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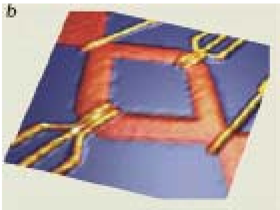
Explicit and hidden symmetries in Quantum Dots and Quantum molecules

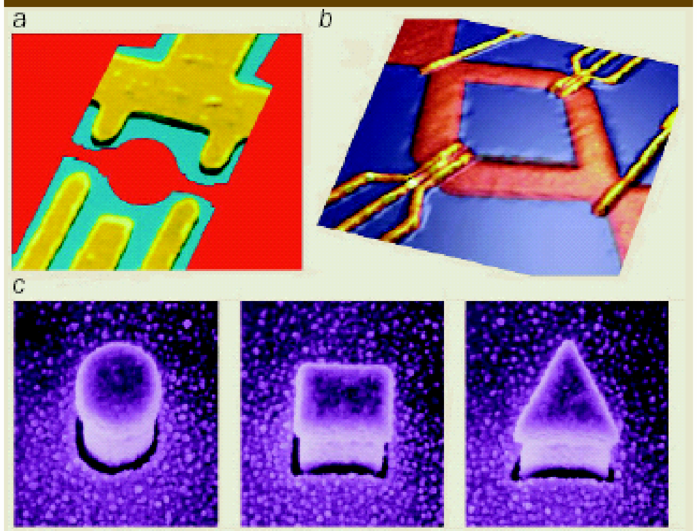
Outline



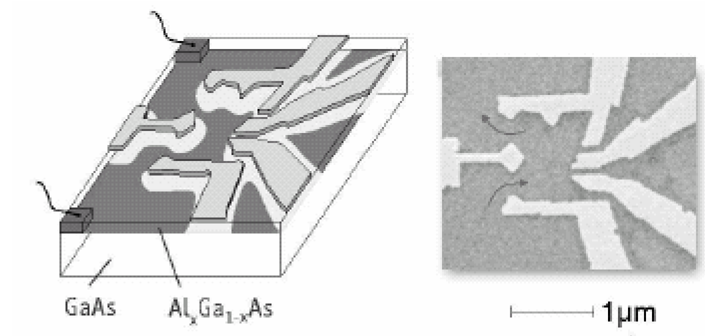
- Quantum Dot Devices
- Kondo effect in Quantum Dots
- Exotic Kondo effect in complex dots
- From quantum dots to quantum chains
- Large quantum dots: level statistics
- Stoner instability in mesoscopic systems
- Perspectives

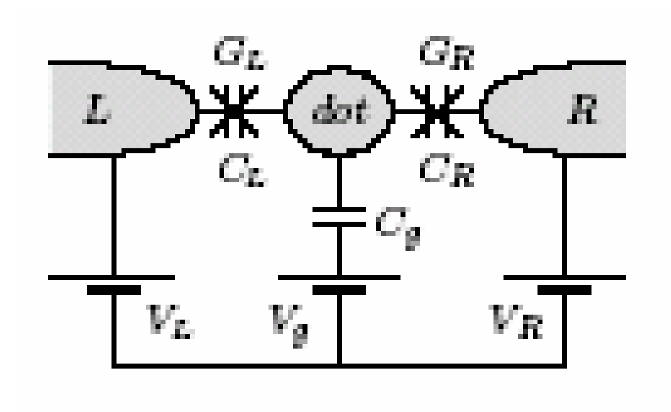
Quantum Dot devices

4 Quantum-dot devices



(*a*) A quantum dot can be defined by applying voltages to the surrounding gate electrodes (yellow). The tunnelling between the dot and the external electrodes (top left) is controlled by changing the voltages on the lower-left and lower-right gates. This coupling defines the lifetime broadening, Γ, of the quantum state in the dot. The number of electrons and the energy levels are tuned by the voltage on the lower-central gate. The puddle of electrons (confined red region) is about 0.5 microns in diameter. (*b*) Quantum dots can be placed in both arms of a two-slit interference device. Such a device has been used to investigate whether this scattering destroys the interference pattern. (*c*) Three quantum dots that have been used to compare the Kondo effect for singlet, doublet and triplet spin-states.

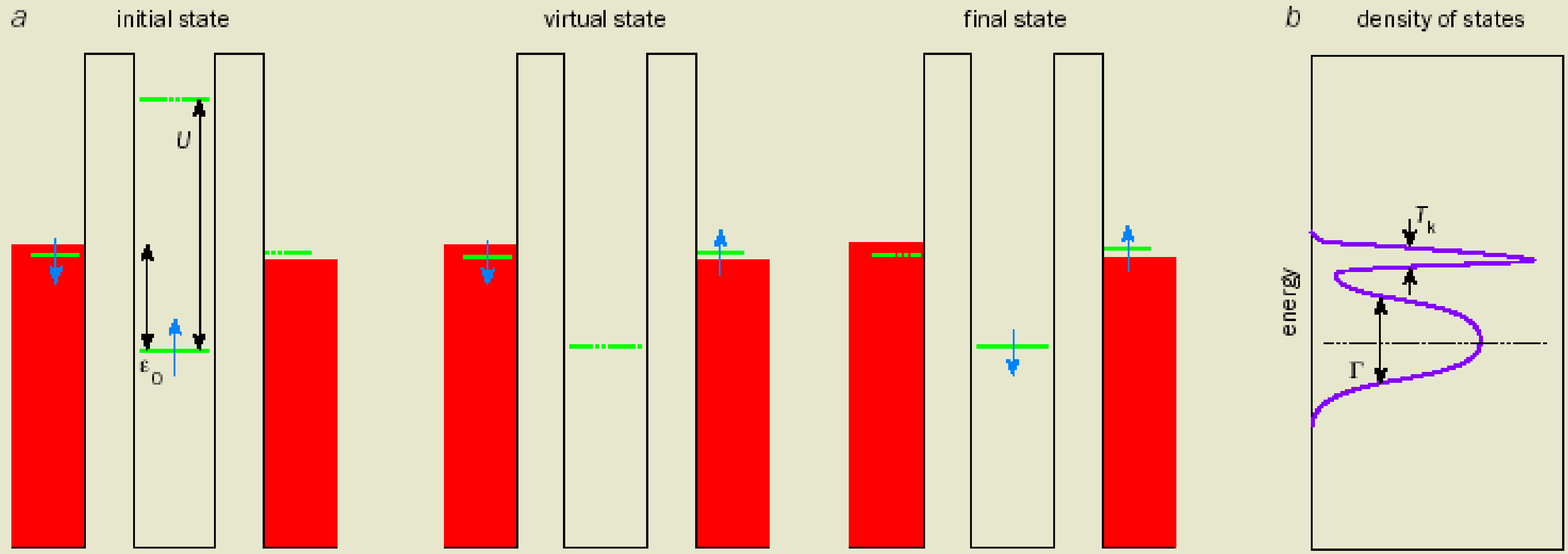




Artificial atoms

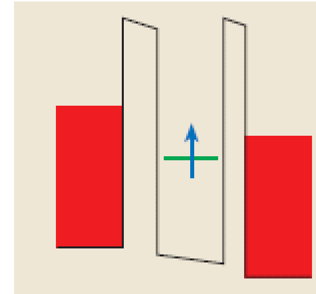
Kondo Effect in Quantum Dots

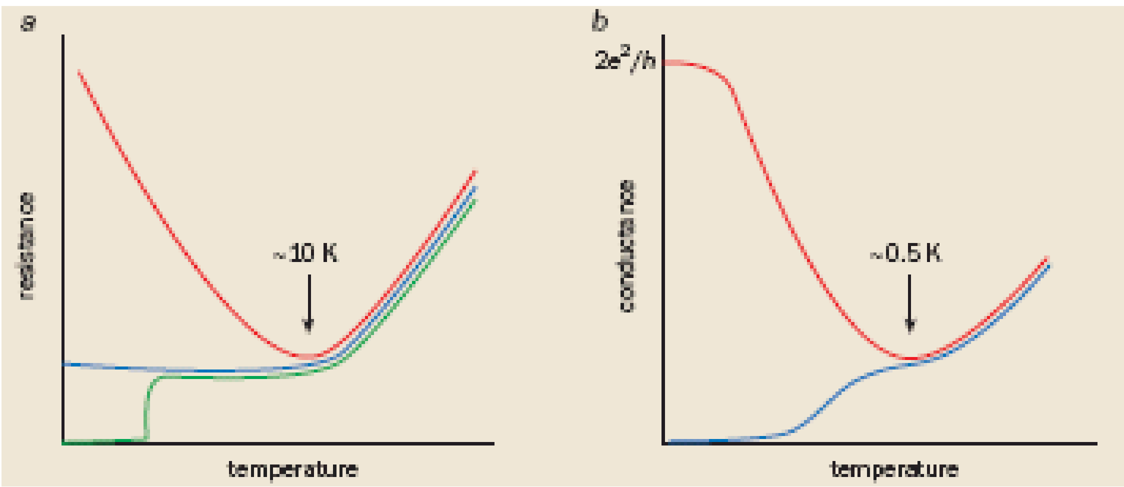
2 Spin flips



(a) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy ε_0 below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, U, while it would cost at least $|\varepsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden "virtual state" outside the impurity, and then be replaced by an electron from the metal. This can effectively "flip" the spin of the impurity. (b) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.

