physica status solidi

www.interscience.wiley.com

reprints

www.pss-rrl.com

www.pss-c.com

www.pss-b.com

www.pss-a.com

WILEY-VCH

REPRINT

Phys. Status Solidi C **6**, No. 9, 2024 – 2028 (2009) / **DOI** 10.1002/pssc.200881756

physica status solidi c
www.pss-c.com
current topics in solid state physics

Quantum tunneling through moving Kondo objects

Konstantin Kikoin*[,1], Michael N. Kiselev[2], and Jean Richert[3]

[1] School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel
[2] The Abdus Salam International Centre for Theoretical Physics, 34014, Trieste, Italy
[3] Laboratoire de Physique Théorique, Université Louis Pasteur and CNRS, 67084 Strasbourg Cedex, France

Received 17 September 2008, revised 15 January 2009, accepted 1 February 2009
Published online 14 July 2009

PACS 72.15.Qm, 73.23.Hk

* Corresponding author: e-mail konstk@post.tau.ac.il, Phone: +972-3-6406588, Fax: +972-3-6422979

We show how the time-dependent modulation of charge input signal applied to the gate electrode in a double quantum dot may be converted to a pulse in the Kondo tunneling current, which is predominantly spin response of this nano-device in a strong Coulomb blockade regime. The stochastic component of the input signal reveals itself as the infrared cutoff of Kondo tunneling transmission.

1 Introduction Among many possibilities provided by nanotechnologies one of the most challenging is the possibility of study quantum tunneling through nanoobject moving in space. This motion may be classical (shuttling induced by nanomechanical or electrostatic forces) or quantum (due to excitation of vibration modes). In particular, such motion affects zero bias anomalies in the Kondo resonance regime. Kondo anomalies in tunnel conductance of molecular complexes may be induced by phonon absorption and emission assisting single-electron tunneling [1]. Kondo shuttling through double quantum dot (DQD) with even occupation arises due to specific dynamical symmetry of spin multiplets characterizing magnetic state of these nanoobjects [2]. This dynamical symmetry allows also conversion of time-dependent charge input signal applied to the gate into Kondo response in tunnel conductance [3].

In this paper we show that the charge input signal induced by the motion of DQD in T-shape geometry induced by the time-dependent force applied to the side dot. Coherent component of this signal transforms into a pulse in the Kondo cotunneling current, whereas the incoherent component results in stochastization of the spin response of DQD.

2 Transformation of coherent signal We study the mechanism of activation of internal spin degrees of freedom by means of a time-dependent charge signal induced in a side valley of DQD in a contact with metallic leads in the T-shape geometry (Fig. 1). The electrons in

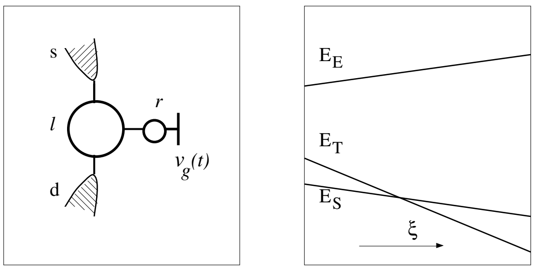

Figure 1 Left panel: DQD in a T-shape two-terminal geometry; right panel: evolution of energy levels in DQD as a function of scaling parameter $\xi = \ln D_0/D$ (see text).

this device are described by the Hamiltonian

$$H = \sum_{j=l,r;\sigma} \left(\varepsilon_j n_j + Q_j n_j^2\right) + V \sum_{ij\sigma}^{i\neq j} (d^\dagger_{i\sigma} d_{j\sigma}) + \sum_{b=s,d} \varepsilon_{kb} n_{kb\sigma} + W \sum_{k\sigma} (c^\dagger_{k\sigma} d_{l\sigma} + \text{h.c.}) + V_g(t) n_r \quad (1)$$

where the four first terms represent electrons in the DQD, electrons in the leads, and the lead-dot tunneling. The last time-dependent term stands for shuttling component of potential relief in the side dot r. Here $n_{k\sigma} = c^{\dagger}_{kb\sigma} c_{kb\sigma}$ is the occupation number for band electrons with the wave vector k and spin σ in the source and drain lead, $n_{j\sigma} = d^{\dagger}_{j\sigma} d_{j\sigma}$ is the electron number in the left and right dot, and $Q_{(l,r)}$ are the Coulomb blockade parameters. The tunneling Hamiltonian involves only electrons in the left dot. Only the the even standing wave $c_{k\sigma} = (c_{ks\sigma} + c_{kd\sigma})/\sqrt{2}$ enters the tunneling Hamiltonian.

