Systemic risk and financial complexity

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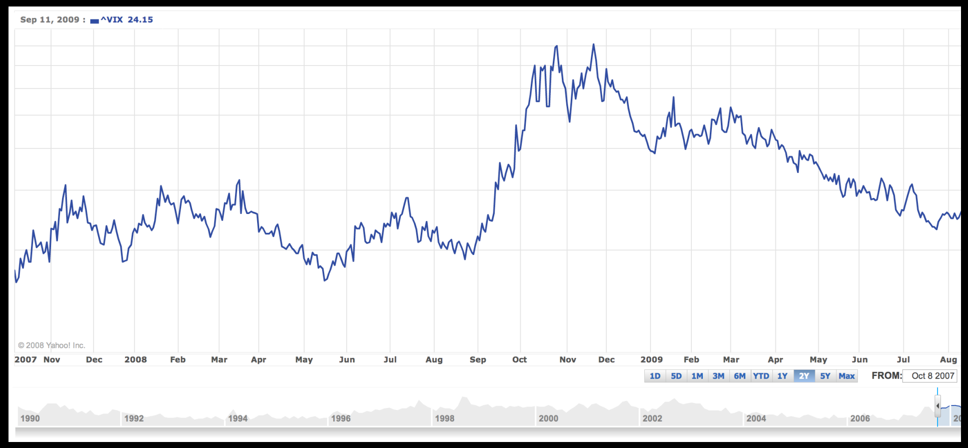
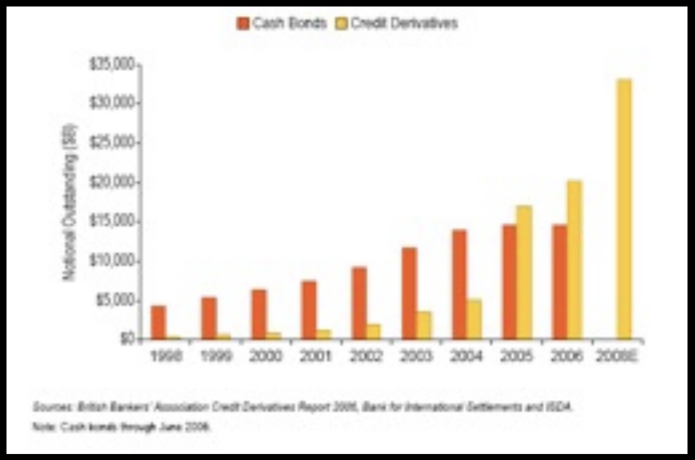
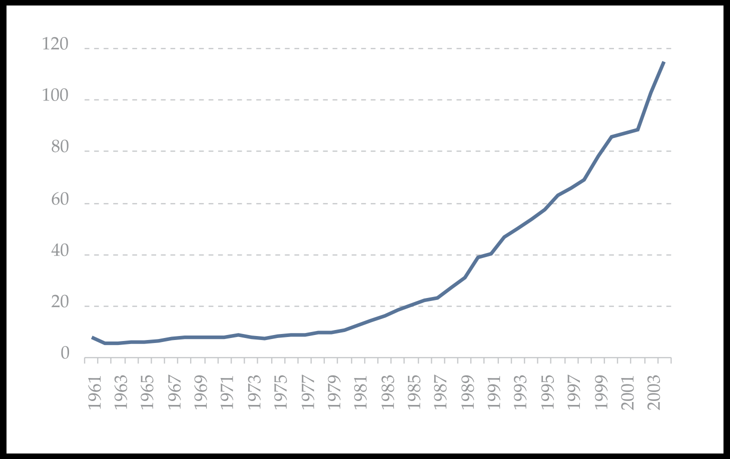
Bad guys or bad theories



Financial instability: price volatility (CBOE-VIX index)



Volumes traded within financial industry (e.g. credit derivatives)



Size of financial markets/GDP

Financial systemic risk: networks, contagion, etc...

Can this happen even without market imperfections, misaligned incentives, toxic assets, contagion effects, etc?

The financial innovation spiral

(Merton and Bodie 2005)

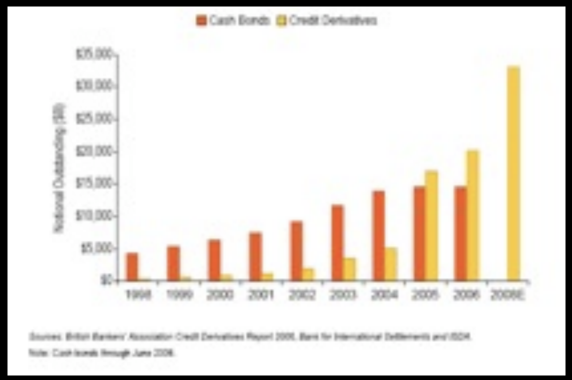
"As products such as futures, options, swaps, and securitized loans become standardized [...] the producers (typically, financial intermediaries) trade in these new markets and volume expands; increased volume reduces marginal transaction costs and thereby makes possible further implementation of more new products and trading strategies by intermediaries, which in turn leads to still more volume [...] and so on it goes, spiraling toward the theoretically limiting case of zero marginal transactions costs and dynamically complete markets."

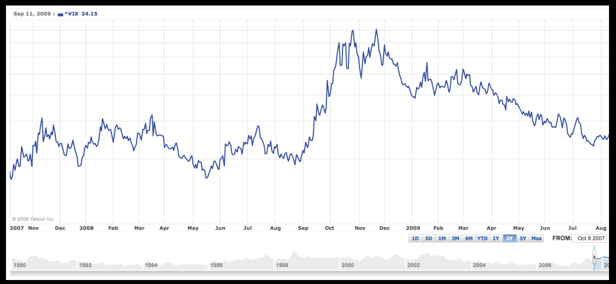
"When particular transaction costs or behavioral patterns produce large departures from the predictions of the ideal frictionless neoclassical equilibrium for a given institutional structure, new institutions tend to develop that partially offset the resulting inefficiencies. In the longer run, after institutional structures have had time to fully develop, the predictions of the neoclassical model will be approximately valid for asset prices and resource allocations."

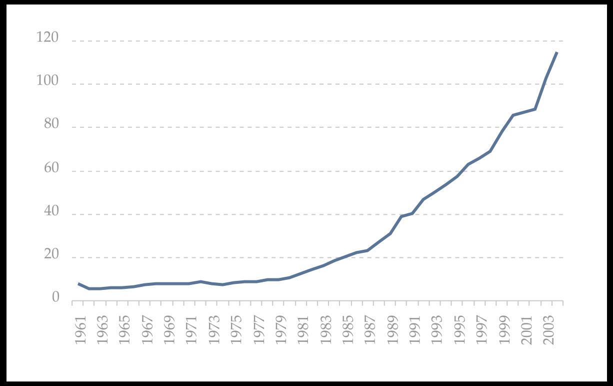
(see also R. J. Shiller, "The Subprime Solution" 2008)

