



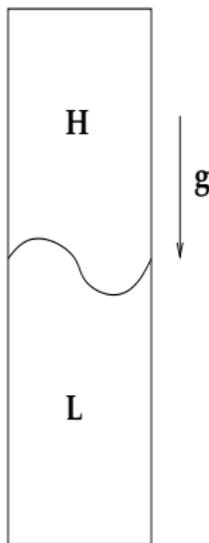
Oscillatory behavior in the compressible Rayleigh–Taylor Instability

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The Rayleigh–Taylor Instability



Hypotheses:

- Perfect Gases,
- 2 hypotheses:
 - compressible perfect fluids,
 - miscible heat conductor Newtonian fluids.

6 or 7 variables:

$u, v, w, P, \rho, T, (c$ in miscible Newtonian fluids)

Equations:

- Compressible Navier-Stokes
- Energy transport
- Perfect gas equation of state.
- Boundary conditions
- Concentration transport in miscible Newtonian fluids

Simplification: Invariance by rotation around the Z-axis.

RTI has 5 or 6 variables and 6 or 9 parameters

$h_L, h_H, A_t, S_r, \gamma_H, \gamma_L$ adding R_e, P_r, S_c in miscible Newtonian fluids.



- 1 What is the Rayleigh–Taylor instability?
- 2 Linear analysis of the Rayleigh–Taylor Instability
 - Normal mode analysis
 - Numerical Methods
 - Classification of known results
- 3 Perfect Fluids: TS case
- 4 Miscible compressible Newtonian fluids: TG case
- 5 Conclusion



- Linear?
 - Characteristic of the short time growth
 - Mixing layer growth absorbs selected frequencies
 - Turbulence properties have a direct link with the linear.
- Systematic investigation of the linear RTI operator spectrum.
 - Stationary isotherm base state $G = G_0(z) + g(x, z, t)$
 - Normal mode decomposition $g(x, z, t) = \hat{g}(z)e^{nt-ik}$
 - temporal study $n = f(k, \dots)$



- Spectral discretization = high order scheme
 - Global polynomial discretization,
 - Exponential convergence to the solution with the numbers of discretization points,
 - High order quadrature formula : N points \rightarrow exact integration of a $2N - 1$ order polynom.
 - Chebychev in z direction
- Multidomain
 - Adaptive discretization to refine high variation zones,
 - Parallel computing efficiency.
- Spectrum
 - Generalized eigenvalues problem diagonalization,
 - Eigenvalue and eigenvector refinement: Rayleigh iterations.

Classification of the various R-T problems:

- G, general case,

and two limits:

- Isothermal T,
- Isentropic S.

These 3 cases are possible for both base flow and perturbations.

Hydrostatic Equilibrium type	Perturbation type		
	Isothermal (T) $R_e P_r \rightarrow 0, R_e \rightarrow \infty$	Isentropic (S) $R_e \rightarrow \infty$	General (G)
	TI-case Blake, Mathews-Blumenthal Baker, Ribeyre <i>et al.</i> Depends on A_t, S_r , and $h_{H,L}$ γ -independent	TS-case Bernstein-Book, Livescu Depends on $A_t, S_r, \gamma_{H,L}$ and $h_{H,L}$	TG-case Lafay <i>et al.</i> Depends on $A_t, S_r, \gamma_{H,L},$ $h_{H,L}, R_e, S_c$ and P_r
Isothermal (T) $R_e P_r \neq 0$			
Isentropic (S) $R_e \rightarrow \infty$	□	SS-case Lezzi-Prosperetti Depends on $A_t, S_r, \gamma_{H,L}$ and $h_{H,L}$	□
General (G)	Has to be done	Has to be done	Has to be done