The time independent part $H^{(0)}_{dot}$ of the dot Hamiltonian may be diagonalized in the two-electron charge sector $\mathcal{N} = 2$, $H^{(0)}_{dot} = \sum_{\Lambda} E_{\Lambda} |\Lambda\rangle\langle\Lambda|$. The three lowest states at $V_g = 0$ are spin triplets $|T\nu\rangle$ with spin projections $\nu = 0, \pm 1$, a singlet $|S\rangle$ and a charge transfer singlet exciton $|E\rangle$ with energies [4]

$$\begin{aligned} &E_T = \varepsilon_l + \varepsilon_r, \; E_S = E_T - 2\beta V, \\ &E_E = 2\varepsilon_l + Q_l + 2\beta V, \end{aligned} \tag{2}$$

where $\beta = V/\Delta_{ES}$ and $\varepsilon_{(l,r)}$ are the electron energy levels. The static part $V_g(0)$ of the gate voltage is incorporated in ε_r. Eqs. (2) are obtained for $Q_r \gg Q_l$, $\beta \ll 1$. The ground state is a spin singlet E_S.

We study the influence of charge perturbation containing both coherent and stochastic components

$$V_g(t) - V_g(0) = v_g(t) = \tilde{v}_g(t) + \delta v_g(t) \tag{3}$$

on the current through DQD. Here $\tilde{v}_g(t) = \tilde{v}_g \cos \Omega t$ is a coherent (deterministic) contribution and $\delta v_g(t)$ a noise component which is determined by its moments

$$\overline{\delta v_g(t)} = 0, \;\; \overline{\delta v_g(t) \delta v_g(t')} = \overline{v^2} f(t - t')\,. \tag{4}$$

The overline stands for the ensemble average and the characteristic function $f(t - t')$ will be specified below. We incorporate $V_g(t)$ into the energy levels (2) by means of a canonical transformation [3,5]

$$\begin{aligned} \widetilde{H}_{\rm dot} &= e^{-i\Phi_1(t) n_r} H_{\rm dot} e^{i\Phi_1(t) n_r} \\ &\quad - i\hbar e^{-i\Phi_1(t) n_r} \left(\frac{\partial}{\partial t} e^{i\Phi_1(t) n_r} \right); \\ \Phi_1(t) &= \frac{1}{\hbar} \int^t dt' v_g(t'). \end{aligned} \tag{5}$$

As a result, in the lowest orders in V_g, the time dependent part of the dot Hamiltonian acquires the form

$$\delta H_{\rm dot}(t) = -V \left(i\tilde{\Phi}_1(t) \mathcal{S}^{(1)}_{lr} + \frac{1}{2} \overline{\Phi_1(t)^2} \mathcal{S}^{(2)}_{lr} \right) \tag{6}$$

where $\mathcal{S}^{(p)}_{lr} = \sum_{\sigma} [d^{\dagger}_{r\sigma} d_{l\sigma} + (-1)^p d^{\dagger}_{l\sigma} d_{r\sigma}]$,

$$\overline{\Phi_1(t)^2} = \frac{\overline{v^2}}{\hbar^2} \int^t dt' \int^t dt'' f(t' - t''). \tag{7}$$

