Large scale flows in natural and mixed convection

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Motivation

- Convective turbulence in closed volumes is associated with large-scale circulations of the flow (LSC).
- LSC and the amount of heat are determined by proper orthogonal decomposition (POD) of the field.
- The most energetic POD modes give insight into the dynamic dominance of coherent flow and temperature patterns: How much do they contribute to the global momentum & heat transfer?
- Influence of the geometry and inflow conditions needs to be clearly understood.

Numerical model

$$u_t + u \cdot \nabla u + \nabla p = Gr^{-1/2} \nabla^2 u + T \boldsymbol{e}_z$$

$$\nabla \cdot u = 0$$

$$T_t + u \cdot \nabla T = Gr^{-1/2} Pr^{-1} \nabla^2 T$$

with Parameters:

$$Gr = \alpha g D^3 \Delta T / \nu^2, \quad Pr = \nu / \kappa = 0.7, \quad Ra = Gr\, Pr = 10^7 - 10^9$$

POD ANALYSIS:

- For the Fluctuations $\boldsymbol{v} = (\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{\theta})$, we seek $\boldsymbol{\Phi}$, such that $\dfrac{\langle |(v, \Phi)|^2 \rangle}{\|\Phi\|^2}$ is maximized

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- **The SOLUTION is given by**

$$K\Phi = \iint K(\boldsymbol{x},\boldsymbol{x}')\Phi(\boldsymbol{x}')\,d\boldsymbol{x}'^3 = \lambda\Phi(\boldsymbol{x})$$

$$K_{ij}(\boldsymbol{x},\boldsymbol{x}') = \langle v_i(\boldsymbol{x})\,v_j{*}(\boldsymbol{x}')\rangle$$

- **THE METHOD OF SNAPSHOTS:**

$$\Phi(\boldsymbol{x}) = \sum_{n=1}^{M} \alpha_n \boldsymbol{v}^{(n)}(\boldsymbol{x}) \quad \Rightarrow \quad \left(\frac{1}{M}\iiint_V \sum_{i=1}^{4} v_i^{*(m)} v_i^{(n)}\,d\boldsymbol{x}^3\right)\alpha(n) = \lambda\,\alpha(m)$$

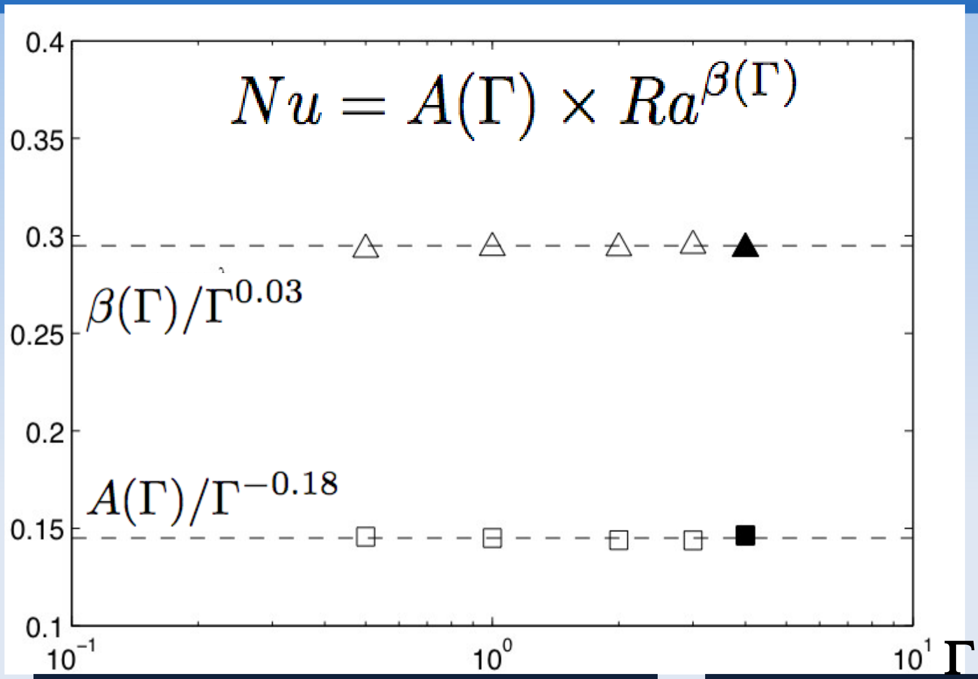
- **TIME DEPENDENT COEFFICIENTS:**

$$\boldsymbol{v}(\boldsymbol{x},t) = \sum_n a_n(t)\,\boldsymbol{\Phi}^{(n)}(\boldsymbol{x})$$

$$a_n(t) = (\boldsymbol{\Phi}^{(n)}(\boldsymbol{x}),\boldsymbol{v}(\boldsymbol{x},t))$$

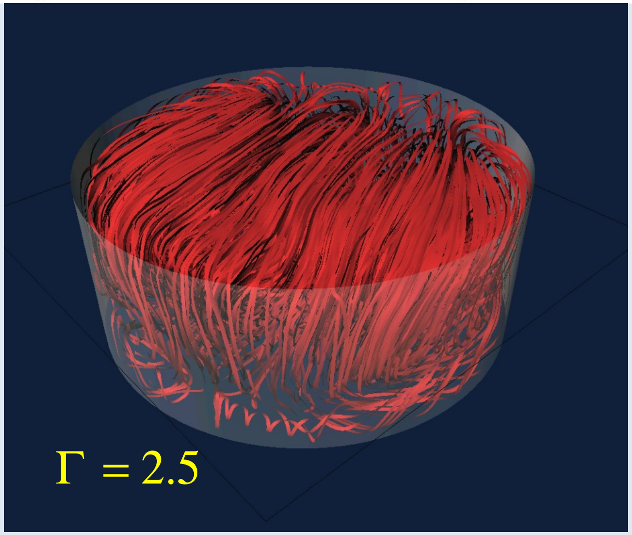
$$a_n(t) = \iiint_V \Phi_i^{*(n)}(\boldsymbol{x})\,v_i(\boldsymbol{x},t)\,d\boldsymbol{x}^3$$

