

# Compensation driven superconductor-insulator transition

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ICTP, 20<sup>th</sup> May, 2009

# Superconductivity and Coulomb interactions

## Clean, granular systems

Josephson junction arrays  
(Fazio, Schön, ...)

$E_J > E_C \rightarrow$  Superconductivity

$E_J < E_C \rightarrow$  Insulator

Simply exponential **pair** transport (see K. Efetov's talk)

At lowest T:

$$\begin{array}{l} E_J > E_C \rightarrow G = G_0 \exp(T_0 / T) \\ E_J < E_C \rightarrow R = R_0 \exp(T_0 / T) \end{array}$$

What if there is strong disorder (generic)?  $\delta E_C \sim E_C$

Insulator: gap is destroyed  $\rightarrow$  a priori no simple activation!

What if there are no pre-structured grains?

Do “effective grains” form due to the disorder configuration?

SIT in strong disorder:

Localization and delocalization of  
Cooper pairs  
in Coulomb disorder

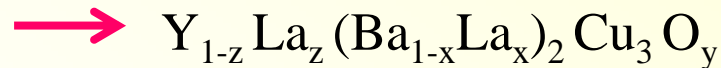
Similar analysis for neutral cold atoms in random disorder potentials, see  
Falko, Nattermann and Pokrovski (08); Shklovskii (08)

# Compensated high Tc materials

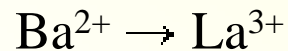
*K. Segawa and Y. Ando, PRB 74, 100508 (2006)*



Doping n-type carriers by La-substitution for Ba



n-type doping controlled by  $x$



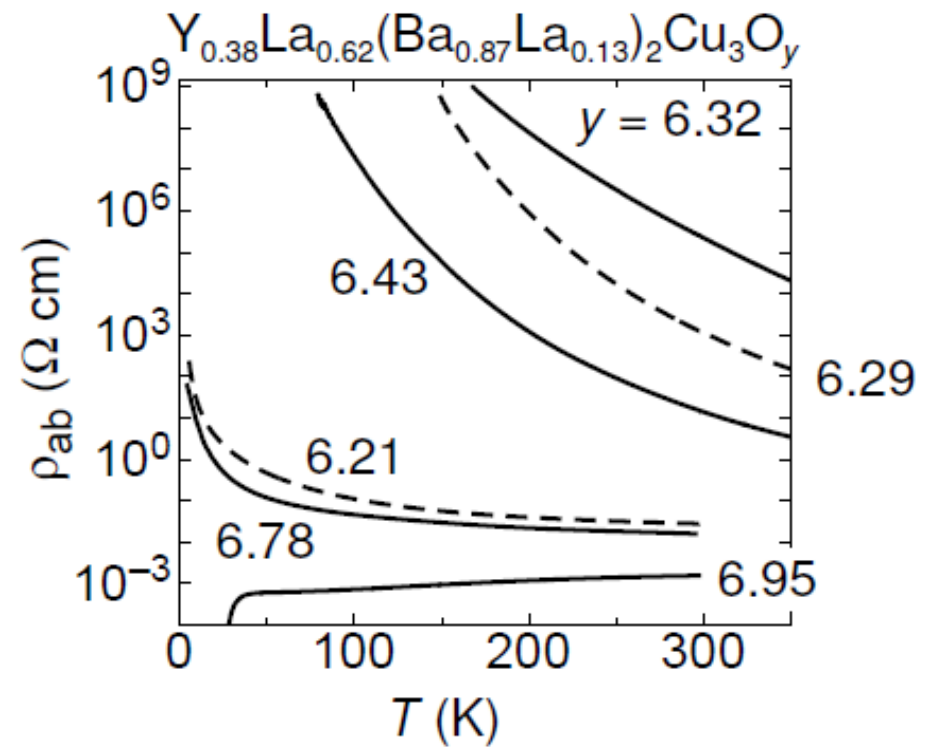
Vary p-type doping by  
annealing oxygen content  $y$