Thus, the time-dependent charge input signal induces coherent and stochastic interdot tunneling in DQD described by two terms in (6). One may formally include this term in the definition of the levels (2) and rewrite the dot Hamiltonian as $H_{dot}(t) = \sum_{\Lambda} E_{\Lambda}(t) |\Lambda\rangle\langle\Lambda|$ where the level E_T remains intact because the charge fluctuations do not influence spin degrees of freedom, whereas

$$\begin{aligned} &E_S(t) = E_S - \delta_{\rm ad}(t) - \delta_{{\rm st},S}(t), \\ &E_E(t) = E_E + \delta_{\rm ad}(t) + \delta_{{\rm st},E}(t). \end{aligned} \tag{8}$$

where the coherent part of renormalization $\delta_{\rm ad}(t) = (2V^2/\Delta_{ES}) \Phi_1^2(t)$ may be considered adiabatically under the condition $\delta_{\rm ad}/\Delta_{ES} \ll 1$, whereas the stochastic correction $\delta_{{\rm st},\Lambda}(t) = (V^2/4\Delta_{\Lambda T}) \overline{\Phi_1^2(t)}$, needs a more refined treatment. Here $\Delta_{\Lambda\Lambda'} = |E_{\Lambda} - E'_{\Lambda}|$.

Coherent and stochastic components appear also in the cotunneling part of the effective Hamiltonian $H_{\rm cot}$ which may be derived by means of the time-dependent Schrieffer-Wolff (SW) transformation [3,5]. Unlike in the standard case of spin 1/2 quantum dots [5], the SW transformation applied to DQD with the spectrum (8) intermixes the states $|\Lambda\rangle$. This intermixing is described by the operators $|\Lambda\rangle\langle\Lambda'|$, which form together with diagonal operators$|\Lambda\rangle\langle\Lambda|$ the set of generators of the $SO(5)$ group [3]. Ten generators are organized in three vectors $\mathbf{S}$, $\mathbf{P}$, $\mathbf{M}$ and one scalar A. Here $\mathbf{S}$ is the usual $S = 1$ spin operator, $\mathbf{P}$ and $\mathbf{M}$ are the vectors describing transitions between spin triplet $|T\mu\rangle$ and two singlets $|S\rangle$ and $|E\rangle$, respectively, $A = -i\mathcal{S}^{(1)}_{lr}/\sqrt{2}$ intermixes the latter states under the constraint imposed by the Casimir operator $\mathcal{C} = \mathbf{S}^2 + \mathbf{P}^2 + \mathbf{M}^2 + A^2 = 4$ (see [3] for further details). The Hamiltonian $H(t) = H_{\rm dot}(t) + H_{\rm cot}(t)$ may be rewritten in terms of the above group generators:

$$\begin{aligned} H_{\rm dot}(t) &= \frac{1}{2} \left(E_T \mathbf{S}^2 + \widetilde{E}_S \mathbf{P}^2 + \widetilde{E}_E \mathbf{M}^2 \right) - \mu(\mathcal{C} - 4); \\ H_{\rm cot}(t) &= J_0^T \mathbf{S} \cdot \mathbf{s} + \widetilde{J}^S \mathbf{P} \cdot \mathbf{s} + \widetilde{J}^E \mathbf{M} \cdot \mathbf{s}. \end{aligned} \tag{9}$$

The $SO(5)$ symmetry is preserved by the last term in $H_{\rm dot}$ by means of the Lagrange multiplier μ. Tilted coupling parameters are time-dependent. The charge-spin conversion mechanism under discussion is a manifestation of the $SO(5)$ dynamical symmetry of DQD. The Hamiltonian (9) describes Kondo cotunneling through DQD in presence of time-dependent perturbations (3) Its coherent component is responsible for the conversion of the charge signal $\tilde{v}_g(t)$ into a Kondo-type zero bias anomaly in tunnel current response, which arises in the spin channel. Its stochastic component $\delta v_g(t)$ results in the correction of charge noise into incoherent corrections to Kondo cotunneling.

