Graphene: Strong coupling physics in a nearly perfect quantum liquid

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Outline

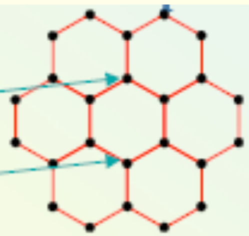
- Relativistic physics in graphene, quantum critical systems and conformal field theories
- Strong coupling features in collision-dominated transport
- Comparison and similarities with strongly coupled fluids (via AdS-CFT)
- Graphene: a super-low viscosity, i.e., “almost perfect” quantum liquid!

Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms

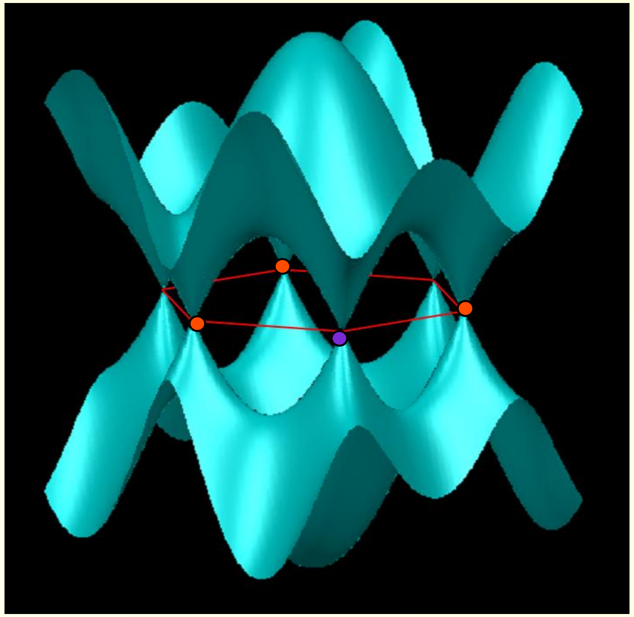
$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$



$$\hat{H} = v\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix} = v\vec{\sigma}\cdot\vec{p}$$

$$\mathbf{p} = \mathbf{k} - \mathbf{K} \quad \rightarrow \quad E_{\mathbf{p}} = \mathrm{v}_F|\mathbf{p}|$$

Tight binding dispersion



2 massless Dirac cones in the Brillouin zone:
(Sublattice degree of freedom ↔ pseudospin)

Fermi velocity (speed of light")