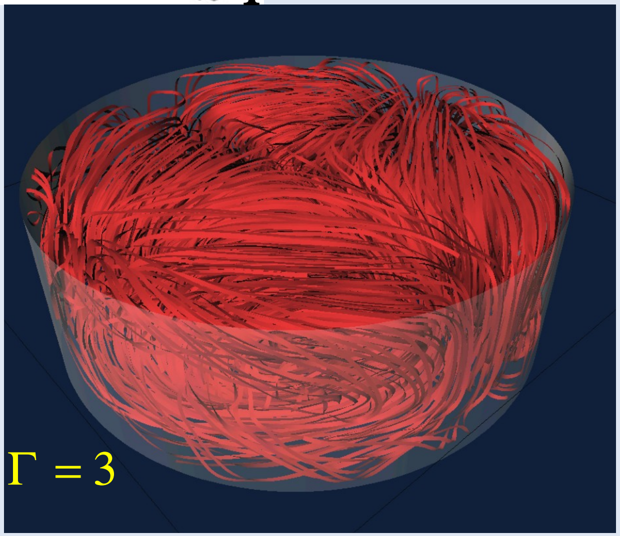
Previous work: geometry dependence

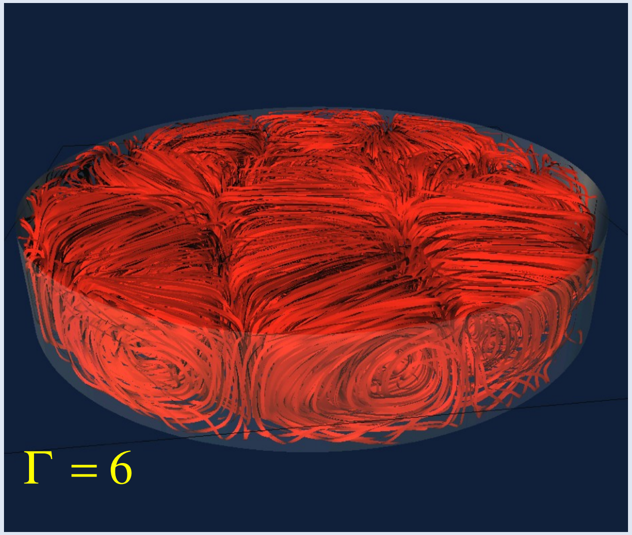


□ △ **Bailon-Cuba, Emran & Schumacher, J. Fluid Mech., submitted (2009)**

■ ▲ **Niemela & Sreenivasan, J. Fluid Mech. (2006)**





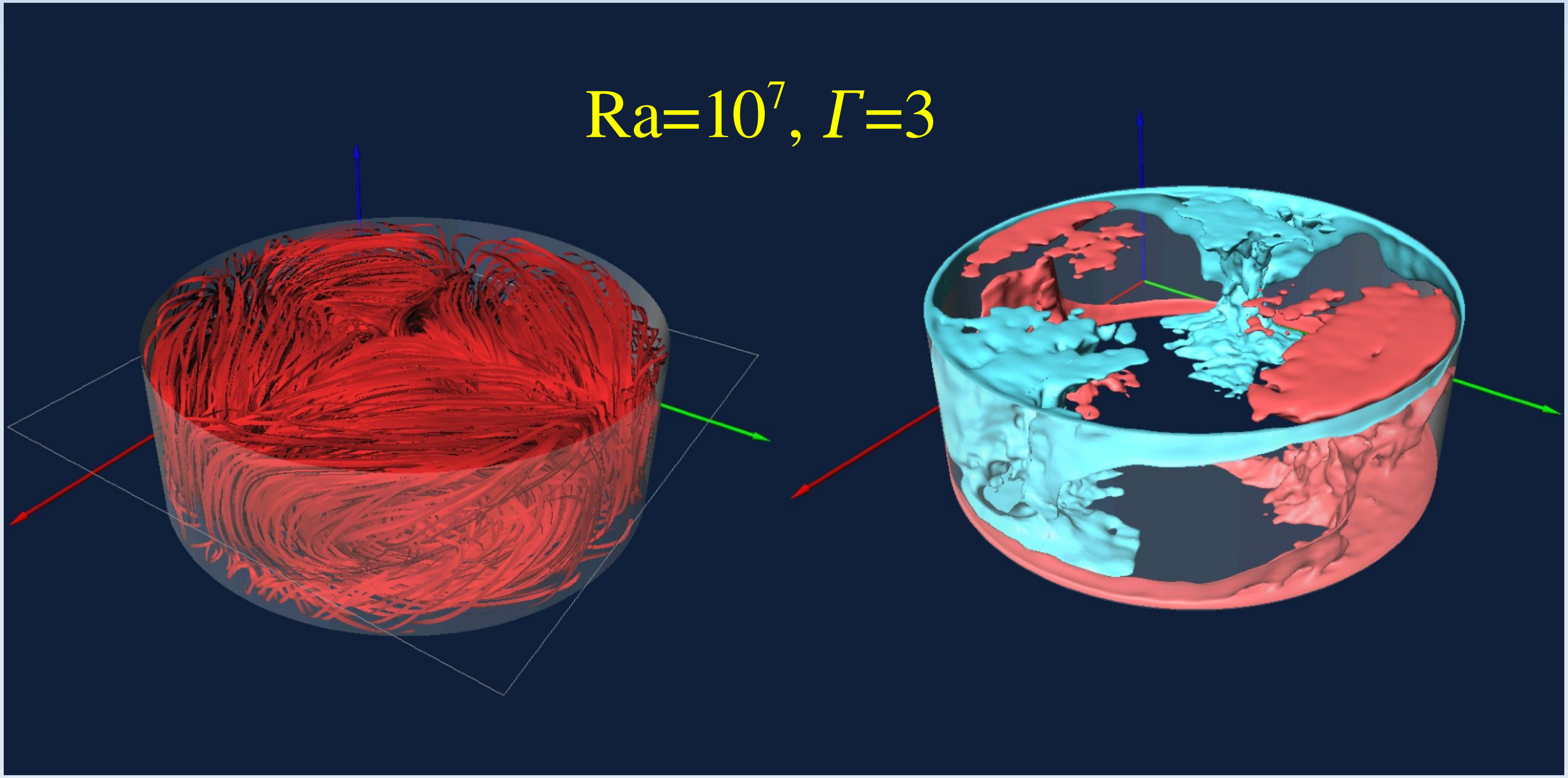


Systematic dependence of LSC and global heat transport with the aspect ratio Γ ($Ra=10^7$)

POD analysis of free convection

Boundary Conditions:

- No-slip at horizontal plates $u=0$, & at a fixed temperature
- Side walls adiabatic & no-slip, $u=0$ and $\partial T/\partial n=0$



Streamlines of first flow mode

Iso-surfaces of first temperature mode

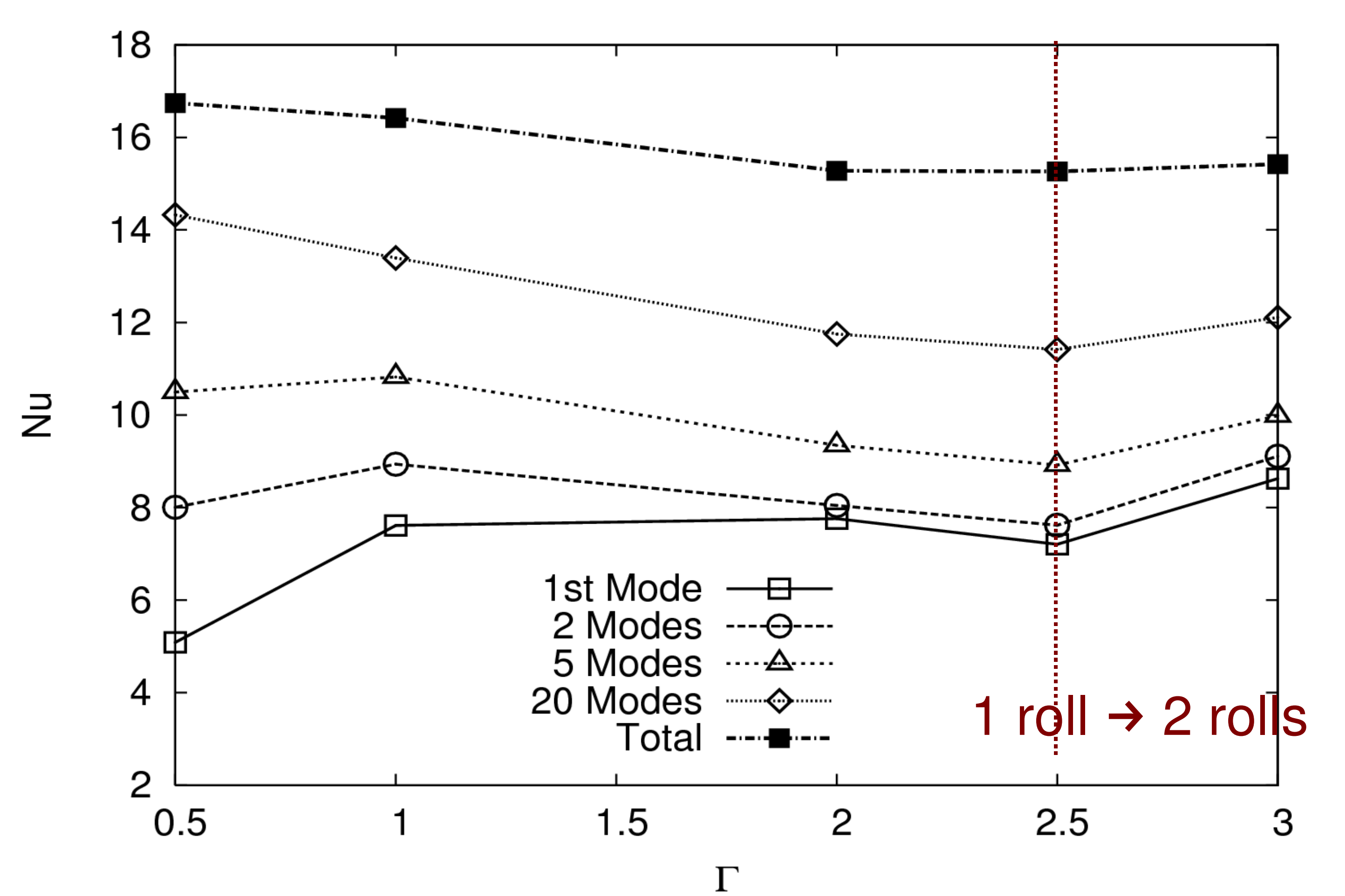
Turbulent heat transfer

- Nusselt Number

$$Nu = 1 + \frac{H}{\kappa \Delta T} \sum_{m,n=1}^{M} \langle a_m(t) \Phi_3^{(m)}(\boldsymbol{x}) a_n(t) \Phi_4^{(n)}(\boldsymbol{x}) \rangle_{V,t}$$

