# Anisotropic large-scale circulation and transport in zonostrophic turbulence

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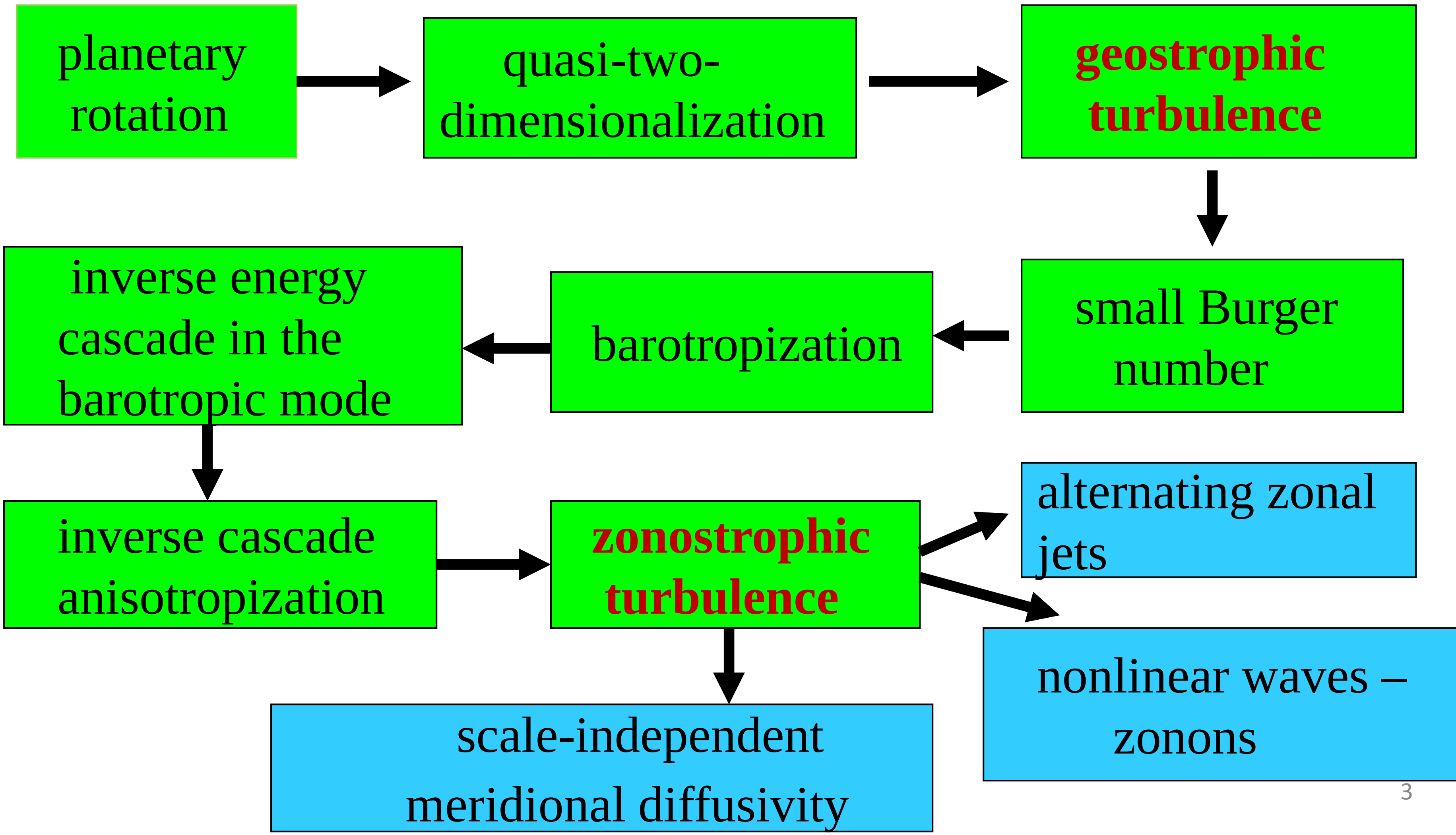
# Presentation outline:

- Motivation, basic physics, barotropization
- Unforced and forced problems
- Unsteady and steady-state problems
- Classification of different regimes
- Zonostrophic  turbulence
- Rossby waves and nonlinear waves - zonons
- Meridional diffusion in zonostrophic turbulence
- Conclusions

**Motivation** – understanding of planetary and terrestrial zonal flows from the viewpoint of anisotropic turbulence with waves

## Background physics



# Basic equations

$$\frac{Dq}{Dt} \equiv \frac{\partial q}{\partial t} + J(\psi, q) = 0$$

Inviscid conservation of quasi-geostrophic, 3D, potential vorticity

$$q = \nabla^2 \psi + f + \frac{\partial}{\partial z}\left[\frac{f_0^2}{N^2(z)} \frac{\partial \psi}{\partial z}\right].$$

Potential vorticity

$$J(\psi, q) = \frac{\partial \psi}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial q}{\partial x}$$

Nonlinear term

$$Pr = f_0/N, \qquad \tilde{z} = z/Pr$$

Prandtl ratio and rescaled vertical coordinate

$$\tilde{E} = \int |\nabla_3 \psi|^2 \, \mathrm{d}V, \quad \tilde{Z} = \int (\nabla_3^2 \psi)^2 \, \mathrm{d}V$$

3D energy and enstrophy invariants

$$\nabla_3 = (\partial/\partial x, \partial/\partial y, \partial/\partial \tilde{z}).$$

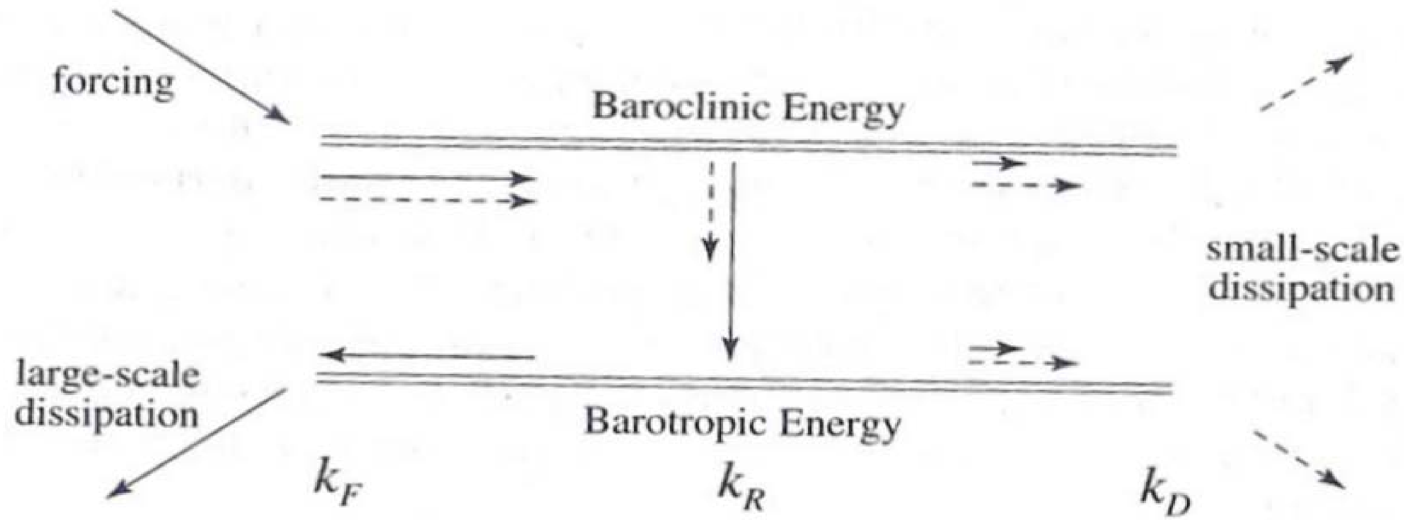
Rescaled 3D Laplacian

# 3D inverse energy cascade

3D inverse cascade transfers energy to smaller $k_x$ and $k_y$ but also to smaller $k_z$ → lengthening of the vertical scales, **barotropization**