$$6.21 < y < 6.95$$

$y < 6.32$  : n-type doping

$y = 6.32$ : fully compensated

$6.32 < y$  : p-type doping

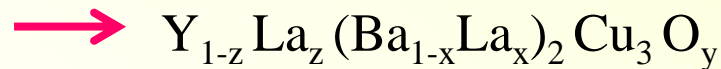


# Compensated high Tc materials

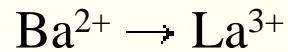
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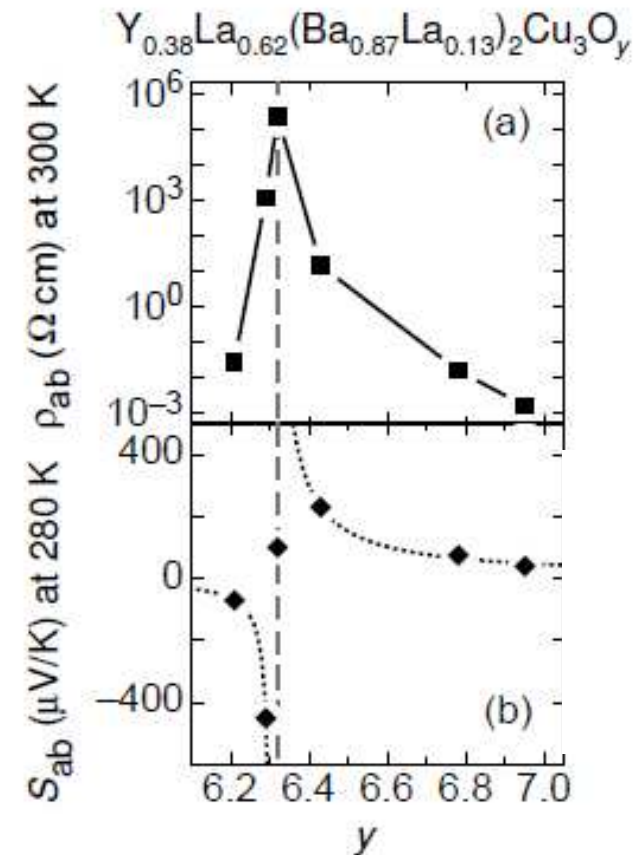
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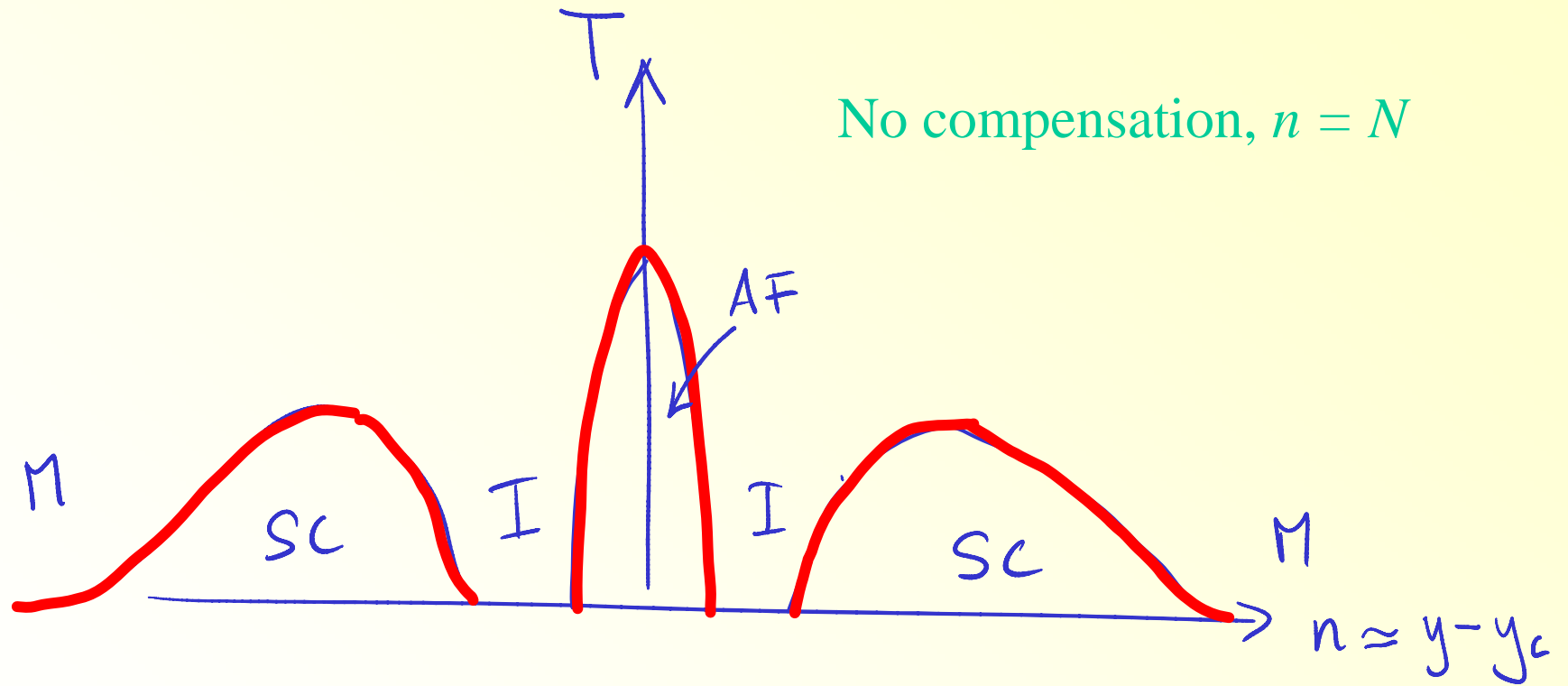
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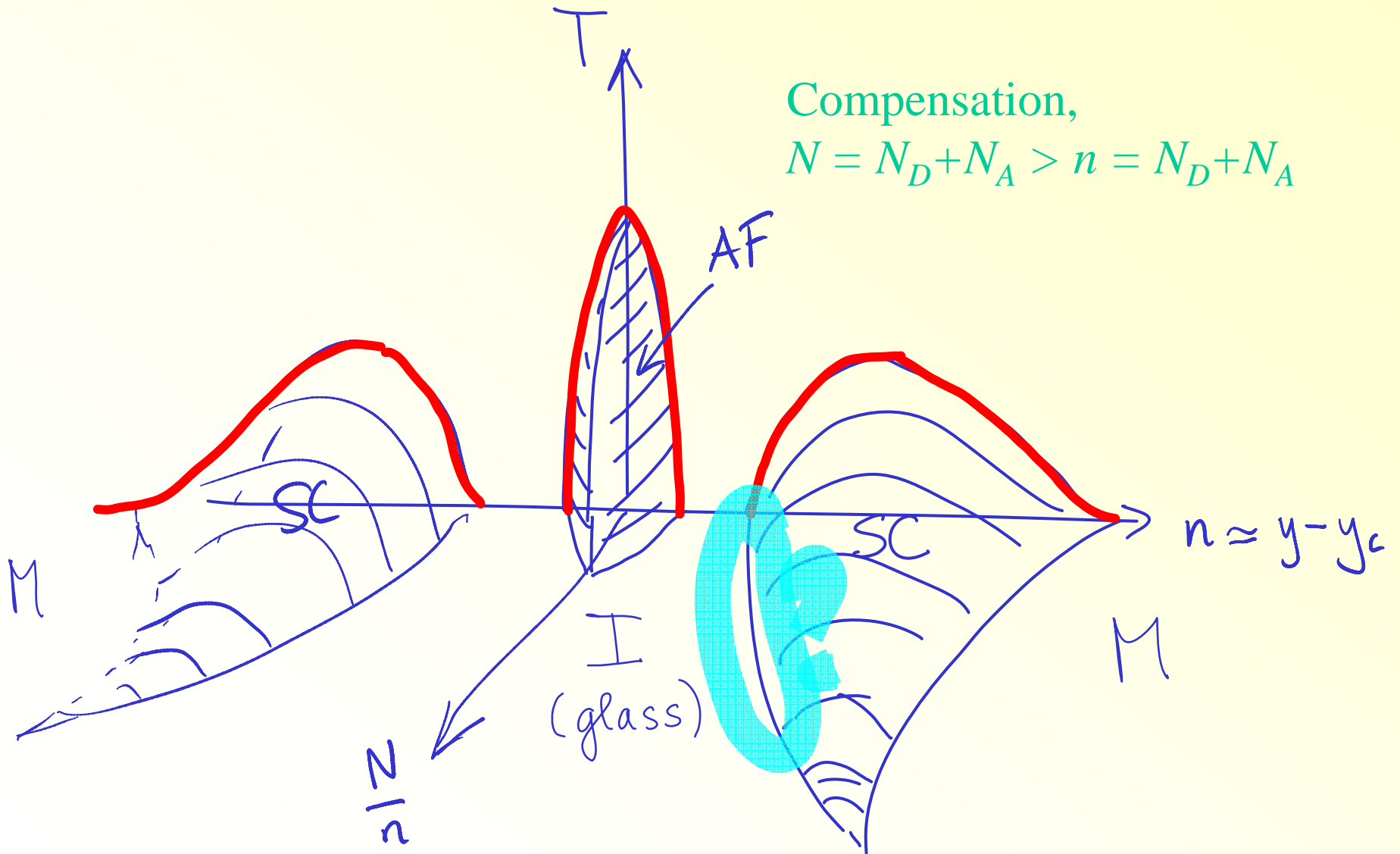