The coherent part of the time-dependent Kondo problem can be solved in adiabatic approximation [3,5]. As a result of a time dependent Schrieffer-Wolff transformation and elimination of high-energy states, additional renormalizations $M_{\Lambda}(\xi)$ of the levels E_{Λ} arise where $\xi = \ln(D_0/D)$ is the scaling variable [4,6] Then the singlet-triplet gap transforms into

$$\Delta_{ST}(t) = \Delta^0_{ST} + M_T - M_S - \delta_{\rm ad}(t) \tag{10}$$

and the singlet-triplet crossover (Fig. 1, right panel) takes place, provided the self-consistent condition $T_K(\Delta_{ST}, t) >$

$\Delta_{ST}(t)$ is satisfied [4,6–9]. T_K is a sharp function of Δ_{ST} with a maximum T_{K0} at $\Delta_{ST}=0$ (inset in the right panel of Fig. 2). Its right slope is described by the ratio

$$\frac{T_K(\Delta_{ST},t)}{T_{K0}} = \left[\frac{T_{K0}}{\Delta_{ST}(t)}\right]^{\eta}, \tag{11}$$

valid for intermediate asymptotic positive values of Δ_{ST} at $T_{K0}/\Delta_{ST} \le 1$ (dotted part of the curve $T_K(\Delta)/T_K(0)$ in the inset). Here $\eta < 1$ is a universal constant. This sharp

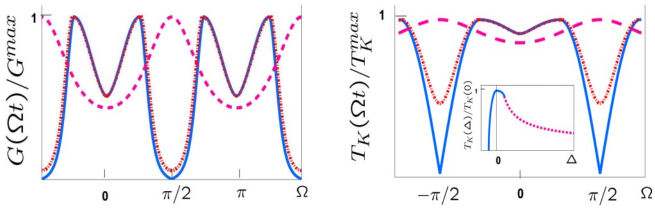

Figure 2 (Color online) Left panel: Oscillations of ZBA in tunnel conductance due to time-dependent potential $\tilde{v}_g \cos \Omega t$ corresponding to the ST-gap oscillating around large positive $\Delta^0_{ST} \gg T_{K0}$ (dashed line), around zero Δ down to $T_K^{min} = T_{K0}/2$ (dash-dotted line) and down to $T_K^{min} = 0$ (solid line), respectively. Right panel: time dependent T_K (notations are the same as for the left panel); insert: T_K as a function of Δ_{ST}. Calculations are performed at $T/T_{K0} = 1.4, \eta = 0.5$.

dependence is a key to the transformation mechanism of charge input into Kondo conductance response.

We estimate the influence of $\tilde{v}_g(t)$ on the tunnel conductance $G(T,t)$ at given $T > T_K$ in a situation where the adiabatic temporal variations of $T_K(\Delta_{ST},t)$ take place. Then the tunnel conductance obeys the law $G/G_0 \sim \ln^{-2}(T/T_K)$. Using (11) one gets

$$G(t)/G_0 \sim \left(\ln(T/T_{K0}) - \eta \ln(T_{K0}/\Delta_{ST}(t))\right)^{-2} \tag{12}$$

The curves $G(\Omega t)$ shown in Fig. 2 (left panel) describe transformations of an oscillating signal in the charge perturbation $\tilde{v}_g(t)$ in shuttling side dot into oscillations of Kondo-type zero bias anomaly (ZBA) of tunnel conductance (see also [2]). The transformation effect is especially distinct when the oscillations of $v_g(t)$ change the sign of Δ_{ST}, i.e. induce an $S \to T$ crossover (solid and dash-dotted curves in Fig. 2 corresponding to the solid part of $T_K(\Delta)$ curve in the inset).

3 Transformation of stochastic signal The mechanism which converts a stochastic component $\delta v_g(t)$ of the input signal into a stochastic spin response is quite unusual. Instead of dephasing due to time-dependent spin flip processes [5], stochastization of the energy spectrum of DQD results in the loss of a Curie-type spin response at some characteristic energy ζ. This effect is related to the time dependence of the factor $\tilde{\mu}(t)$ in the Hamiltonian (9). Indeed, inserting (8) into (9), one may write the stochastic part of $H_{\rm dot}$ as

$$H^{\rm st}_{\rm dot} = [\delta_{{\rm st},S}(t)\mathbf{P}^2 - \delta_{{\rm st},E}(t)\mathbf{M}^2]/2 \tag{13}$$