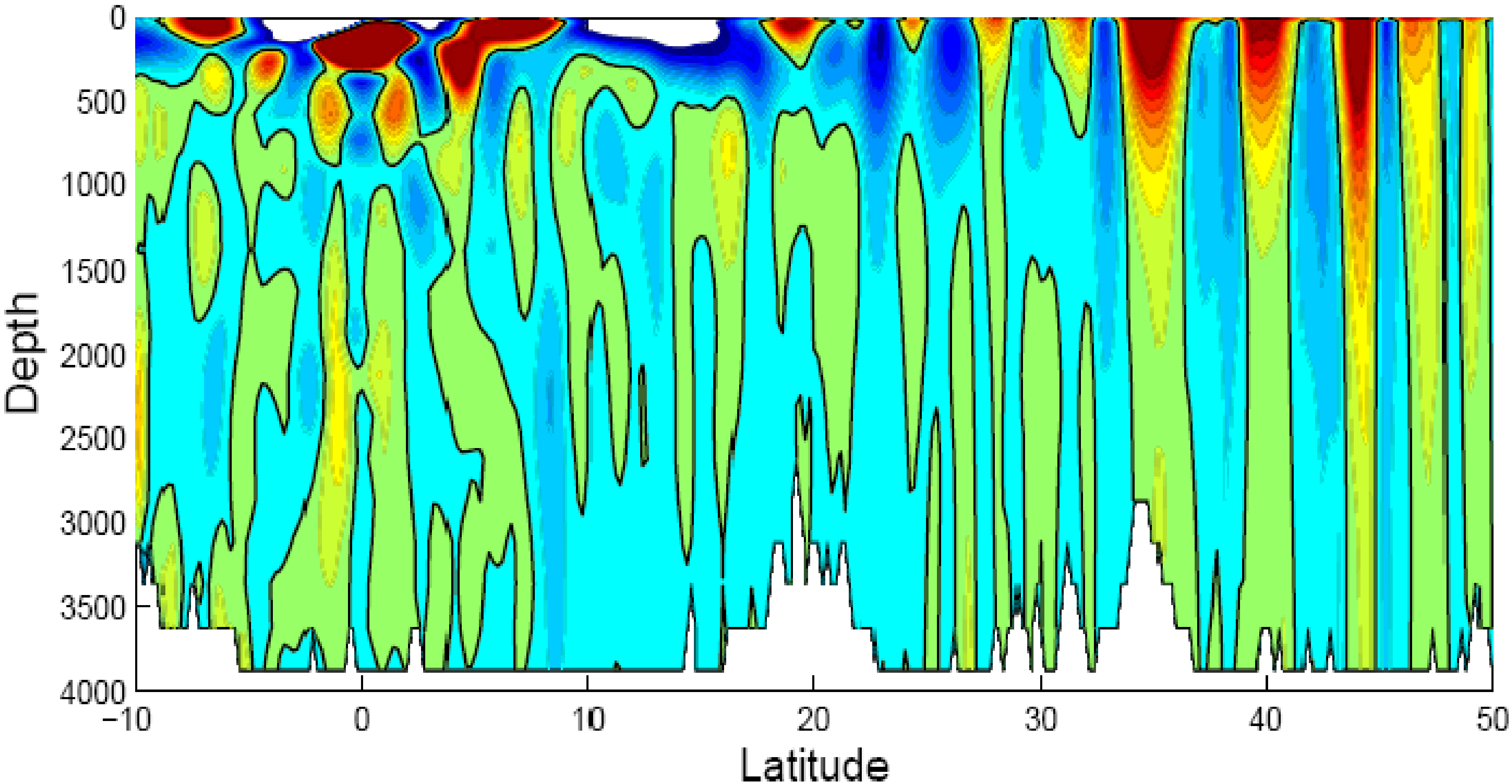
# The barotropization paradigm

**Rhines (1979), Salmon (1998), Read (2001), Vallis (2006)**




**In flows with baroclinic instability, barotropization and inverse cascade are possible for Bu≡$(L_d/R)^2$ <<1, Bu is the Burger number ($L_d$ is the first baroclinic Rossby radius; R is the planetary radius)**

# Barotropization in oceanic flows




Zonal velocity component along 180°E averaged over 3 years from the climatological run of the POP model as a function of latitude and depth. Color saturates at -0.06 $ms^{-1}$ (blue) and 0.06 $ms^{-1}$ (red). Zero contour is given be a black line. From Richards, Maximenko, Bryan, and Sasaki (2006) GRL

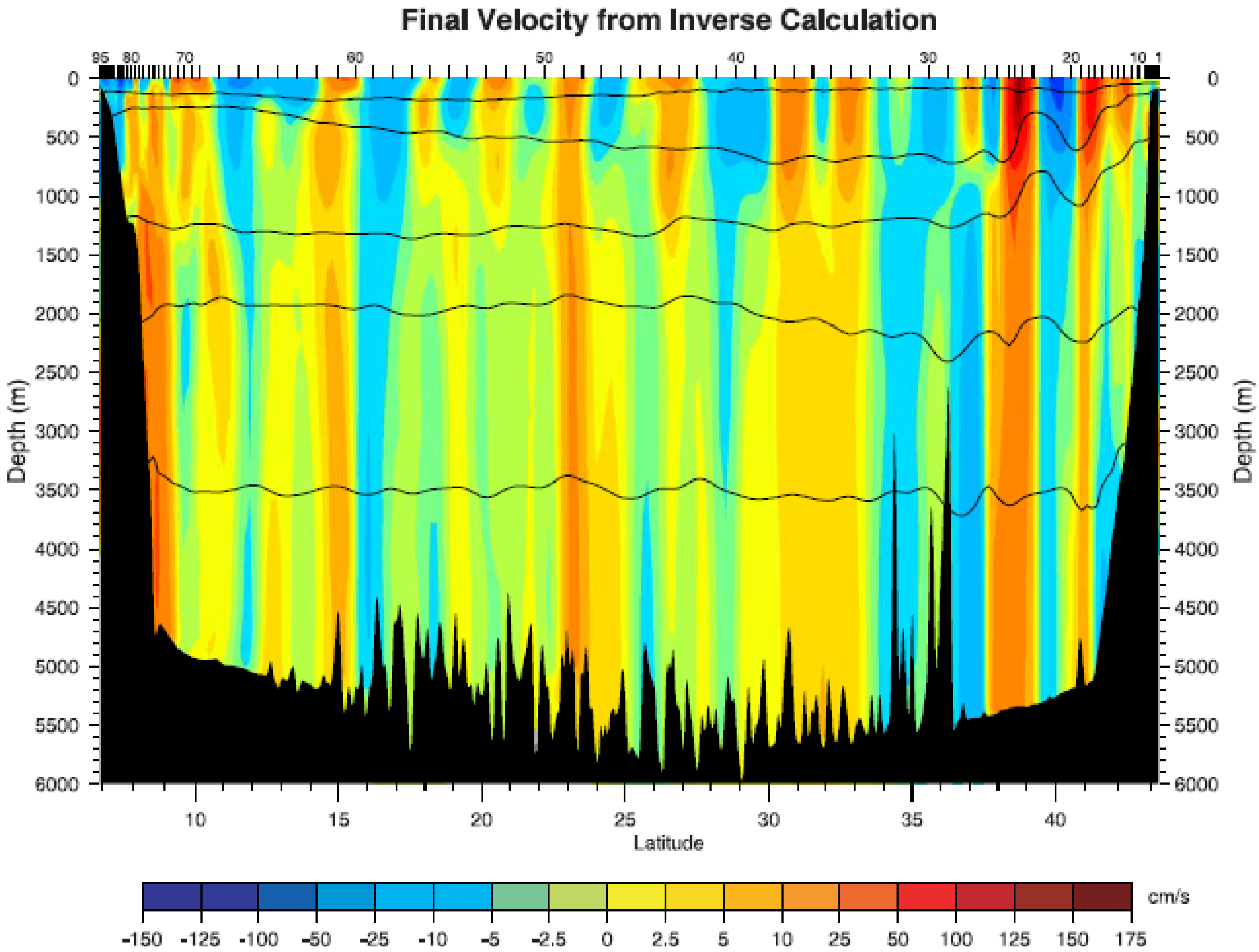


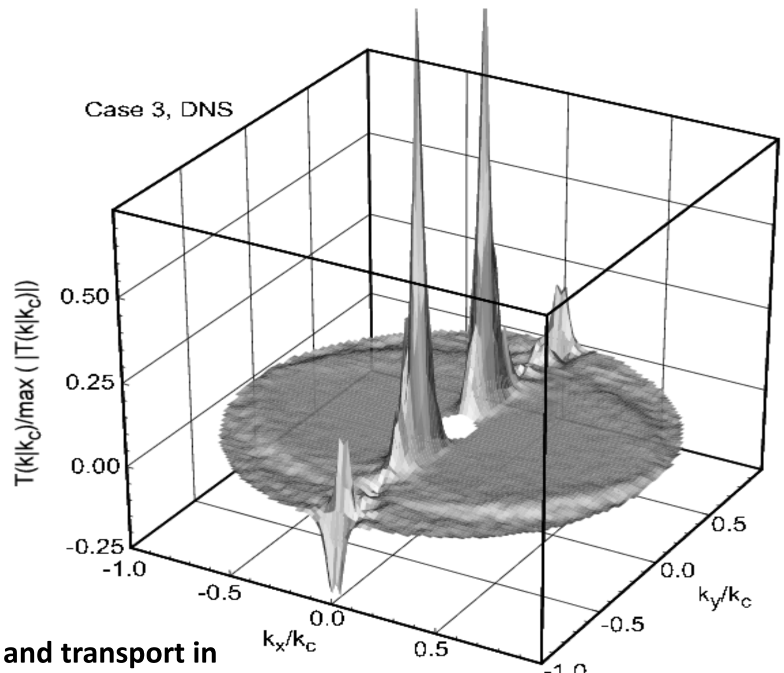

**Figure 12.** Adjusted geostrophic velocities for 52°W section, after initializing to ADCP data and using inverse model with mass and silicate constraints. Sign convention same as in Figure 2.

Anisotropic circulation and transport in zonostrophic turbulence

# 2D turbulence and Rossby waves – the basics

- Turbulence - Rossby wave interaction was considered by Rhines (1975) using *unforced* barotropic flows on a β-plane
- Triad interaction requires simultaneous resonances in wave numbers and frequencies which impedes and anisotropizes the inverse cascade
- Energy is transferred to modes with smaller frequencies and smaller wave numbers ➔ anisotropization and zonation

Spectral energy transfer computed from DNS data by Chekhlov et al. (1996)




**Anisotropic circulation and transport in zonostrophic turbulence**

# The basics - continuation

- Some studies confused unforced and forced flows; forced flows include an additional parameter, $\varepsilon$
- In forced $\beta$-plane turbulence, turbulent eddies gain anisotropy at a scale whose characteristic time is equal to the Rossby wave period ➔ the transitional wave number $k_\beta=(\beta^3/\varepsilon)^{1/5}$ (Vallis and Maltrud, 1993)
- $k_R^{-1}=(\beta/2U)^{-1/2}$, U is the rms velocity ➔ the Rhines's scale – large-scale characteristic. Has different physical meanings in different regimes
- Various combinations of $k_R$ and $k_\beta$ correspond to different flow regimes
- These regimes have been classified in simulations of barotropic 2D turbulence on the surface of a rotating sphere

# Forced 2D turbulence on the surface of a rotating sphere - simulations

Barotropic vorticity equation with small-scale forcing, dissipation, and large-scale drag

$$\frac{\partial \zeta}{\partial t} = -J(\psi, \zeta + f) + D + \xi$$

Spectral model is employed

$$\Psi(\mu, \phi, t) = \sum_{n=1}^{N} \sum_{m=-n}^{n} \psi_n^m(t) Y_n^m(\mu, \phi)$$

- R-truncation; R133 and R240 resolutions
- Random energy injection with the constant rate ε at about $n_\xi = 100$
- Linear large-scale drag (not to disturb inverse cascade)
- Very long-term integrations in a steady-state to compile long records for statistical analysis
- Analyze anisotropic spectrum

$$E(n) = \frac{n(n+1)}{4R^2} \sum_{m=-n}^{n} \langle |\psi_n^m|^2 \rangle$$

$$E(n) = E_Z(n) + E_R(n)$$

zonal (m=0) residual

# Forced flows with the Rayleigh drag: A steady case

- Linear large-scale drag acts upon all wave numbers; the minimum of them, with the maximum energy, is $n_{fr}$
- $n_{fr}$ depends on the drag coefficient, $\lambda$, and $\beta$ and $\varepsilon$:

$$n_{fr} = f(\lambda,\beta,\varepsilon)$$

- According to the Buckingham's $\Pi$-theorem, this system is fully characterized by two non-dimensional parameters, for instance,