$$\mathrm{v}_F \approx 1.1\cdot 10^6\ \mathrm{m/s} \approx \frac{c}{300}$$

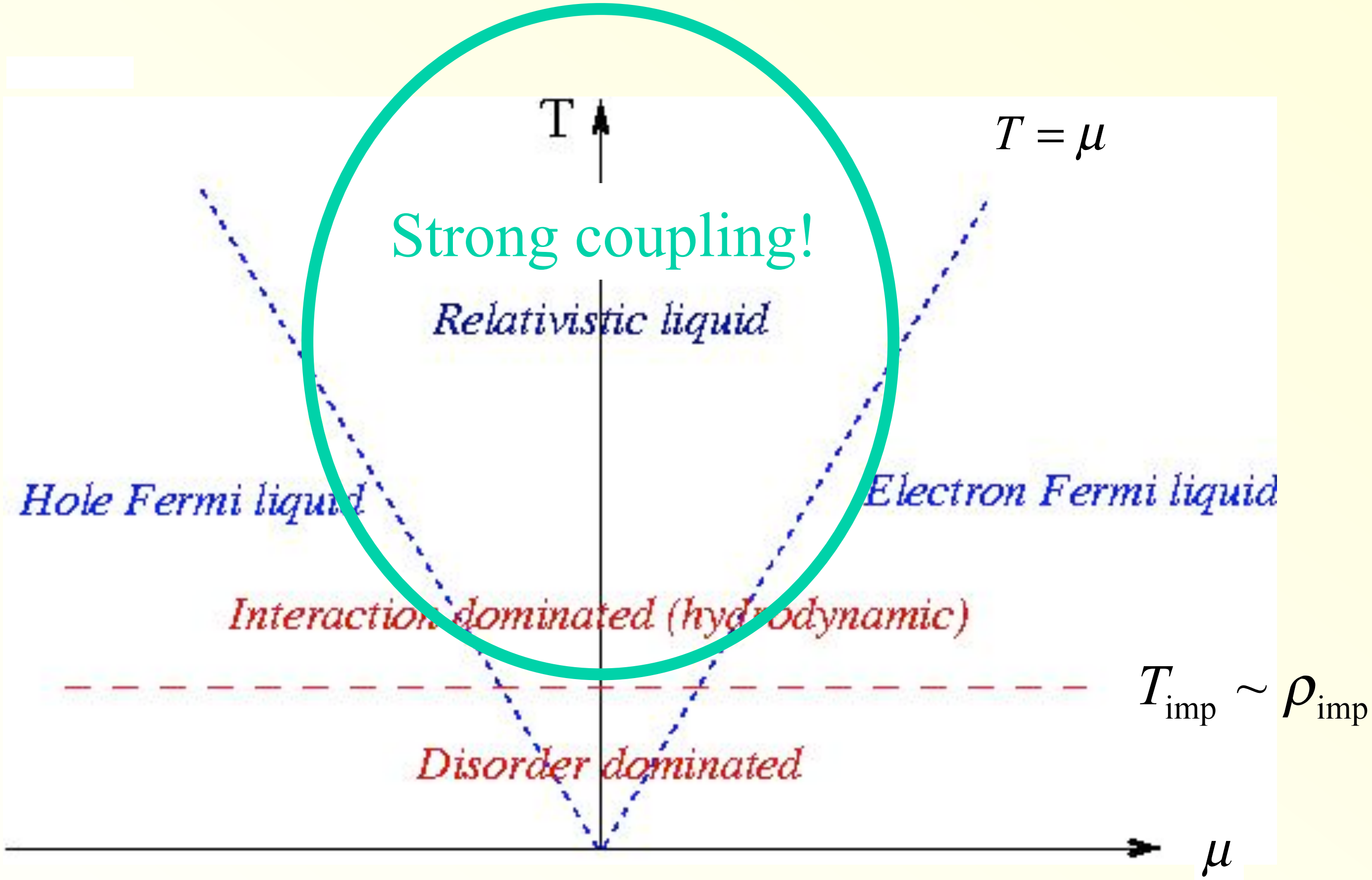
Coulomb interactions: Fine structure constant

$$\alpha \equiv \frac{e^2}{\varepsilon\,\hbar \mathrm{v}_F} = \frac{2.2}{\varepsilon}$$

Relativistic fluid at the Dirac point

D. Sheehy, J. Schmalian, Phys. Rev. Lett. 99, 226803 (2007).

- Relativistic plasma physics of interacting particles and holes!
- Strongly coupled, nearly quantum critical fluid at $\mu = 0$



Very similar as for **quantum criticality** (e.g. SIT) and in their associated CFT's

Other relativistic fluids:

- Bismuth (3d Dirac fermions with very small mass)

- Systems close to quantum criticality (with z = 1)

Example: Superconductor-insulator transition (Bose-Hubbard model)

Maximal possible relaxation rate!

$$\tau_{rel}^{-1} \approx \frac{\hbar}{k_B T}$$

Damle, Sachdev (1996)
Bhaseen, Green, Sondhi (2007).
Hartnoll, Kovtun, MM, Sachdev (2007)

- Conformal field theories (critical points)

E.g.: strongly coupled Non-Abelian gauge theories (akin to QCD):
→ Exact treatment via AdS-CFT correspondence!

C. P. Herzog, P. Kovtun, S. Sachdev, and D. T. Son (2007)
Hartnoll, Kovtun, MM, Sachdev (2007)

Are Coulomb interactions strong?

Fine structure constant (QED concept)

$$\alpha \equiv \frac{e^2}{\varepsilon \hbar v_F} = \frac{2.2}{\varepsilon}$$

Large!

r_s (Wigner crystal concept)

$$r_s \equiv \frac{E_{Cb}(n)}{E_F(n)} = \frac{\sqrt{n}\, e^2/\varepsilon}{\hbar v_F \sqrt{\pi n}} = \frac{\alpha}{\sqrt{\pi}}$$

Small!?

n-independent! (↔ Cb is marginal)

- The coupling strength α depends on the scale.
- Different systems exhibit different scale behavior!

α is the high energy limit of the coupling.
But we care about α(T)!

Are Coulomb interactions strong?

Coulomb interactions:
Unexpectedly strong!
$\longrightarrow$ nearly quantum critical!