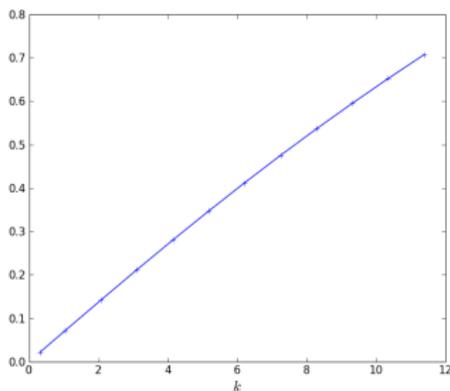
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The dispersion relation in the TS case take the following form:

$$\left[(1 \pm A_t) \left(\frac{n^2}{\gamma} \frac{S_r}{1 \pm A_t} + k^2 \right)^{-1} \left[n^2 q_{-}^H F_{H,L}(h_{H,L}) + k^2 \right] \right]_{H,L} = 0$$

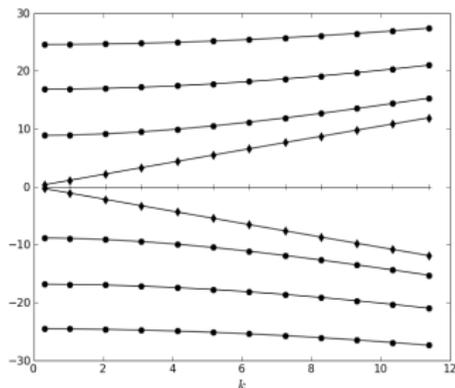
- Equation in terms of n^2

$n^2 > 0$



the pair of classical eigenvalues

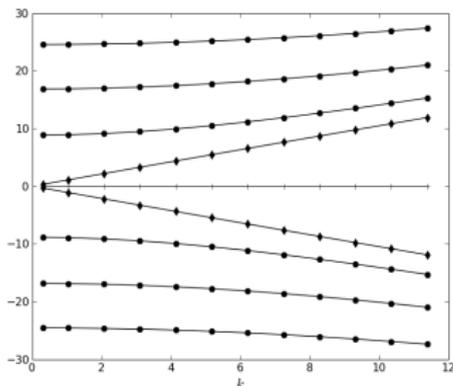
$n^2 < 0$



an countable infinity of
imaginary eigenvalues

Various behaviors \rightarrow necessity of a nomenclature:

Imaginary parts of the growth rate



- **NV-A-1,2,3,...**(Non-viscous type A): Pairs of complex conjugate eigenvalues, non vanishing at $k = 0$.
- **NV-B**(Non-viscous type B): Pairs of linearly growing complex conjugate eigenvalues with k , null at $k = 0$.
- **G** (Gravity - Classical) : Pure real eigenvalue.



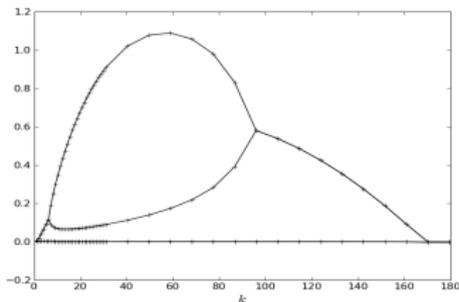
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Dispersion curves:

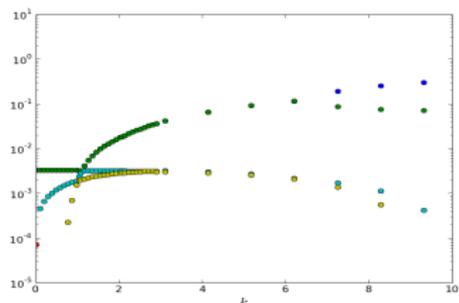
For the reference case in thin layers and different $\gamma_{H,L}$