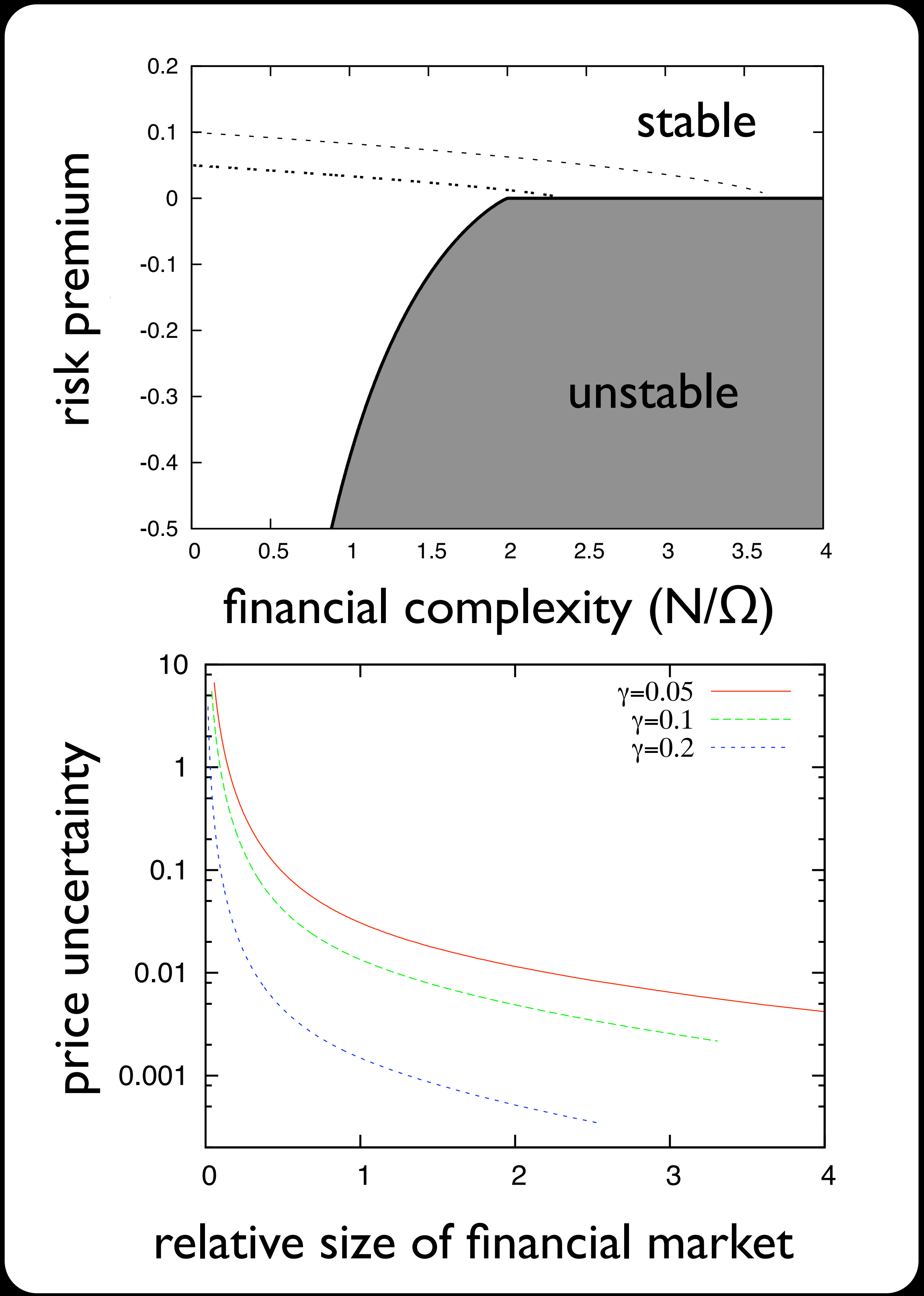
Main result

- As markets approach completeness:
  - allocations become more and more unstable
  - The size of the financial market grows unbounded wrt the ``real economy”
- Stability vs size diagram







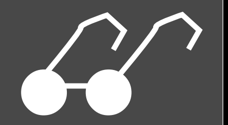


Outline

- The basic intuition: 2 assets, 2 states
- A complex asset market in an equilibrium economy: N assets, Ω states: N, $\Omega \to \infty$
- Spiraling toward complete markets in a competitive financial industry
- How big can the financial market be?
- Illiquid markets and information efficiency
- Conclusions

Intuition: Asset allocation in a risky world

- Tomorrow: rain or sun?
  wait and buy sunglasses or umbrella
  Inefficient, if e.g. tomorrow
  price of sunglasses > price of umbrella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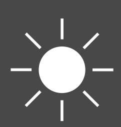
	No	Yes

- Contingent commodity markets:
  markets and prices, open today for
  (sunglasses if rain), (sunglasses if sun), (umbrella if rain), (umbrella if sun)
  Today: shopping in contingency commodity markets
  Tomorrow: delivery and consumption

- Optimal allocation under perfect competition

What if contingent commodity markets do not exist?

- Financial market: 1 riskless B_t and 1 risky S_t assets
  Today $B_0=S_0=1$
  Tomorrow $B_1=1$, $S_1=1+u$ if sun, $S_1=1-d$ if rain

- I want to have C^{rain} euros to buy an umbrella if it rains and C^{sun} euros to buy sunglasses if it is sunny. Can I do that? How much does it cost?

- Yes! Buy a portfolio z_B units of B and z_S units of S such that

$$z_B + (1+u)z_S = C^{\text{sun}}$$
$$z_B + (1-d)z_S = C^{\text{rain}}$$

- How much does it cost?

$$C_0 = z_B + z_S = \frac{d}{u+d}C^{\text{sun}} + \frac{u}{u+d}C^{\text{rain}} = E_q[C_{t=1}]$$

- This can be done for any contingent claim C^w. Independent of probability!

- Assumptions:
  i) perfect competition
  ii) full information
  iii) no-arbitrage: ud>0
  iv) complete market: what if there are three states? (e.g. sun, cloud, rain)

An equilibrium economy: N assets, Ω states

The market (returns)

demand

Investors

Financial industry (banks)

$$
\text{Investors} \Leftarrow
\begin{array}{c}
z_1 \\ \vdots \\ z_k \\ \vdots \\ z_N
\end{array}
\Leftarrow
\begin{array}{ccccc}
r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\
\vdots & \ddots & \vdots & & \vdots \\
r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\
\vdots & & \vdots & \ddots & \vdots \\
r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega
\end{array}
\Leftarrow \text{Financial industry (banks)}
$$

$$\Downarrow$$

$$\max_{\vec{z} \geq 0} E\left[u\left(c(\vec{z})\right)\right]$$

$$r_{N+1}^1 \quad \cdots \quad r_{N+1}^\omega \quad \cdots \quad r_{N+1}^\Omega$$

financial innovation

$$N, \Omega \to \infty, \quad n = \frac{N}{\Omega}$$

Optimizing consumers

Solution of optimal consumption problem:

$$\max_{\vec{z}\geq 0} E\left[u\left(c(\vec{z})\right)\right] \qquad c^\omega = \frac{w_0 + \sum_i z_i r_i^\omega}{p^\omega}$$

w_0 = initial wealth
p^ω = price of goods in state ω

First order conditions:

$$\frac{\partial}{\partial z_i} E_\pi\left[u\left(c^\omega\right)\right] = \sum_\omega \pi^\omega \frac{u'(c^\omega)}{p^\omega} r_i^\omega \quad \begin{cases} = 0 & \Leftrightarrow \quad z_i > 0 \\ < 0 & \Leftrightarrow \quad z_i = 0 \end{cases}$$

i) investors select the assets which are traded (z_i>0) and those who are not (z_i=0)

ii) they determine the Equivalent Martingale Measure (EMM)

$$q^\omega = \pi^\omega \frac{u'(c^\omega)}{Qp^\omega}, \qquad Q = \sum_\omega \pi^\omega \frac{u'(c^\omega)}{p^\omega}$$

A creative financial sector

- Financial instruments are drawn at random from a probability distribution with

$$E_\pi\left[r_i\right] = \sum_\omega \pi^\omega r_i^\omega = -\frac{\epsilon}{\Omega}, \qquad \mathrm{Var}\left[r_i\right] = \frac{1}{\Omega}, \quad i = 1, \ldots, N$$

- Key variables:
  - financial complexity: $n=N/\Omega$
  - risk premium: ε
- Note: Successful innovations ($z_i>0$) are not independent draws

Theory: statistical mechanics

Typical behavior of self-averaging quantities

(De Martino et al. Macroecon. Dyn. 2007)

$$\lim_{\Omega\to\infty}\left\langle \max_{\vec{z}\geq 0} E[u(c^{\omega})]\right\rangle_{\vec{p},\hat{a}} = \lim_{\beta\to\infty}\lim_{\Omega\to\infty}\frac{1}{\beta}\langle \log Z(\beta)\rangle_{\vec{p},\hat{a}}$$

1- The partition function $Z(\beta) = \sum_{\{\vec{z}\geq 0\}} e^{\beta u[c^{\omega}(\vec{z})]}$

2- The replica trick $\langle \log Z\rangle_{\vec{p},\hat{a}} = \lim_{r\to 0}\frac{1}{r}\log\langle Z^r\rangle_{\vec{p},\hat{a}}$