Unlike the adiabatic part of time dependent energy levels $E_\Lambda(t)$ incorporated in (10), this term describes fluctuations due to the dynamical symmetry of DQD. At energies $\sim T_K \ll E_E$ the state $|E\rangle$ is frozen out. Then instead of the exact Casimir constraint $\mathcal{C}$ for the $SO(5)$ group, one deals with a fluctuating constraint $\tilde{\mathcal{C}} = \mathbf{S}^2 + \mathbf{P}^2$ for the reduced group $SO(4)$ describing an ST multiplet where the fluctuating part may be written in the form $\mu_{\rm st}(t)\mathbf{P}^2 = -\mu_{\rm st}(t)\mathbf{S}^2$ with $\mu_{\rm st}(t) = \delta_{{\rm st},S}(t)/2$. At $T \to 0$ where the singlet is also frozen out, one arrives to the effective dot Hamiltonian

$$\begin{aligned} H_{\rm dot}(t) &= \frac{1}{2}E_T\mathbf{S}^2 - \mu_{\rm ad}(\mathbf{S}^2-2) - \mu_{{\rm st},S}(t)\mathbf{S}^2 \\ &= \sum_{\nu=0,\pm1}[\varepsilon - \mu(t)]f^\dagger_\nu f_\nu \end{aligned} \tag{14}$$

where the fermion representation for S=1, $|T\nu\rangle\langle T\nu'| = f^\dagger_\nu f_{\nu'}$, is used in the second line. Here $\varepsilon = E_T/2$, and the time-dependent chemical potential for spin fermions is defined as $\mu(t) = \mu_{\rm ad} - \mu_{{\rm st},S}(t)$. The stochastic component of μ may be treated as a random potential in the time domain which describes the fluctuations of the global fermionic constraint [3]. Then the propagation of spin fermions in the random time-dependent potential $\mu_{{\rm st},S}(t)$ may be studied by means of the "cross technique" developed for the calculation of electron propagation in a field of impurities randomly distributed in real space.

The frequency Ω in $\tilde{v}_g(t)$ is the slowest frequency in our problem, and $\mu_{{\rm st},S}(t)$ also varies slowly in time, so that the relaxation time $\tau = \hbar/\gamma$ in the noise correlation function $D(t-t') = \hbar^2\langle\mu(t)\mu(t')\rangle \sim \exp[-\gamma(t-t')]$ is a longest time in the model. Then one may take the limit

$$D(\omega) = \lim_{\gamma\to 0}\frac{2\zeta^2\gamma}{\omega^2+\gamma^2} = 2\pi\zeta^2\delta(\omega) \tag{15}$$

for its Fourier transform, ζ^2 being the variance of the Gaussian. In this limit the averaged spin propagator describes the ensemble of states with chemical potential $\mu_{\rm st}= const$ in a given state, but this constant is random in each realization [10]. Thus the problem of decoherence of the spin state in stochastically perturbed DQD is mapped on the so-called Keldysh model [11–13] originally formulated for δ-correlated impurity scattering potentials in momentum space. The problem can be solved exactly and the decoherence time is fixed by the variance ζ^2.

The solution of the Dyson equation $G^{-1}(\epsilon) = \epsilon + \mu_0 - \Sigma(\epsilon)$ (Fig. 3) [14], for the Fourier transform of the spin-fermion propagator $G^R_{T\nu}(t-t') = \langle f_\nu(t)f^\dagger_\nu(t')\rangle_R = -i\langle[f_\nu(t)f^\dagger_\nu(t')]_+\rangle$, is given by

$$\Sigma(\epsilon) = \int\frac{d\omega}{2\pi}\Gamma(\epsilon,\epsilon-\omega;\omega)G(\epsilon-\omega)D(\omega) = \zeta^2\Gamma(\epsilon,\epsilon;0)G(\epsilon)$$

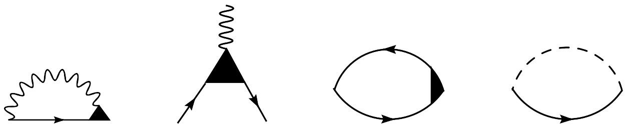

Figure 3 Feynmann diagrams for the self energy Σ with vertex corrections, triangular vertex Γ, spin susceptibility χ and parquet Kondo loops K (from left to right). Solid, dashed and wavy lines denote bare spin-fermion propagator G, conduction electron propagator g and correlation function D.