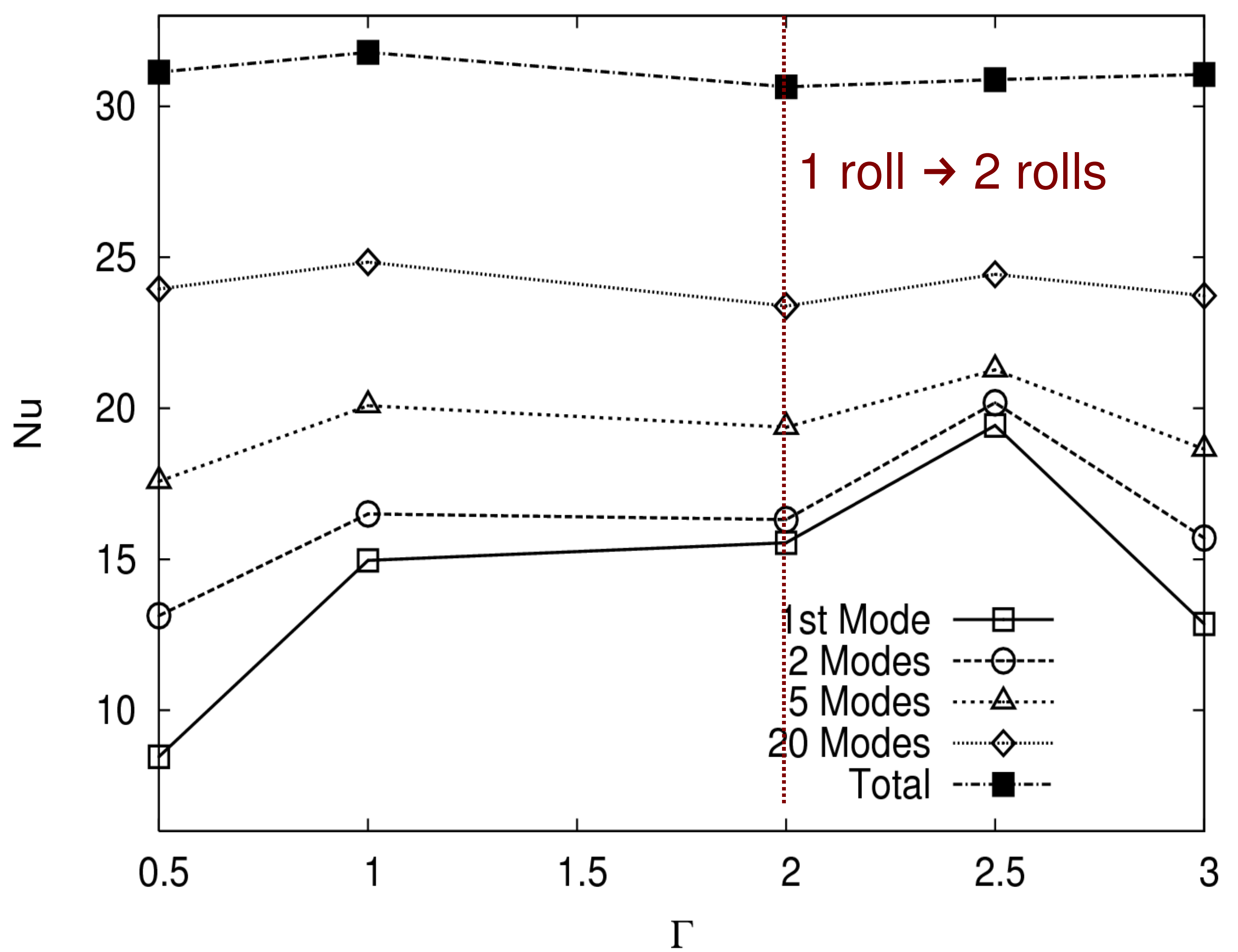
- $a_m(t)$ correspond to the projection of turbulent flow field at time *t* to mode $\phi^{(m)}(\boldsymbol{x})$

(Ra = 10^7)
Bailon-Cuba, Emram & Schumacher, JFM, submitted (2009).



Turbulent heat transfer²

(Ra = 10^8)
Bailon-Cuba, Emram & Schumacher, JFM, submitted (2009).



Rectangular geometry: canonical problem

- $\Gamma = L_x/L_z = L_y/L_z = 2$
- **BC's: Periodic in x & y**

  At z=0, L_z (free slip): $w = T = \partial u/\partial z = \partial v/\partial z = 0$
- Ra =9.9×10^5, 60 Snapshots: 128x128x65

- Fourier transform in x,y and Fourier Kernel

$$F_j(n_x, n_y; z) = \sum_x \sum_y v_j(x, y, z)\, e^{2\pi i (n_x x/L_x + n_y y/L_y)}$$

$$\kappa_{ij}(n_x, n_y; z, z') = \langle F_i(n_x, n_y; z) F_j{*}(n_x, n_y; z') \rangle$$