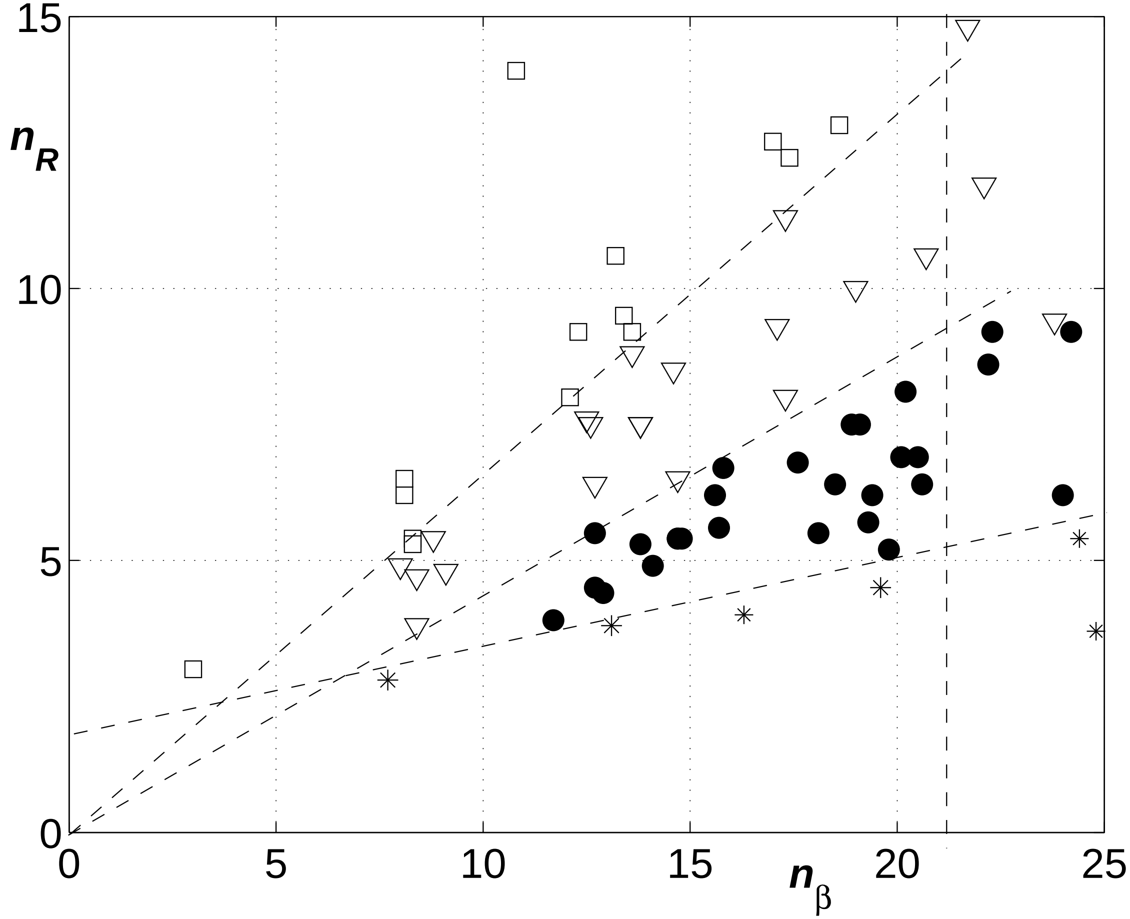
$$\frac{n_{fr}}{n_R} = f\left(\frac{n_\beta}{n_R}\right)$$

- A new nondimensional parameter, the zonostrophy index **$R_\beta = n_\beta/n_R$**, classifies various flow regimes

# Four steady regimes of 2D turbulence on the surface of a rotating sphere

**Friction-dominated**

$R_\beta < 1.5$




**Transitional**

$R_\beta \sim 1.5$ to 2

**Zonostrophic Turbulence**

$R_\beta > 2$

**Condensation**

$$n_R = (\Omega/2UR)^{1/2}$$

The Rhines wave number

$$n_\beta = 0.5(\Omega^3/\epsilon R^3)^{1/5}$$

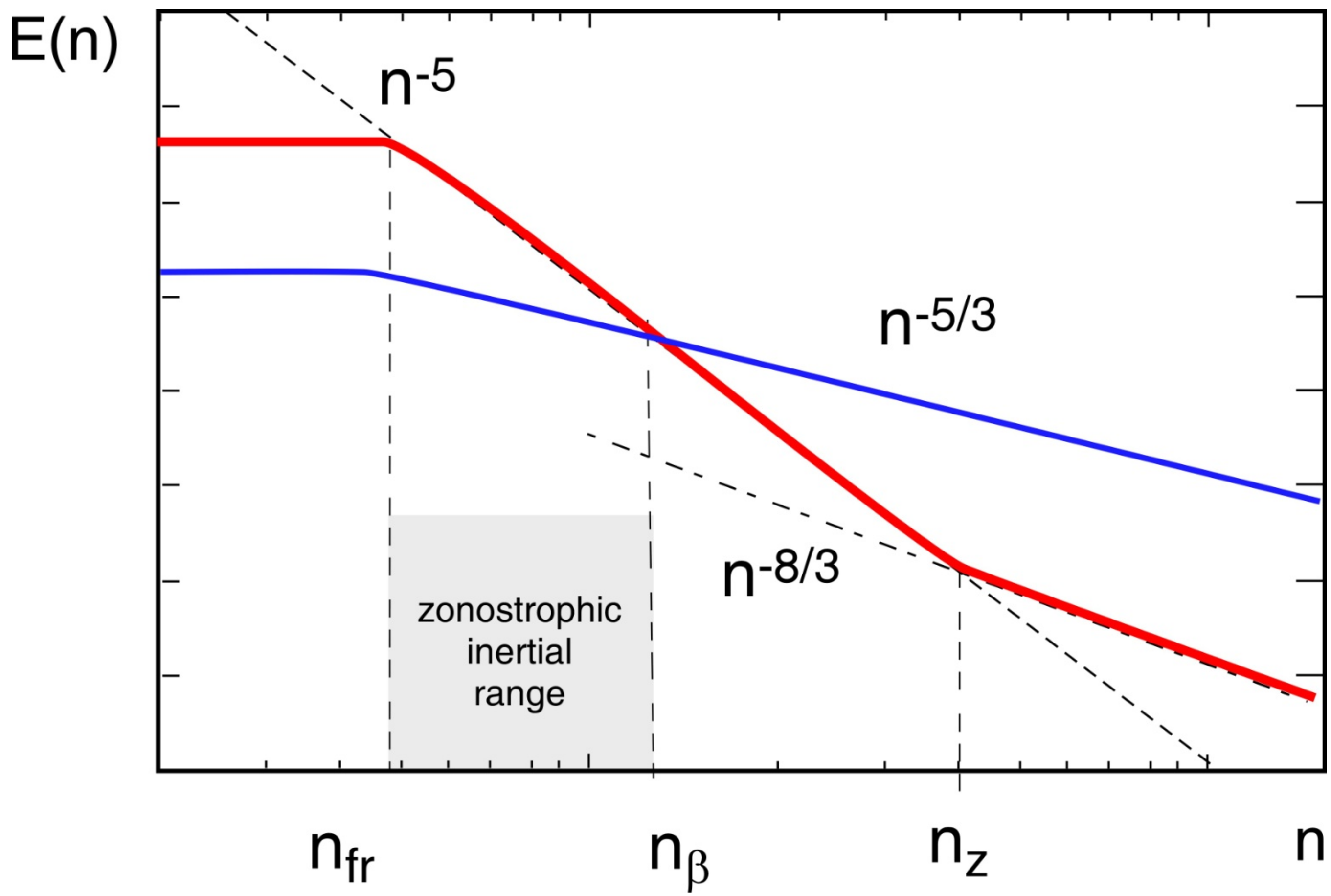
Transitional wave number

# Zonostrophic turbulence: $n_\xi \gtrsim 4n_\beta \gtrsim 8n_R \gtrsim 30$

(from Greek ζωνη - band, belt, and στροφη - turning)