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon|\mathbf{q}|}$$

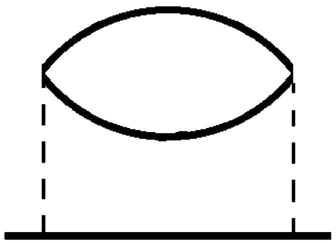
$$H_1 = \frac{1}{2}\int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2q}{(2\pi)^2}\Psi_a^\dagger(\mathbf{k}_2-\mathbf{q})\Psi_a(\mathbf{k}_2)V(\mathbf{q})\Psi_b^\dagger(\mathbf{k}_1+\mathbf{q})\Psi_b(\mathbf{k}_1)$$

RG: ($\mu = 0$)

$$\frac{d\alpha}{d\ell} = -\frac{\alpha^2}{4} + \mathcal{O}(\alpha^3)$$

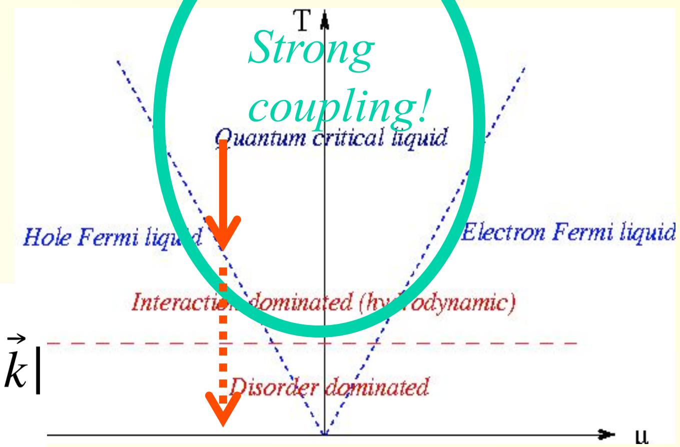
$$\alpha \equiv \frac{e^2}{\varepsilon\hbar v_F} = O(1)$$

$$\alpha(T) = \frac{\alpha^0}{1+(\alpha^0/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$



$$\text{Im}\,\Sigma(\omega,\vec{k}) = \frac{1}{48}\left(\frac{e^2}{\epsilon_0\hbar v_F}\right)^2 \hbar v_F|\vec{k}|$$

RG flow of v_F $\leftrightarrow$ RG flow of α



Gonzalez et al., PRL 77, 3589 (1996)
Two loop: Vafek+Case

Coulomb only marginally irrelevant for $\mu = 0$!

But: ($\mu > 0$)

For $T < \mu$: screening kicks in, short ranged Cb irrelevant

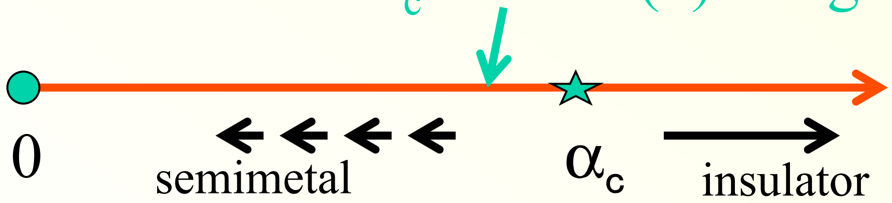
Are Coulomb interactions strong?

• Several studies suggest proximity of a quantum critical point around $\alpha_c = O(1)$ between a Fermi liquid and a gapped insulator.

- 2-loop RG *(Vafek+Case, Herbut et al.)*
- large N expansion (N = 4 = 2*2 flavors) *(Son, Herbut)*
- Gap generation at strong coupling *(Khveshchenko et al)*
- Lattice simulations *(Drut+Lähde, Hands+Strothos)*

• Fractional QHE in suspended graphene indicates rather strong Coulomb interactions.

Approach taken here: $\alpha_c > \alpha = O(1)$ marginally irrelevant



Strong coupling in undoped graphene

MM, L. Fritz, and S. Sachdev, PRB '08.

Inelastic scattering rate
(Electron-electron interactions)

μ > T: standard 2d Fermi liquid

$$\tau_{ee}^{-1} \sim \alpha^2 \frac{k_B T^2}{\hbar \mu}$$

Relaxation rate ~ T,
like in quantum critical systems!
Fastest possible rate!