3- For integer r

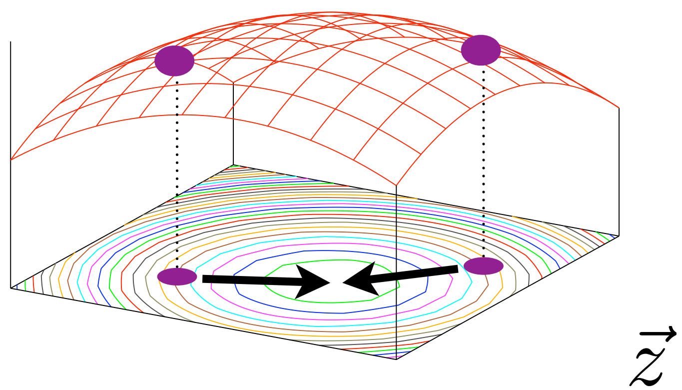
$$\begin{aligned}\langle Z^r\rangle_{\vec{p},\hat{a}} &= \sum_{\{\vec{z}_1\geq 0\}}\cdots\sum_{\{\vec{z}_r\geq 0\}}\left\langle e^{\beta\sum_{a=1}^{r} u[c^{\omega}(\vec{z}_a)]}\right\rangle_{\vec{p},\hat{a}} \\ &= \int d\hat{\Phi}\, e^{r\beta\nu(r,\beta,\hat{\Phi})}\end{aligned}$$

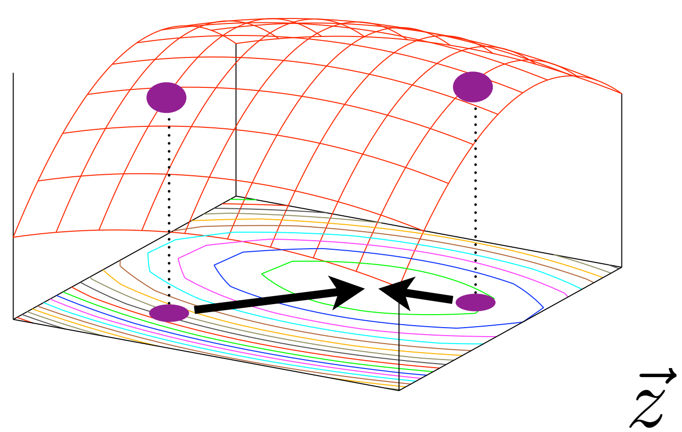
$\hat{\Phi}$ = order parameters

4- Saddle point:

$$\lim_{\Omega\to\infty}\left\langle \max_{\vec{z}\geq 0} E[u(c^{\omega})]\right\rangle_{\vec{p},\hat{a}} = \lim_{\beta\to\infty}\lim_{r\to 0}\max_{\hat{\Phi}}\nu(r,\beta,\hat{\Phi})$$

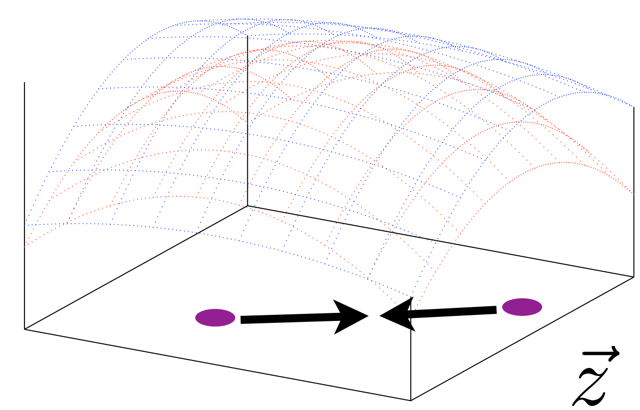
Theory: intuition





Average on

samples



Two approximate solutions converge to the same point which depends on the sample

Two approximate solutions attract each other

The typical behavior

- Observables:

susceptibility $\chi = \lim_{\beta\to\infty} \frac{\beta}{2N}\sum_{i=1}^{N}(z_{i,a}-z_{i,b})^2 = \frac{1}{N}\sum_i \frac{\delta z_i}{\delta p_i^0}$