**$R_\beta > 2$**

**Bu $< 10^{-2}$**




$$E_Z(n) = C_Z(\Omega/R)^2 n^{-5}, \quad C_Z \simeq 0.5$$

$$E_R(n) = C_K \epsilon^{2/3} n^{-5/3}, \quad C_K \simeq 4 \text{ to } 6$$

$$n_z = (2n_\beta/\Delta m)^{3/7} n_\beta$$

Intersection of -5 and -8/3 zonal spectra

# Examples of zonostrophic turbulence - the ocean-Jupiter connection

L13303 GALPERIN ET AL.: ZONAL JETS ON GIANT PLANETS AND IN OCEAN L13303

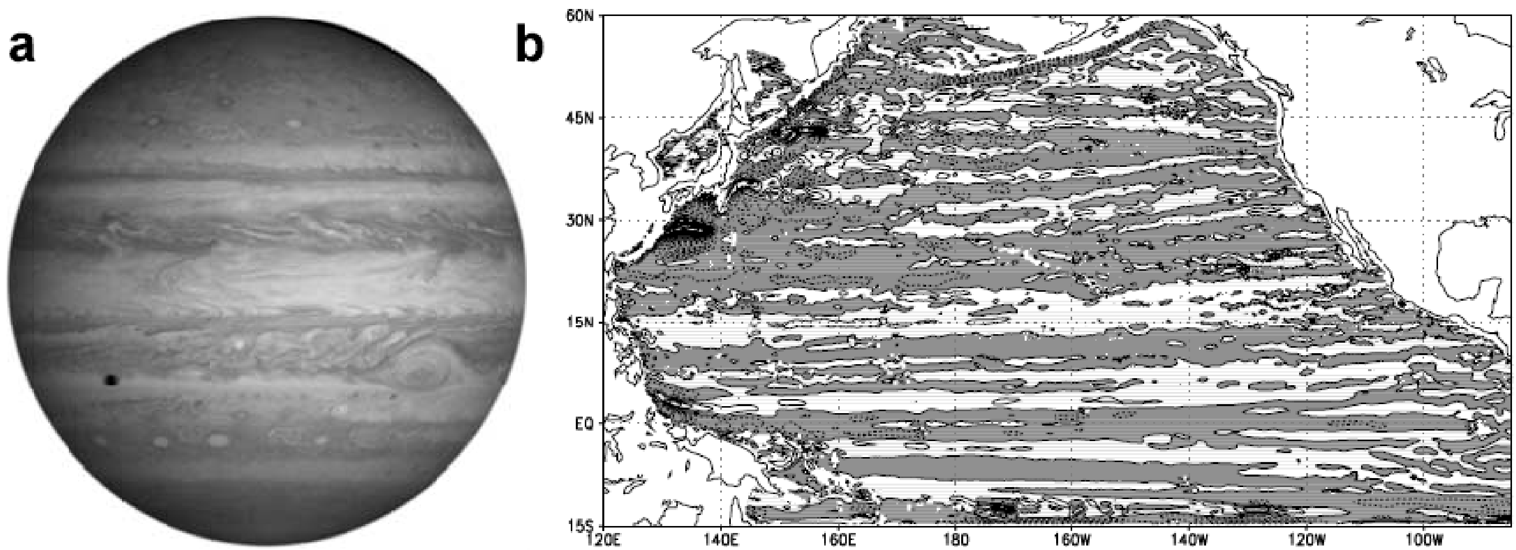



**Figure 1.** (a) Composite view of the banded structure of the disk of Jupiter taken by NASA's Cassini spacecraft on December 7, 2000 (image credit: NASA/JPL/University of Arizona); (b) zonal jets at 1000 m depth in the North Pacific Ocean averaged over the last five years of a 58-year long computer simulation. The initial flow field was reconstructed from the Levitus climatology; the flow evolution was driven by the ECMWF climatological forcing. Shaded and white areas are westward and eastward currents, respectively; the contour interval is 2 cm $s^{-1}$.

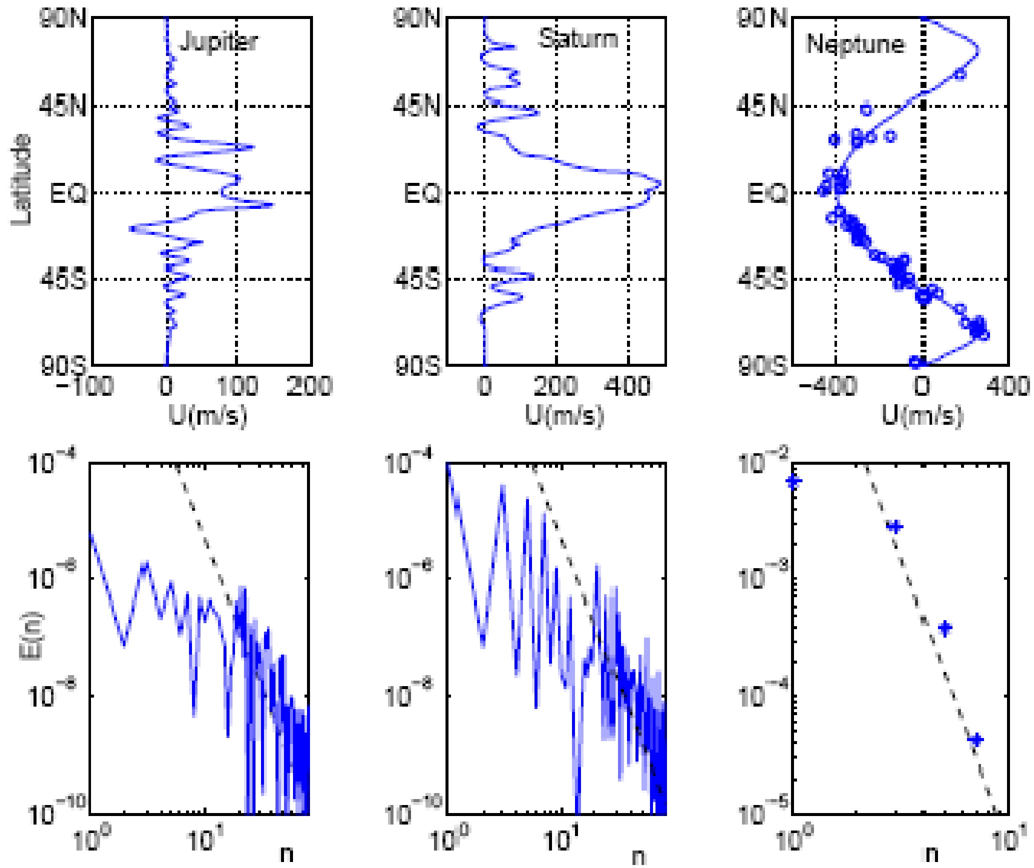
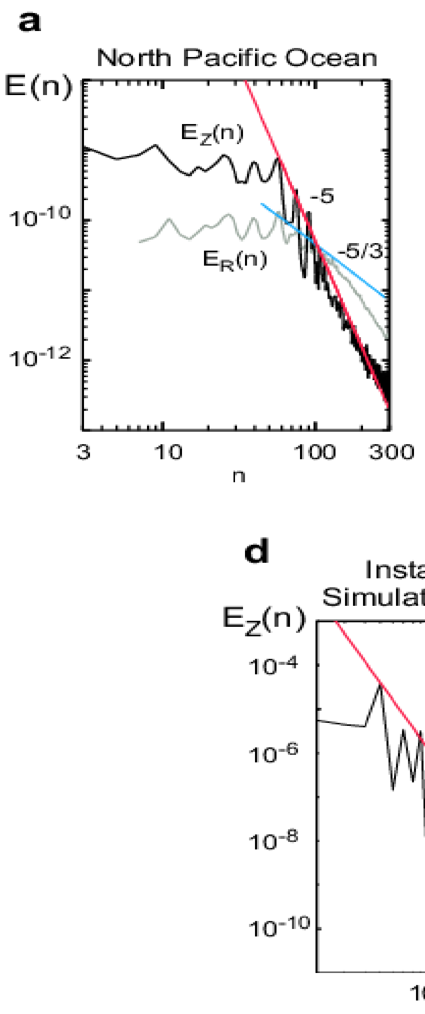



**Fig. 8.** Top row: observed zonal profiles deduced from the motion of the cloud layers (Garcýa-Melendo and Sánchez-Lavega, 2001; Sánchez-Lavega et al., 2000; Hammel et al., 2001; Sromovsky et al., 2001); bottom row: observed zonal spectra (solid lines and asterisks) and theoretical zonal spectra, Eq. (7) (dashed lines) on the giant planets [all spectra are normalized with their respective values of $(\Omega/R)^2$].

# The zonal and residual spectra in the ocean, on giant planets and in simulations are indicative of zonostrophic turbulence




**Anisotropic circulation and transport in zonostrophic turbulence**

# Rossby-Haurwitz waves

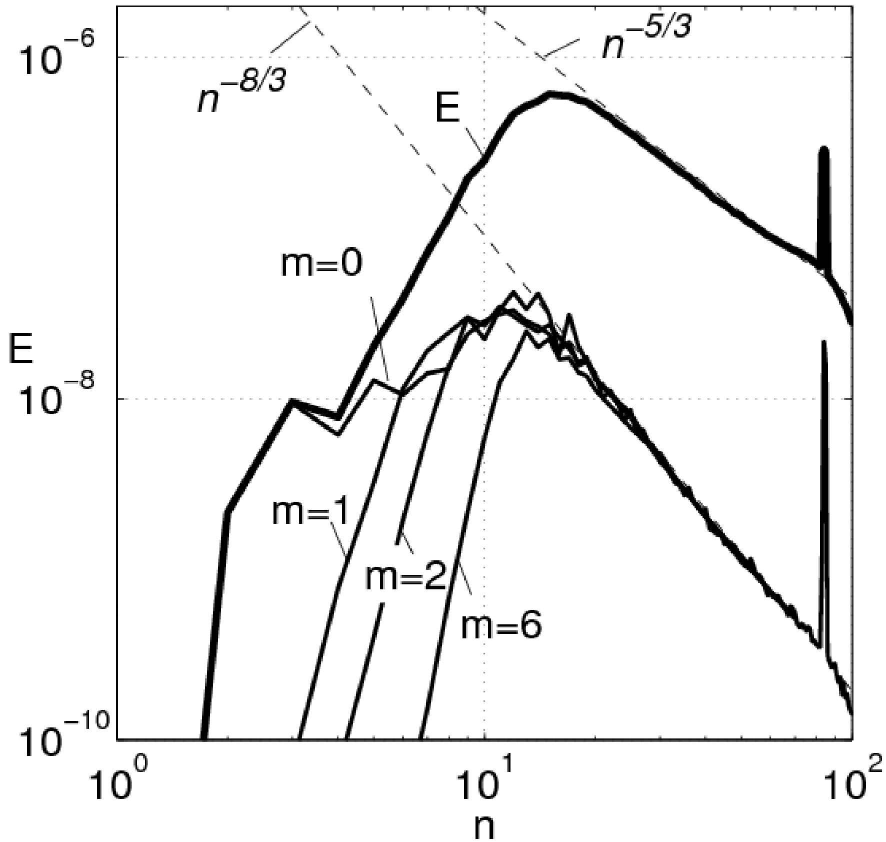
**Linear dispersion relation** $$\omega_{m,n} = -2 \; \frac{\Omega}{R} \; \frac{m}{n(n+1)}$$

To understand this anisotropic turbulent system with waves, we would like to know:

1. Can the waves coexist with turbulence?
2. Can the large-scale waves emerge from the small-scale noise?
3. How the waves interact with turbulence and among themselves in different flow regimes? (i.e., how the answers depend on the zonostrophy index?)

# The friction-dominated regime

$n_R = 9.2$
$n_\beta = 14.3$
$R_\beta = 1.55$




The total spectrum, E(n), obeys the classical Kolmogorov-Batchelor-Kraichnan scaling

$$E(n) = C_K\, \varepsilon^{2/3}\, n^{-5/3}, \quad C_K \cong 6$$

# Rossby-Haurwitz waves and turbulence

Are RHWs present in the fully nonlinear equation and, if yes, how are they affected by the nonlinearity?

