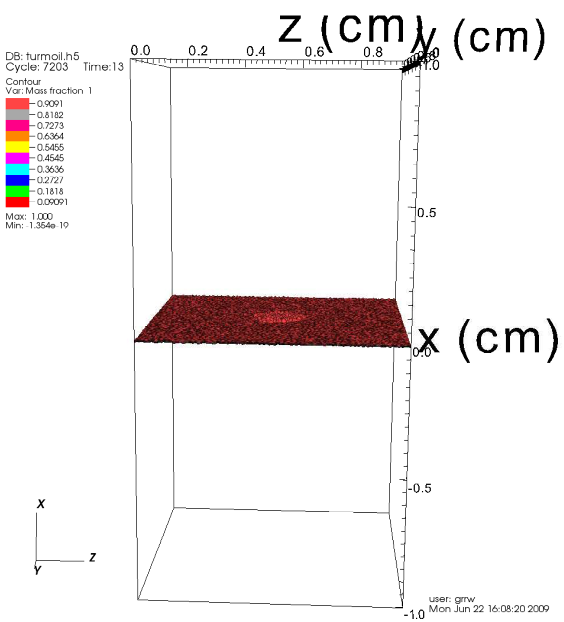
Statistics of peaks

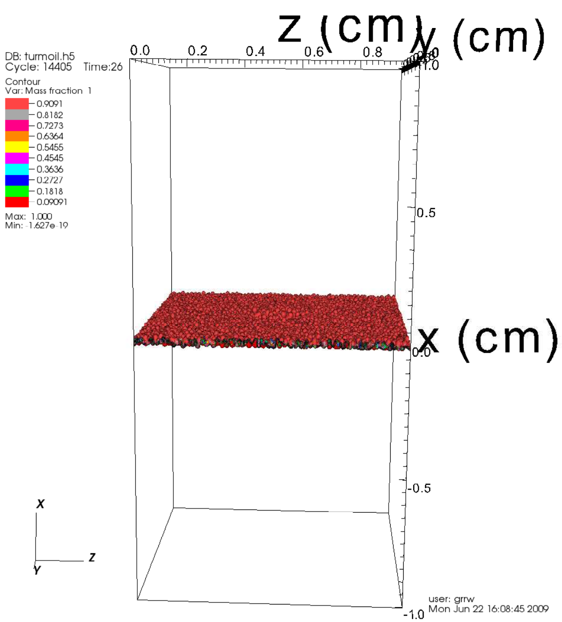
- If mode coupling not significant, hierarchy of structures can be treated by the 'statistics of peaks' (Bardeen et al), including merger history (Cole, Kaufman).
- Low-pass filter initial conditions, and look for when local 'height' predicts nonlinearity to infer scale of dominant structures.
- Obvious analogy to Ramaprabhu et al. analysis.
- 2D column density data integrated perpendicular to initial interface captures linear-mode growth of instability, and can be analysed with Fourier techniques.

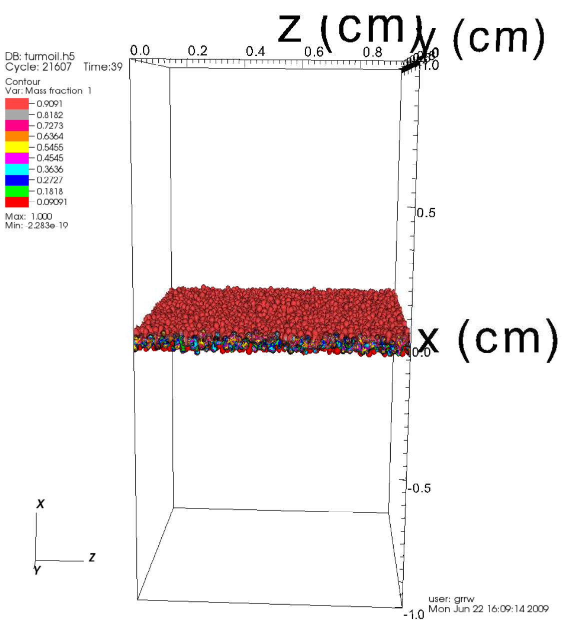
Simulations of perturbed RT growth

Using TURMOIL3D code (Youngs) – staggered-grid 3D hydrodynamics code implements MILES methodology.
Here take narrowband, medium-scale perturbations and multiply the amplitude in a localized region (using a smooth-edged weighting).