EMM dispersion $\sigma = |q-\pi|$

market completeness $\phi = |\{i: \ z_i > 0\}|/\Omega$

volume (or revenue) $V = \sum_i z_i$

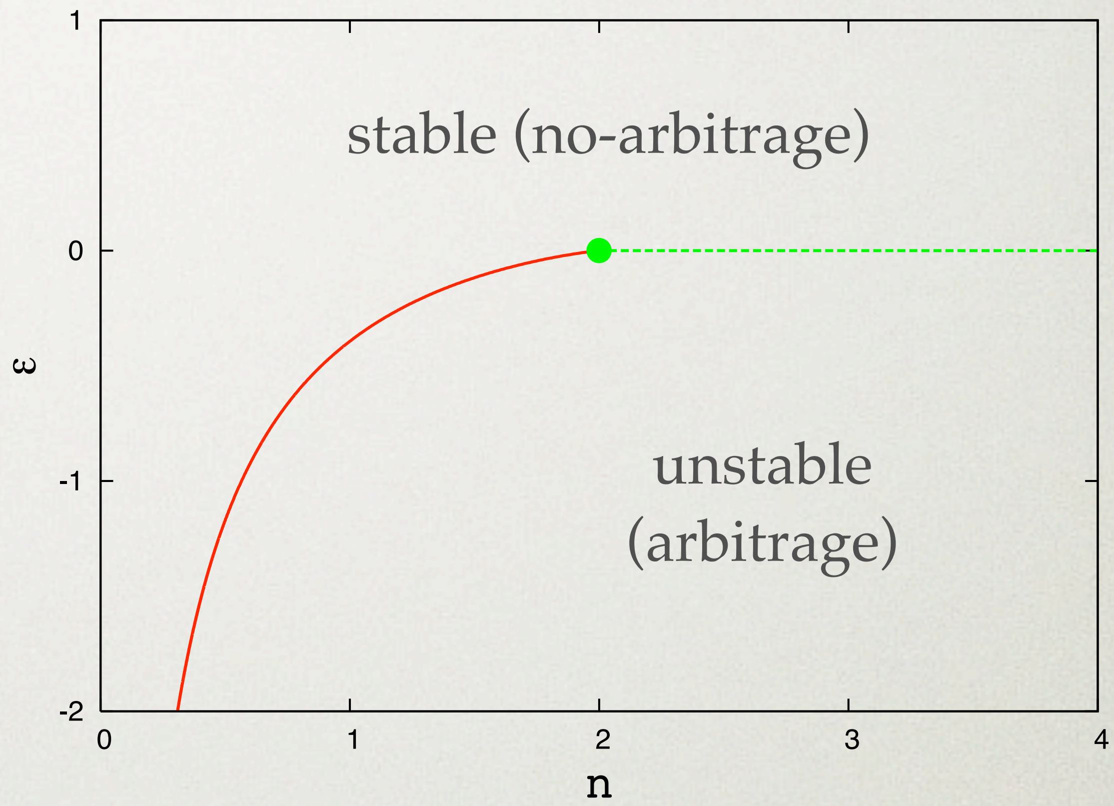
- Consistency relations

Conservation $1 = \langle c^* p\rangle_{t,p} + \epsilon n \langle z^*\rangle_t$

no-arbitrage $E_q[c^\omega p^\omega] = E_q[1] = 1$

Phase diagram

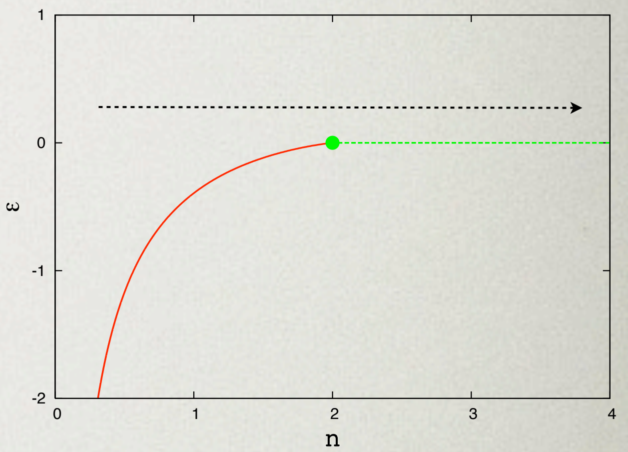
Independent of u(c) & p

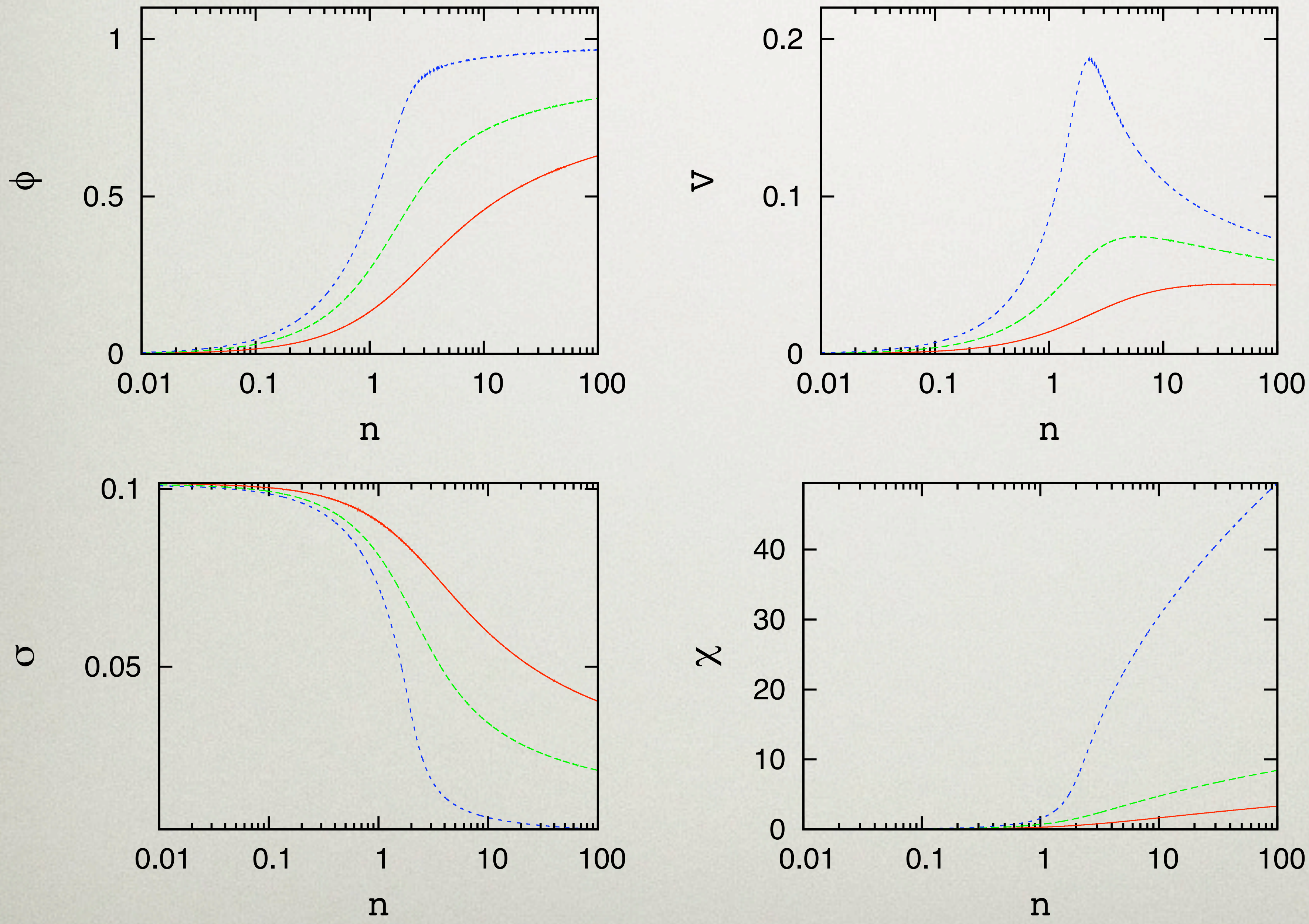


- $\chi \longrightarrow \infty \; \forall \; \varepsilon$
- $\sigma \longrightarrow 0$ for $\varepsilon > 0$
- $\sigma \longrightarrow \infty$ for $\varepsilon < 0$
- For $\varepsilon > 0$
  singularity = complete market ($\varepsilon = 0$, $n > 2$)
- For $\varepsilon < 0$
  singularity < complete market