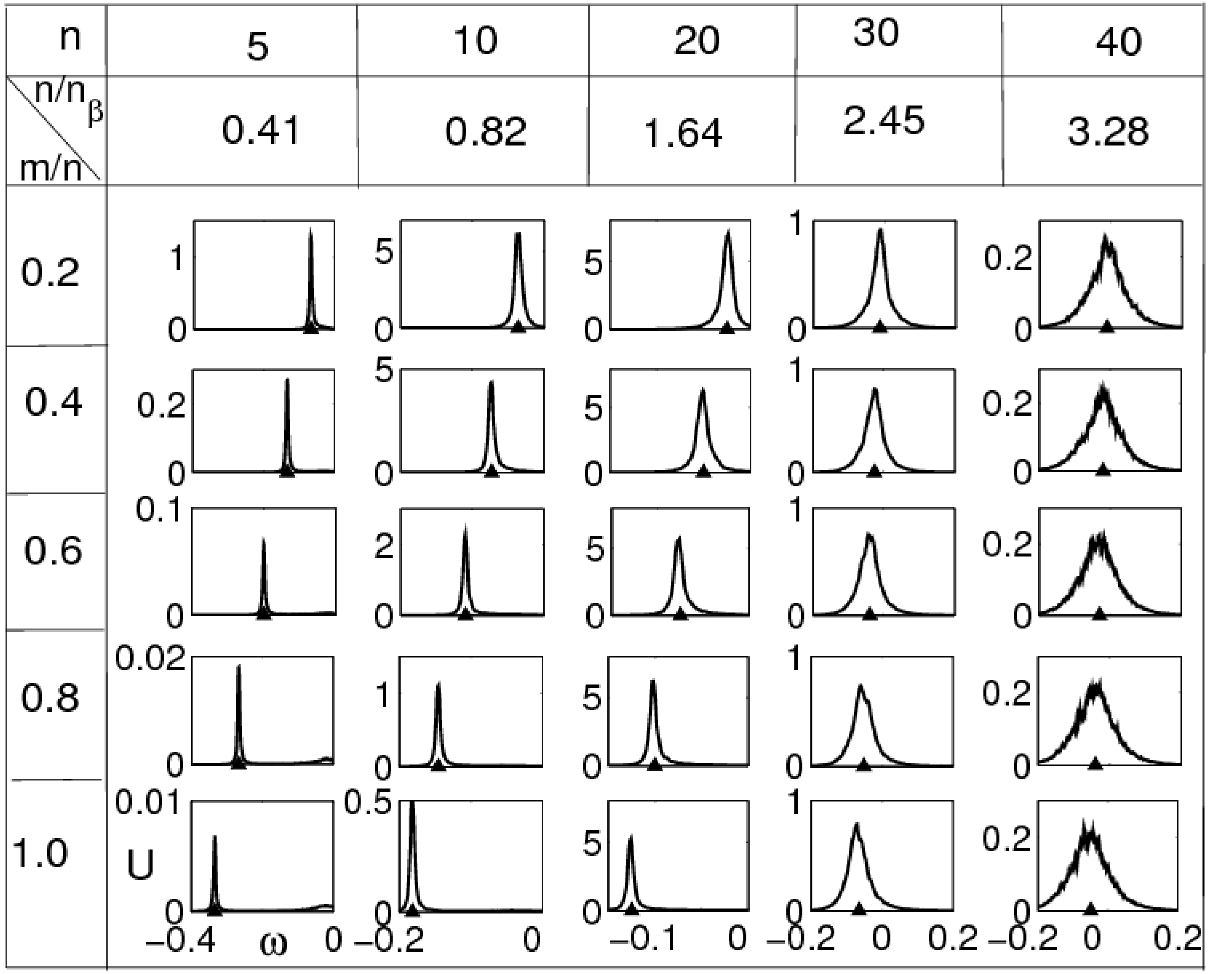
Fourier-transform of the velocity autocorrelation function

$$U(\omega, m, n) = \frac{n(n+1)}{4R^2} \langle \left|\psi_n^m(\omega)\right|^2 \rangle$$

$\psi(\omega)$ is a time Fourier transformed spectral coefficient $\psi(t)$

**Spikes of $U(\omega,m,n)$ correspond to the dispersion relation → the correlator $U(\omega,m,n)$ is a convenient diagnostic tool for finding waves in data and in simulations.**

$n_R = 9.2$
$n_\beta = 12.3$
$R_\beta = 1.34$

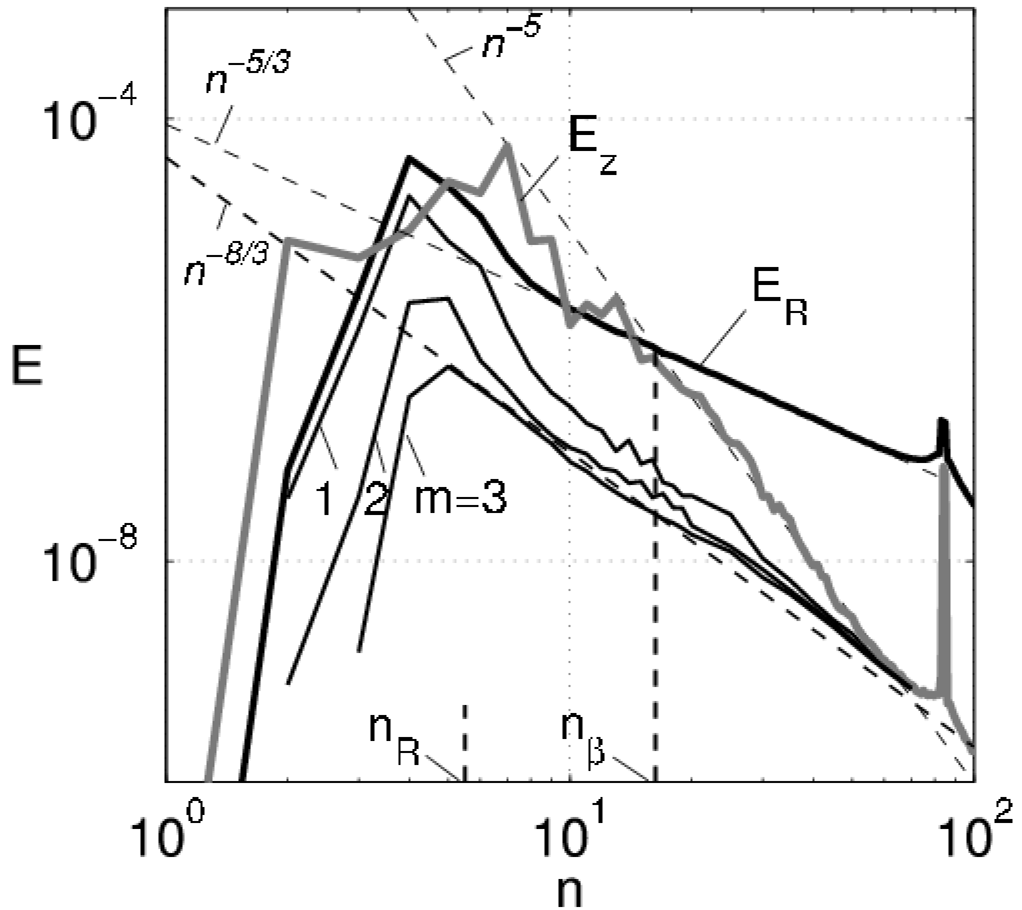



The velocity correlator, $U(\omega,m,n) \times 10^{-7}$, for the friction-dominated regime. The filled triangles correspond to the RHWs dispersion relation.

- The large-scale modes are populated by linear RHWs
- Waves at natural frequencies hog considerable energy
- A strong RHW signature is present even on scales with $n/n_{\beta} > 2$
- On the smallest scales, the RHW peaks are broadened by turbulence
- Even though the flow dynamics is dominated by strong nonlinearity the flow features linear RHWs
- This result is strikingly counter-intuitive!
- In a truly linear system, the excitation of every RHW would require a specific source. In the nonlinear framework, all possible RHWs emerge from the random noise of diverse nature

# Kinetic energy spectra for a zonostrophic regime

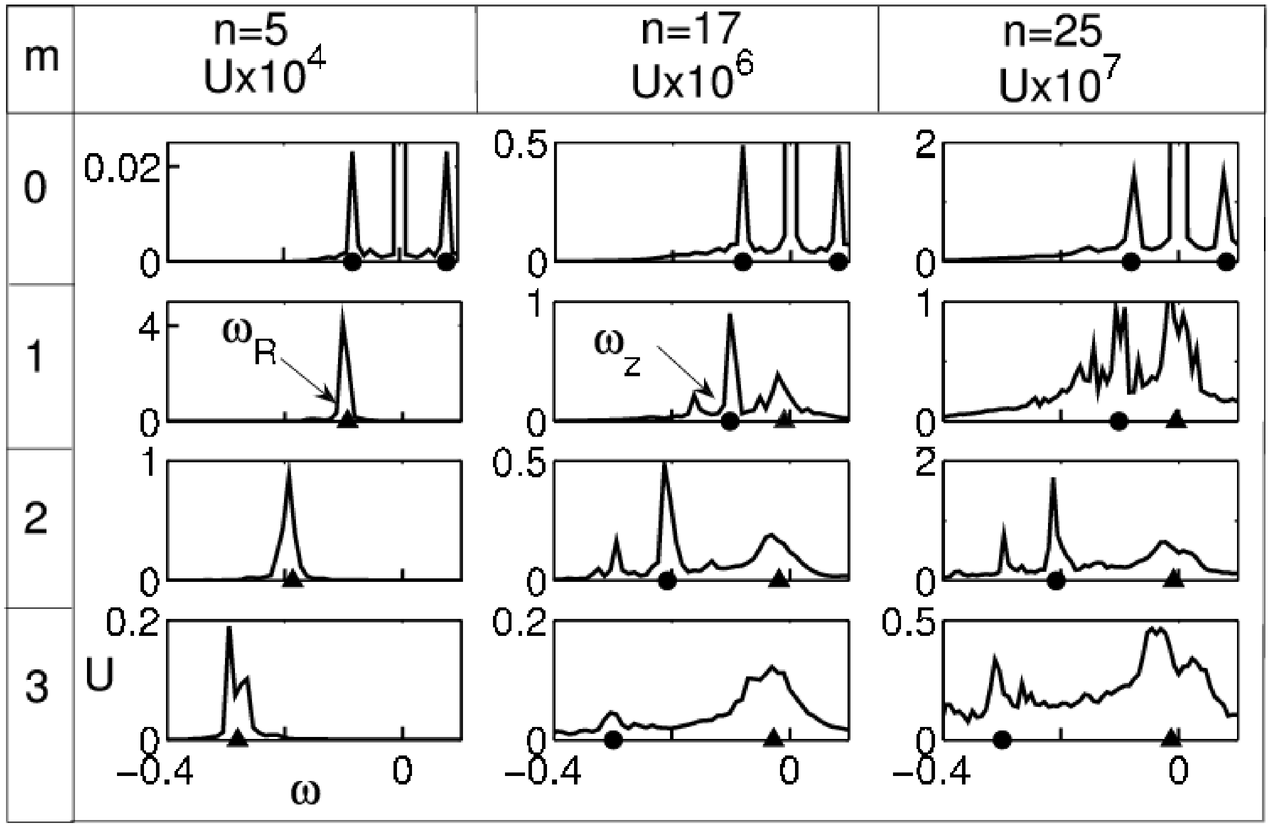
$n_R = 5.5$
$n_\beta = 16.2$
$R_\beta = 2.95$




The thin lines show the modal non-zonal spectra.