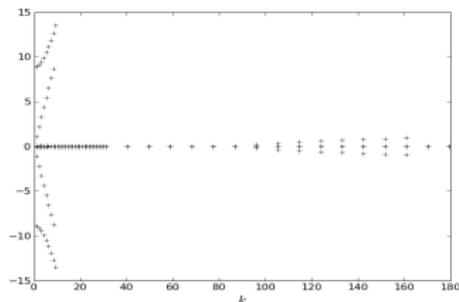
Real Parts



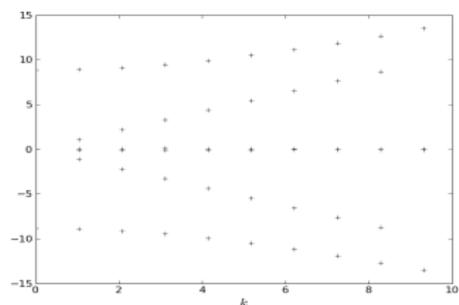
Zoom on the real parts



Imaginary Parts



Zoom on the imaginary parts



Nomenclature:

What's identical to perfect fluids for the same set of parameters

- **NV-A** Same imaginary part between viscous and perfect fluids. Non zero real part, dominant at low k .
- **NV-B** Same imaginary part between viscous and perfect fluids. Non zero real part, lower than the first NV-A's.

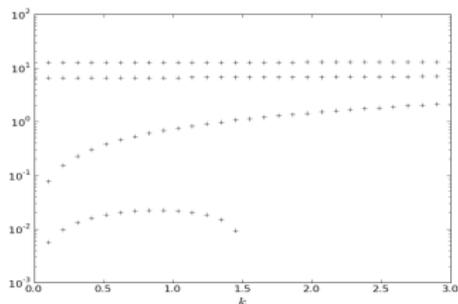
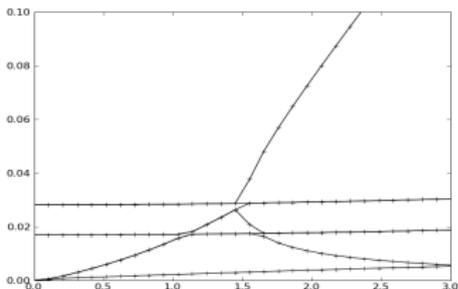
What's different: Eigenvalue nature (real/complex) variation with the $k \rightarrow$ Refinement of the nomenclature:

- **G-R1** (Gravity R1): Most growing real mode, the classical one.
- **G-R2** (Gravity R2): Real mode with a lower growth rate.
- **G-CC1** (Gravity complex conjugate): pair of complex conjugate, vanishing at $k = 0$.
- **G-CC2** (Gravity complex conjugate): pair of complex conjugate.

For the reference case:

- **NV-A-1:** Dominant mode for $k < 1$.
- **NV-B:** Dominant mode $n^o 2$ for $k < 1$.

Extending this zone with γ_L variation:

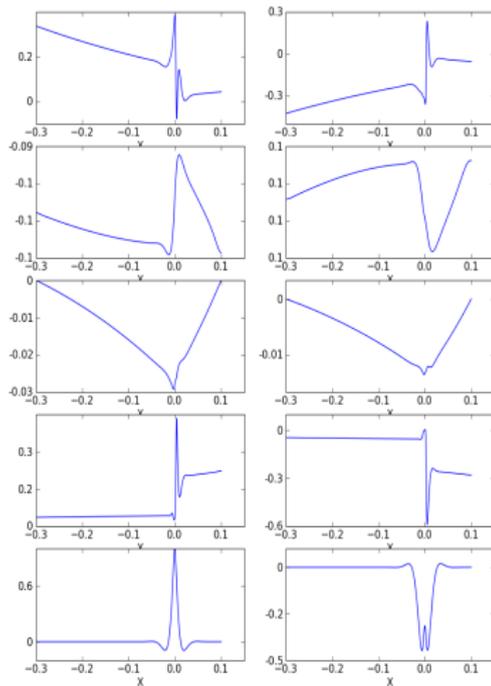
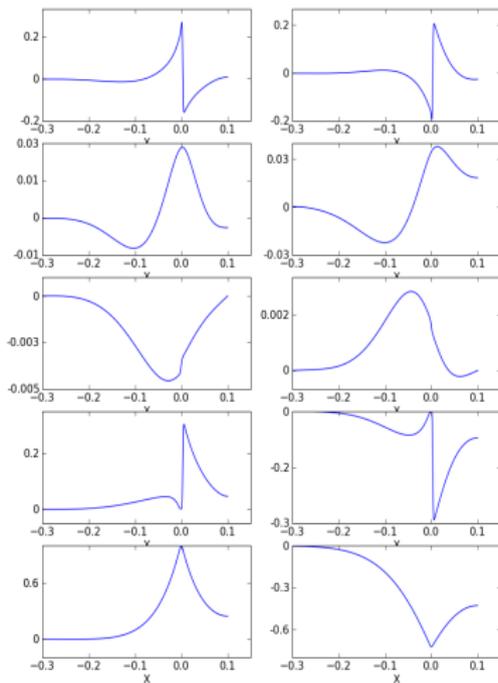