Increasing financial complexity

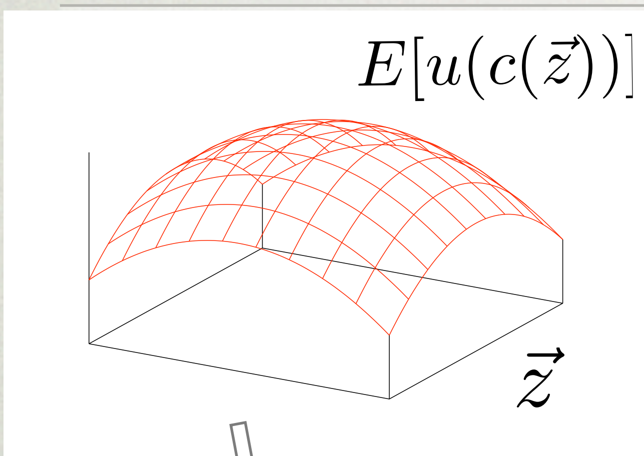
$\epsilon = 0.01, 0.05, 0.10$

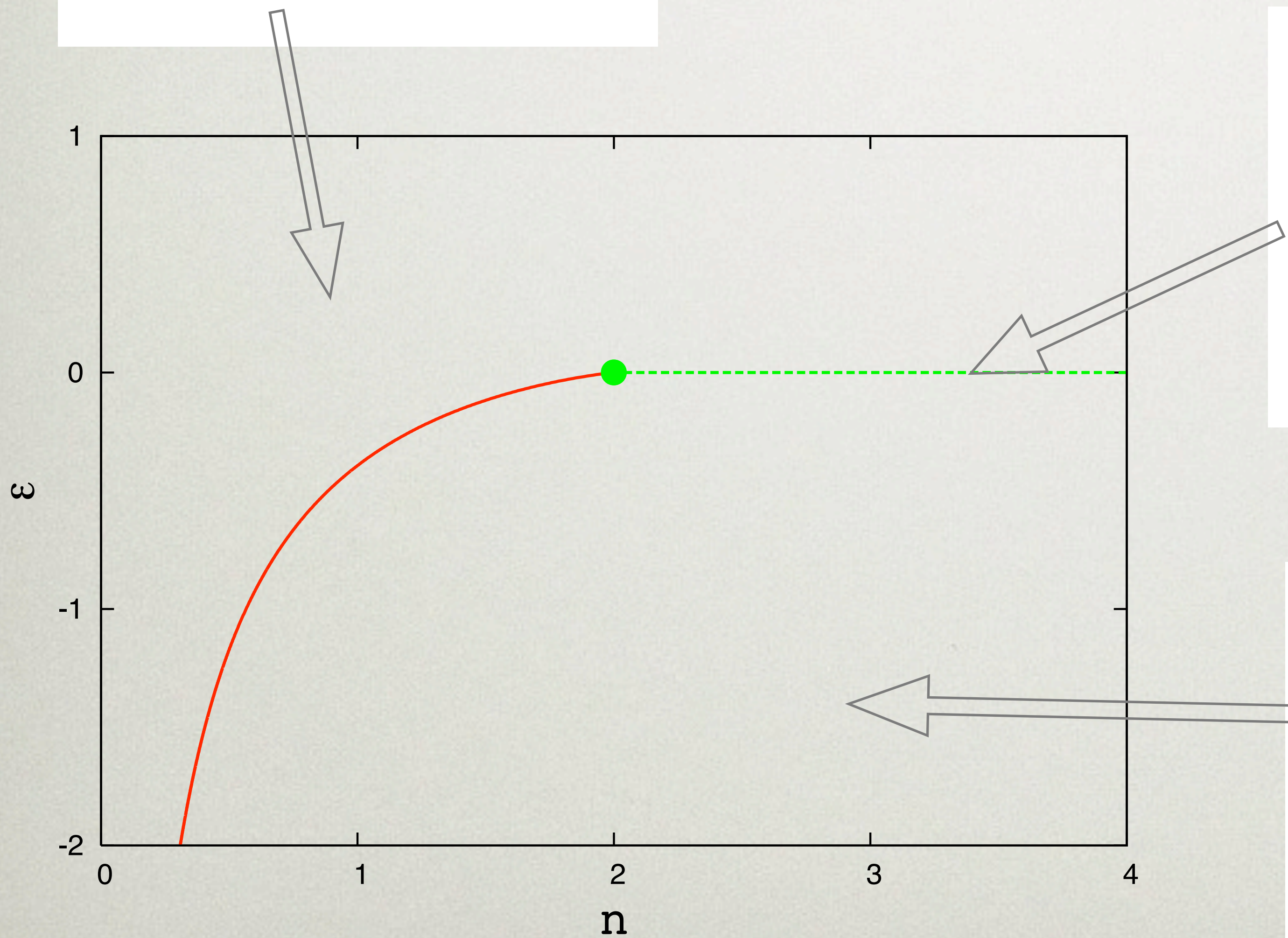


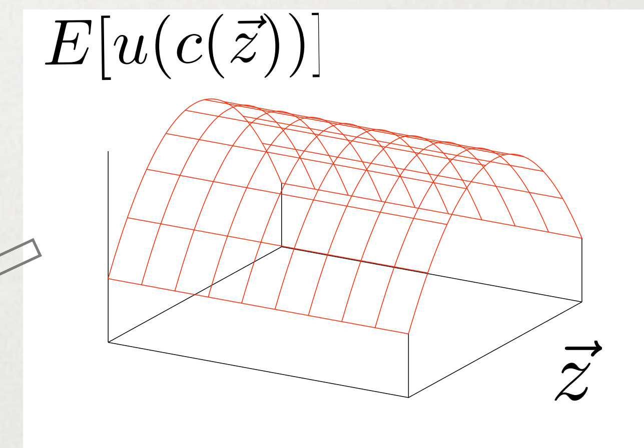


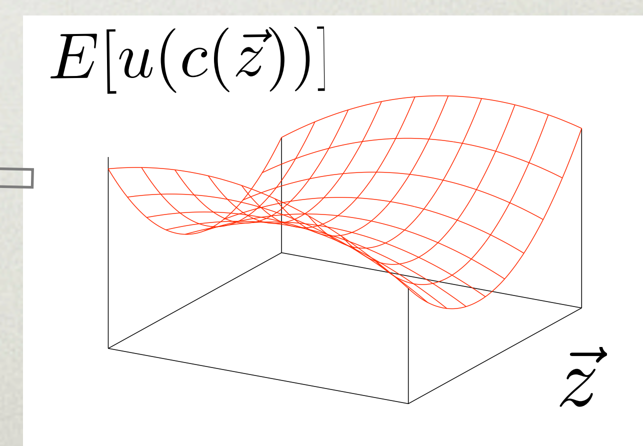
Intuition: landscape E[u]

- Ω free variables ($z_i>0$), Ω constraints
  $\varepsilon<0 \Rightarrow$ unstable directions can appear (arbitrages)