# Expected phase diagram



# Expected phase diagram

Compensation,  
 $N = N_D + N_A > n = N_D + N_A$



Analysis of the SIT in terms of a  
scaling analysis

-

All numerical prefactors will be  
neglected



# The compensation driven metal-insulator transition (fermions)

Uncompensated semiconductors (3d): Mott's criterion

Effective Bohr radius: 
$$a = \frac{\hbar^2 \kappa}{me^2}$$

Metal-insulator transition (MIT): 
$$n_{\text{MIT}} a^3 = N_{D,\text{MIT}} a^3 = 0.02 = \mathcal{O}(1)$$

Overlapping hydrogen-like wavefunctions  $\rightarrow$  delocalization

With BCS instability in the metal  $\rightarrow$  SIT:

$$n_{\text{SIT}} a^3 \approx n_{\text{MIT}} a^3 \sim 1$$

# The compensation driven metal-insulator transition (fermions)

Metal-Insulator transition in strongly compensated semiconductors

Non-trivial regime:  $Na^3 \gg 1$   
 Heavy doping  $N = N_D + N_A$

Most carriers are captured by doping ions  $\rightarrow$   
 Excess carriers in the conduction band:  $n = N_A - N_D \ll N$

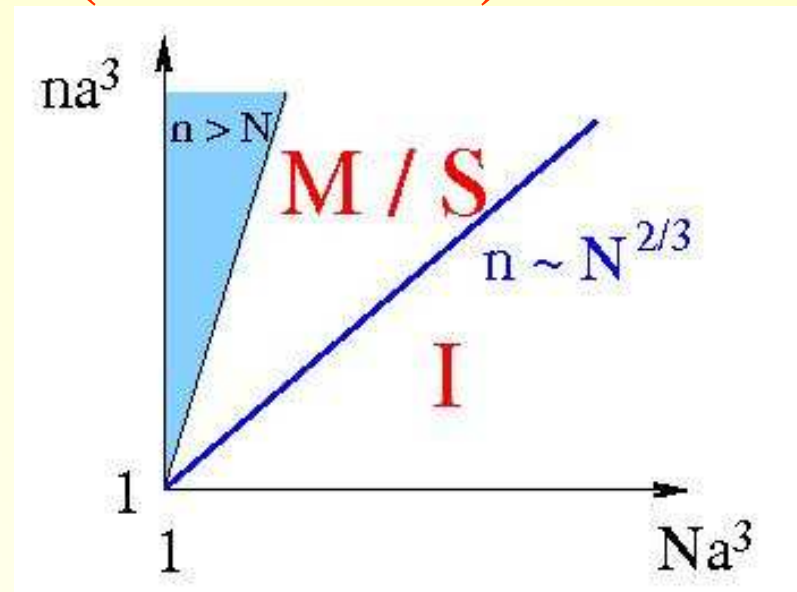
$\rightarrow$  Strong disorder from  $N$  random charged impurities!