μ < T: strongly coupled relativistic liquid

$$\tau_{ee}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar}$$

"Heisenberg uncertainty principle for well-defined quasiparticles"

$$E_{qp}\left(\sim k_B T\right) \geq \Delta E_{\text{int}} = \hbar \tau_{ee}^{-1} \sim \alpha^2 k_B T$$

As long as α(T) ~ 1, energy uncertainty is saturated, scattering is maximal
→ Nearly universal strong coupling features in transport,
similarly as at the 2d superfluid-insulator transition *[Damle, Sachdev (1996, 1997)]*

Consequences for transport

1. -Collisionlimited conductivity σ in clean undoped graphene;
   -Collisionlimited spin diffusion D_s at any doping
2. Graphene - a perfect quantum liquid: very small viscosity η!
3. Emergent relativistic invariance at low frequencies!
   Despite the breaking of relativistic invariance by
   - finite T,
   - finite μ,
   - instantaneous 1/r Coulomb interactions

Collision-dominated transport → relativistic hydrodynamics:
a) Response fully determined by covariance, thermodynamics, and σ, η
b) Collective cyclotron resonance in small magnetic field (low frequency)

Hydrodynamic regime:
(collision-dominated)

$$\tau_{ee}^{-1} >> \tau_{imp}^{-1}, \omega_c^{typ}, \omega_{AC}$$

Collisionlimited conductivities

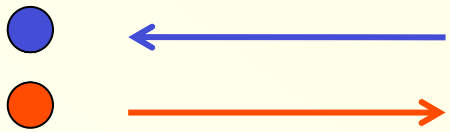
Damle, Sachdev, (1996).
Fritz et al. (2008), Kashuba (2008)

Finite charge [or spin] conductivity in a pure system (for μ = 0 [or B = 0]) !

- Key: Charge or spin current without momentum

(particle [spin up])

(hole [spin down])



$$\vec{J}_{s,c} \neq 0, \text{ but } \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite collision-limited conductivity! $\sigma(\mu = 0) < \infty \quad ; \quad \sigma(\mu \neq 0) = \infty$
- Finite collision-limited spin diffusivity! $D_s(\mu; B = 0) \propto v_F^2 \tau_{ee} < \infty,$
- Only marginal irrelevance of Coulomb: Maximal possible relaxation rate ~ T

$$\tau_{ee}^{-1} \approx \alpha^2 \frac{k_B T}{\hbar}$$

→ Nearly universal conductivity at strong coupling

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \to \sigma(\mu = 0) \sim \frac{e}{k_B T / \mathrm{v}^2}\left(e\frac{(k_B T)^2}{(\hbar \mathrm{v})^2}\right)\frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

Expect saturation as α →1, and eventually phase transition to insulator

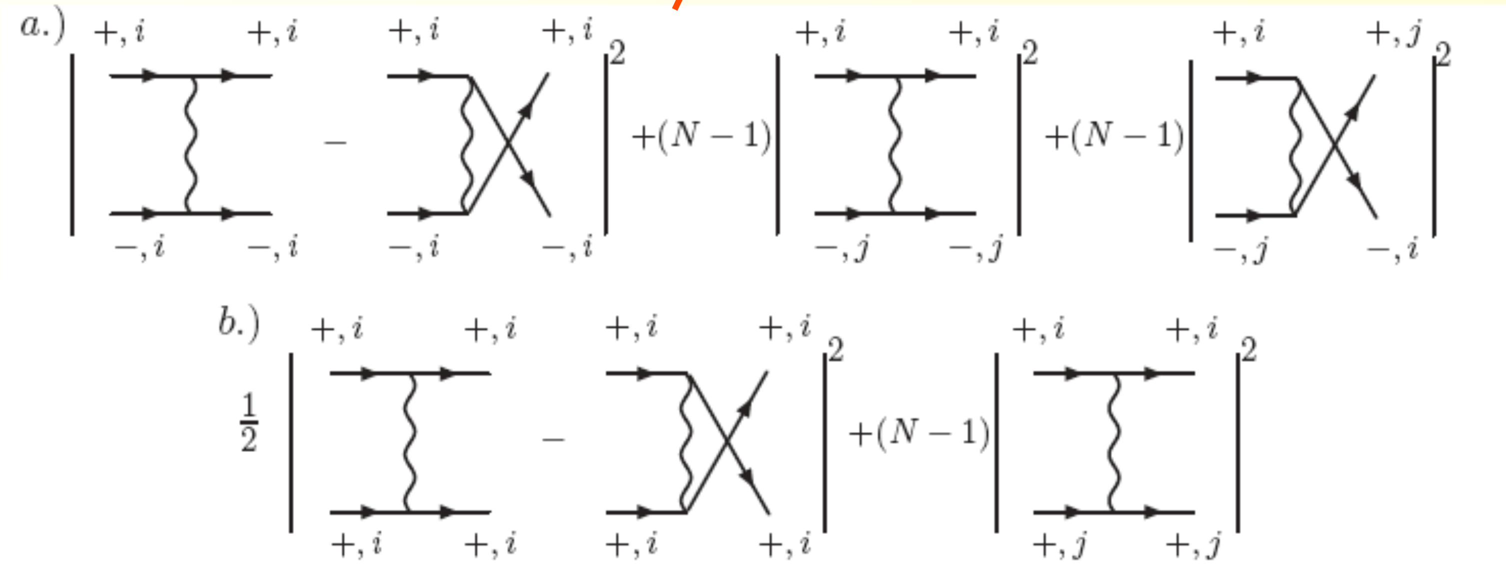
Marginal irrelevance of Coulomb: $\alpha \approx \frac{4}{\log(\Lambda/T)} < 1$