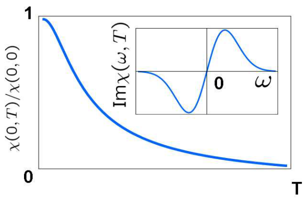

Figure 4 (Color online) Static susceptibility $\chi(0,T)$. Inset shows a frequency dependence of Imχ with a maximum at $\omega \sim \zeta$.

(the index ν is omitted, since the fluctuations of μ are related to the global $U(1)$ symmetry). Using the Ward identity for a triangular vertex shown in Fig. 3, $\Gamma(\epsilon,\epsilon;0) = dG^{-1}(\epsilon)/d\epsilon$, we transform the Dyson equation into differential equation $\zeta^2 dG/dx + xG - 1 = 0$ where $x = \epsilon + \mu_0$ and the boundary condition reads: $G(x \to \infty) = 1/x$. The solution of this equation given by [12]

$$G^R(x) = \frac{1}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2\zeta^2} \frac{dz}{x - z + i\delta} \qquad (16)$$

represents the set of spin states with stochastic chemical potential averaged with a gaussian exponent characterized by the variance ζ^2. Remarkably, $G^R(x)$ has no poles, singularities or branch cuts.

To check the spin properties of stochasticized DQD, we calculate its spin response at finite T determined as $\chi(i\omega_m) = T\sum_n \mathcal{G}(i\omega_m + i\epsilon_n)\mathcal{G}(i\epsilon_n)\Gamma(i\epsilon_n, i\epsilon_n + i\omega_m; i\omega_m)$ where $\mathcal{G}(i\epsilon_n)$ is the Matsubara continuation of (16) on to the imaginary axis (see Fig. 3). The same Ward identity provides the exact equation for the vertex $\Gamma^R(\epsilon,\epsilon;0) = (\epsilon G^R - 1)/\zeta^2 (G^R)^2$, which allows to calculate the static susceptibility

$$\chi(\omega = 0, T) = \frac{1}{\sqrt{8\pi}\zeta} \int_{-\infty}^{\infty} dx e^{-x^2/2} x \tanh\left(\frac{x\zeta}{2T}\right) \qquad (17)$$

From its temperature dependence plotted in Fig. 4 we see that the the Curie-type spin response $\chi(0,T) \sim 1/T$ at high $T \gg \zeta$ transforms into a constant at zero T, $\chi(0,0) \sim 1/\zeta$. This means that the DQD loses the characteristics of a localized spin at $\{\omega, T\} \ll \zeta$ due to stochastization, and therefore it cannot serve as a source of Kondo screening at low energies. The dynamic susceptibility Im$\chi(\omega > \zeta, T)$ is given by expression (see Fig. 4, inset)

$$\mathrm{Im}\chi(\omega, T) = \frac{1}{\zeta} \tanh\left(\frac{\omega}{2T}\right) \exp\left(-\frac{\omega^2}{4\zeta^2}\right)$$
$$\int_0^{\infty} \frac{dx e^{-x^2/4}}{\cosh(x\zeta/(2T))/\cosh(\omega/(2T)) + 1}. \qquad (18)$$

When deriving this equation, we neglected the vertex corrections which become important only at small frequencies $\omega \ll \zeta$. Dramatic change of dynamic spin correlations of the DQD affects the Kondo response described by H_{cot} (9).