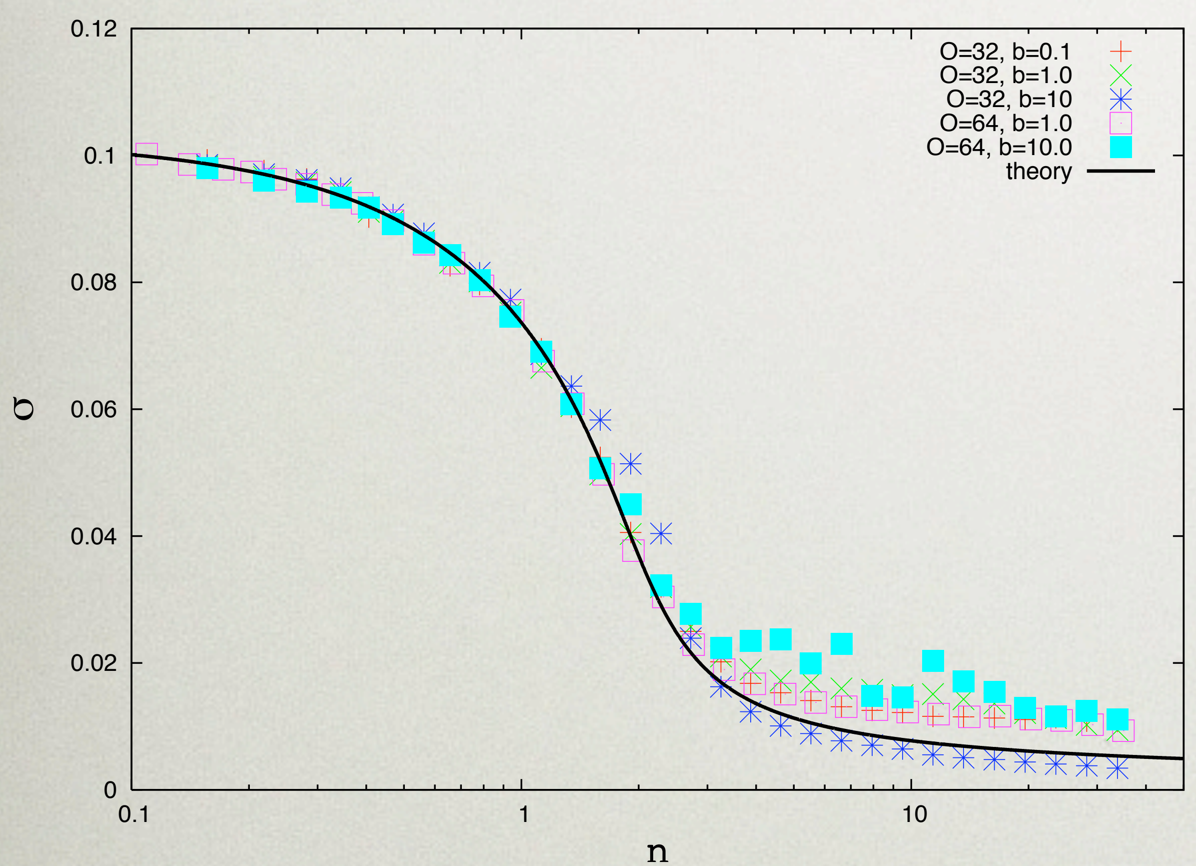


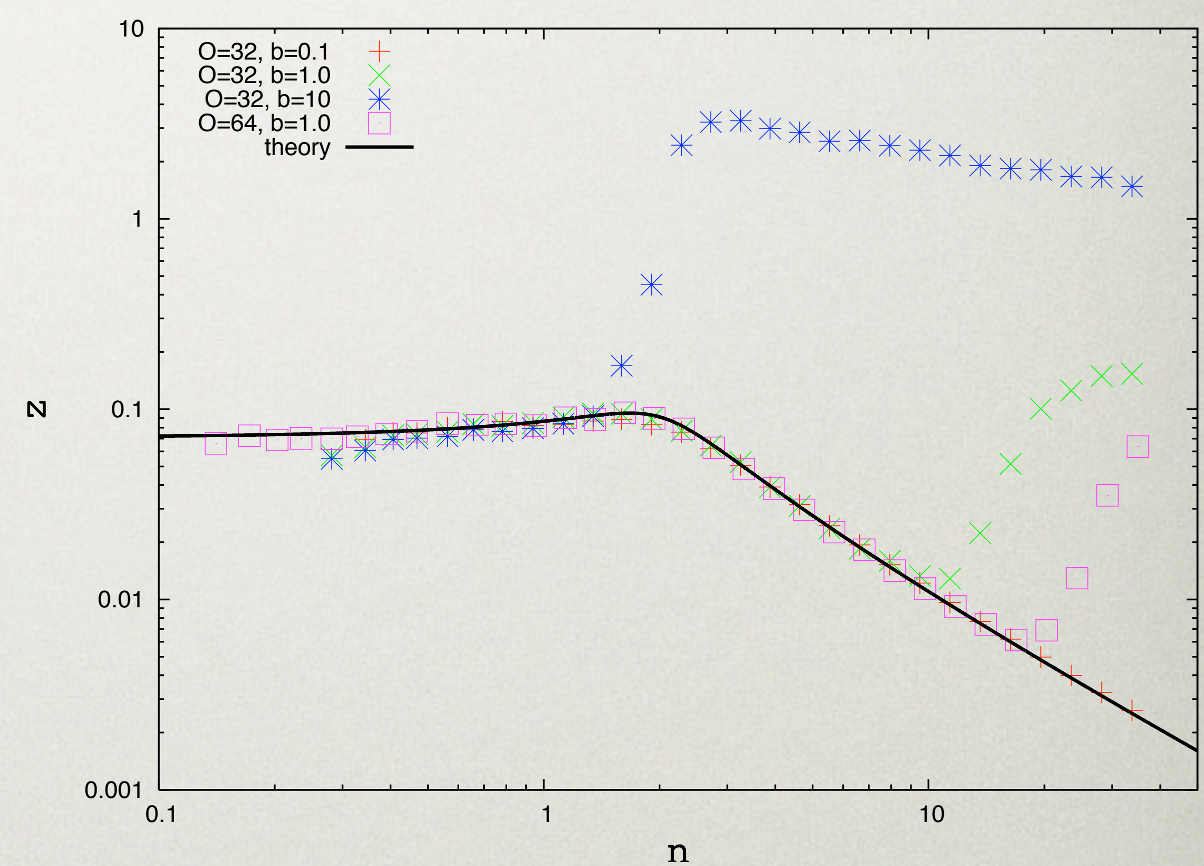
Learning to invest

$$\epsilon = 0.01, \quad \gamma = 0.5, \quad \Omega = 32$$

Hard to learn when market is nearly complete

(cfr Brock, Hommes, Wagener, 2006)





$$\sigma^2 = \frac{1}{\Omega} \sum_{\omega} (q^\omega - \bar{q})^2 \qquad \langle z \rangle = \frac{1}{N} \sum_i z_i$$

A competitive Financial Industry

- Part of the risk of a new instrument can be hedged buying existing instruments
- Residual risk

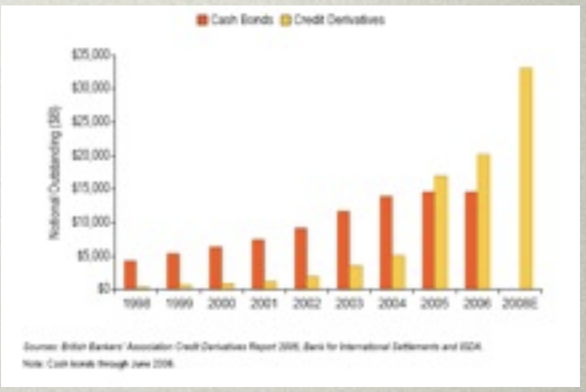
$$\Sigma = \min_{\vec{u}} \mathrm{Var}\left[r^{\omega}_{\mathrm{new}} - \sum_i v_i r^{\omega}_i \right] = 1 - \phi$$

- In competitive market, risk premium vanishes as $\phi \to 1$ e.g. Mean Variance profit function

$$\Rightarrow \quad \epsilon = \frac{\gamma}{2}(1-\phi)$$

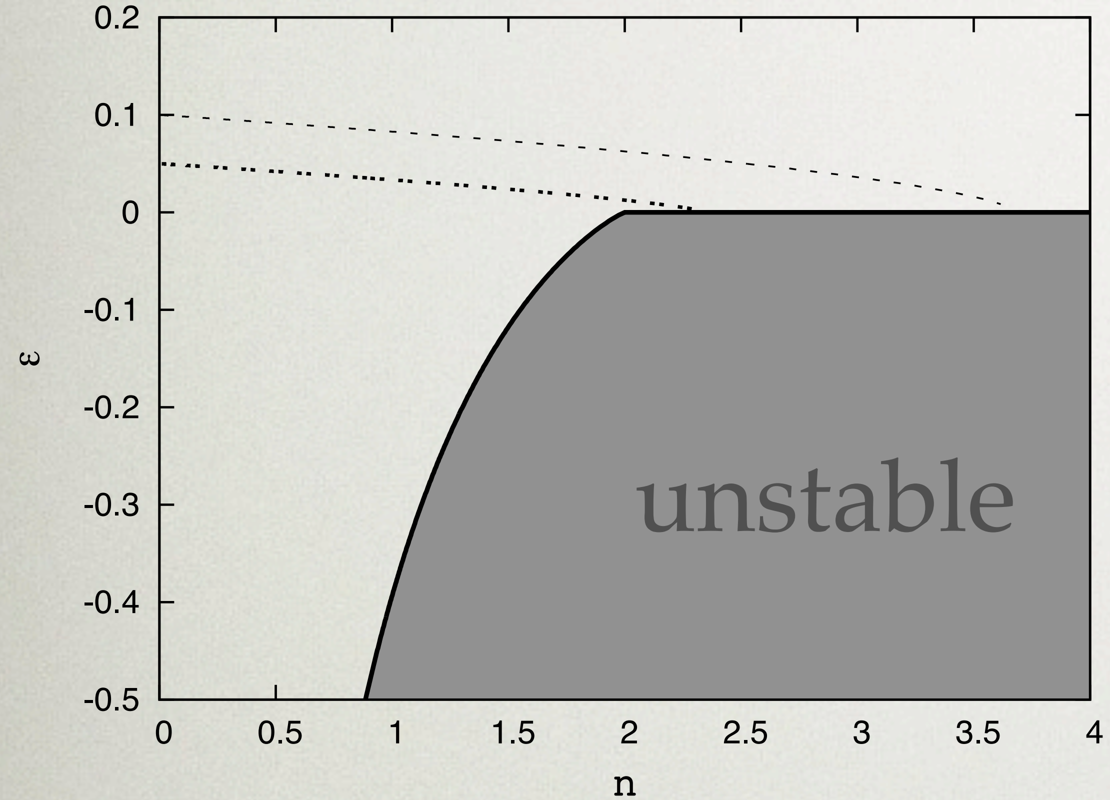
- The weights of portfolios used to hedge each instrument diverges as $\phi \to 1$