Boltzmann approach

L. Fritz, J. Schmalian, MM, and S. Sachdev, PRB 2008

Boltzmann equation in Born approximation

$$\left(\partial_t + e\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right] \propto \alpha^2(T)$$



→ Collision-limited conductivity: $\sigma(\mu=0) = \dfrac{0.76}{\alpha^2(T)}\dfrac{e^2}{h}$

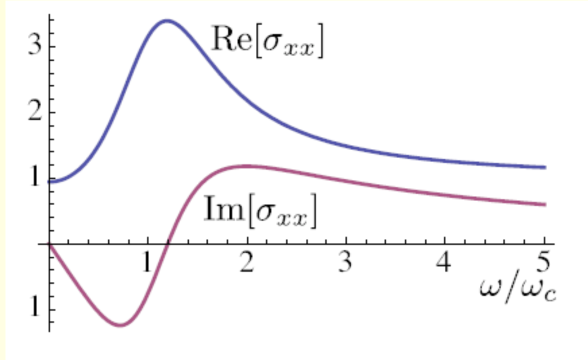
→ Obtain collision-dominated transport: Emergent relativistic (covariant!) hydrodynamics confirmed! *MM, L. Fritz, and S. Sachdev, PRB 2008*

Collective cyclotron resonance

MM, and S. Sachdev, PRB 2008

Relativistic magnetohydrodynamics: pole in AC response

$$\sigma_{xx}(\omega) = \sigma_Q \frac{\omega\left(\omega + i\gamma + i\omega_c^2/\gamma\right)}{(\omega+i\gamma)^2 - \omega_c^2}$$

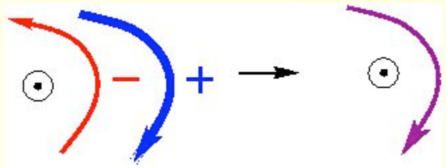


Pole in the response

$$\omega^* = \pm\omega_c - i\gamma$$

Collective cyclotron frequency of the relativistic plasma

$$\omega_c = \frac{\rho\, B/\mathrm{c}}{(\varepsilon + P)/\mathrm{v}_\mathrm{F}^2} \leftrightarrow \omega_c^{\mathrm{FL}} = \frac{e\, B/\mathrm{c}}{m}$$



Broadening of resonance:

$$\gamma = \sigma_Q \frac{(B/c)^2}{(\varepsilon + P)/\mathrm{v}_\mathrm{F}^2}$$

Observable at room temperature in the GHz regime!

Transport beyond weak coupling approximation?

Recall:

Collision-limited conductivity:

$$\sigma(\mu = 0) = \frac{0.76}{\alpha^2(T)} \frac{e^2}{h}$$

Was obtained in weak coupling (Boltzmann quasiparticle approximation)

[Similar to ε-expansion in 3-ε for quantum critical superfluid-insulator systems] *(Damle and Sachdev)*

Can one do any better, at least in some cases? Yes - AdS/CFT !

Transport beyond weak coupling approximation??