- **NV-A-1:** Dominant mode for $k < 1.5$.
- **NV-A-2:** Dominant mode over **G-CC1:** for $k < 1$.
- **NV-B:** Dominant mode over **G-CC1:** for $k < 0.15$.

Eigenfunctions

G-CC1, $k = 2.07$
 $n = 0.23e - 1 + 0.15e - 1i$

NV-B, $k = 3.00$
 $n = 0.52e - 02 + 2.17i$

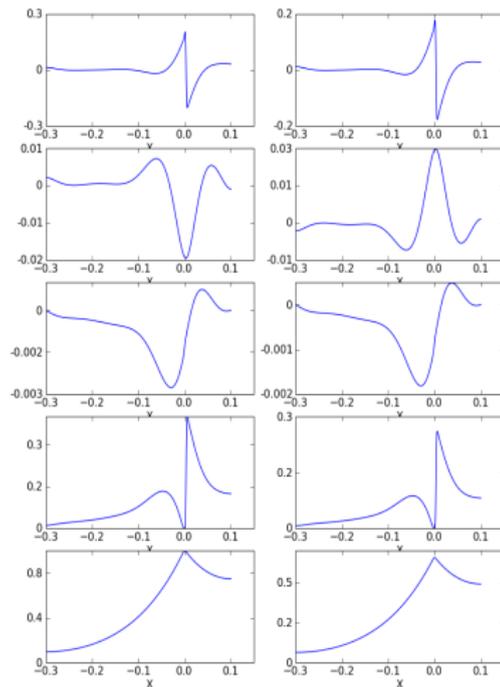
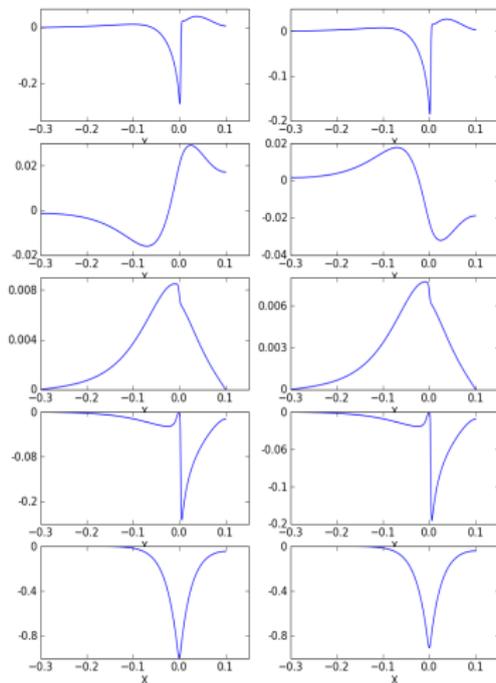


G-R1 et G-R2, $k = 3$.