$$\sum_i v_i^2 = \frac{\phi}{1-\phi}$$

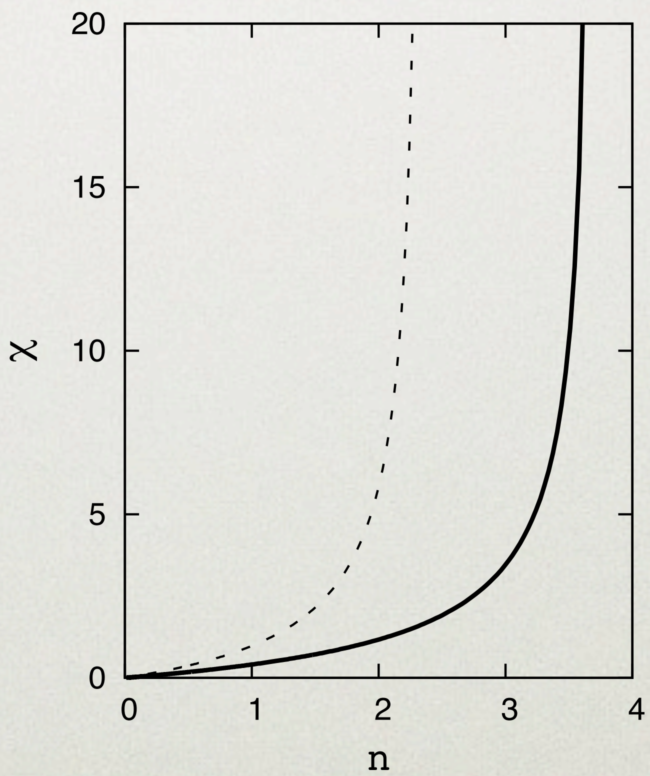


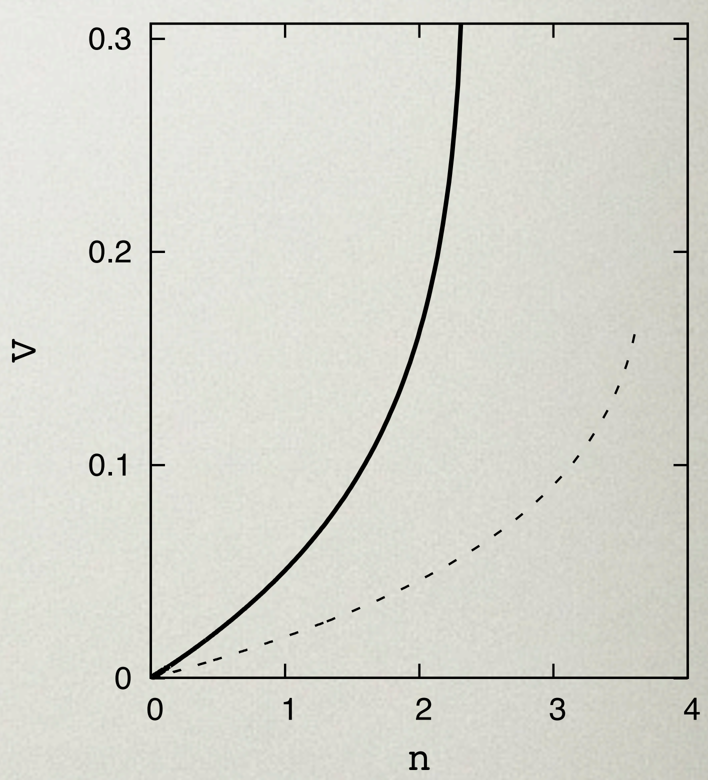
- Susceptibility in the interbank market also diverges

Mean variance banks $\epsilon = \frac{\gamma}{2}\Sigma$



Consumer market:
infinite susceptibility, finite volume





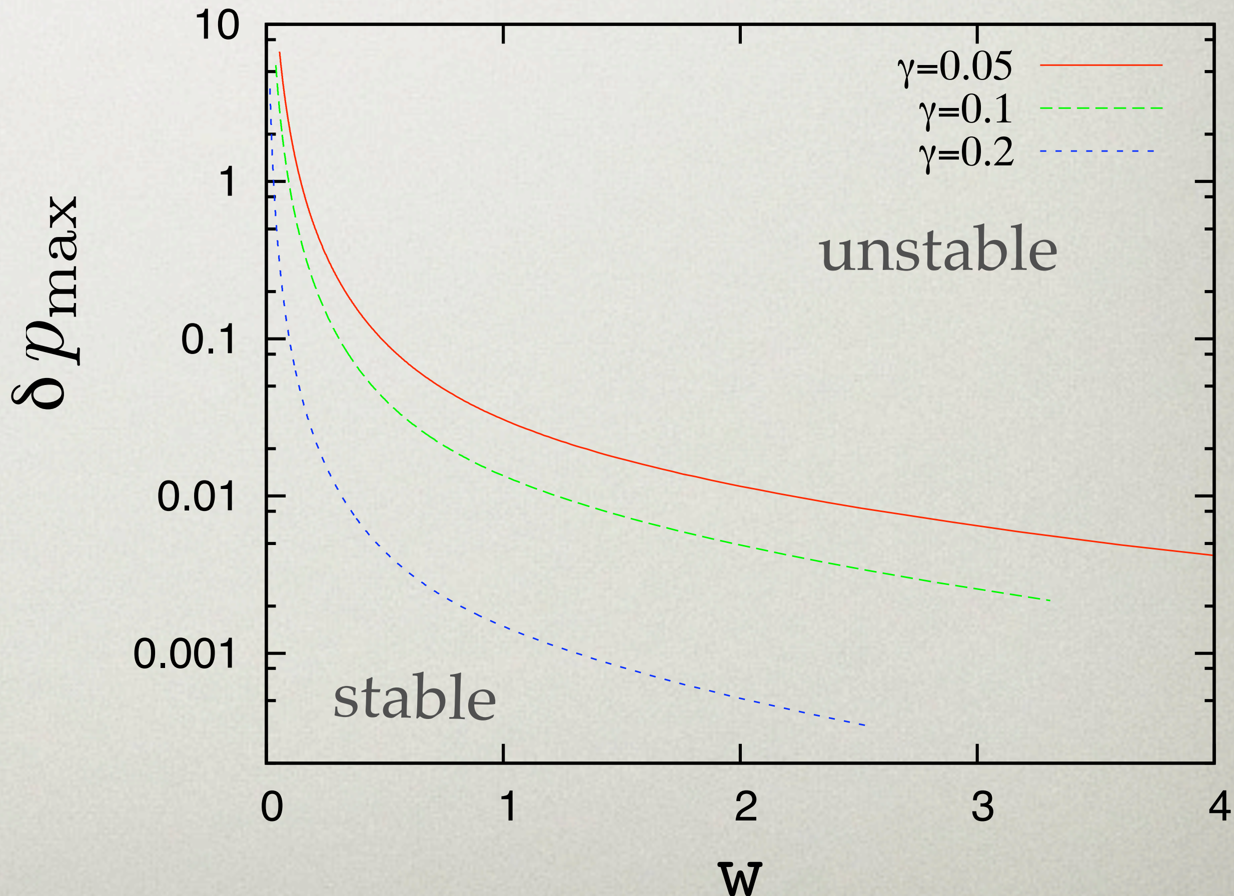
Interbank market:
both susceptibility and volumes diverge as $\phi \to 1$

Stability and the size of financial markets

- Relative size of financial markets ≈ volume of trading for hedging one unit of a new asset $w = \sqrt{\sum_i v_i^2}$
- Financial stability: → price uncertainty $$\frac{\delta z}{z} = \frac{1}{z}\frac{\delta z}{\delta p}\delta p = \frac{\chi}{z}\delta p \ll 1$$

$$\delta p_{\max} = \frac{z}{\chi}$$

- Stability diagram on a given trajectory in (n,ε) plane.



Toward illiquid markets: underlying and derivatives

$$
\begin{array}{cccccc}
\text{demand} & & & & & \\
z_1 & r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\
\vdots & \vdots & \ddots & \vdots & & \vdots \\
z_k & r_k^1 & \cdots & r_k^\omega & \cdots & r_k^\Omega \\
\vdots & \vdots & & \vdots & \ddots & \vdots \\
z_N & r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \\
\zeta_1 & f_1^1 & \cdots & f_1^\omega & \cdots & f_1^\Omega \\
\vdots & \vdots & & \vdots & & \vdots \\
\zeta_H & f_H^1 & \cdots & f_H^\omega & \cdots & f_H^\Omega
\end{array}
$$

Derivatives:

$$f_h^\omega = F_h(r_1^\omega, \ldots, r_N^\omega) - f_h^0$$

Return of underlying:

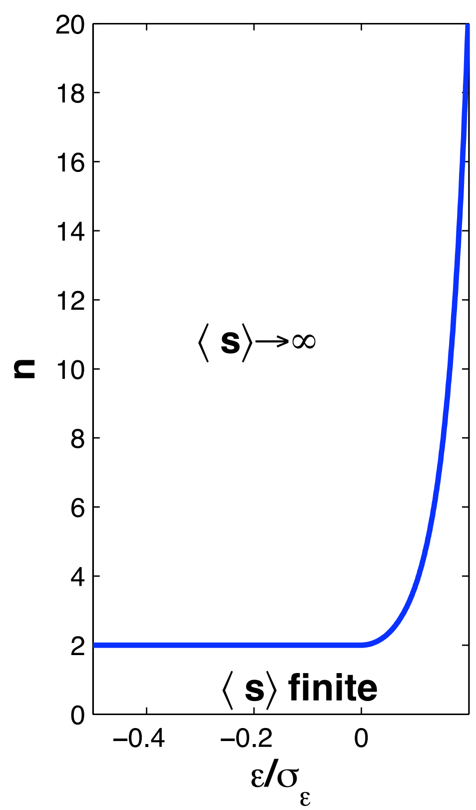
$$r_k^\omega = \rho(z_k, \zeta_1, \ldots, \zeta_H)$$