Universal Scaling

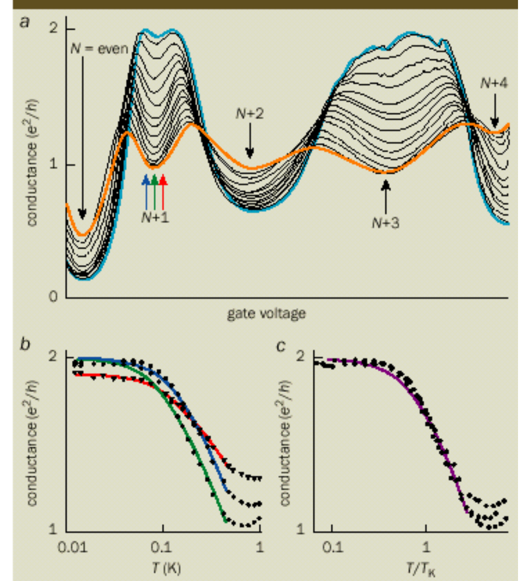




$$G/G_0 \propto \ln^{-2}\left(\max[T/T_K]\right)$$

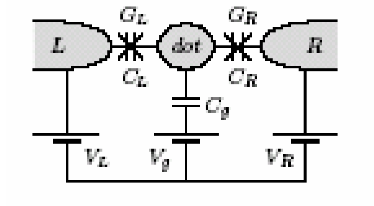
$$T_K = \frac{1}{2}(\Gamma U)^{1/2} \exp\left(\pi\varepsilon_0 \frac{\varepsilon_0 + U}{\Gamma U}\right)$$

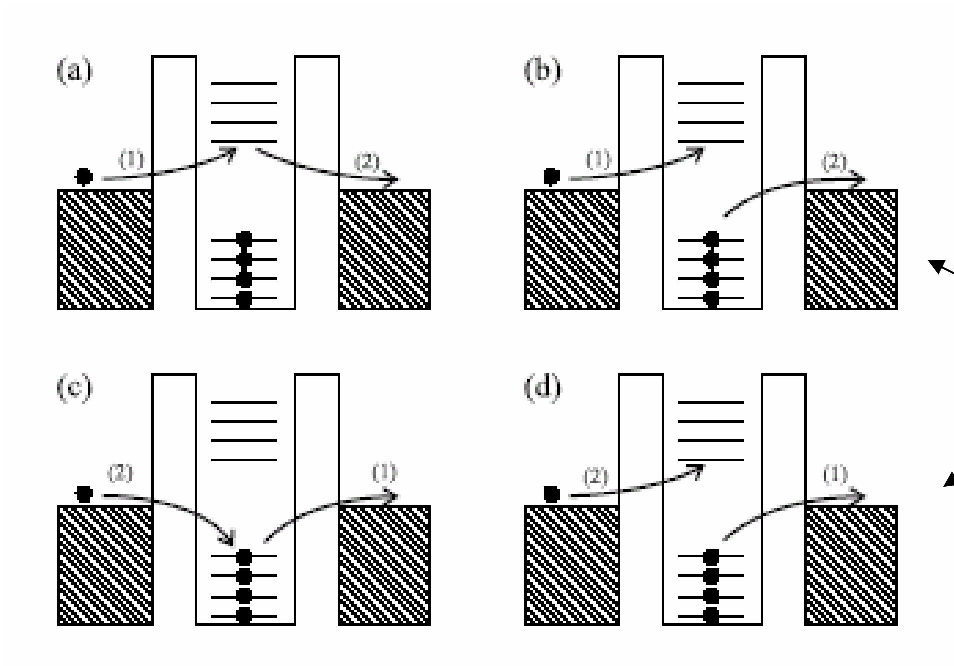
5 Universal scaling

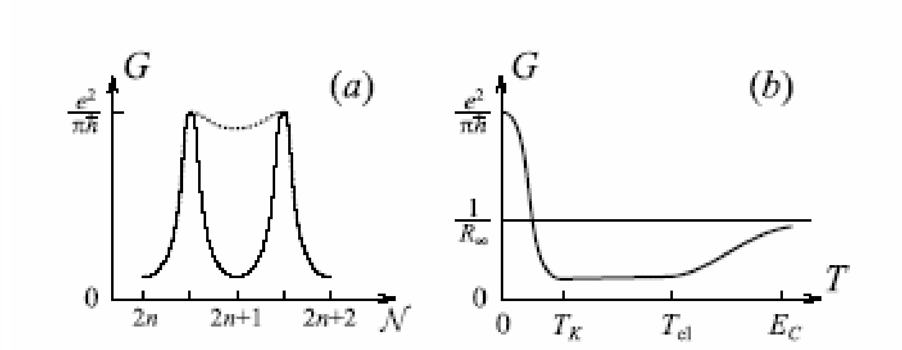


(a) The conductance (y-axis) as a function of the gate voltage, which changes the number of electrons, N, confined in a quantum dot. When an even number of electrons is trapped, the conductance decreases as the temperature is lowered from 1 K (orange) to 25 mK (light blue). This behaviour illustrates that there is no Kondo effect when N is even. The opposite temperature dependence is observed for an odd number of electrons, i.e. when there is a Kondo effect. (b) The conductance for N + 1 electrons at three different fixed gate voltages indicated by the coloured arrows in (a). The Kondo temperature, T_K, for the different gate voltages can be calculated by fitting the theory to the data. (c) When the same data are replotted as a function of temperature divided by the respective Kondo temperature, the different curves lie on top of each other, illustrating that electronic transport in the Kondo regime is described by a universal function that depends only on T/T_K.

Tunneling and co-tunneling



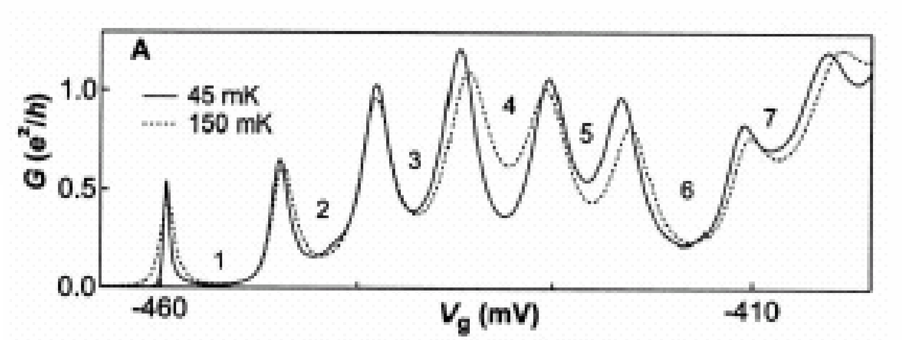




Electron-like

co-tunneling

Hole-like



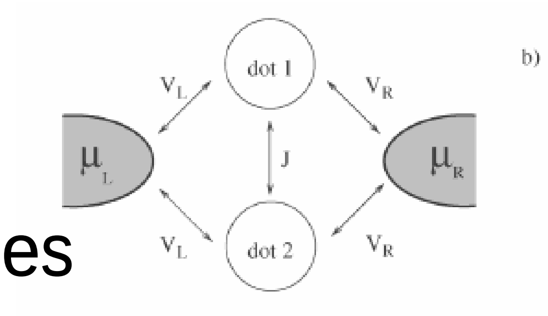
Elastic

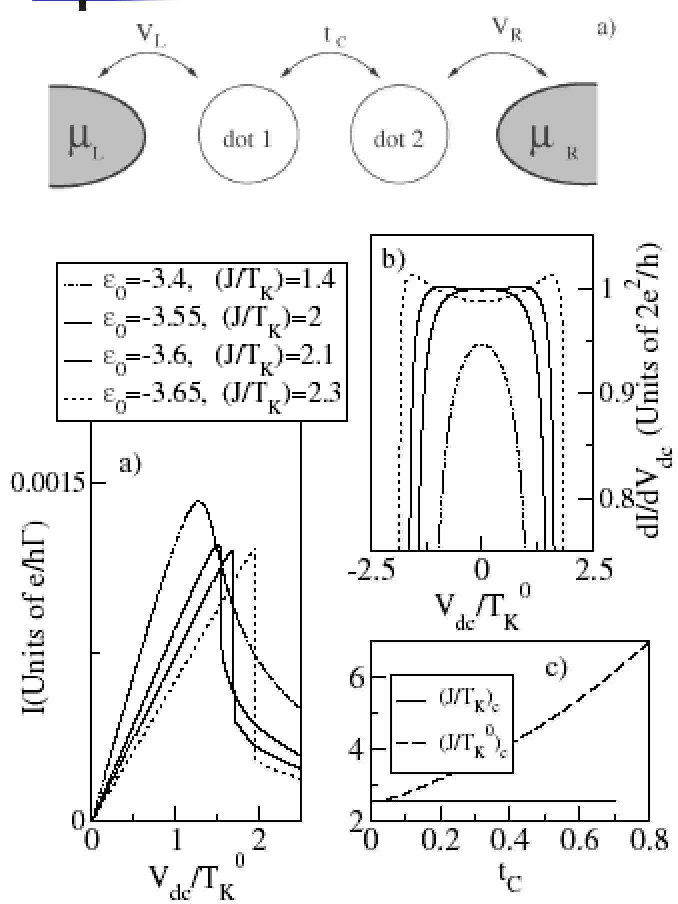
Inelastic

Common sense:

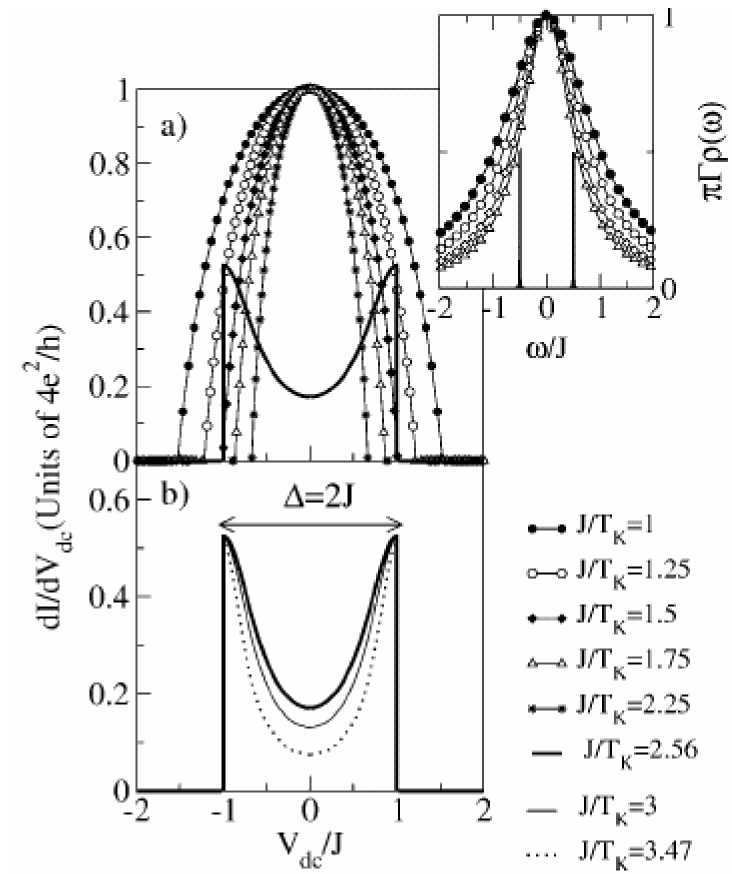
- Kondo effect exists if the total number of electrons in a dot is odd
- Kondo effect is destroyed by external magnetic field
- Relaxation effects associated with the non-equilibrium conditions eliminate the Kondo peak

Parallel and Serial Dots:
from artificial atoms to molecules



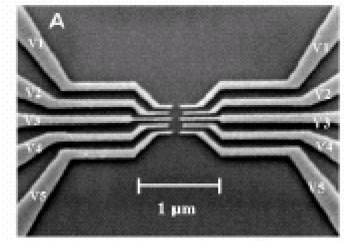


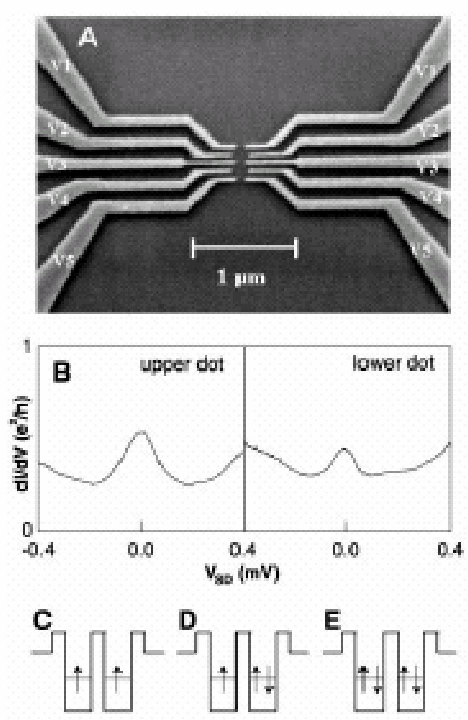
R.Lopez, R.Aguado, and G.Platero, PRL 2002

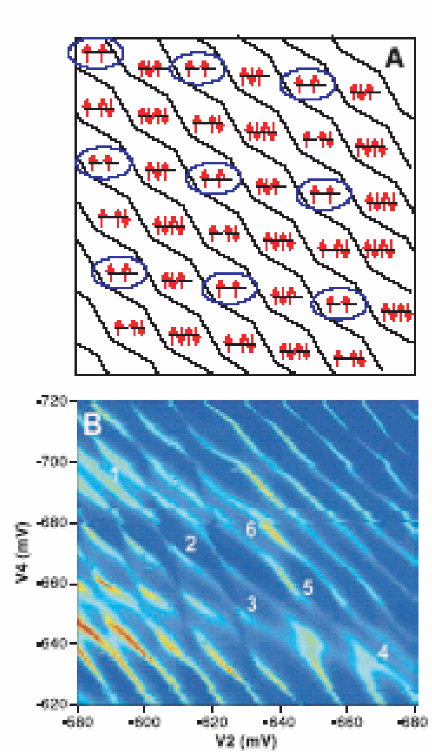


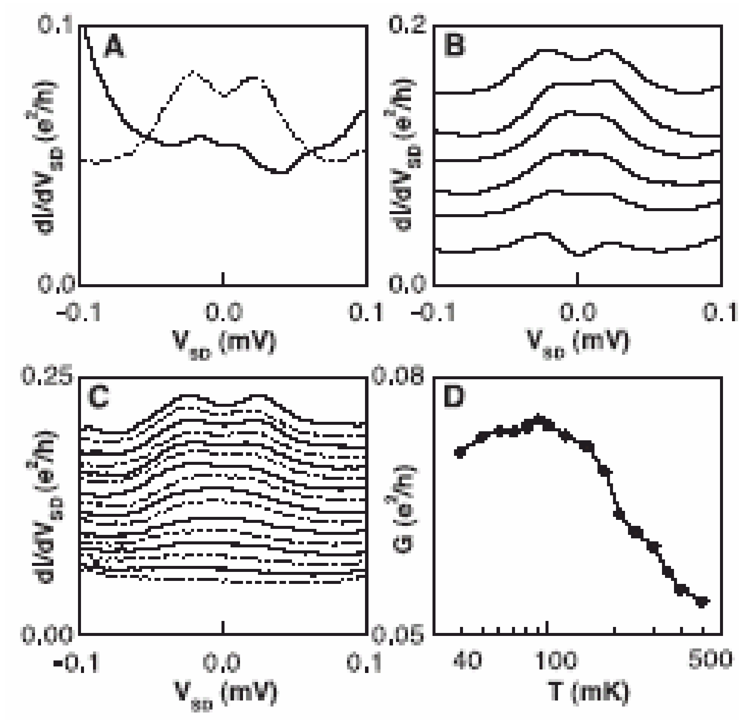
Competition between Kondo and RKKY interactions

Kondo effect in an Artificial Quantum Dot Molecule



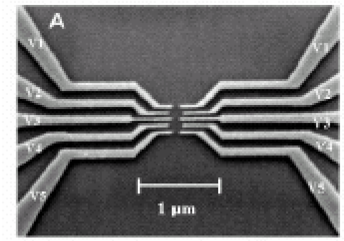






Zero-bias maximum

Exotic symmetries



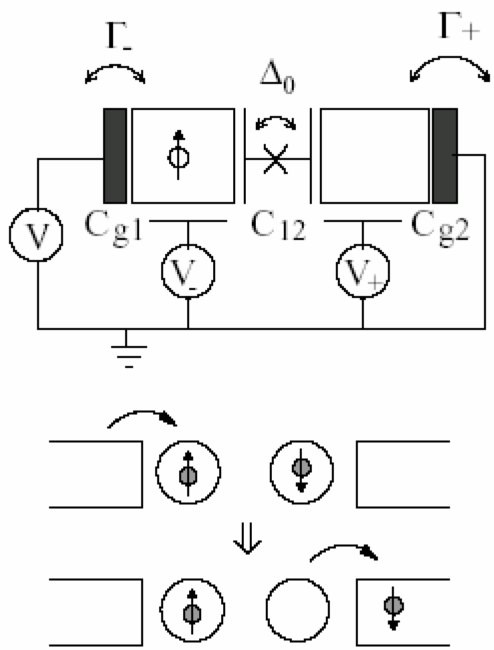
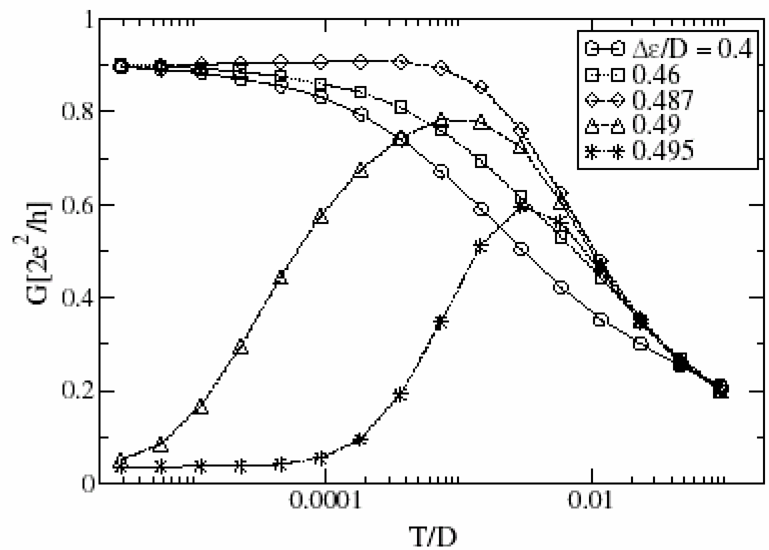


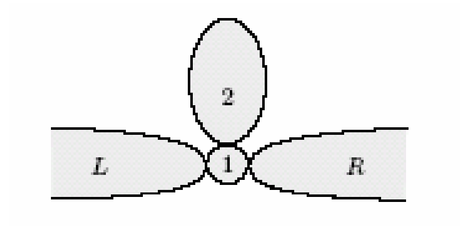
FIG. 1. Upper part: Schematics of the DD device. Lower part: Virtual process leading to 'spin-flip assisted tunneling' as described in Eq. (4)

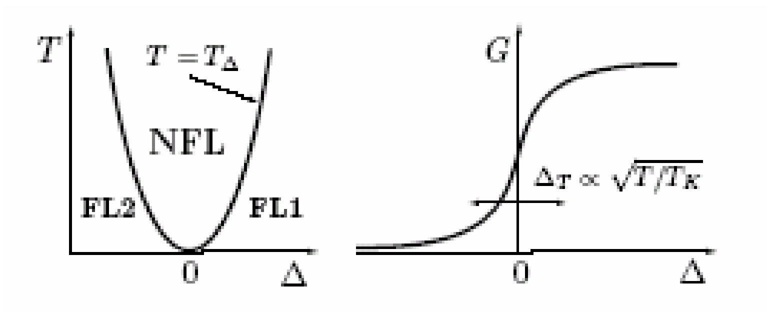


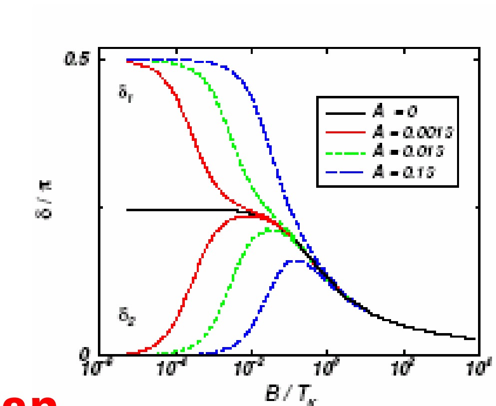
J. Von Delft et al (2003)