Graphene transport

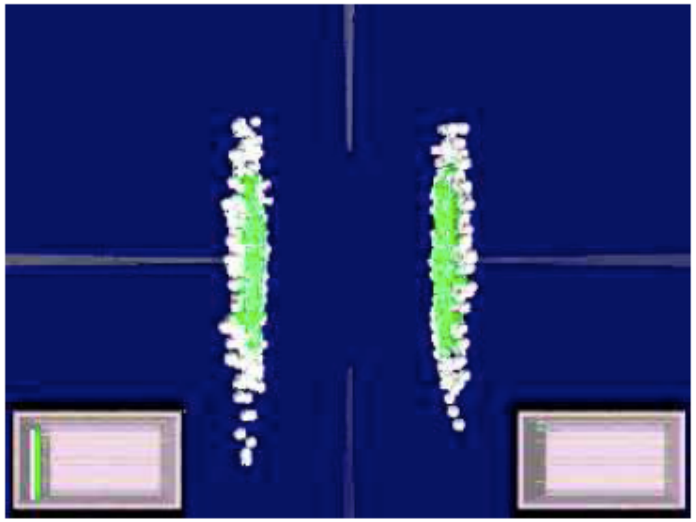
$\leftrightarrow$

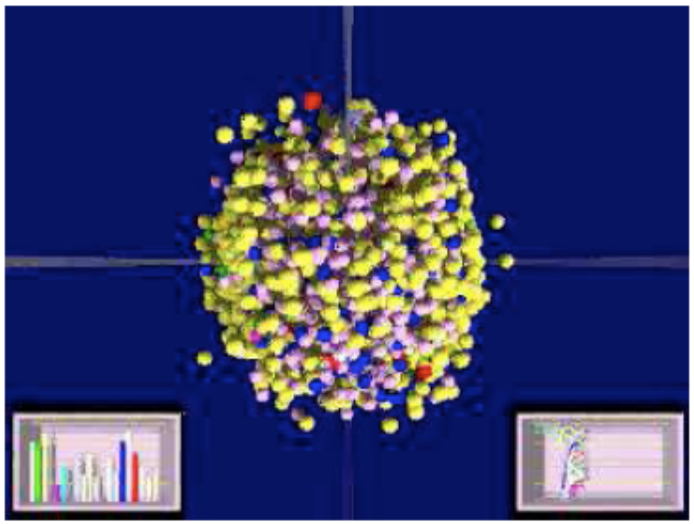
Very strongly coupled, critical relativistic liquids?

Solvable via AdS – CFT correspondence

Reviews: S. Sachdev, MM (2009), S. Hartnoll (2010)

Motivation: Nucleus collisions (RHIC)





Quark-gluon plasma

A strongly coupled relativistic fluid described by QCD

–

Experimental observation:
Very low-viscosity fluid!!
(a “perfect fluid”?)

Compare graphene to Strongly coupled relativistic liquids

S. Hartnoll, P. Kovtun, MM, S. Sachdev (2007)

Idea:

- Take SU(N) Yang Mills theory (relativistic and strongly coupled!)
- Obtain exact results via string theoretical AdS–CFT correspondence

  [Mapping a 2+1 CFT (quantum critical) onto a 3+1 gravity system]

  Duality: strong coupling to weak coupling
- Compare **phenomenology** with graphene [or generally: quantum critical systems]

→

- Confirm the results of hydrodynamics: response functions σ(ω), resonances
- Calculate the transport coefficients for a strongly coupled theory!

SU(N) Yang Mills: $\sigma(\mu=0)=\sqrt{\frac{2}{9}}N^{3/2}\frac{e^2}{h}; \quad \frac{\eta_{shear}}{s}(\mu=0)=\frac{1}{4\pi}\frac{\hbar}{k_B}$

Interpretation: $N^{3/2}$ effective degrees of freedom, strongly coupled: $\tau T = O(1)$

Graphene – a nearly perfect liquid!

$\longleftrightarrow$ *MM, J. Schmalian, and L. Fritz, (PRL 2009)*

Anomalously low viscosity (like quark-gluon plasma)

"Heisenberg" $$\frac{\eta}{s} \sim E_{qp}\tau \geq 1$$ → Measure of strong coupling:

Conjecture from AdS-CFT:

$$\frac{\text{shear viscosity}}{\text{entropy density}} = \frac{\eta}{s} > \frac{\hbar}{k_B}\frac{1}{4\pi}$$

• For a liquid there is a limit to its "perfection" with regard to its viscosity!