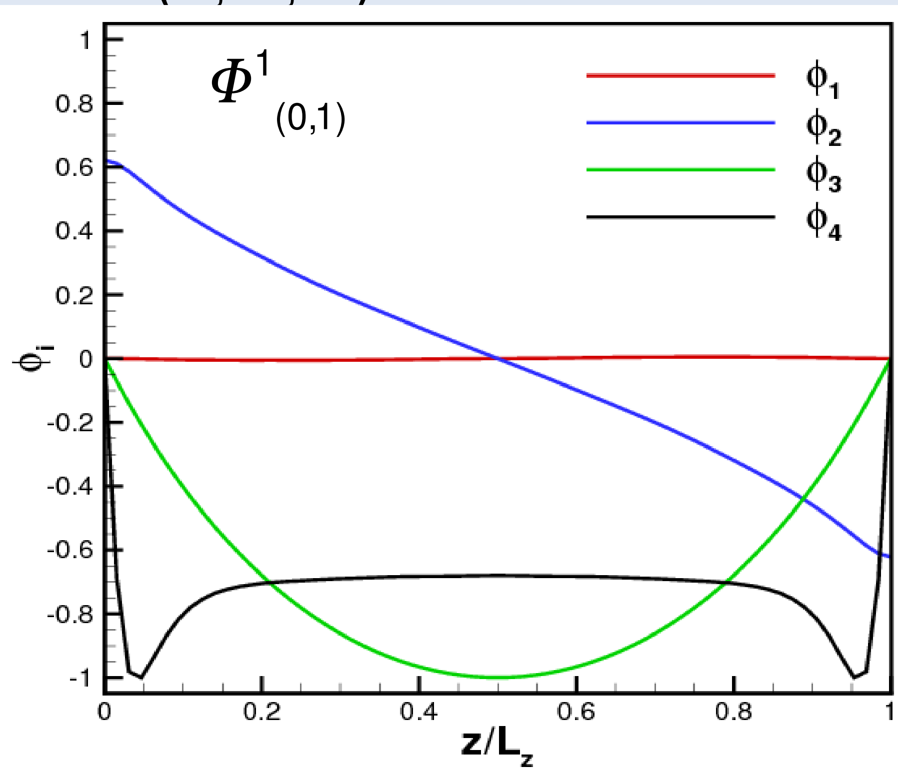
- Method of Snapshots

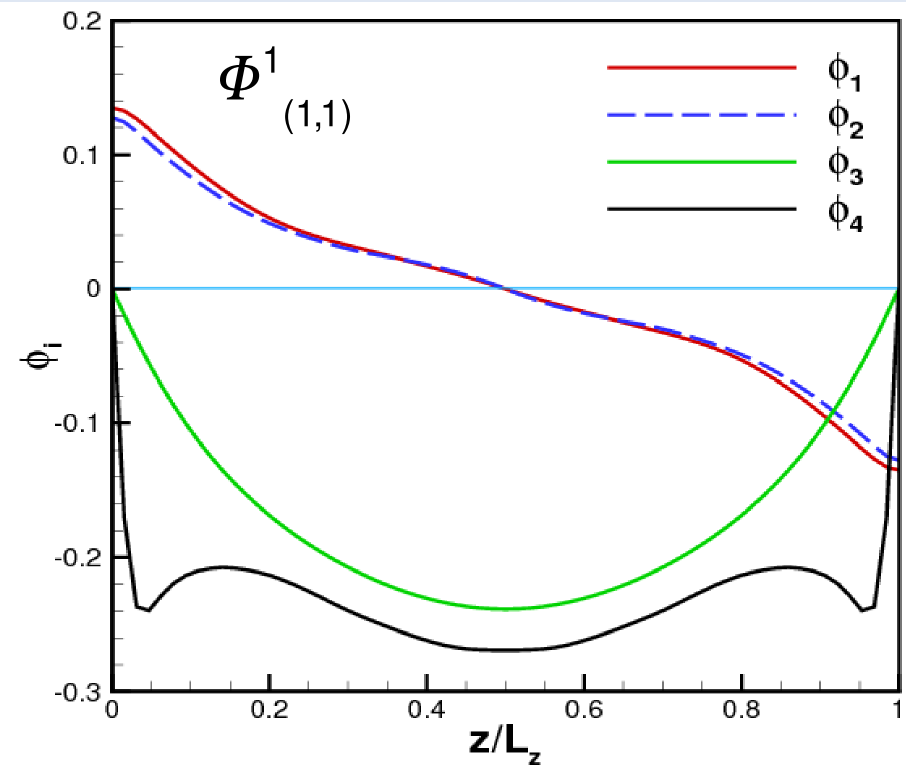
$$\left(\int_0^{L_z} \frac{1}{M} \sum_{i=1}^{4} F_i^{*(m)}(n_x, n_y; z') F_i^{(n)}(n_x, n_y; z')\, dz' \right) \alpha_n(n_x, n_y) = \lambda(n_x, n_y)\, \alpha_m(n_x, n_y)$$

Results

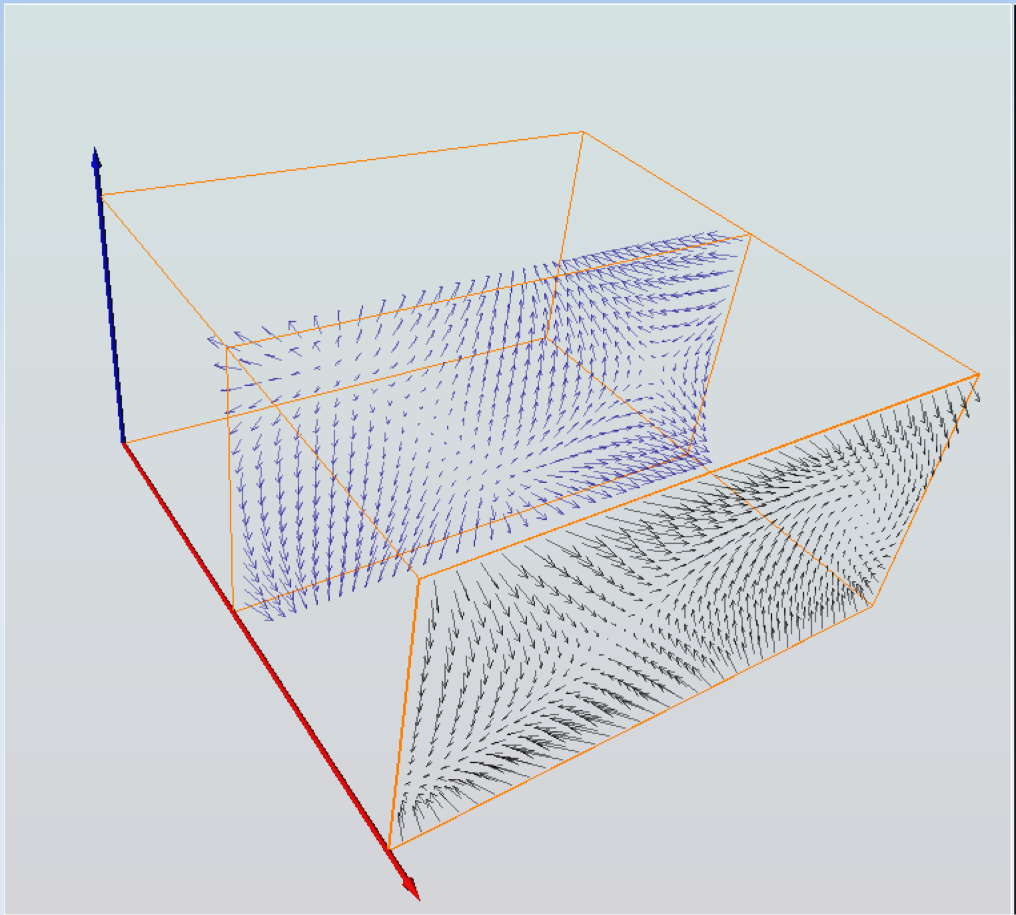
k	(n_x, n_y, n)	$\lambda^n_{nx,ny}$	Degeneracy*	% Energy
1	(0, 1, 1)	1.4130	4	30.078
2	(0, 1, 2)	0.1846	4	3.929
3	(0, 1, 3)	0.1694	4	3.605
4	(1, 1, 1)	0.1678	4	3.571
5	(0, 2, 1)	0.1303	4	2.774
6	(1, 2, 1)	0.0583	8	2.482
7	(0, 2, 2)	0.1077	4	2.293

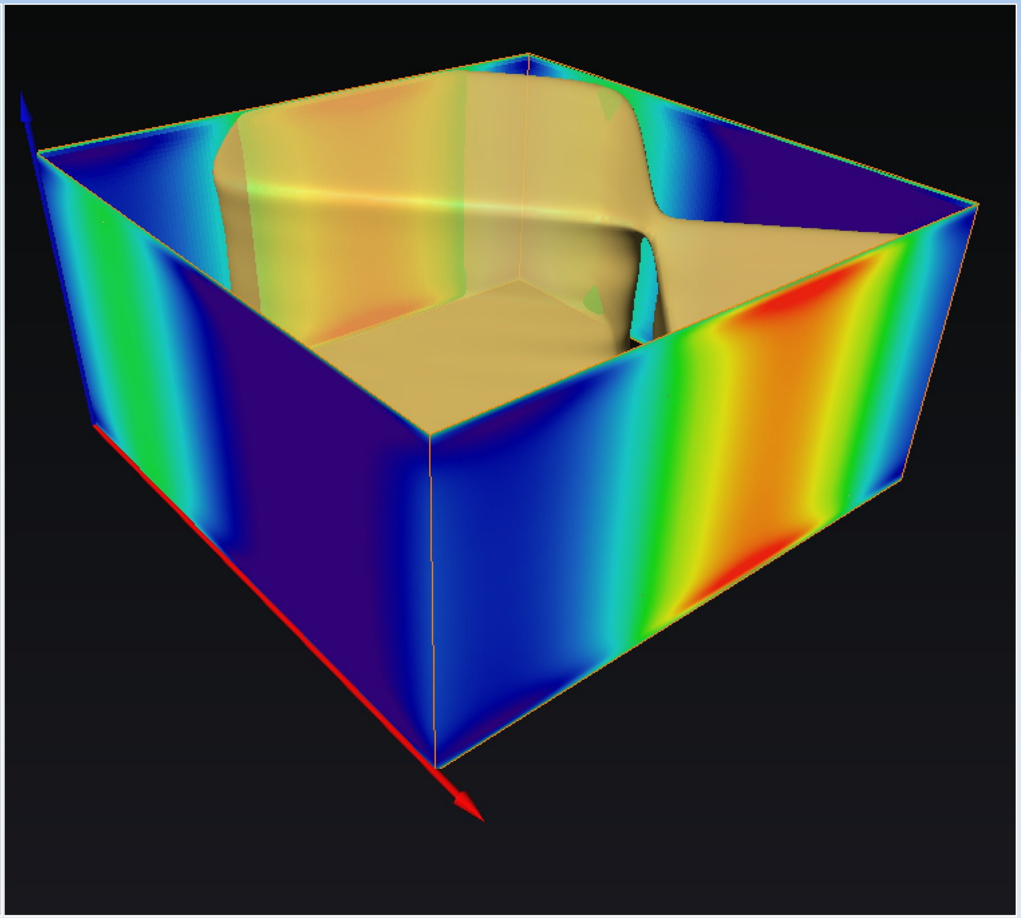
∗ Due to The 16 discrete symmetries: reflections & rotations