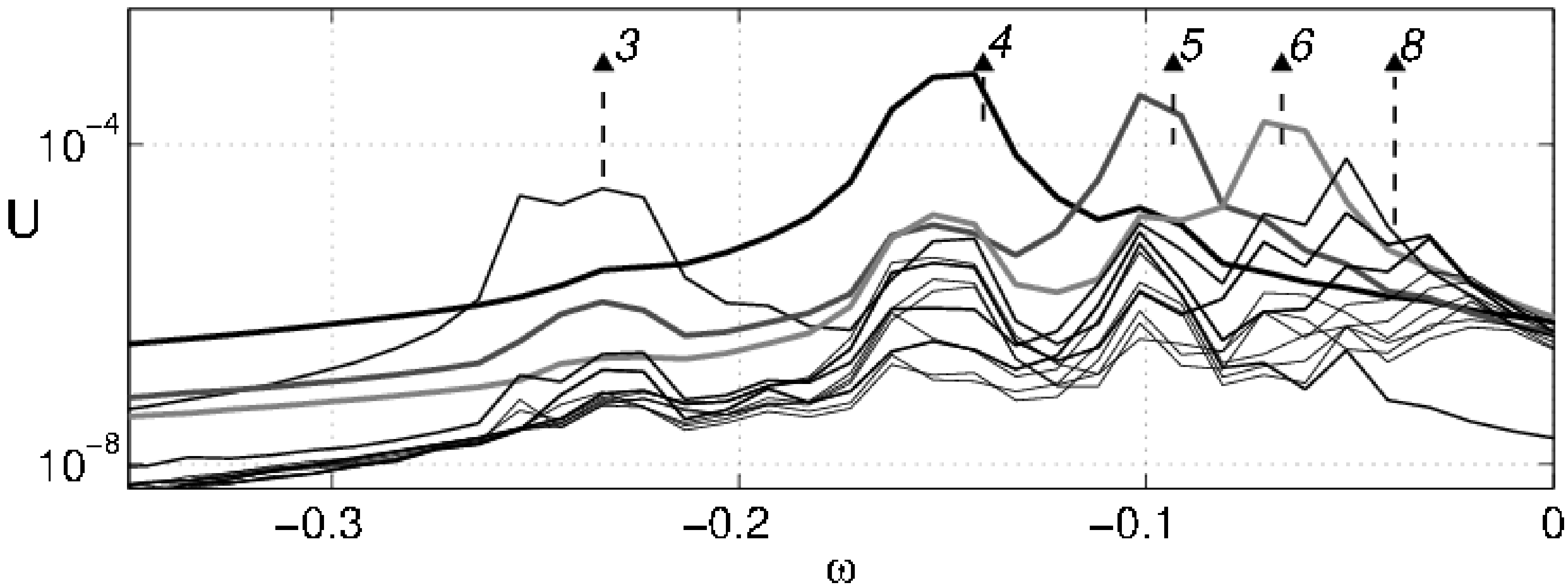
# Waves in zonostrophic turbulence

$n_R = 5.5$
$n_\beta = 16.2$
$R_\beta = 2.95$




❖U(ω,m,n) is the velocity correlator
❖Filled triangles ➔ RHWs dispersion relation
❖Filled circles ➔ zonons

# The physical nature of the zonons

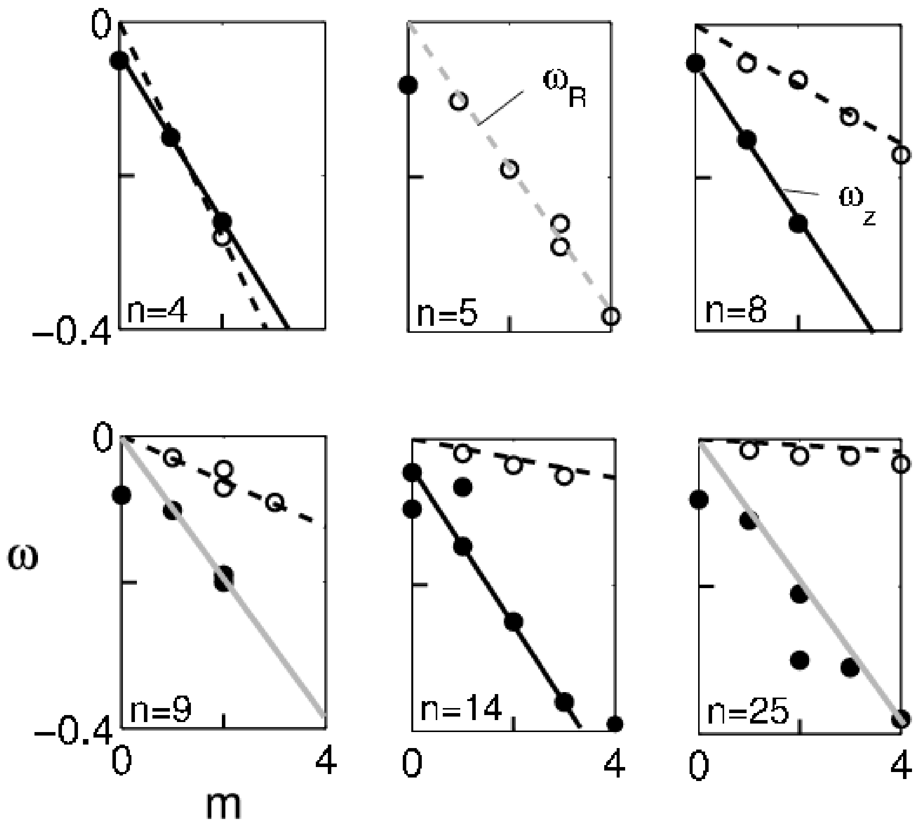



❖Correlator U(ω,1,n) (n = 3 through 15) for a zonostrophic regime. The filled triangles show the RHWs dispersion relation.

❖Up to n = 6, each U(ω,1,n) exhibits the dominant peak at ω of its own RHW frequency. The secondary peaks are zonons.

❖Zonons are **forced** oscillations excited by RHWs in modes with the same m and practically all other n. Frequencies of the forced waves are equal to the frequencies of the corresponding "master" RHWs and are n-independent

# Frequencies of RHWs and zonons as functions of m for different n



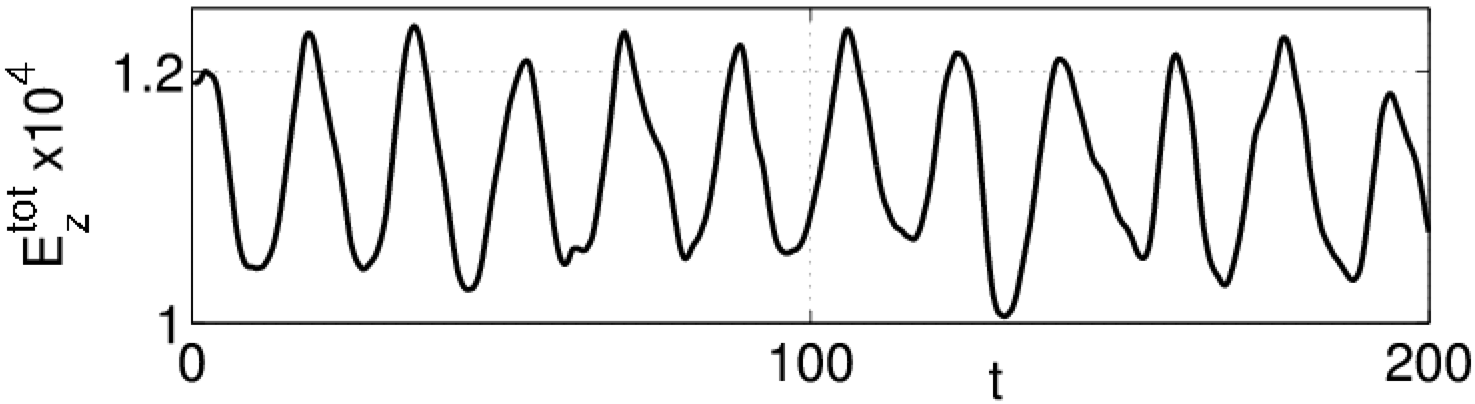

- RHWs are evident in all modes including $n > n_\beta$
- Along with RHWs, one discerns zonons excited by the most energetic RHWs with n = 4 and 5
- $\omega_z(n,m) \propto m$ and are independent of n for all zonons

❖Zonons form wave packets

❖Their zonal speeds are $c_z = \omega_z(m,n)/m$

❖$c_z$ are equal to the zonal phase speeds of the corresponding master RHWs

❖There exist **zonal** zonons, $\omega_z(n,0)\neq 0$ while $\omega_R(n,0)=0\ \forall\ n$




❖Total energy of zonal modes oscillates in time as a superposition of waves with two frequencies corresponding to the zonal zonons