Delocalization transition upon tuning  $n$ :

$$n_{\text{MIT}}(N) = \frac{N}{(Na^3)^{1/3}} = \frac{N^{2/3}}{a}$$

$$\rightarrow n_{\text{MIT}} a^3 = (Na^3)^{2/3} \gg 1$$

$$\rightarrow n_{\text{MIT}} \ll N$$



Experimentally confirmed in compensated Ge

# MIT: Derivation

*Efros and Shklovskii (1971)*

## 1. Non-linear screening of the disorder

Random charge density in volume  $R^d$ :  $n_{\text{net imp}}(R) \sim \frac{(NR^3)^{1/2}}{R^3}$

Non-linear screening scale  $R_s$ :  $n_{\text{net imp}}(R) \sim n \rightarrow R_s(n) \sim \frac{N^{1/3}}{n^{2/3}}$

Roughness of the disorder potential:  $eV_{\text{dis,Cb}}(R_s) \sim \frac{e^2}{\kappa R_s} (NR_s^3)^{1/2} \sim \frac{e^2 N^{2/3}}{\kappa n^{1/3}}$

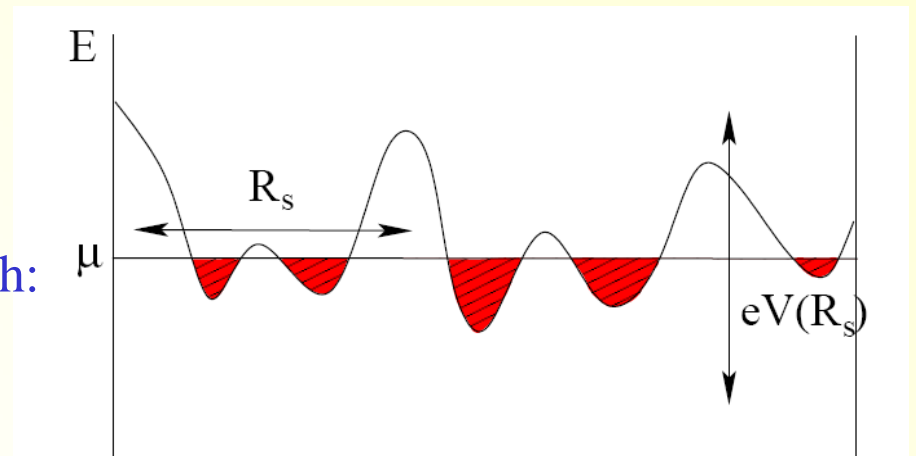
## 2. Delocalization condition

Fermi energy of excess carriers:

$$E_F \sim \hbar^2 n^{2/3} / m$$

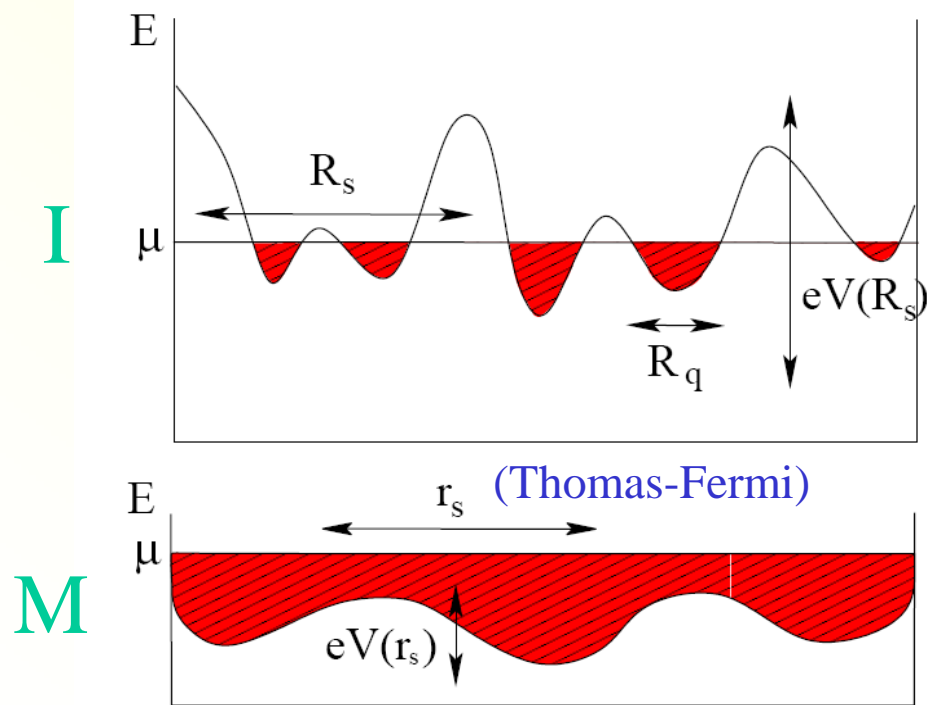
Delocalization if disorder is weak enough:

$$E_F \geq V(R_s) \Leftrightarrow \boxed{n_{\text{MIT}} \sim \frac{N}{(Na^3)^{1/3}}}$$



Consistency condition for scaling analysis:  $NR_s^3 \gg 1 \Leftrightarrow Na^3 \gg 1$

# Droplets in the insulator



In the insulator:  $n < n_{\text{MIT}}$

Small droplets (Fermi lakes in the deepest wells) of size  $R_q < R_s$  !

Typical size of the droplets:

$$\mu(n = (N / R_q^3)^{1/2}) = eV_{\text{Cb}}(R_q)$$

$$\rightarrow R_q = \frac{a}{(Na^3)^{1/9}} = R_s(n) \cdot (n / n_{\text{MIT}})^{2/3}$$

Transport: Variable range hopping between the droplets!

# Compensated superconductors with preformed pairs

Assume strong coupling mechanism (“glue”)

→ preformed Cooper pairs of size  $\xi$   
(finite pairing energy  $E_{\text{pair}}$  - no nodal quasiparticles)

Possible systems with preformed pairs:

- Underdoped high  $T_c$  materials
- Bipolarons
- [Anderson pseudospins (doubly occupied localized wavefunctions)]

- How do bosons modify non-linear screening and delocalization?
- How does the BEC-BCS crossover manifest itself?

$$n\xi^3 = 1$$

- What is the transport on the insulating side?

# BEC – BCS crossover

BEC – BCS crossover :

$$n_{\text{SIT}} \xi^3 \sim 1$$

- For  $n_{\text{SIT}} \xi^3 > 1$  the transition remains the same as with fermions
- Distinctly “bosonic” behavior occurs at the SIT when