3D-structure of POD modes

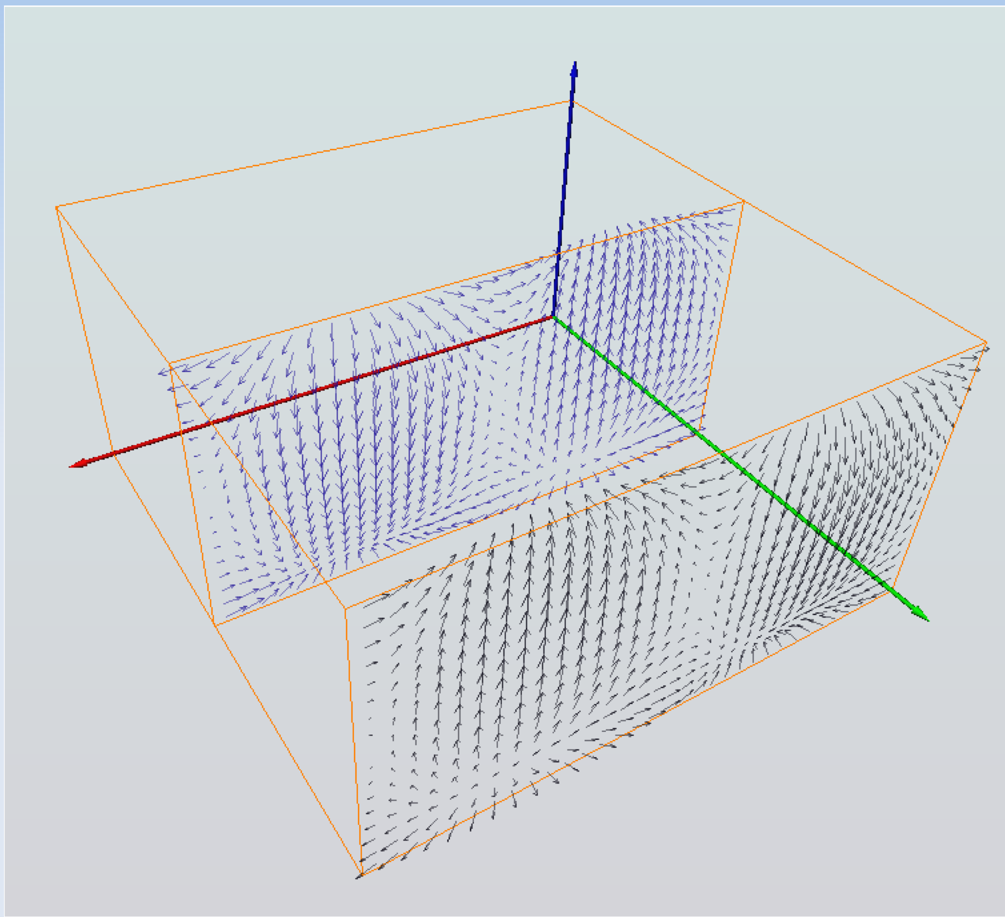




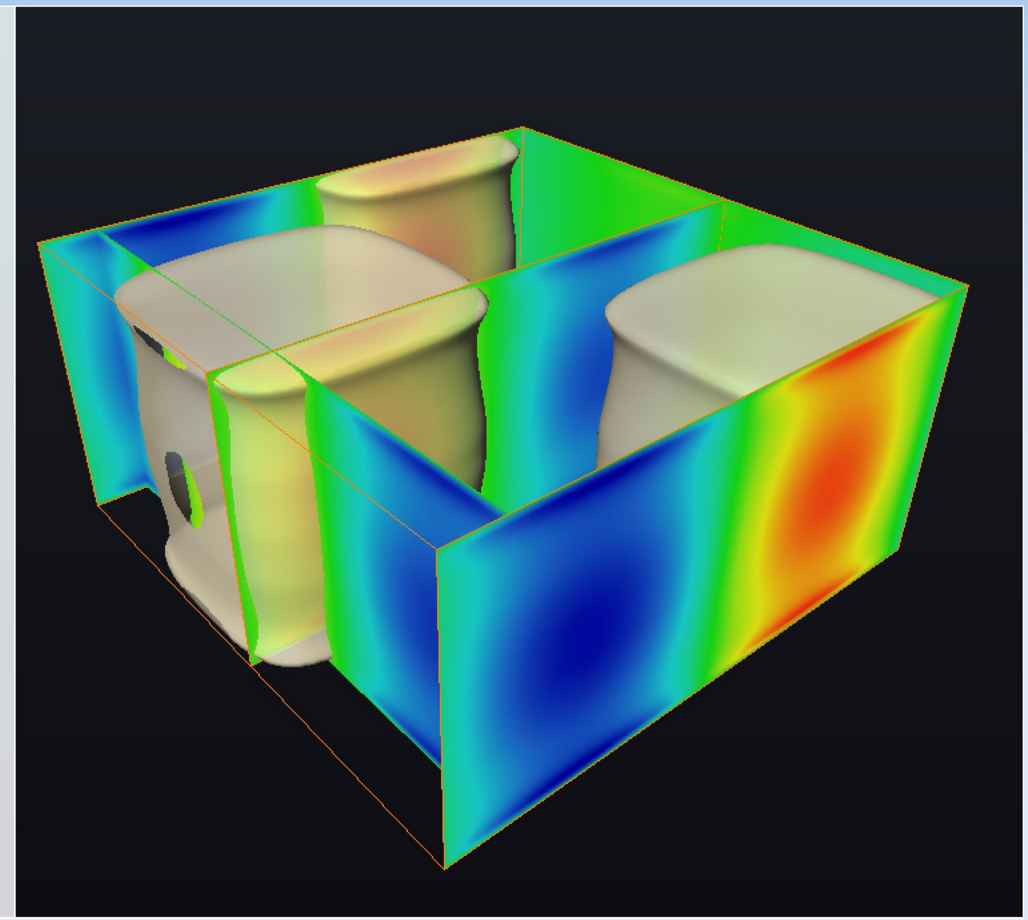
$\Phi^1_{(0,1)}$-Velocity

$\Phi^1_{4(0,1)}$-Temperature

3D-structure of POD modes[2]



$\Phi^1_{(1,1)}$-Velocity



$\Phi^1_{4(1,1)}$-Temperature

Mixed convection

We consider a complex rectangular setting which mimics indoor ventilation problems as present in a passenger cabin of an airplane (or/and concert hall)

Air plane cabin

Concert hall

Rectangular geometry

Boundary Conditions:

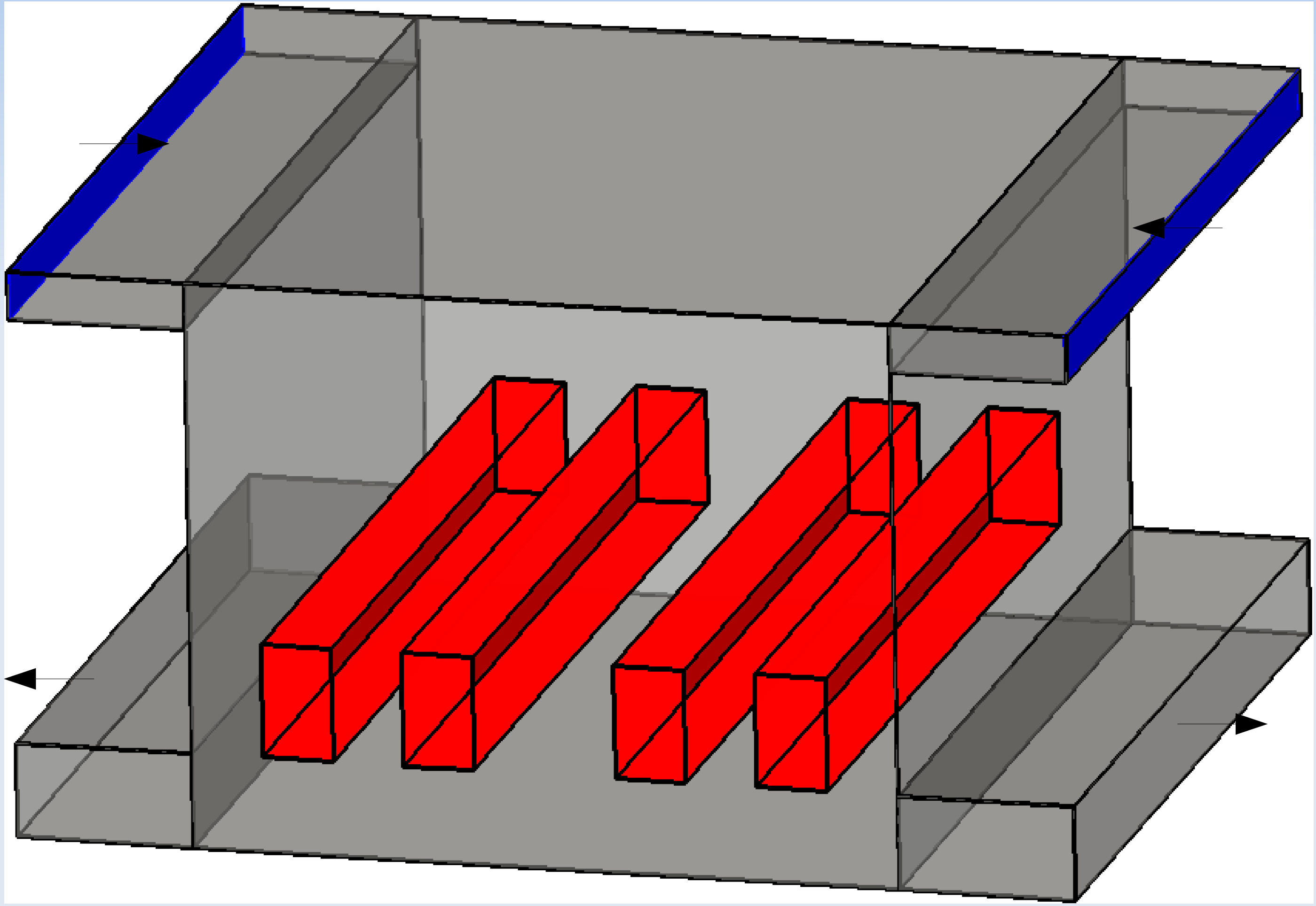
$$\left.\begin{array}{l} T=-0.5 \\ u=5\, m/s \end{array}\right\}$$ at the Inlet

$T=0.5$ at the Obstacles

$\partial T/\partial \boldsymbol{n}=0$ Boundaries of Outlet Ducts & Walls

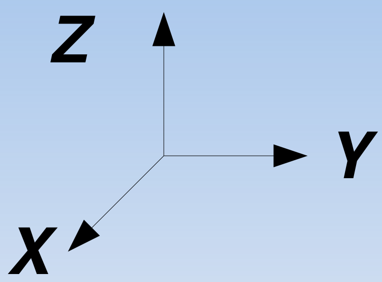
$\partial u/\partial \boldsymbol{n}=0$ Boundaries of Outlet Ducts

$u=0$ At the walls



Domain for the DNS by Shishkina & Wagner.

Rectangular geometry: symmetries



$\{ I, R^2, X, Y \}$ **Isomorphic to Abstract Group D2**

whose elements are the rotation by 180°

$R^2(x,y,z,u,v,w,\theta) = (-x,-y,z,-u,-v,w,\theta)$

and the reflections in x & y

$X(x,y,z,u,v,w,\theta) = (-x,y,z,-u,v,w,\theta)$

$Y(x,y,z,u,v,w,\theta) = (x,-y,z,u,-v,w,\theta)$

The set of snapshots is extended Four Times:

$$\left(\frac{1}{4\mathrm{M}} \iiint_V \sum_{i=1}^{4} v_i^{*(m)} v_i^{(n)} d\,\boldsymbol{x}^3 \right) \alpha(n) = \lambda\, \alpha(m)$$

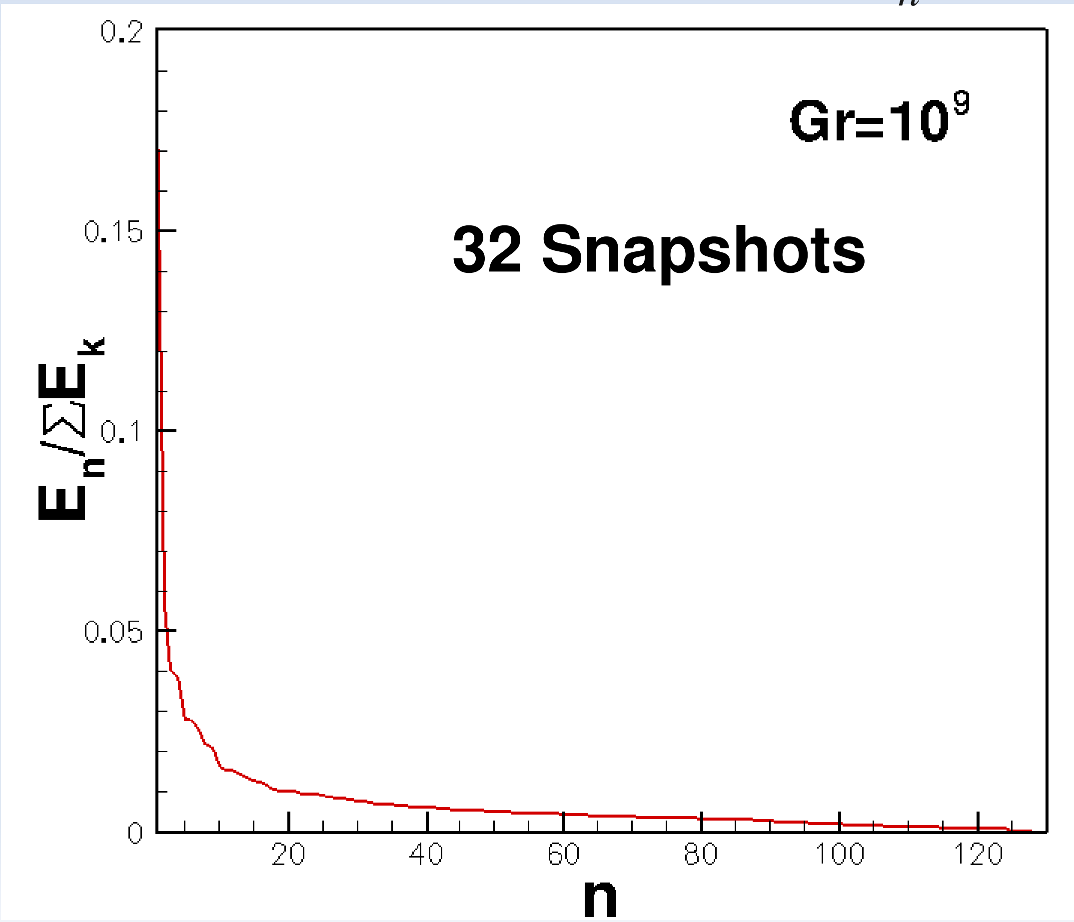
$$m, n = 1, 2, 3, \ldots 4M$$

Energy content of the POD modes

$$u'_i(\boldsymbol{x},t)=u_i(\boldsymbol{x},t)-\overline{U}(x,z)=\sum_n a_n(t)\boldsymbol{\Phi}_i^{(n)}(\boldsymbol{x})$$