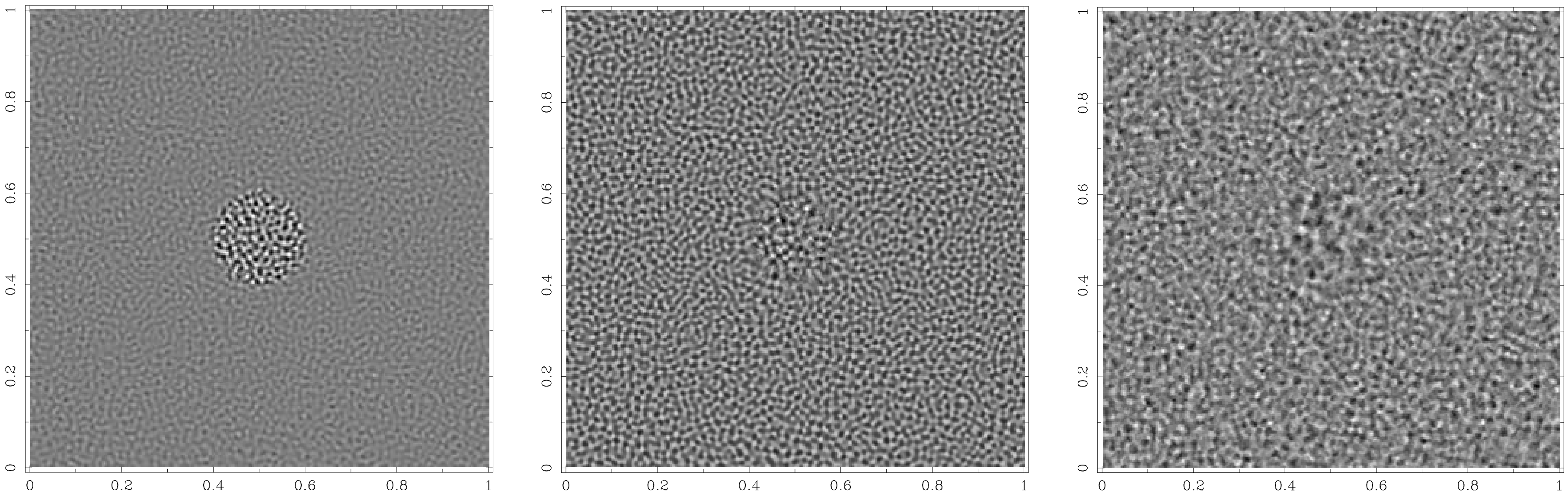
Enhanced roughness patch







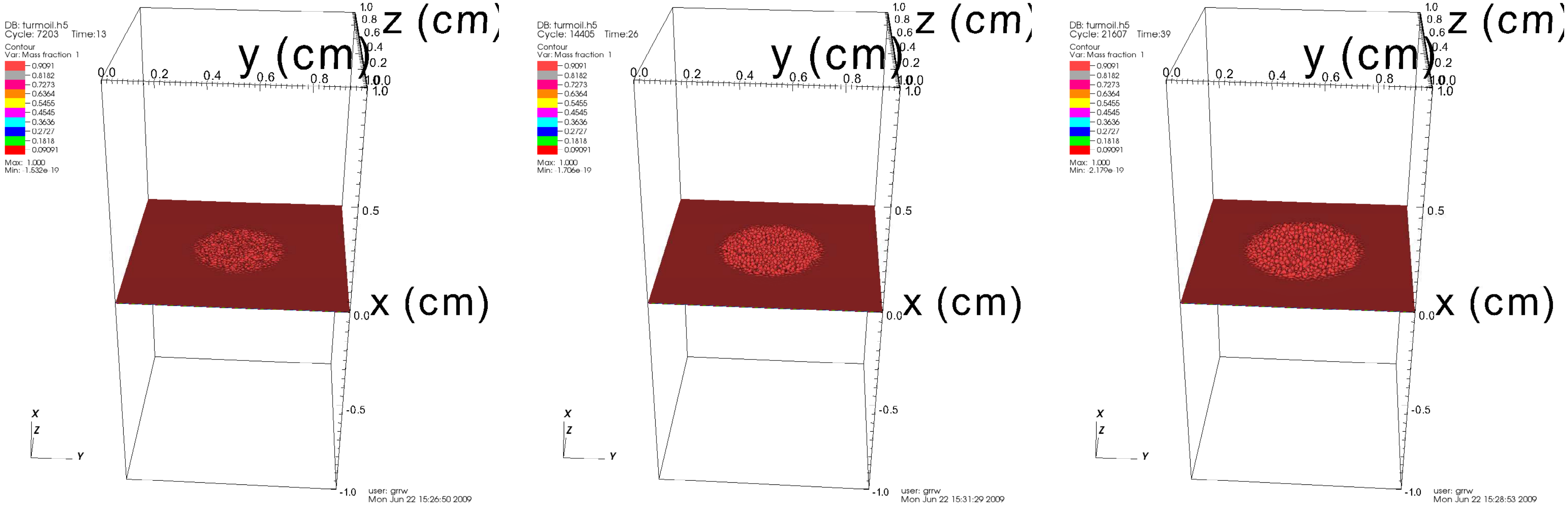
Column densities



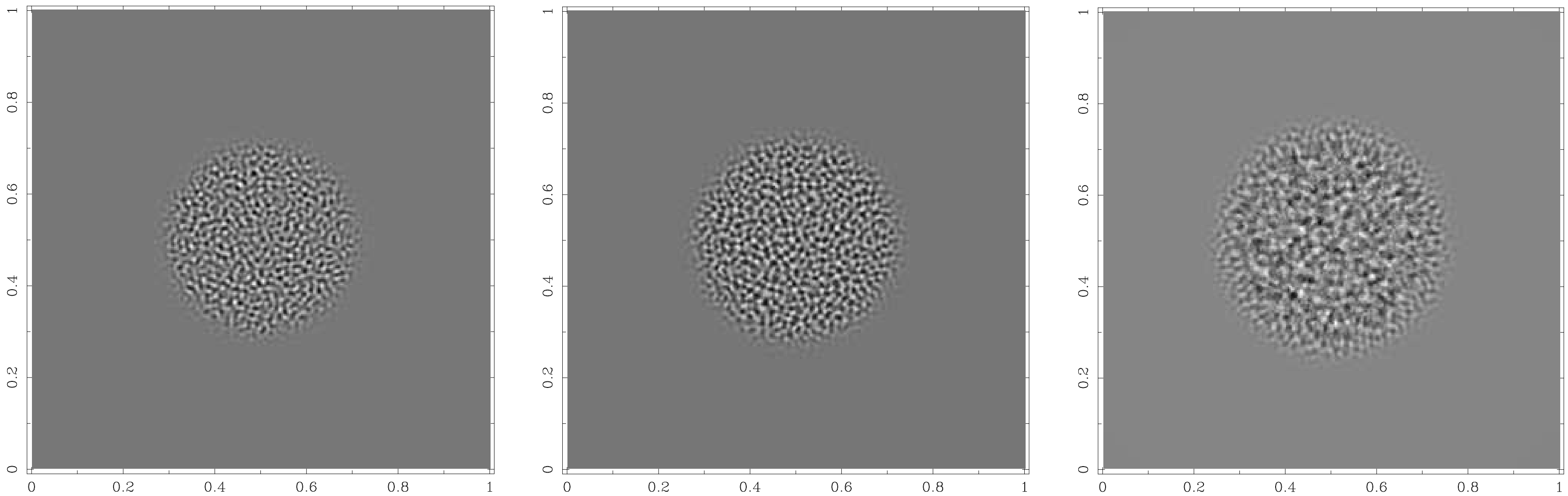
Enhanced roughness – comments

- Early growth most rapid in enhanced roughness patches
- Amount that these patches lead other areas soon reduces (due to logarithmic dependence on initial perturbation amplitude)
- Growth of perturbations: initial linear-mode growth fastest in high-amplitude regions. But small-scale features go non-linear earlier: after this, the growth-rate advantage is smaller.
- At late time (when width of the mixing region is comparable to the size of the roughened patch), expect the effects to again become apparent – requires higher degree of perturbation than in the current runs.