SU(4) symmetry

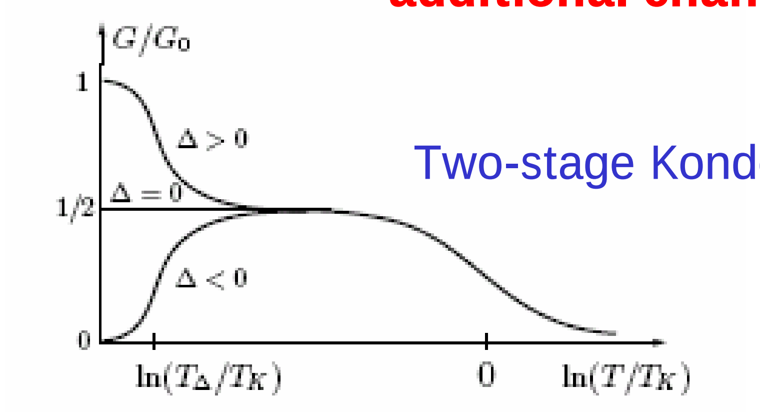
Quantum Phase Transition in a two-channel quantum dot



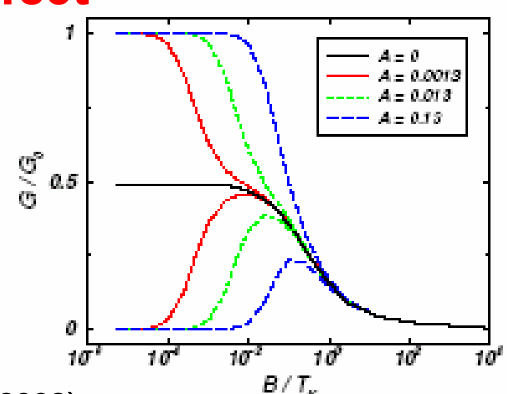




Electrons in large dot provide an additional channel for Kondo effect

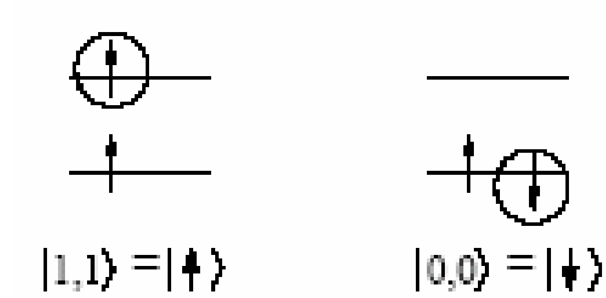


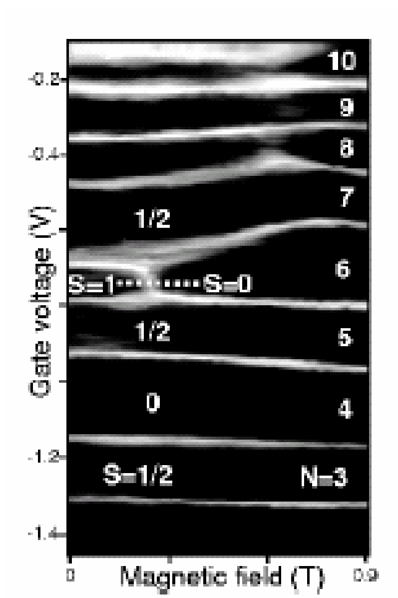
Two-stage Kondo effect

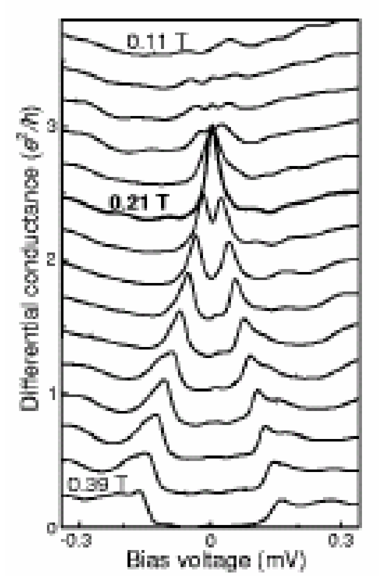


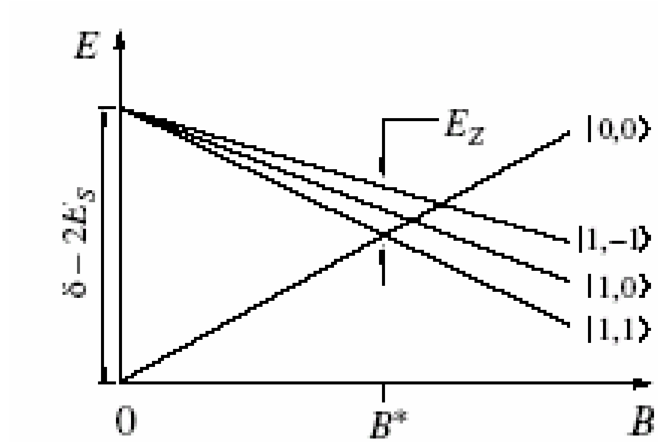
L.Glazman et al (2003)

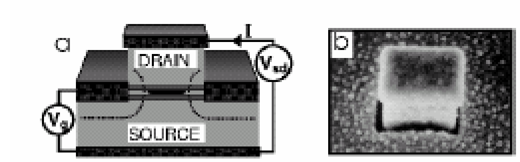
Singlet-triplet transition in a magnetic field