❖Nonlinear interactions between zonal and nonzonal modes cause energy oscillations around the saturated value

❖All zonons are "slave" waves excited by RHWs

❖Their dispersion relations differ from RHWs ➔ zonons should be recognized as an entity completely different from RHWs

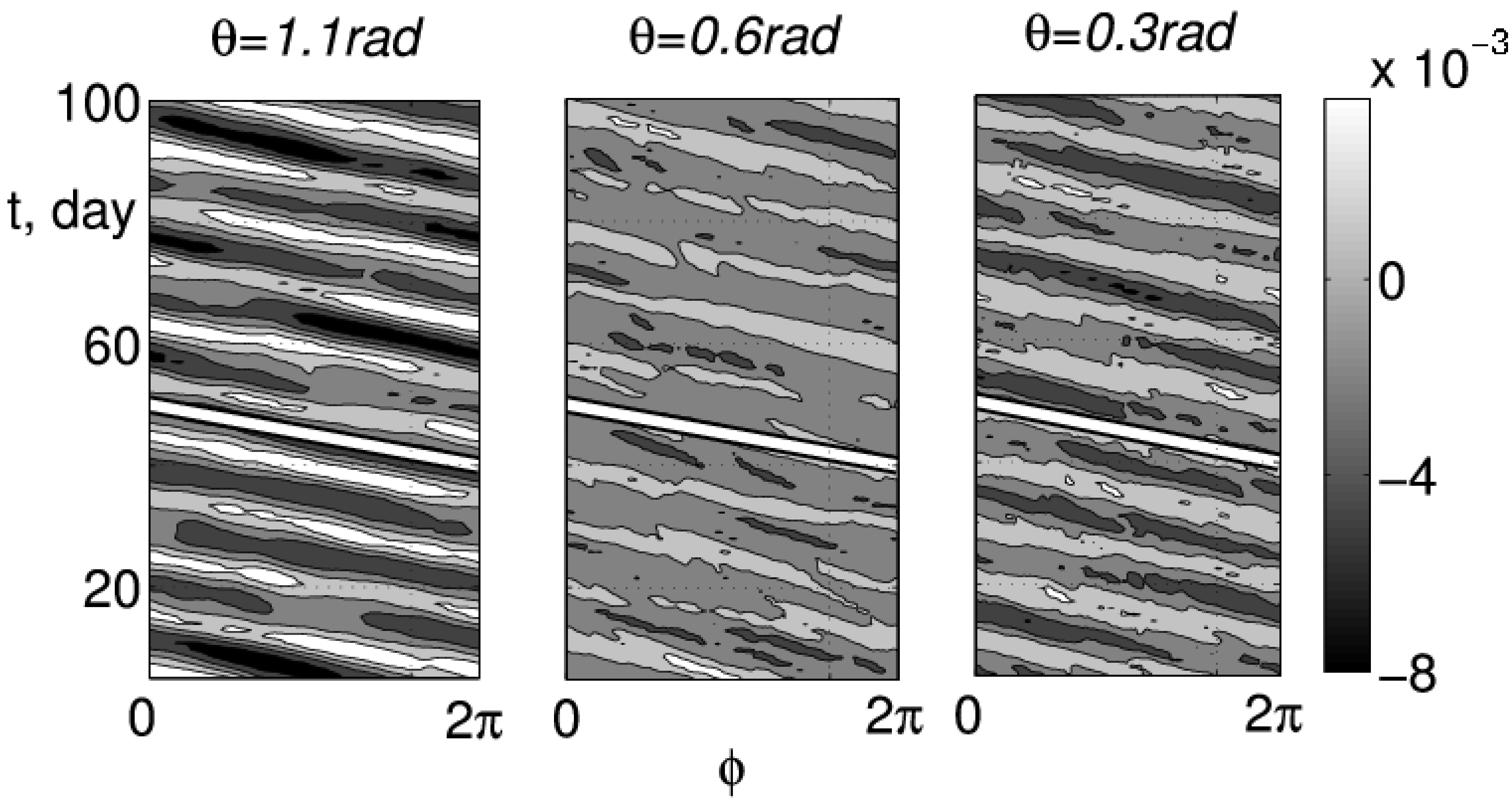
❖How do the zonons appear in the physical space? The RHWs with $n = 4$ (denoted $n_E$) are the most energetic ➔ their respective packets of zonons are dominant in physical space and are easiest to observe

❖The zonal speed of these packets is $\omega_R(n_E,m)/m = c_{RE}$

❖In physical space, these zonon packets are expected to form westward propagating eddies detectable in the Hovmoller diagrams

❖The slope of the demeaned diagrams yields a velocity of the zonally propagating eddies relative to local zonal flows

❖If eddies are indeed comprised of zonons, their zonal phase speed should be equal to $c_z = c_{RE}$

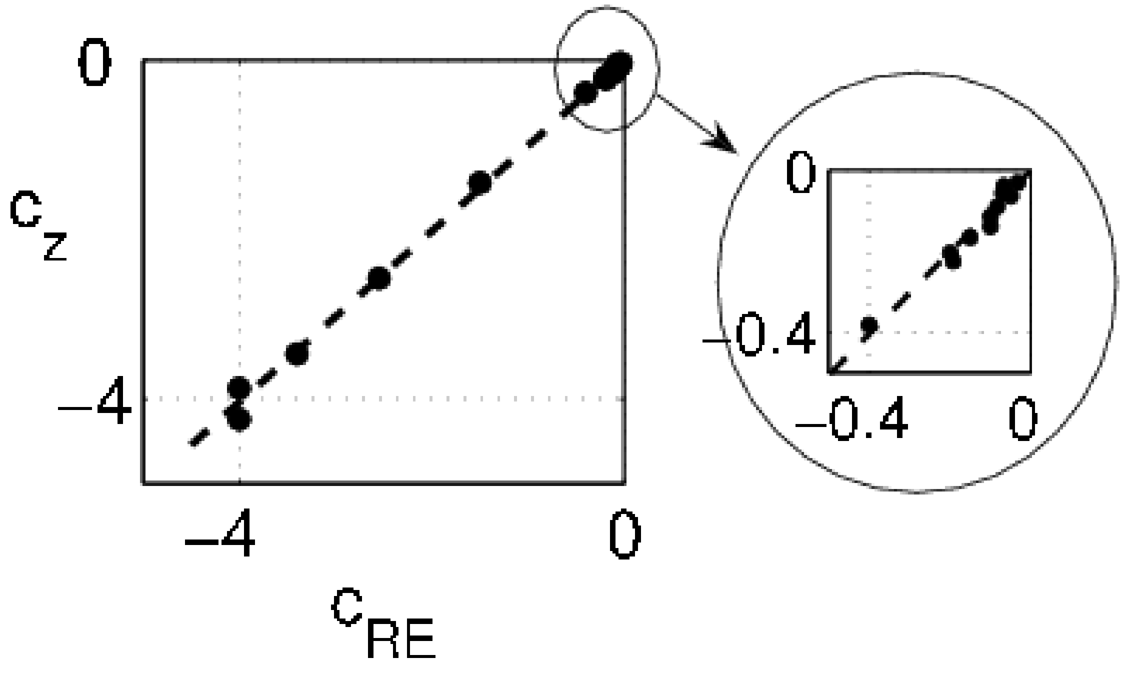



❖The Hovmoller diagrams indeed reveal westward propagating eddies at three different off-equatorial latitudes at which the zonal jets have their maximum, minimum, and zero velocity

❖This figure demonstrates that $c_z = c_{RE}$ at all three latitudes

❖The white lines show the slope used to calculate the angular velocities

# Comparison of zonal phase speeds from the Hovmoller diagrams and the RHW dispersion relation for most energetic RHWs for different simulations




❖The westward propagating eddies can be identified with the energetic zonon packets moving largely independently of the zonal flows

# Eddies in the ocean

GRL, 2007

## Global observations of large oceanic eddies

Dudley B. Chelton,[1] Michael G. Schlax,[1] Roger M. Samelson,[1] and Roland A. de Szoeke[1]

"… more than 50% of the variability over much of the World Ocean is accounted for by eddies with amplitudes of 5–25 cm and diameters of 100–200 km. These eddies propagate nearly due west at approximately the phase speed of nondispersive baroclinic Rossby waves… The vast majority of the eddies are found to be nonlinear. "

u/c (nonlinear advection/linear wave propagation) is the measure of non-linearity.

We found that for the ocean, $R_\beta \cong 1.5$, the circulation is marginally zonostrophic. Other studies confirmed the presence of the inverse energy cascade in both barotropic and baroclinic modes. It is possible that these eddies are zonons but more work is needed.

# Anisotropic waves, turbulence and transport

- **Turbulent transport is fundamentally affected by a β-effect, Rossby-Haurwitz waves, 2D turbulence, inverse energy cascade**
- **Even though such flows exhibit the negative Laplacian viscosity phenomenon [Starr, 1968], their turbulent diffusivity is positive and obeys the Richardson diffusion law (ε is now the rate of the inverse cascade)**

$$\kappa \propto \varepsilon^{\frac{1}{3}} k^{-\frac{4}{3}}$$

***$\kappa$ is the eddy diffusivity that accumulated at a scale $k^{-1}$ as a result of contributions from all eddies smaller than $k^{-1}$***

# Meridional diffusion in flows with a β-effect

- **We studied the meridional diffusion in simulations with the barotropic vorticity equation coupled with the diffusion equation**
- **The flow was in a steady state**
- **$R_\beta$ varied between 0 and 4 covering all three flow regimes**
- **A tracer was released from thin zonal rings located at different latitudes, and from the polar caps thus eliminating the effect of the zonal advection while allowing for testing the effect of the zonal jets upon the meridional diffusion**
- **The tracers' evolution was followed until they reached high latitudes from which they could escape via polar regions**