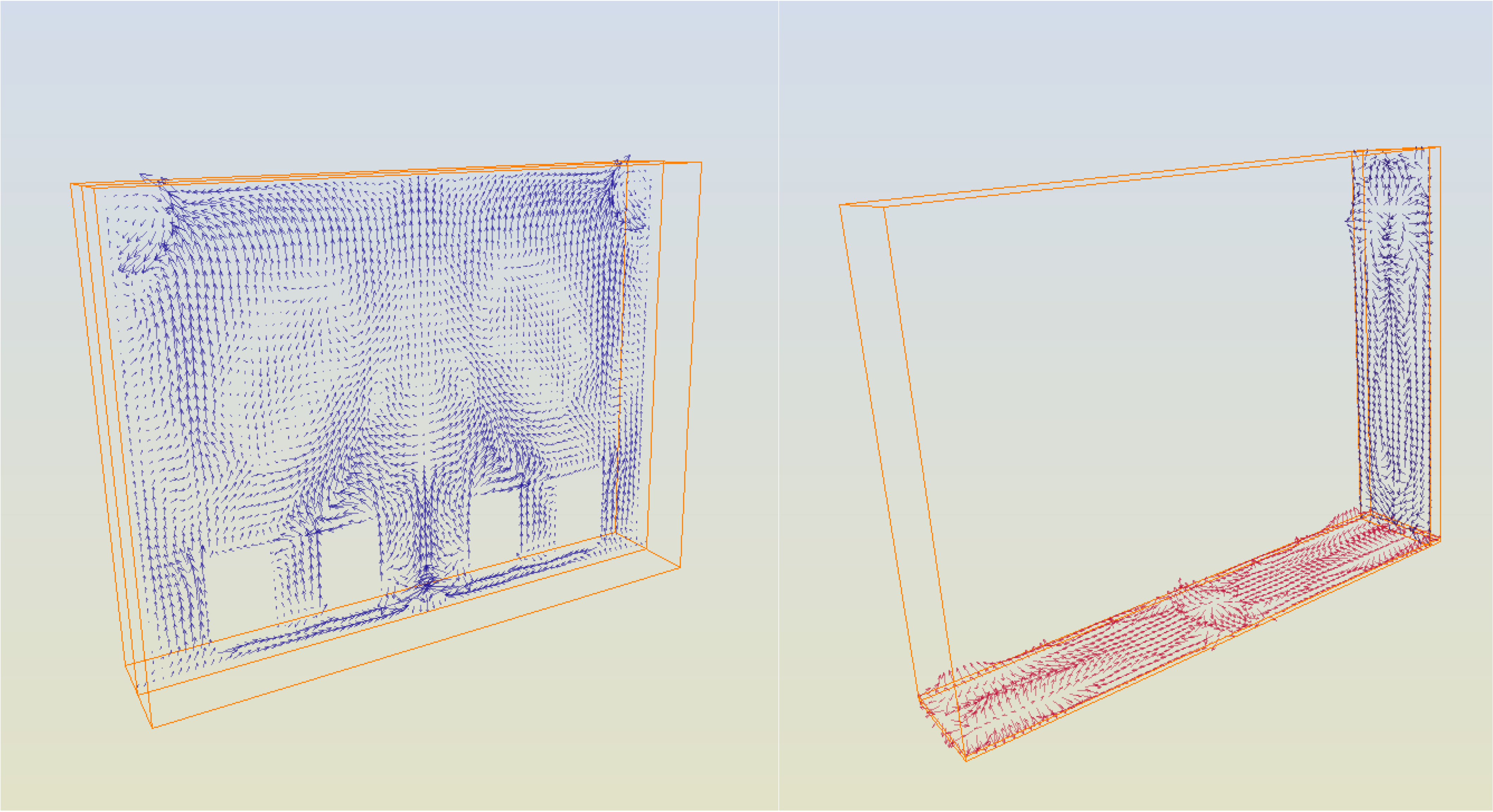
$$i=1,2,3$$

$$\theta(\boldsymbol{x},t)=T(\boldsymbol{x},t)-\overline{T}(x,z)=\sum_n a_n(t)\boldsymbol{\Phi}_4^{(n)}(\boldsymbol{x})$$



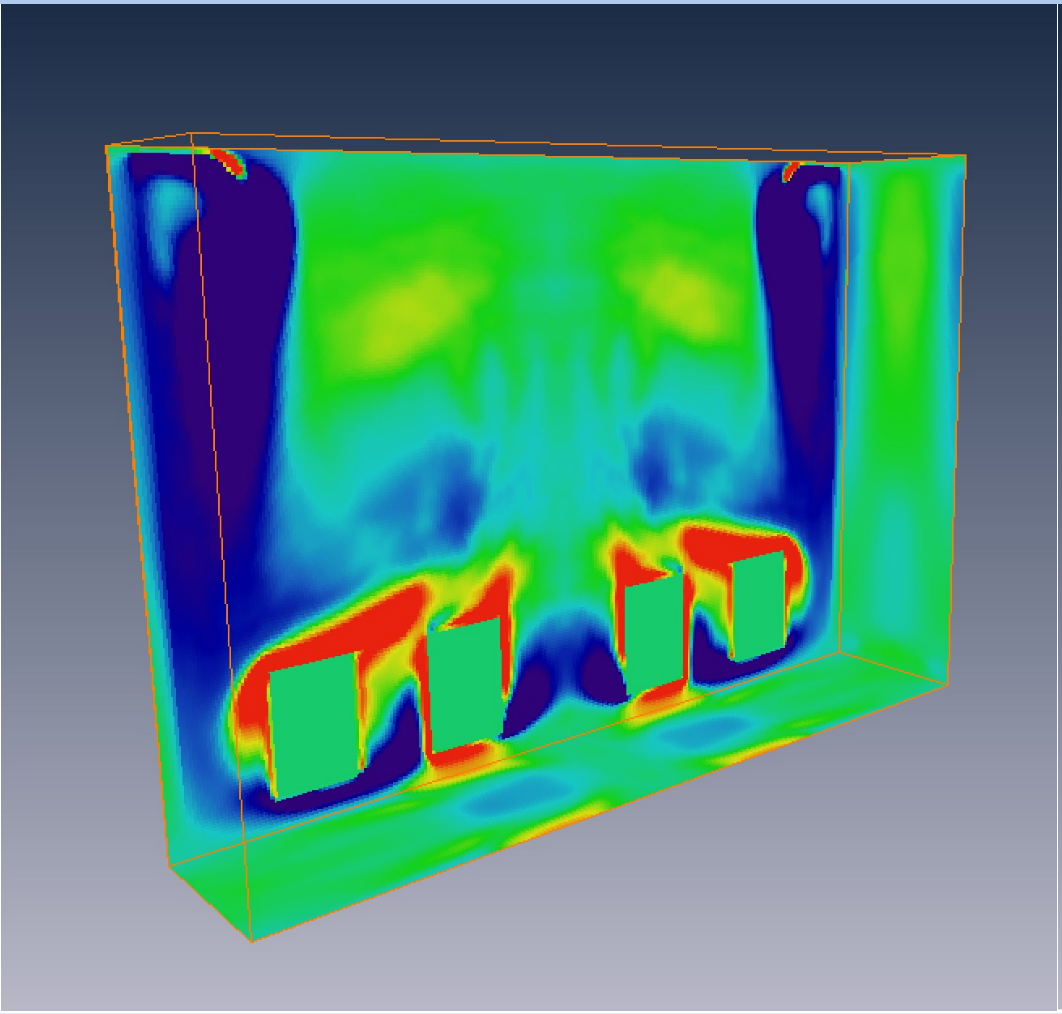
Mode	% Energy
1	18.27
2	5.72
3	4.06
4	3.88
5	2.80
6	2.77
7	2.59
8	2.20
9	2.13
10	1.66
11	1.54
12	1.53