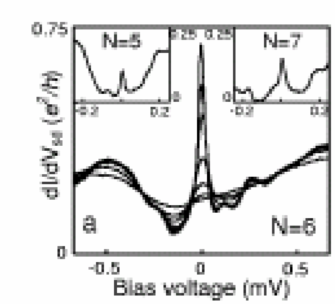


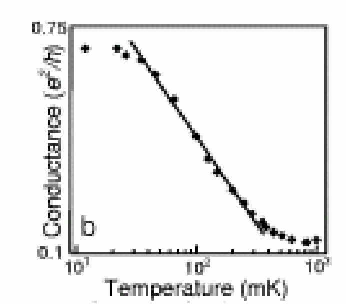


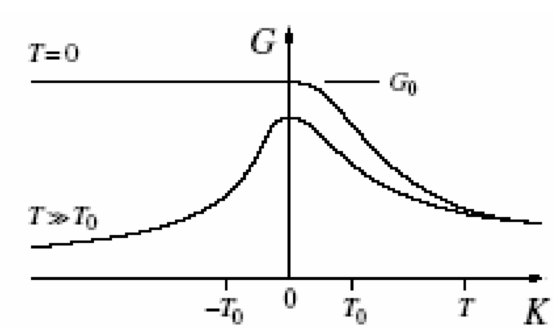






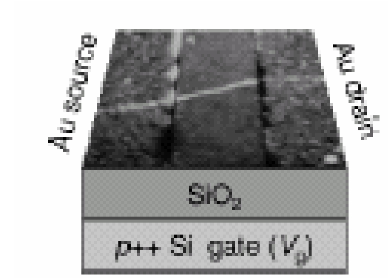


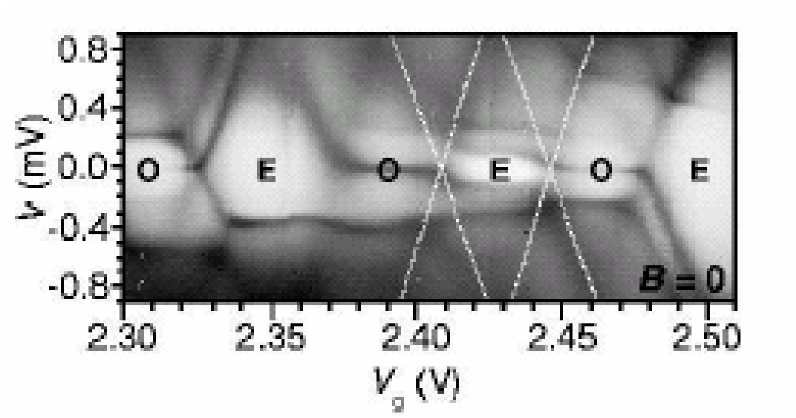


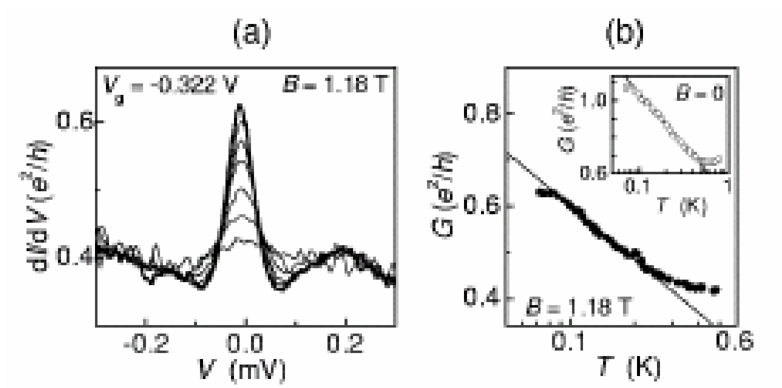


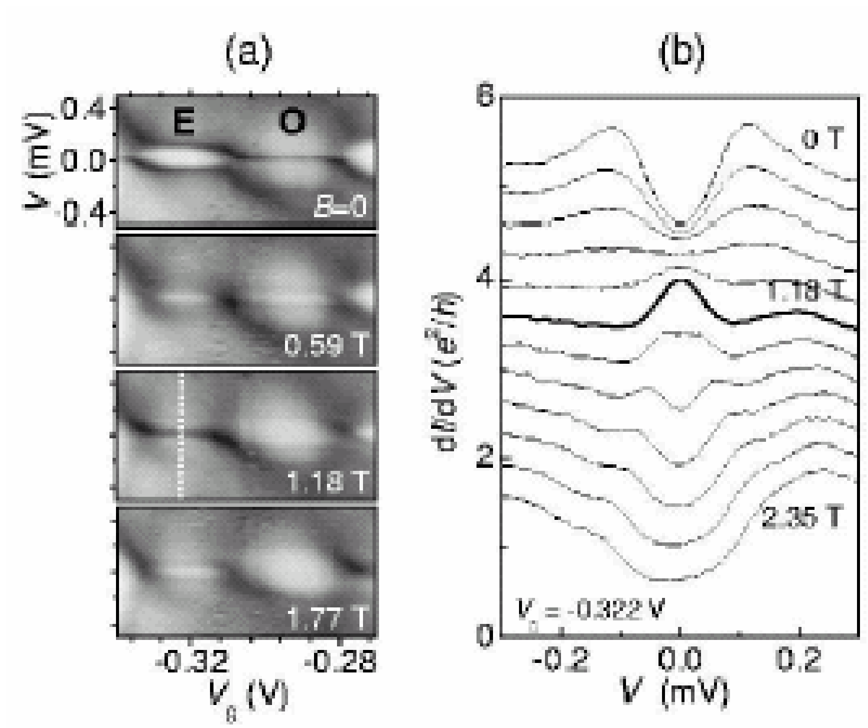
L.Glazman et al (2001)

Single-wall carbon nanotubes



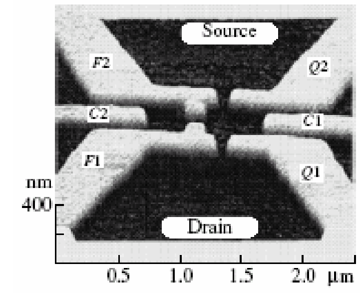






Transition is driven by Zeeman splitting

Parallel Double Quantum Dot



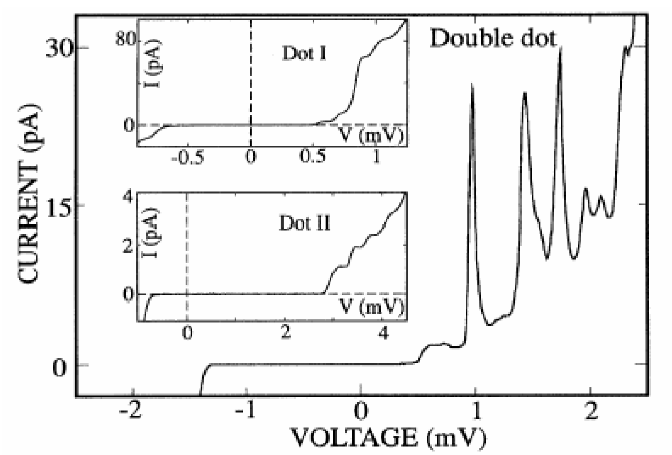
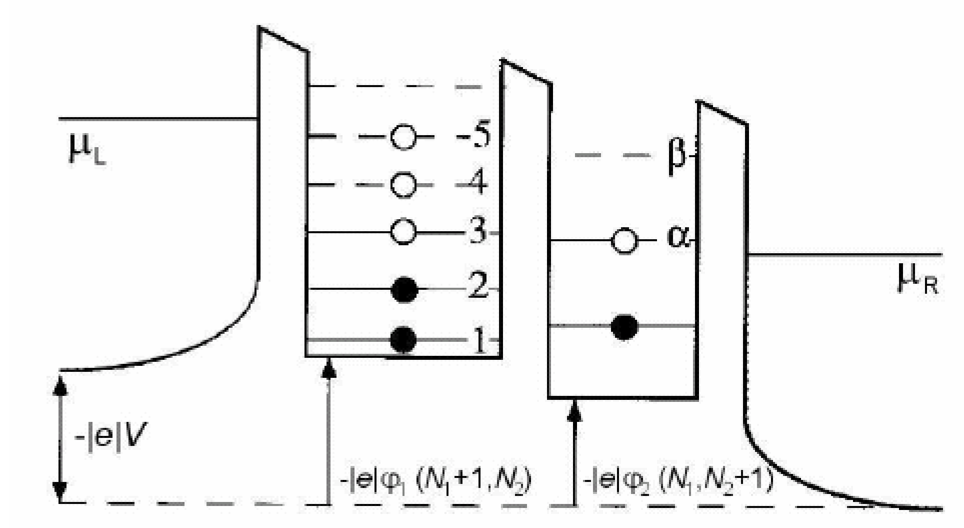
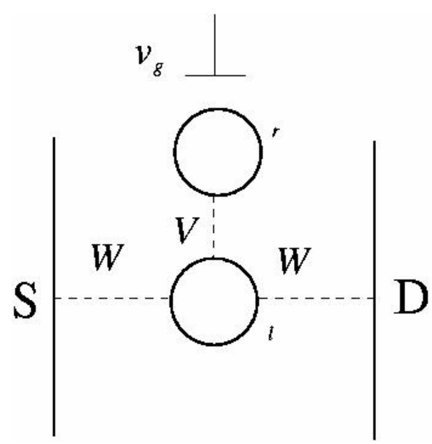


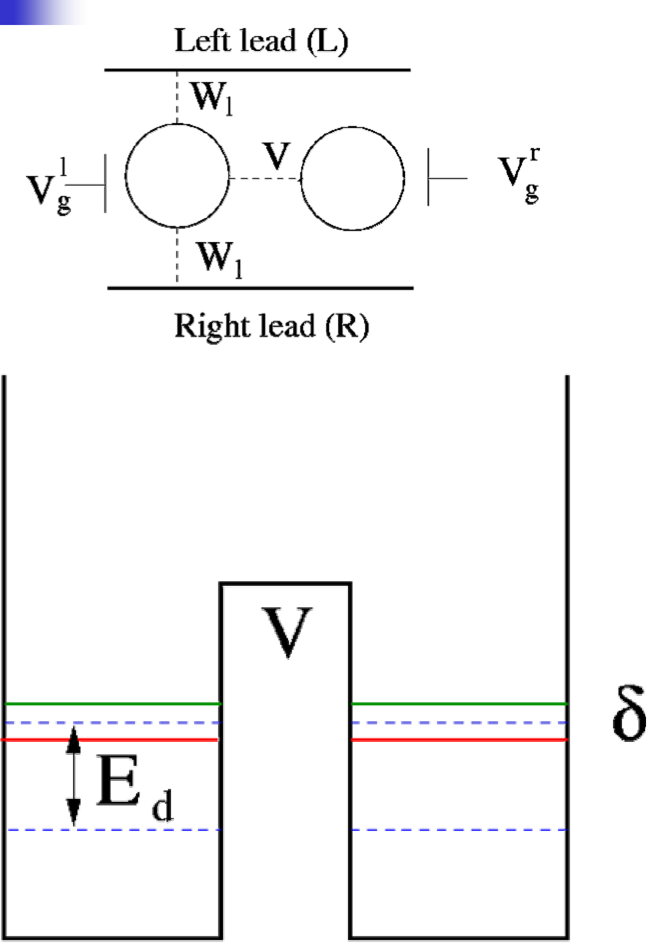
FIG. 2. *I-V* curve of the double dot, showing sharp resonances in the current when two 0D states line up. Upper inset: *I-V* curve of dot I. Lower inset: *I-V* curve of dot II. Both insets show a suppression of the current at low voltages due to the Coulomb blockade and a stepwise increase of the current due to the discrete energy spectrum of the dot.



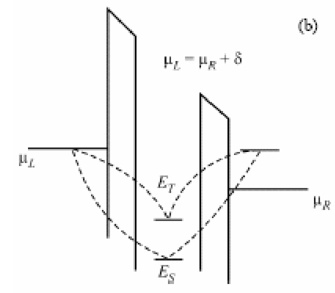


T-shape geometry

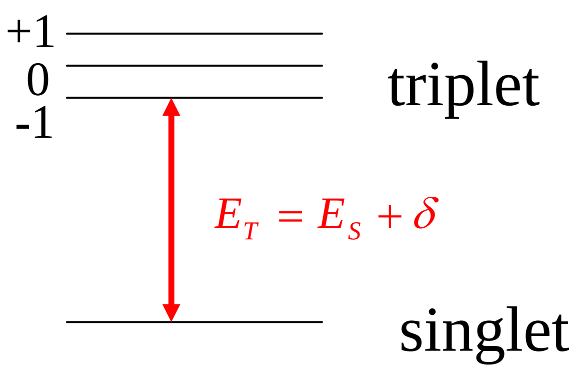
Symmetric and asymmetric double dots



Spin Rotator (SR) Model

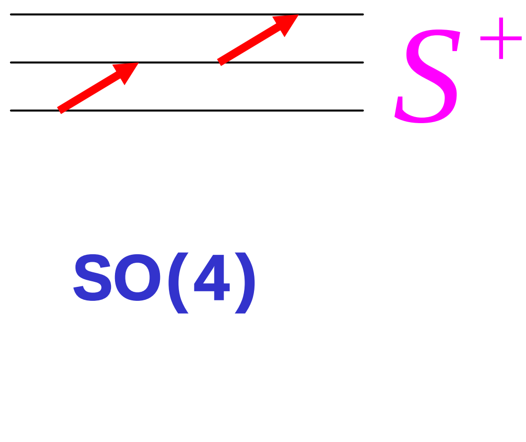


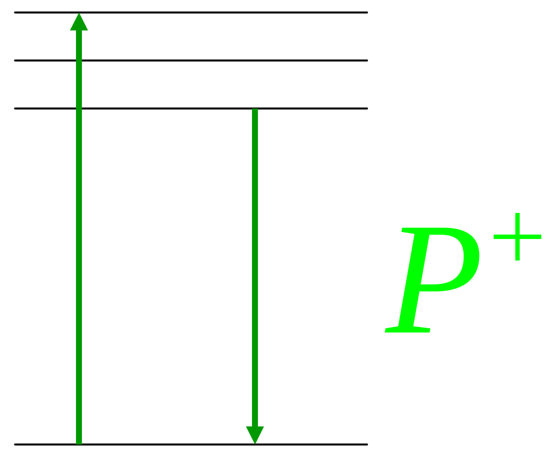
$$H_{\text{int}} = \sum [(J^{TT}_{\alpha\alpha'} \vec{S} + J^{ST}_{\alpha\alpha'} \vec{P}) \cdot \vec{s}_{\alpha\alpha'} + J^{S}_{\alpha\alpha'} X^{SS} n_{\alpha\alpha'}]$$