Price of derivatives:

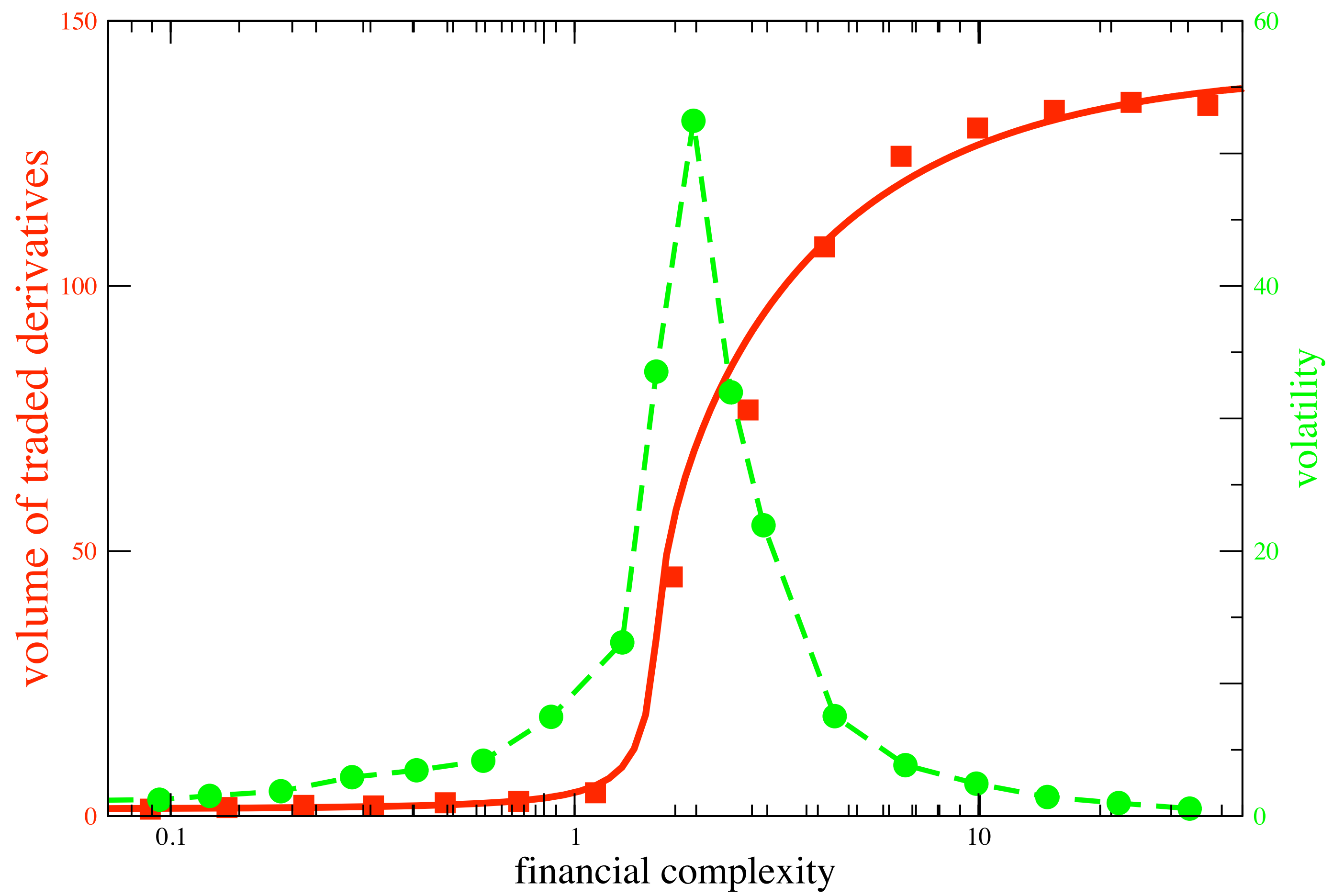
$$f_h^0(z_1, \ldots, z_N, \zeta_1, \ldots, \zeta_H)$$

N derivatives, one underlying

Phase transition
from supply limited
to demand limited



Susceptibility $\chi \to \infty$
instability in underlying market



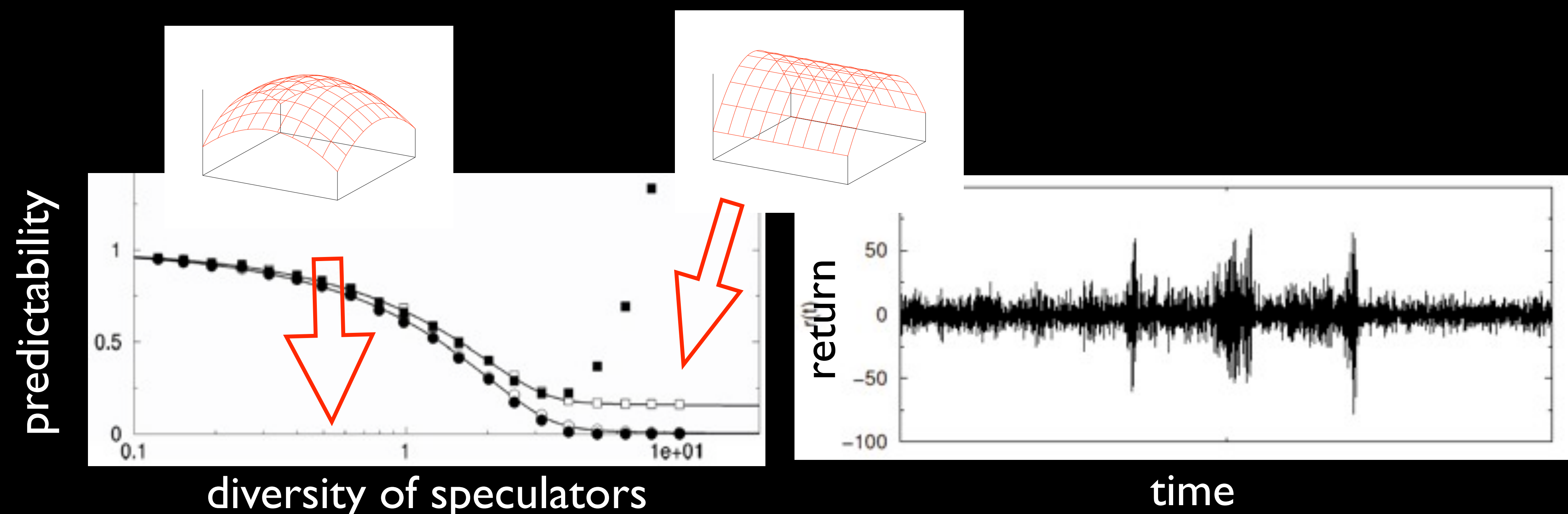
(F. Caccioli, M. Marsili, P. Vivo EPJB 2009)

Conclusions

- The proliferation of financial instruments, even in an ideal world (perfect competition and full information), leads to systemic instability
  - Complete markets lie on a critical line with infinite susceptibility
  - A competitive financial sector is expected to converge to this singularity
  - The volume generated by banks to hedge financial instruments they sell diverges as markets approaches completeness
  - Learning to invest optimally is hard (Brock, Hommes, Wagener 2006)
- The larger (and more complex) the financial market is, the more price indeterminacy is problematic
  - Institution should grow in size with financial complexity
  - Quantitative measure of financial stability based on price indeterminacy and relative size of financial sector?

Financial complexity and market information efficiency

- Markets as information “food chain” (e.g. Minority Games)



Excess volatility as signature of market information efficiency (Challet, MM, Zhang ’05)

Market impact matters (markets are always illiquid)

Financial stability as a public good

- how much are we prepared to pay for
  - Complete markets?
  - Informationally efficient markets?
  - Perfect competition?
  - ...