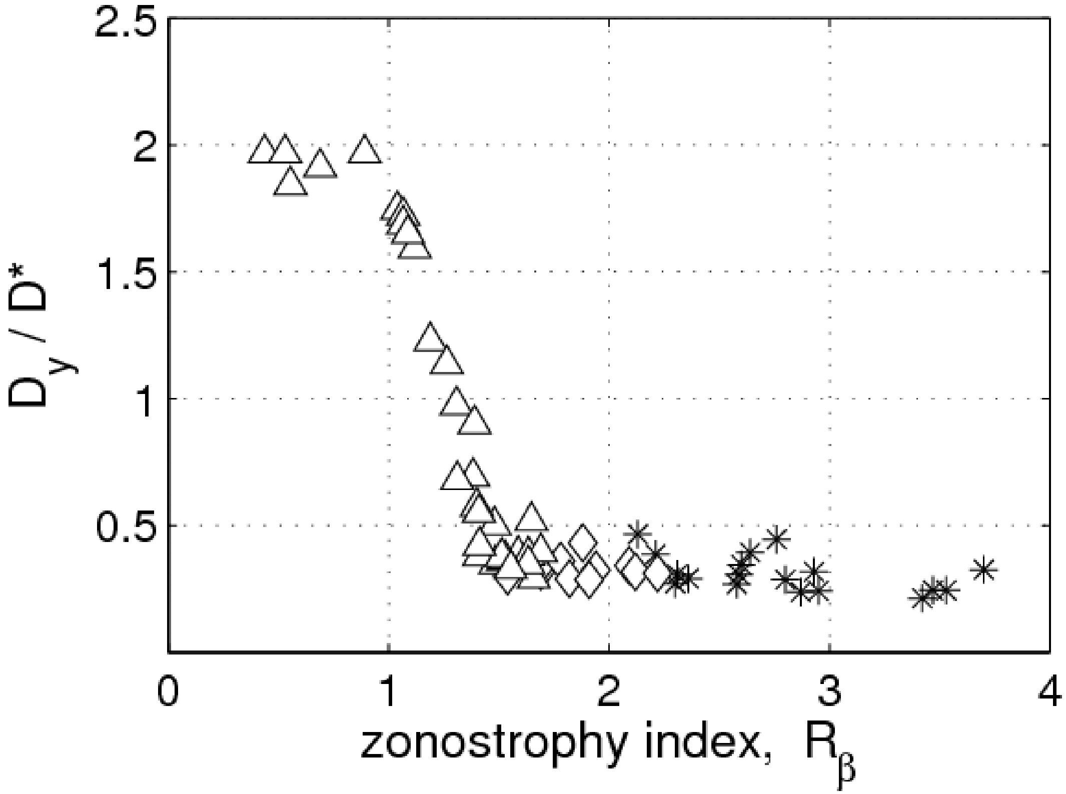
# Simulations and results

**The meridional diffusivity, $D_y$, was calculated by three different methods:**

- **from the meridional spread, $\sigma_y^2$, according to the well known expression, $\sigma_y^2 = 2D_y t$, t being the time elapsed after the tracer release**
- **from the Lagrangian trajectories**
- **from the Reynolds averaging, by dividing the meridional turbulent tracer flux by the mean meridional tracer gradient**

***The results from all three methods were quite similar***

# Lateral diffusion in various regimes




**$D^* = 2\ \varepsilon^{1/3}\ n_E^{-4/3}$ for $R_\beta < 1$, $n_E \propto n_{fr}$ is the wave number with the maximum energy**

**$D^* = 0.3\ \varepsilon^{1/3}\ n_\beta^{-4/3}$ for $R_\beta > 1$; $D_y$ keeps its value throughout the transitional and zonostrophic regimes**

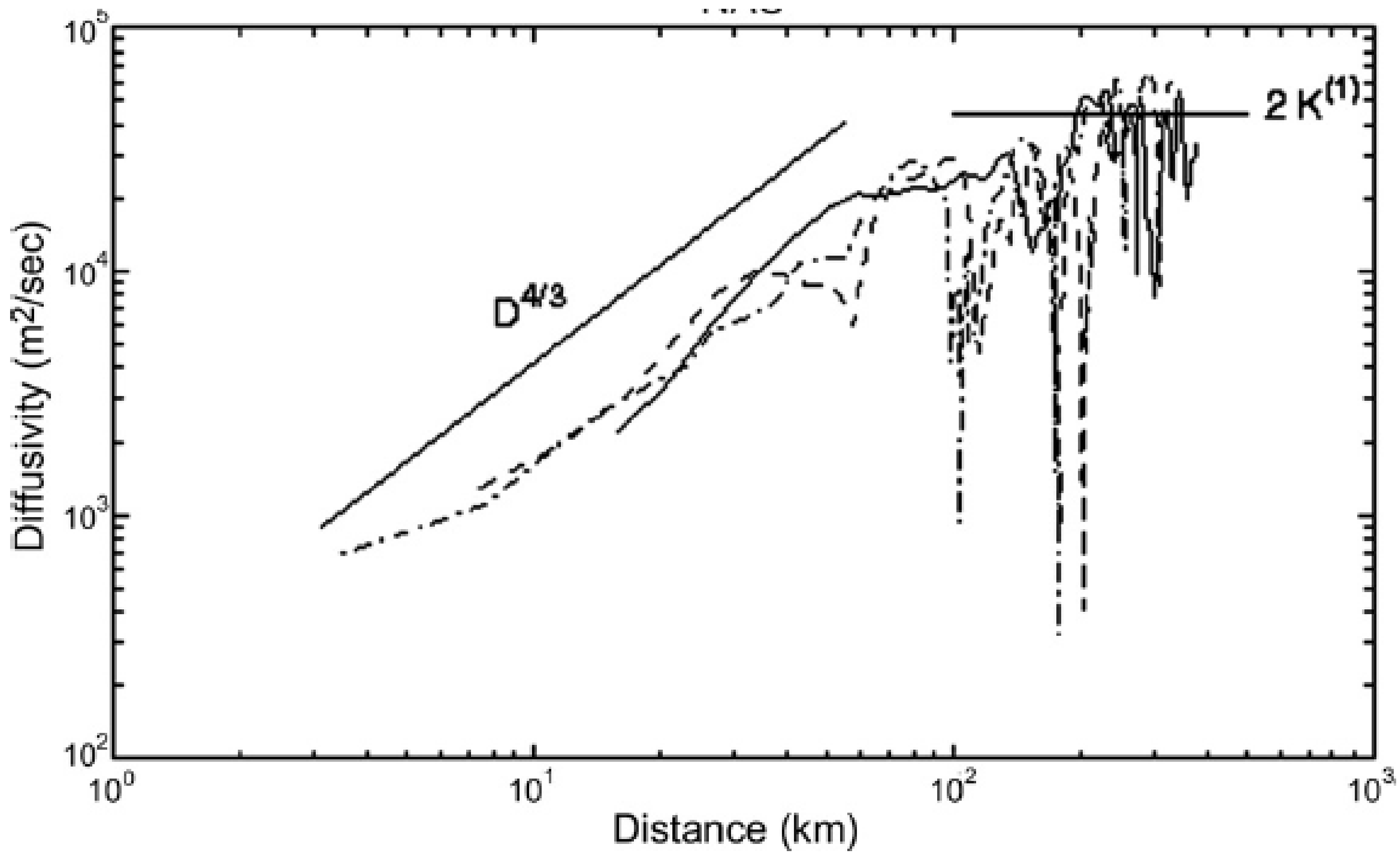
# Waves and lateral diffusion

- **In zonostrophic turbulence, lateral diffusion becomes scale-independent, $D_y = 0.8\, \varepsilon^{3/5}\, \beta^{-4/5}$**
- **This result is rather remarkable:** ***the scales where a $\beta$-effect is important are dominated by the Rossby-Haurwitz waves that make no contribution to the meridional diffusion***
- **The effective transport on these scales is determined by quasi-isotropic turbulent eddies with scales not exceeding $n_\beta^{-1}$**
- **A similar expression was derived by Lapeyre and Held [2003]. This scaling arose from the dimensional considerations assuming that the inverse cascade is halted by a $\beta$-effect on scales $O(n_\beta^{-1})$ so that $D_y$ depends on $\varepsilon$ and $\beta$ only**
- **But we showed that a $\beta$-effect does not halt the inverse cascade; it only causes its anisotropization on scales $O(n_\beta^{-1})$ and larger (Sukoriansky et al. [2007]) and so our result characterizes the interaction between turbulence and waves**

# Tracers in the ocean

- **Galperin et al. [2004] argued that the deep large-scale oceanic circulation is marginally zonostrophic with $R_\beta \sim 1.5$. In this case one could expect a transition from the Richardson diffusion to the scale-independent law $D_y = 0.8\,\varepsilon^{3/5}\,\beta^{-4/5}$**
- **If the long-term average of $\varepsilon$ is taken as a constant, $D_y$ is also a constant**
- **Transitions from the Richardson law to a constant diffusivity were indeed observed in some experiments on relative dispersion of subsurface floats in the North Atlantic [LaCasce, 2008].**
- **The scale of this transition can be estimated as $n_\beta^{-1}$. With $\varepsilon \sim 10^{-10}$ $m^2\,s^{-3}$ [Galperin et al., 2006] and $\beta \sim 10^{-11}$ $m^{-1}\,s^{-1}$, $n_\beta^{-1}$ is appraised at approximately 100 km, in good agreement with the observations.**

# Floats in the western North Atlantic




**Relative diffusivity vs. distance for floats from the western North Atlantic, with initial separations of 7.5, 15 and 30 km. From LaCasce and Bower (2000).**

# Conclusions

- **Zonostrophic turbulence is a subset of geostrophic turbulence; $R_\beta$>2 and Bu<0.01**
- **Zonons are forced oscillations excited by RHWs in other modes via non-linear interactions**
- **Zonons are an integral part of the zonostrophic regime. They emerge in the process of energy accumulation in the large-scale modes and formation of the steep $n^{-5}$ spectrum**
- **The mechanism of zonon generation is accommodated neither in conventional second-moment closure theories nor in theories of wave turbulence and weakly-nonlinear wave interactions**
- **More research is needed to clarify zonons' roles in planetary circulations**
- **Scales dominated by waves do not contribute to the diffusion in the direction of zero frequency**
- **Lateral diffusion in flows with a $\beta$-effect is carried out only by turbulent, nearly-isotropic scales**
- **Diffusion of momentum and scalar are very different because waves can mix momentum but not scalar – an aspect often overlooked. Analogies implying a similarity are dangerous!**
- **Some theories of the oceanic and atmospheric mixing and mixing barriers are decoupled from the dynamics. Is this a valid approach?**

# Acknowledgments

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# The End

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