$n = 0.143$

$n = 0.57e - 2$

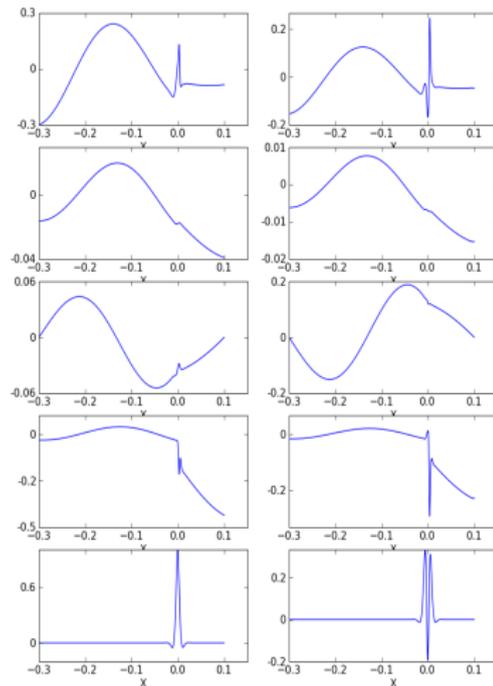
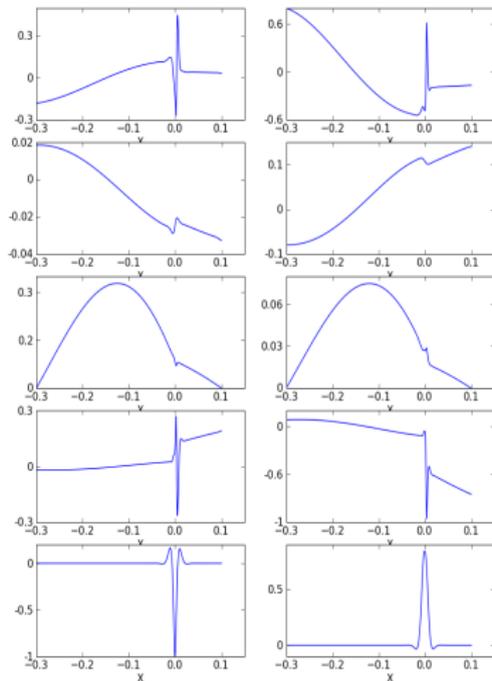


Eigenfunctions

NV-A-1 et NV-A-2, $k = 3.0$

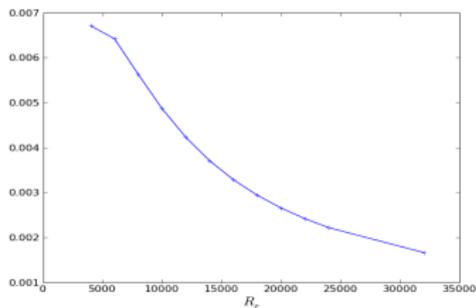
$$n = 0.3e - 1 + 6.96i$$

$$n = 0.18e - 1 - 12.87i$$

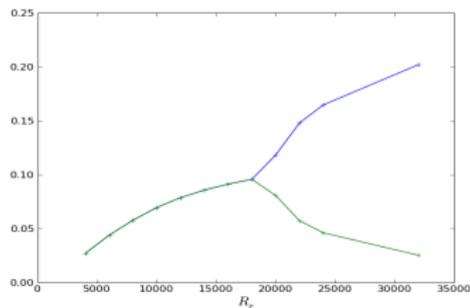


$R_e \rightarrow \infty$: perfect fluid behavior

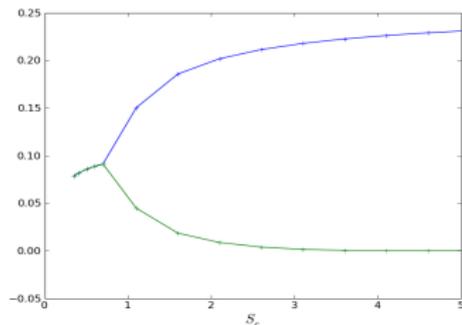
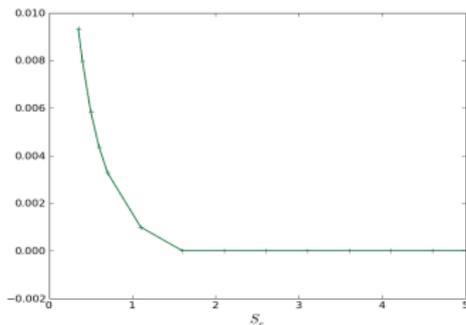
Real part of NV-A-1 à $k \approx 0$