• There are no ideal fluids with $\eta = 0$!

Is there a "most perfect" liquid? Candidates must be quantum!

Graphene – a nearly perfect liquid!

MM, J. Schmalian, and L. Fritz, (PRL 2009)

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Doped Graphene & Fermi liquids:
(Khalatnikov etc)

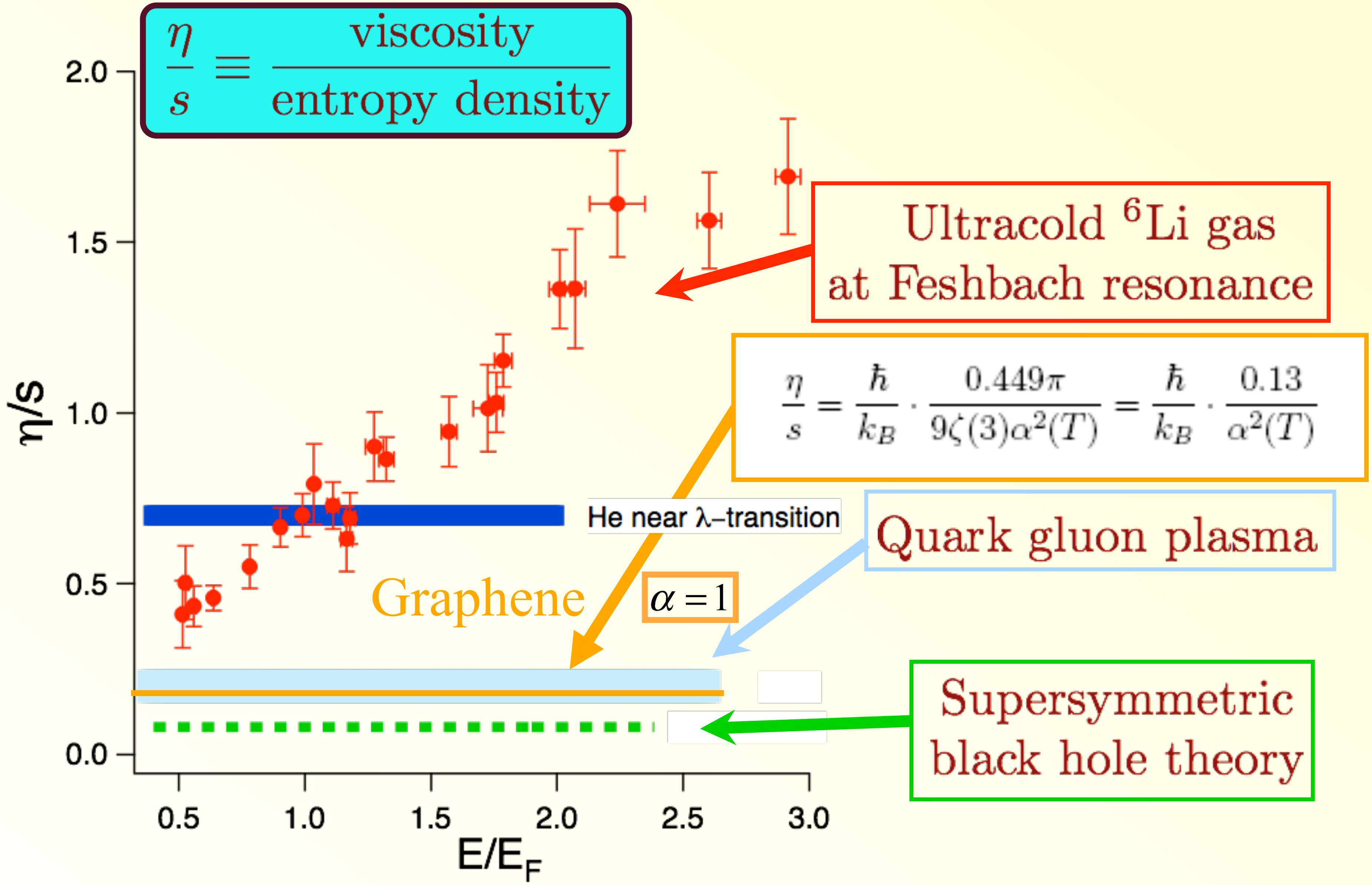
$$\eta \propto n \cdot m\mathrm{v}^2 \cdot \tau \;\rightarrow\; n \cdot E_F \cdot \frac{\hbar E_F}{(k_B T)^2} \qquad s \propto k_B n \frac{T}{E_F} \qquad \frac{\eta}{s} \sim \frac{\hbar}{k_B}\left(\frac{E_F}{T}\right)^3$$