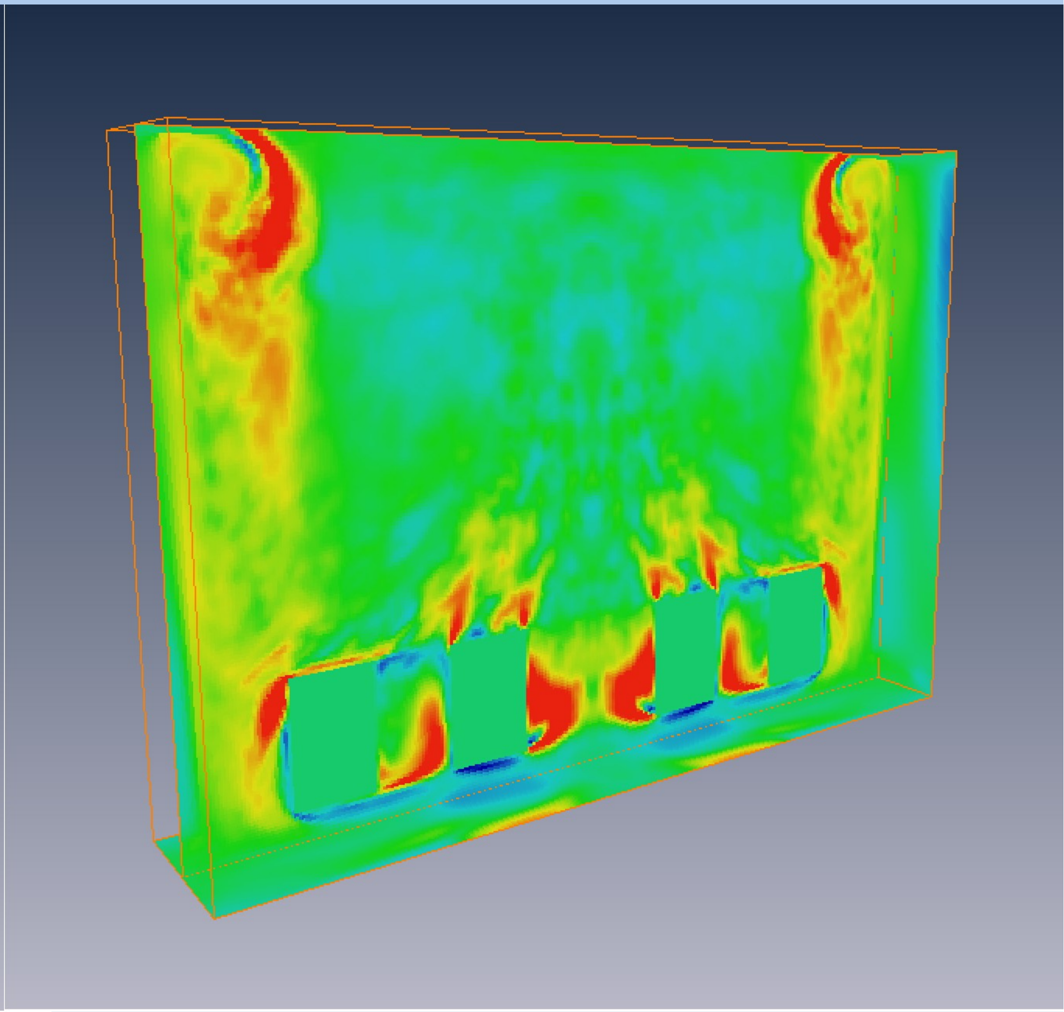
First mode: velocity field



32 Snapshots (DNS by Shishkina & Wagner)

First mode: temperature field





At the wall interface

At the middle plane

32 Snapshots (DNS by Shishkina & Wagner)

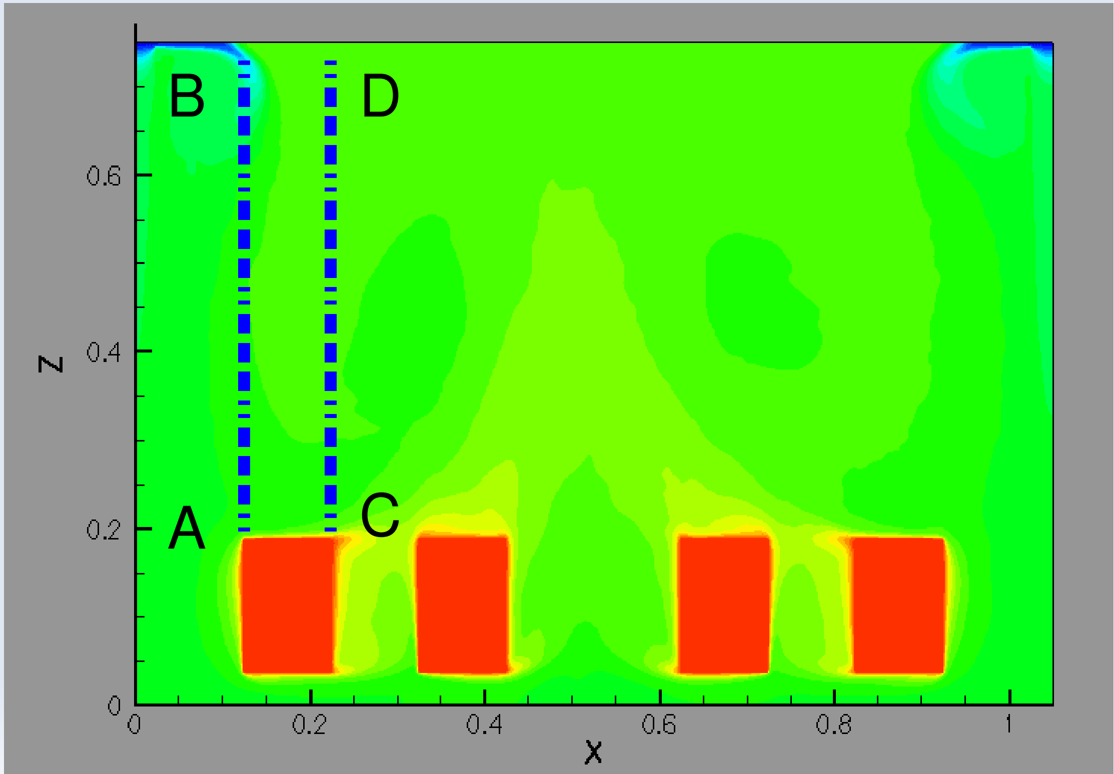
Nusselt number-POD modes dependence

- From the energy equation $$T_t + u \cdot \nabla T = \nabla^2 T$$
- Taking the ensemble average & along *"y"*

$$\frac{\partial \langle uT \rangle}{\partial x} + \frac{\partial \langle wT \rangle}{\partial z} = \frac{\partial^2 \langle T \rangle}{\partial x^2} + \frac{\partial^2 \langle T \rangle}{\partial z^2}$$

- And after horizontal averaging inside the sub-volume ABCD:

$$\frac{d \langle wT \rangle}{d z} = \frac{d^2 \langle T \rangle}{d z^2}$$



Nusselt number-POD modes dependence[2]

$$\frac{d\langle T\rangle_{y,t}}{dz}\Big|_{AC} = \boxed{\overline{\frac{d\langle T\rangle_{y,\Delta x,t}}{dz}}} - \langle wT\rangle_{\Delta V,t}$$

Known from Average Temperature Field

NEXT STEPS

- To complete the study of the *Nu*-POD Modes dependence for *Ra*=10^8 & 10^9
- To identify the large scale flow patterns for these geometries.
- To start a coarse graining procedure in order to determine the integral heat & momentum fluxes.