Isolated rough patch



Isolated rough – column

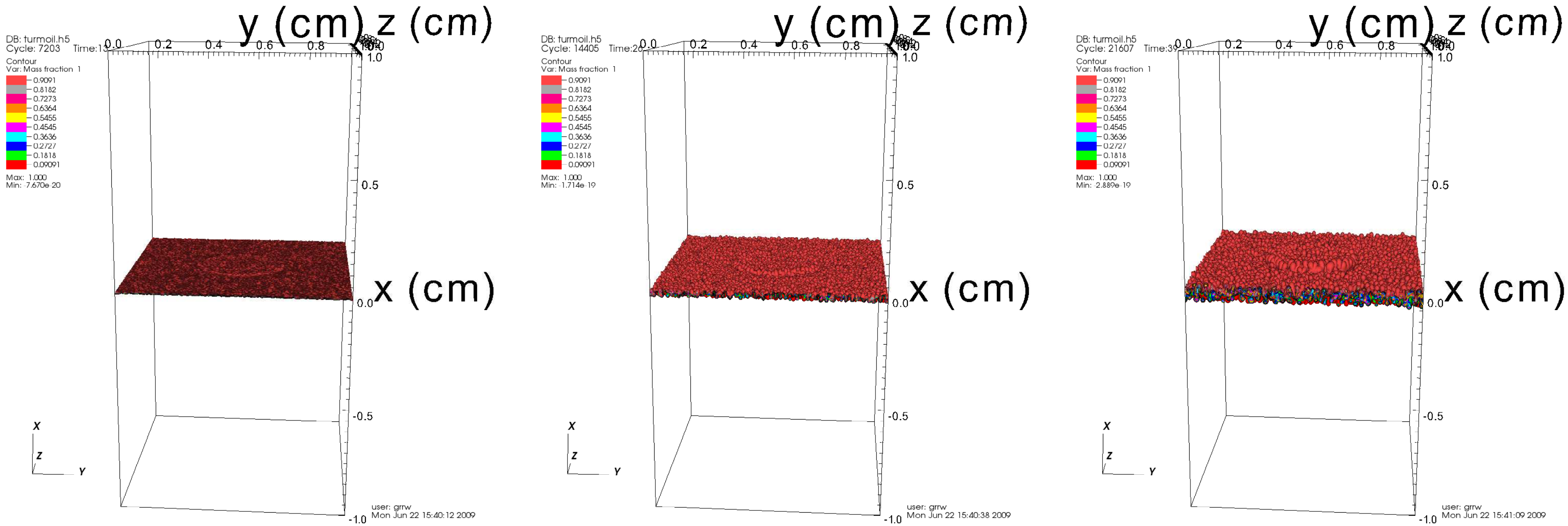


Isolated rough – comments

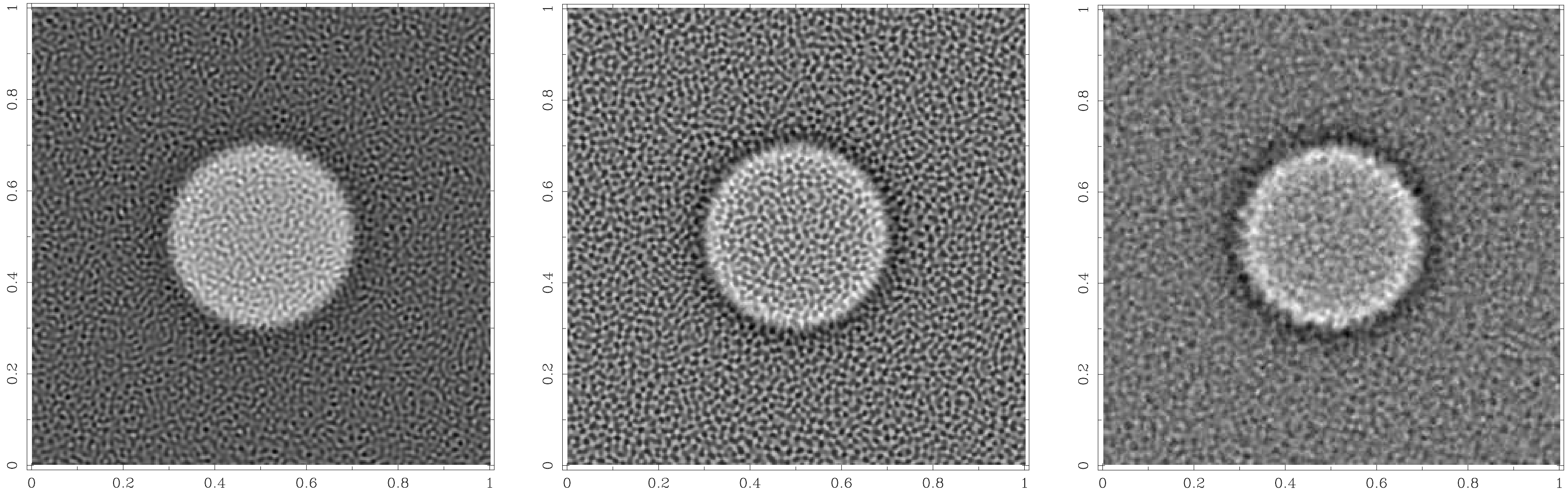
Reduce the perturbation to zero outside roughened patch to investigate limiting behaviour.
Growth is clearly by lateral (but local) spread of the mixing layer rather than any wave-like process, driven by KH instability at the margins of the perturbed region.
For nonlinear processes, it is quite natural that different basis sets are most appropriate in different asymptotic regimes. This is familiar in particle physics, e.g. the properties of the K meson.

Macroscopic perturbation

Raised 'platform' with same local finish as wider surface.



Macroscopic pert'n – column



Macroscopic pert'n – comments

- Mixing region grows laterally due to wakes of nonlinear modes.
- Wakes are strongly damped laterally $\Rightarrow$ lateral spread of the mixing layer slow.
- As in Llor's analysis of isotropic turbulence, nonlinear effects populate the low-wavenumber spectrum as a result of their locality – but they only propagate slowly in real space. (A spatial δ-function has strong low wavenumber tails.)
- Local perturbation growth rate is $n \sim (\mathrm{At}gk)^{1/2}$: for k spreads at $v \sim n/k \sim (\mathrm{At}g/k)^{1/2}$ if the initial amplitudes are comparable.
- $\Rightarrow$ localized perturbations spread by the longest wavelengths which have time to grow to nonlinearity.

Conclusions

- Perturbation-driven growth only roughly predicted from initial conditions using filtering techniques, due to poor separation of scales
- A (weakly) time-varying α value may be useful to describe the development of instability from arbitrary surfaces
- Growth by mode coupling is best described in the spatial, not frequency, domain.