Undoped Graphene:

$$\eta \propto n \cdot m\mathrm{v}^2 \cdot \tau \;\rightarrow\; n_{\mathrm{th}} \cdot k_B T \cdot \frac{\hbar}{\alpha^2 k_B T} = \frac{\hbar}{\alpha^2} n_{\mathrm{th}} \qquad s \propto k_B n_{\mathrm{th}}$$

Boltzmann-Born Approximation:

$$\frac{\eta}{s} = \frac{\hbar}{k_B} \cdot \frac{0.449\pi}{9\zeta(3)\alpha^2(T)} = \frac{\hbar}{k_B} \cdot \frac{0.13}{\alpha^2(T)}$$



T. Schäfer, Phys. Rev. A ***76****, 063618 (2007).*

A. Turlapov, J. Kinast, B. Clancy, Le Luo, J. Joseph, J. E. Thomas, J. Low Temp. Phys. ***150****, 567 (2008)*

Electronic consequences of low viscosity?

MM, J. Schmalian, L. Fritz, (PRL 2009)

Electronic turbulence in clean, strongly coupled graphene?
Reynolds number:

$$\mathrm{Re} = \frac{s/k_B}{\eta/\hbar} \times \frac{k_B T}{\hbar v/L} \times \frac{u_{\mathrm{typ}}}{v}$$

Flow parameters:

L: Typical linear size of system (width of a constriction)
u_{typ}: drift velocity of the driven electron-hole plasma

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MM, J. Schmalian, L. Fritz, (PRL 2009)

Electronic turbulence in clean, strongly coupled graphene?
Reynolds number:

$$\mathrm{Re} = \frac{s/k_B}{\eta/\hbar} \times \frac{k_B T}{\hbar v/L} \times \frac{u_{\mathrm{typ}}}{v}$$

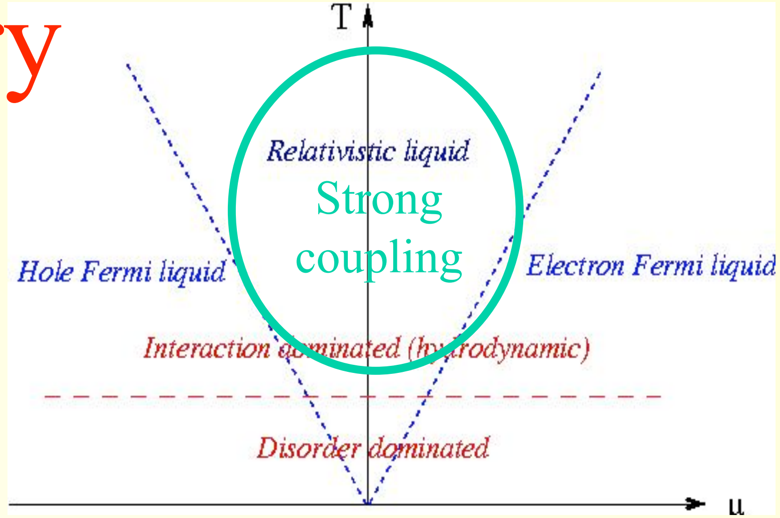
Strongly driven mesoscopic systems: (Kim's group [Columbia])

$L = 1\mu m$
$u_{typ} = 0.1\mathrm{v}$
$T = 100K$

→ $\mathrm{Re} \sim 10 - 100$

Complex fluid dynamics!
(pre-turbulent flow)
Shocks, intermittency etc?
New phenomenon in an electronic system!
Similar effect expected in quantum critical systems!

Summary



- Undoped graphene is strongly coupled in a large temperature window!

- Nearly universal strong coupling features in transport; many similarities with strongly coupled critical fluids (described by AdS-CFT)

- Emergent relativistic hydrodynamics at low frequency

- Graphene: Nearly perfect quantum liquid!
  → Complex (turbulent?) current flow at strong driving