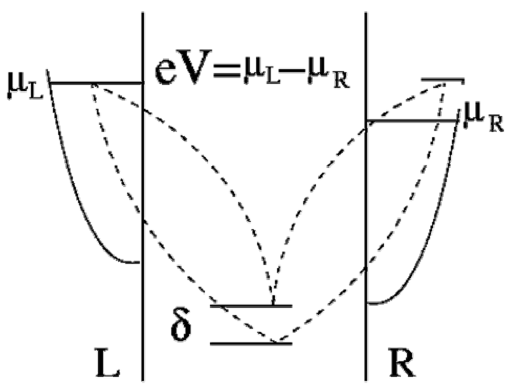


$$s_{\alpha\alpha'} = \sum_{kk'} c^{+}_{k\alpha\sigma} \hat{\tau}_{\sigma\sigma'} c_{k'\alpha'\sigma'}$$

$$n_{\alpha\alpha'} = \sum_{kk'} c^{+}_{k\alpha\sigma} \hat{1}\, c_{k'\alpha'\sigma}$$







SO(4)

Hidden symmetry in a Coulomb problem

Hydrogen atom:

$$H = \frac{p^2}{2\mu} - \frac{e^2}{r} \qquad E_n = -\frac{\mu e^4}{\hbar^2}\frac{1}{2n^2}$$

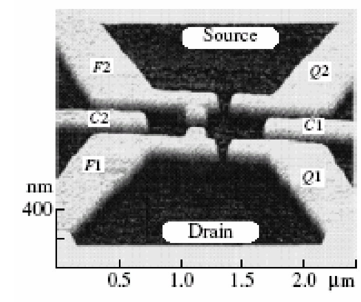
$$\vec{A} = \frac{1}{2\mu}\left(\vec{p}\times\vec{L} - \vec{L}\times\vec{p}\right) - \frac{e^2\vec{r}}{r}$$

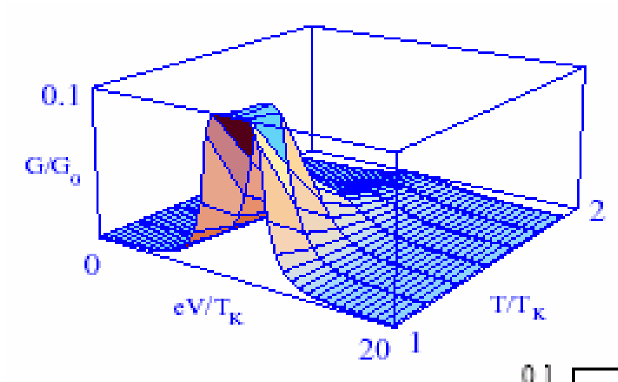
Runge-Lenz vector

$$\left[H,\vec{L}\right] = 0 \qquad \left[H,\vec{A}\right] = 0 \qquad \left(\vec{A}\bullet\vec{L}\right) = 0$$

$$SU(2)\otimes SU(2) = SO(4)$$

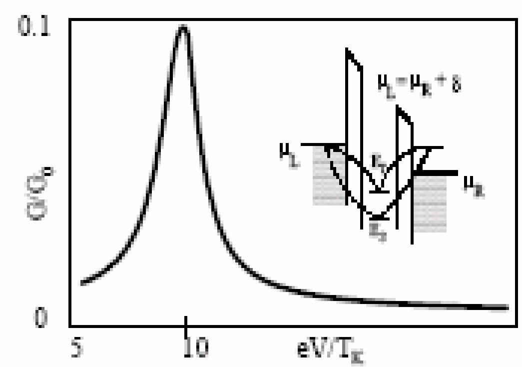
Non-equilibrium Kondo effect in Double Quantum Dot





$$G(eV,T) \propto |J_{LR}^{ST}|^2$$

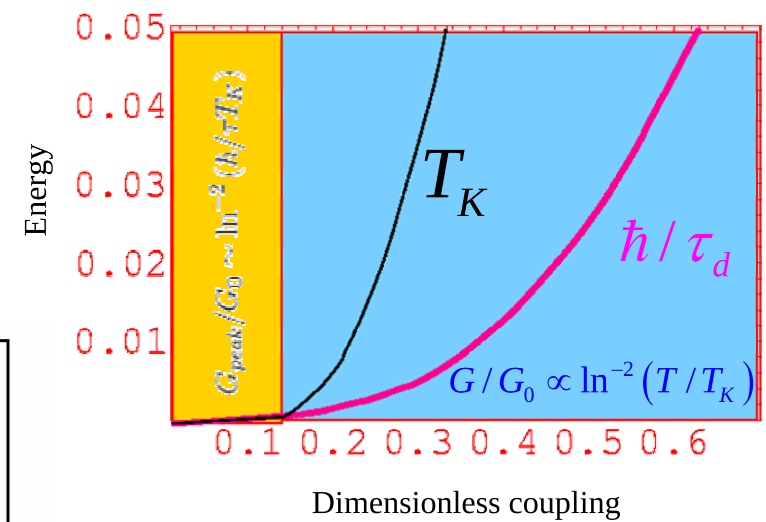
$$G_0 = 2e^2/h$$



DC Voltage

$$G/G_0 \propto \ln^{-2}\left(\max\left[\left(eV-\delta\right),T\right]/T_K\right)$$

MNK, K.Kikoin and L.W.Molenkamp, PRB 68, 155323 (2003)

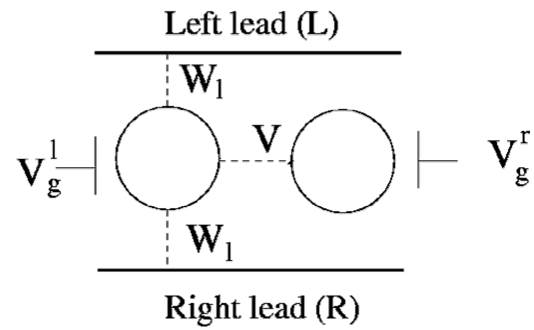


AC Voltage

$$G_{peak} \propto \overline{G\left(V_{ac}\cos\left(\omega t\right)\right)}$$

$$G_{peak}/G_0 \propto \ln^{-2}\left(\frac{\hbar}{\tau T_K}\right)$$

Basic inequalities



$\{eV_{dc}, eV_{dot}, eV_{ac}\} < \{E_d, U - E_d\}$ Validity of effective Hamiltonian

$T_K << eV \leq \delta << D$ Absence of Kondo effect in equilibrium

$|eV - \delta| << T_K$ Condition of Kondo resonance in nonequilibrium

$\delta\left(\frac{\delta}{D}\right)^2 << T_K << \delta$ DC decoherence rate effects are irrelevant

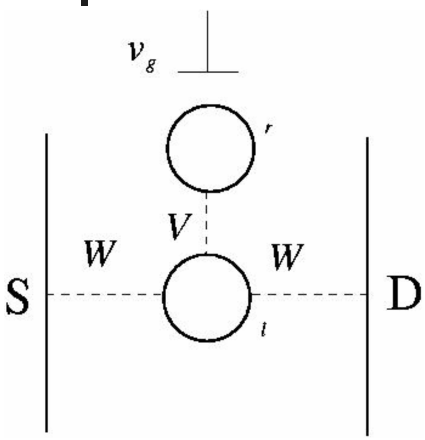
$\hbar / \tau_d << T_K$ AC decoherence rate effects are irrelevant

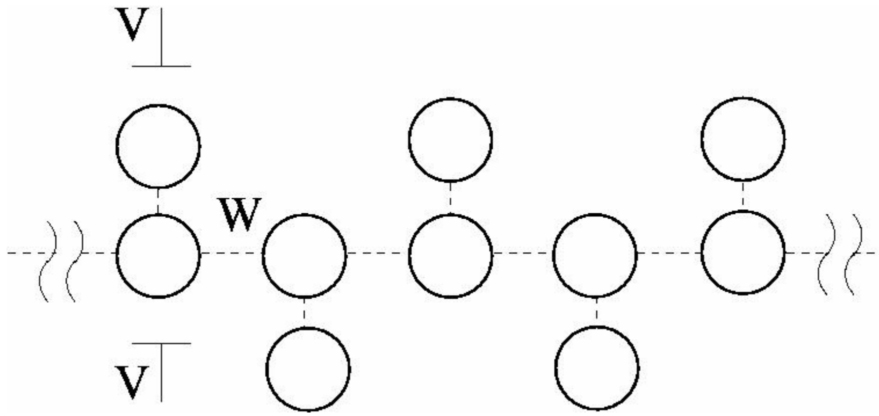
Messages

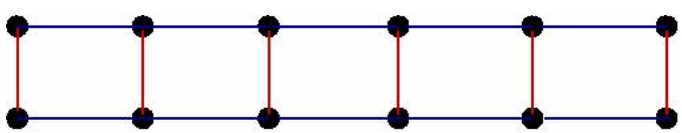
- Kondo effect exists in quantum dots in both situations when the total number of electrons is odd (usual) and even (exotic)
- Kondo effect may not be destroyed by an external magnetic field
- Kondo effect may not be destroyed by the relaxation effects associated with non-equilibrium conditions

