GR HW 1

Due Nov 21 at 11am, in class

Goals: Lorenz transformations, Four-vector and four-tensor notation, Relativistic kinematics, Rest-frame, Energy-momentum conservation, Levi-Civita tensor.

1. Two events x_A^μ and x_B^μ are space-like separated. Find the boost to a Lorentz frame in which the two events are simultaneous.

2. Spaceship is coming back to Earth at $0.995c$. During a TV contact, people on Earth notice that time runs faster onboard (by how much?). How come the astronauts become younger than people on Earth?

3. An ideal photon spaceship (transforming fuel mass to energy of ideally collimated photons with 100% efficiency) goes to the center of the galaxy (10 kpc from Earth) and back. It starts from rest and finishes at rest. The accelerations and decelerations are equal to $a = 10m/s^2$. Calculate:

   a. The trip time by Earth and onboard clocks.

   b. The initial mass of the fuel M. (The mass of the ship without fuel plus the mass of the astronauts is m.)

4. The neutral Sigma baryon, Σ^0, with mass m_Σ, decays into a Lambda baryon, Λ, with mass m_Λ, and a massless photon.

   (a) Find the energy of the photon in the frame in which the Σ^0 is at rest.

   (b) Find the energy of the photon in the frame in which the Λ is at rest.

   Hint: it simplifies the algebra to use four-vectors.

5. $\phi, A^\mu, T^{\mu\nu}$ are scalar, vector and tensor. Which of the following equations are covariant

   a. $\phi = A_0$

   b. $\phi = A^\mu A_\mu$

   c. $\phi = A_0 A^0$

   d. $\phi = T_{\mu\nu} T^{\mu\nu}$

   e. $T_{\mu\nu} = T^{\nu\mu}$

   f. $T_{\mu\nu} = T_{\nu\mu}$

   g. $T^{\mu\nu} = A^\mu + A^\nu$

   h. $T_{\mu\nu} = -T_{\nu\mu}$

   i. $T_\nu^\mu = -T_\mu^\nu$

   j. $T^{\mu\nu} = A^\mu A^\nu$

   k. $\phi = \det T^{\mu\nu}$

   l. $\phi = \det T_\nu^\mu$

6. In d dimensions

$$\varepsilon^{\mu_1 \cdots \mu_d} \varepsilon_{\nu_1 \cdots \nu_d} A_{\mu_1}^{\nu_1} \cdots A_{\mu_d}^{\nu_d} = c \det A_\mu^\nu. \tag{1}$$

   Find c.