Time dependence in the vertex Γ (Fig. 3) results in dephasing of Kondo cotunneling [3], which, however, is exponentially weak at $T \ll \Delta_{ST}$. More significant is the influence of stochastization of the DQD. Inserting (16) into the Kondo loop $K(i\epsilon_n) \sim J^2 T \sum_m \int \frac{d\mathbf{p}}{(2\pi)^3} g(\mathbf{p}, i\omega_m)\mathcal{G}(i\epsilon_n \pm i\omega_m)$ responsible for logarithmic singularity in conventional Kondo scattering (Fig. 3), one obtains a combination of logarithmic, hypergeometric and imaginary error functions

$$K(\epsilon \to 0)/J = \rho_0 J \ln(\sqrt{2C}D/\zeta) \qquad (19)$$
$$+\frac{1}{2}\rho_0 J \left[{}_1F_1\left(\frac{3}{2}, 2, \frac{T^2}{2\zeta^2}\right)\left(\frac{T}{\zeta}\right)^2 - \pi \mathtt{Erfi}\left(\frac{T}{\sqrt{2}\zeta}\right)\right]$$

where $\gamma = \ln C$ is the Euler constant. In two limiting cases of low and high temperatures relative to the dispersion ζ of the noise spectrum, it leads to following expressions

$$K(\epsilon \to 0)/J = \begin{cases} \rho_0 J \ln(D/T), & T \gg \zeta \\ \rho_0 J \ln(D/\zeta), & \zeta \gg T \end{cases} \qquad (20)$$

Thus the noise amplitude plays the role of the infrared cutoff (similarly to Kondo-spin glass problem [15]). This cutoff distorts the coherent pulses in ZBA (Fig. 2) provided ζ is comparable with T_K .

In conclusion, we presented and discussed the conversion mechanism of a time-dependent coherent and stochastic input signal induced by a shuttling side dot in T-shape DQD into a Kondo-type spin response in tunnel conductance. This mechanism is related to the dynamical symmetry of DQD.

Acknowledgements Authors are indebted to F. Marquardt, N. Prokof'ev, M.V. Sadovskii and T. Ziman for valuable advices.

References

[1] K. Kikoin, M. N. Kiselev, and M. R. Wegewijs, Phys. Rev. Lett. **96**, 176801 (2006).
[2] M. N. Kiselev, K. Kikoin, R.I. Shekhter, and V.M. Vinokur, Phys. Rev. B **74**, 233403 (4) (2006).
[3] M. N. Kiselev, K. Kikoin, Y. Avishai, and J. Richert, Phys. Rev. B **74**, 115306 (2006).
[4] K. Kikoin and Y. Avishai, Phys. Rev. Lett. **86**, 2090 (2001); Phys. Rev. B **65**, 115329 (2002).

[5] A. Kaminski, Yu. V. Nazarov, and L.I. Glazman, Phys. Rev. B **62**, 8154 (2000).
[6] F. D. M. Haldane, Phys. Rev. Lett. **40**, 416 (1978).
[7] A. Kogan, G. Granger, M. A. Kastner, D. Goldhaber-Gordon, and H. Shtrikman, Phys. Rev. B **67**, 113309 (2003).
[8] M. Eto and Yu. V. Nazarov, Phys. Rev. Lett. **85**, 1306 (2000); M. Eto, J. Phys. Soc. Jpn. **74**, 95 (2005).
[9] M. Pustilnik and L. I. Glazman, Phys. Rev. Lett. **85**, 2993 (2000); Phys. Rev. B **64**, 045328 (2001).
[10] The analog of this ensemble in real space is the inhomogeneous broadening of a ESR signal.
[11] L. V. Keldysh, Semiconductors in strong electric field, D. Sci. Thesis, Lebedev Institute, Moscow (1965).
[12] A. Efros, Zh. Eksp. Teor. Fiz. **59**, 880 (1970) [Sov. Phys.–JETP **32**, 479 (1971)].
[13] M. V. Sadovskii, Diagrammatica (World Scientific, Singapore, 2005).
[14] Here μ_0 is a constant non-random part of $\mu(t)$.
[15] M. N. Kiselev and R. Oppermann, JETP Lett. **71**, 50 (2000) [Pisma Zh. Eksp. Theor. Fiz. **71**, 359 (2000)].