Reason: explicit and hidden symmetries in complex quantum dots

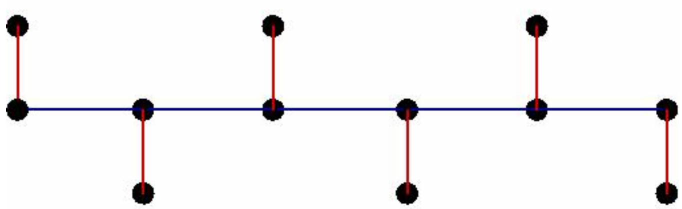
From complex dots to quantum chains



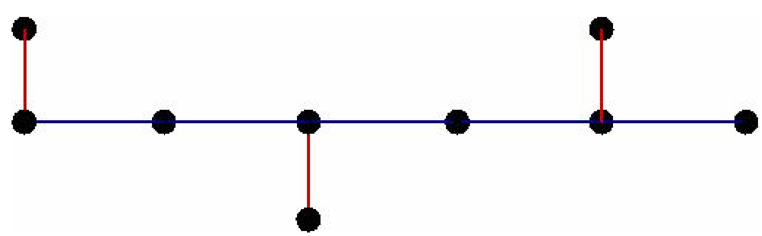




(a)



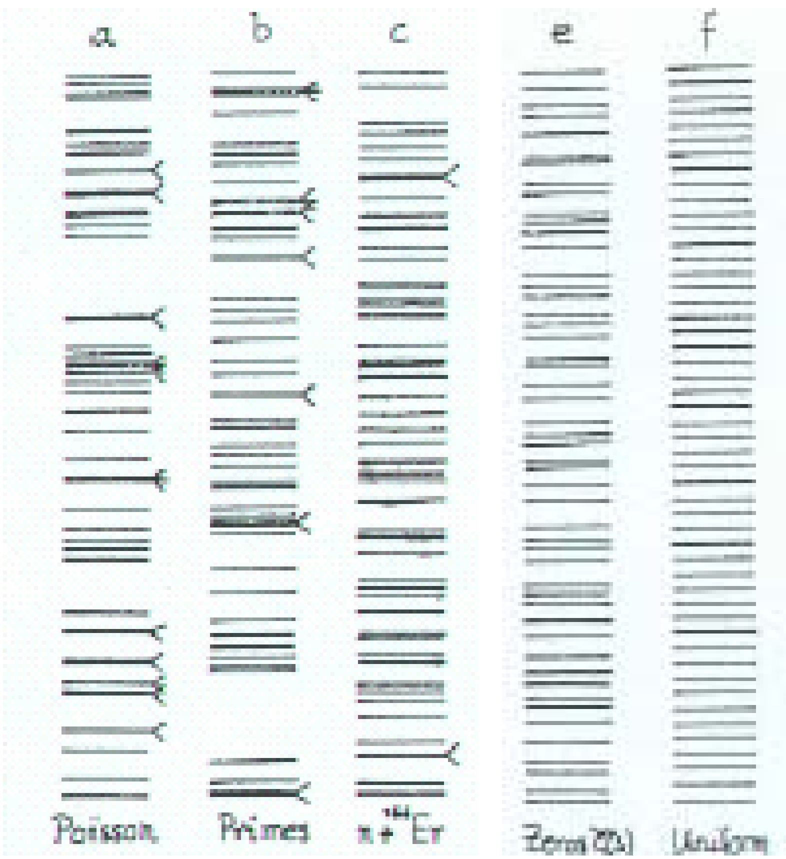
(b)



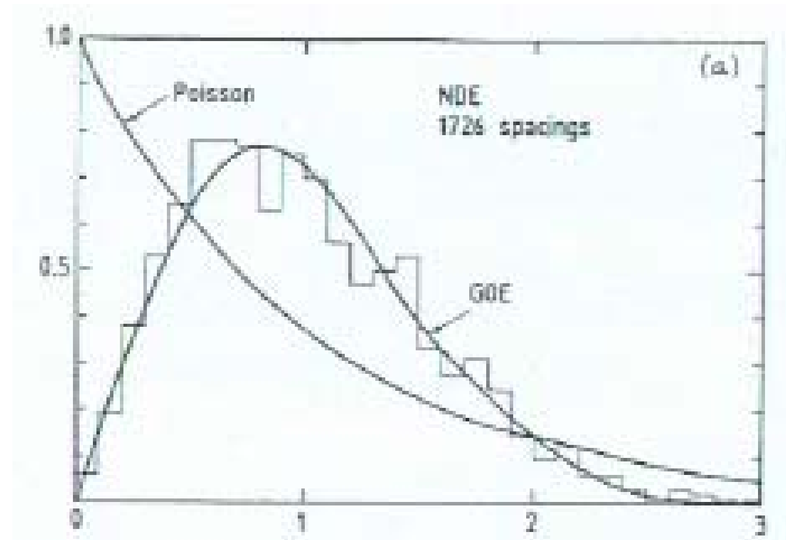
- Haldane Gap
- Charge Density Waves
- Exciton propagation

K.Kikoin, Y.Avishai and MNK cond-mat/0309606

Random Matrices: Wigner-Dyson statistics



GOE GUE



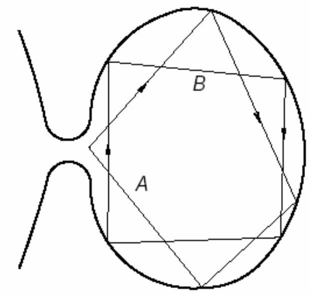
$$P\left(\left\{E_\mu\right\}\right) \propto \exp\left(\frac{\beta}{2}\sum_{\nu\neq\mu}\ln\left[\frac{|E_\mu - E_\nu|}{\delta}\right]\right)$$

$\beta = 1$ Orthogonal (GOE)

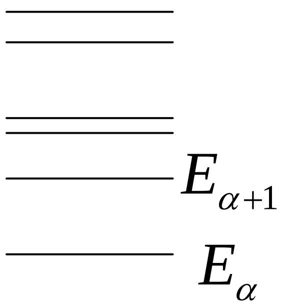
$\beta = 2$ Unitary (GUE)

$\beta = 4$ Simplectic (GSE)

Metallic QD: Universal Hamiltonian



Electron-electron interactions in isolated metallic grains



Mean-level spacing $\delta = \langle E_{\alpha+1} - E_{\alpha} \rangle$ (kinetic energy)

Thouless energy $E_T \sim D \cdot L^{-2}$ diffusive regime

$E_T \sim v_F L^{-1}$ ballistic regime

$$g = E_T / \delta \gg 1$$

GUE

$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2 - \lambda_{BCS} \hat{T}^{+} \hat{T}$$

charge spin Superconducting

$$E_c = \frac{e^2}{2C}$$

Coulomb blockade

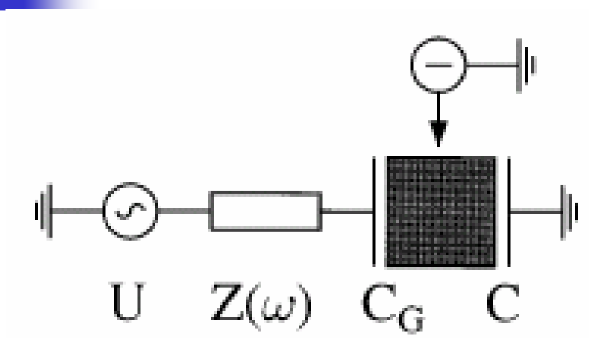
Scaling:

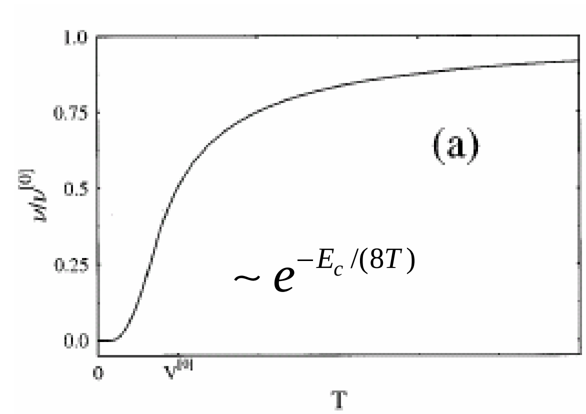
Short-range interaction $E_c \sim |J| \sim \delta$

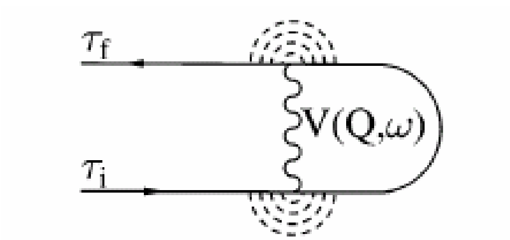
Coulomb interaction $E_c / \delta = r_s (k_F L)^{d-1} \gg |J|$