Real part of G-CC1 à $k \approx 5.17$

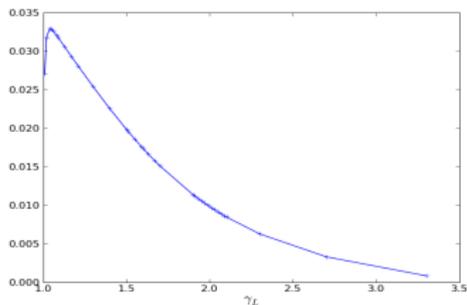


Threshold effect induced by the transport coefficients (for ex. S_c)

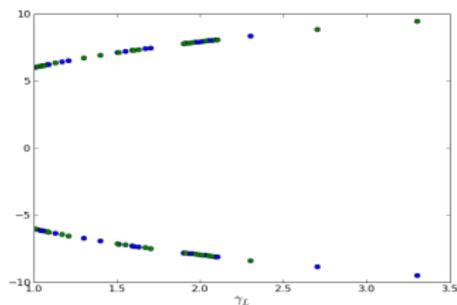


Variation of γ_L

Real part of NV-A-1 at $k \approx 0$

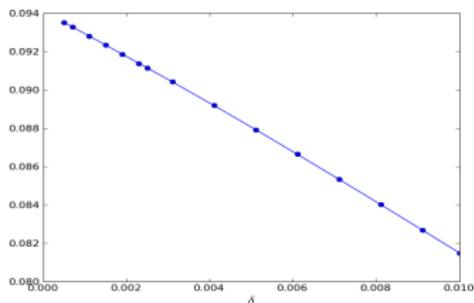


Imaginary part of NV-A-1 at $k \approx 0$

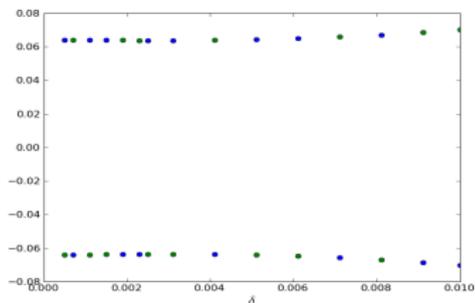


Variation of δ

Real part of G-CC1 at $k \approx 5.17$



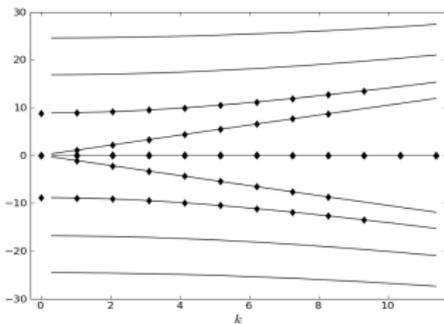
Imaginary part of G-CC1 at $k \approx 5.17$



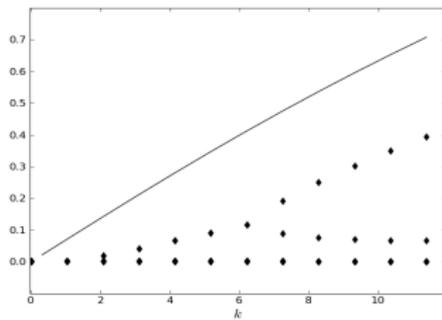


Comparison of the dispersion curves on the equivalent reference case.

Imaginary part



Real part





- Compressibility effects limited within low k ,
- Non-vanishing complex growth rate at large scales ($k \rightarrow 0$),
- NV Instabilities keep their imaginary parts,

Tendances

- $R_e \rightarrow \infty$, $G \rightarrow G-R1$,
- $R_e \rightarrow \infty$, $\mathcal{R}(NV) \rightarrow 0$,
- Threshold effect induced by the transport coefficients.

Perspectives

- Non linear stability,
- Non linear simulation of these oscillatory modes.



Thank you for your attention.