Zero-bias anomaly in finite-size systems

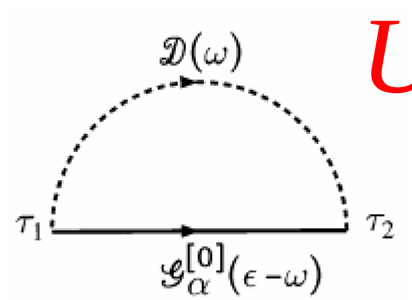
$$H_{\text{int}} = E_c(\hat{n} - N)^2$$



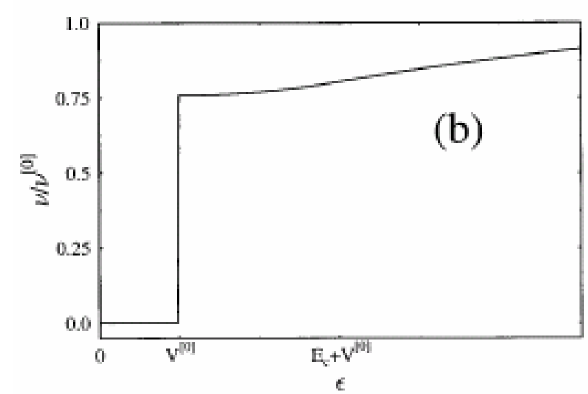




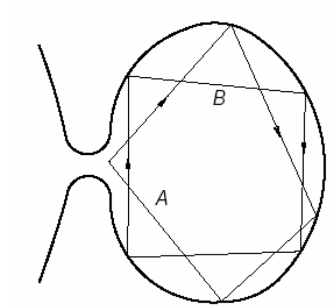
ZBA

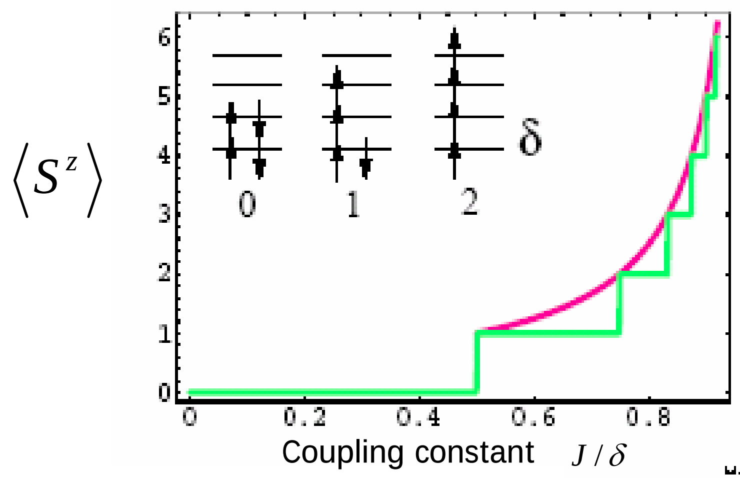


$U(1)$ gauge



Mesoscopic Stoner Instability

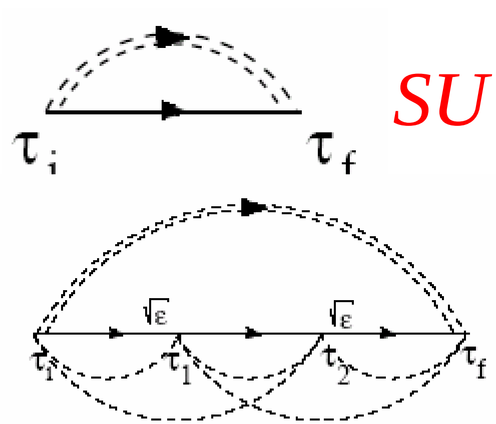




$$H_{\text{int}} = E_c(\hat{n} - N)^2 - J(\vec{S})^2$$

Stoner Instability

Transverse spin fluctuations are important



$SU(2)$

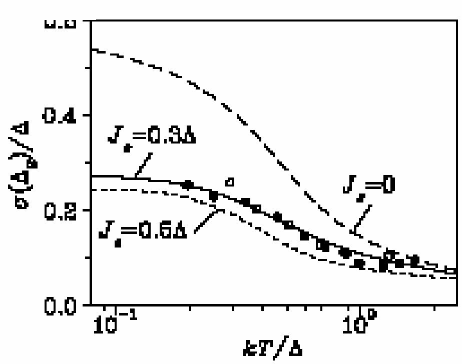
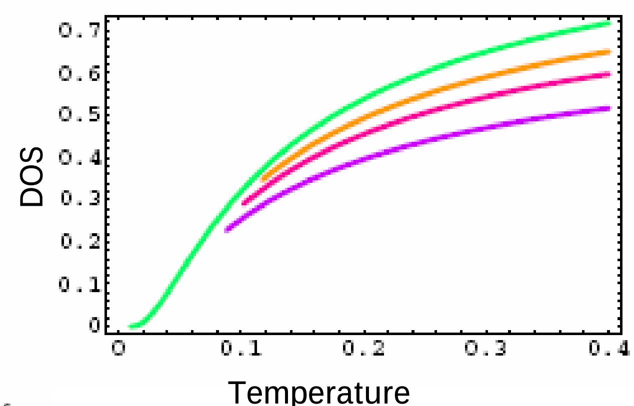


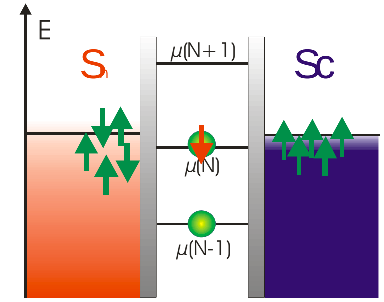
FIG. 1: The width $\sigma(\Delta_2)$ of the peak-spacing distribution for three different values of the exchange-interaction strength J_s. The symbols are the experimental data of Ref. [5].

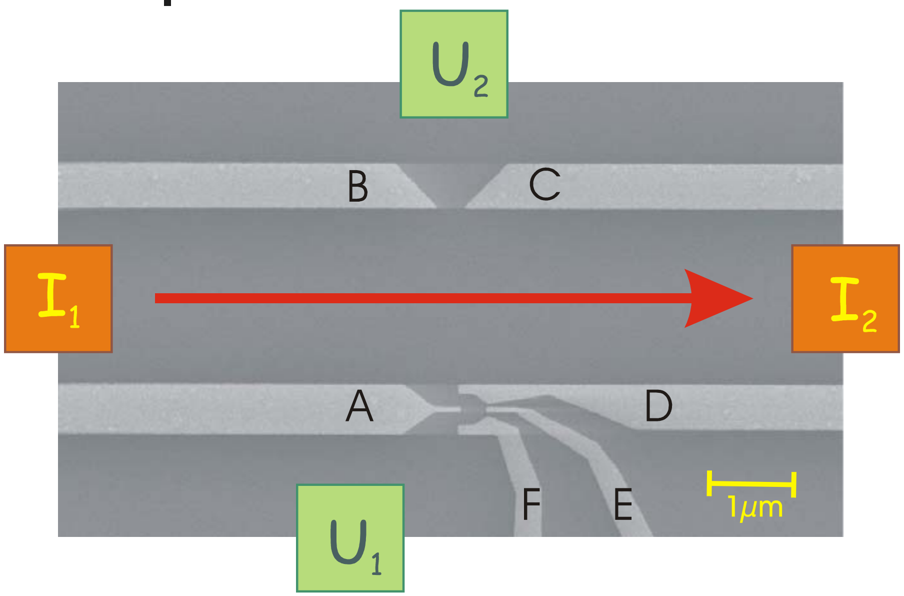
Y.Alhassid and T.Rupp (2003)

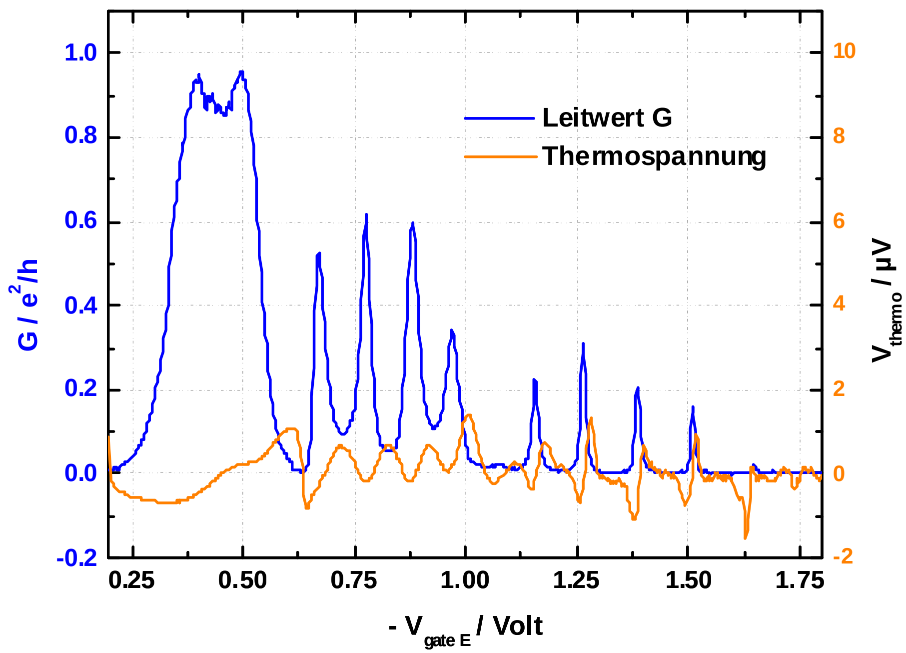


MNK and Y.Gefen (in preparation)

Perspectives: Thermopower of quantum dot







R.Scheibner, H.Buhmann and L.W. Molenkamp (unpublished)

It is a capital mistake to theorise before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts. (Sherlock Holmes in "A scandal in Bohemia" by Conan Doyle)

Summary

- Complex quantum dots possess hidden symmetries responsible for several exotic transport properties of these nano-devices
- Magnetic correlations between electrons in a dot result in many interesting effects (Stoner instability, Kondo effect, Non-Fermi-Liquid behavior etc)
- Transport properties of quantum dots is an interesting object both for experimental and theoretical investigations