$$n_{\text{SIT}} \xi^3 < 1 \quad \leftrightarrow \quad \xi < a$$

→ Needed: small pairs, large Bohr radii



$$n_{\text{SIT}}(N) = ?$$

# BEC regime – 3d SC

1. Nonlinear screening with  $(2e)$ 's instead of  $e$ 's, but otherwise no difference

$$eV_{\text{Cb}}(n) = eV_{\text{Cb}}(R_s) \sim e^2 N^{2/3} / n^{1/3}$$

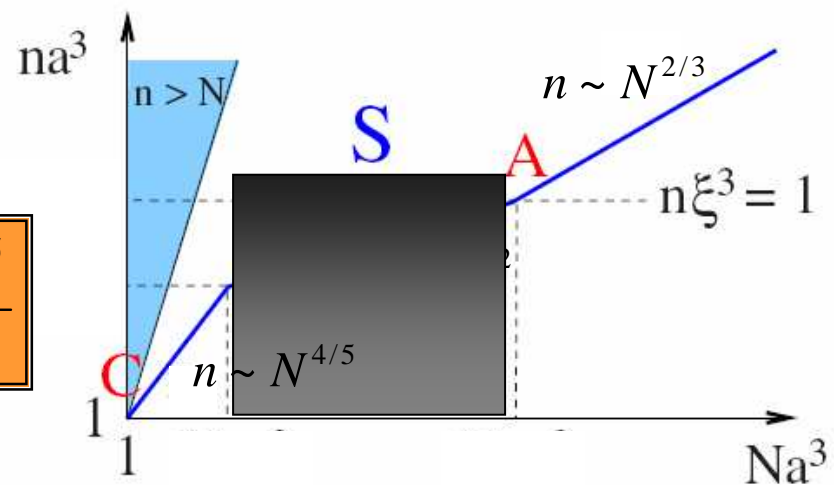
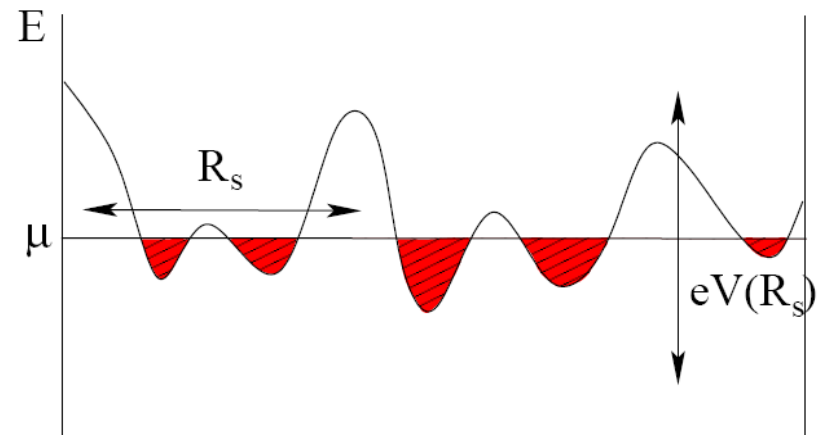
2. Very low density

Energy of confinement to screening volume:

$$\mu(n, R) = \frac{\hbar^2}{mR^2}$$

Delocalization of the BEC condensate:

$$\mu(n, R_s) \sim eV_{\text{Cb}}(n) \rightarrow n_{\text{SIT}}(N) = \frac{N^{4/5}}{a^{3/5}}$$



# BEC regime – 3d SC

1. Nonlinear screening with  $(2e)$ 's instead of  $1e$ 's, but otherwise no difference

$$eV_{\text{Cb}}(n) = eV_{\text{Cb}}(R_s) \sim e^2 N^{2/3} / n^{1/3}$$

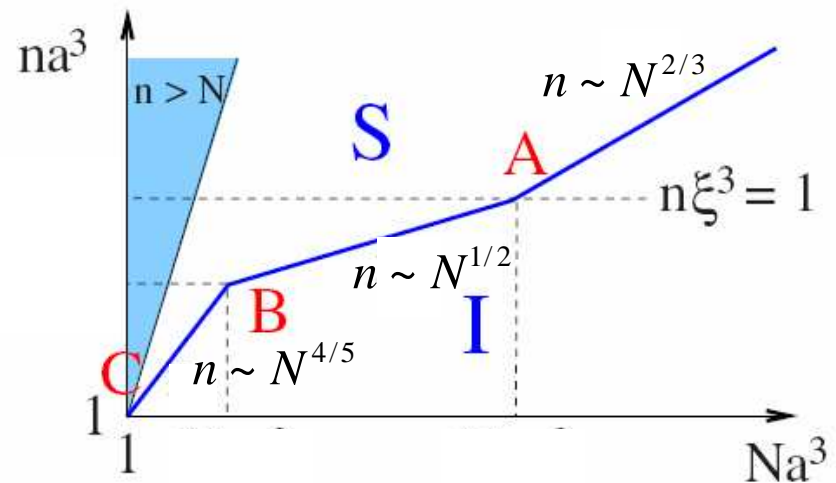
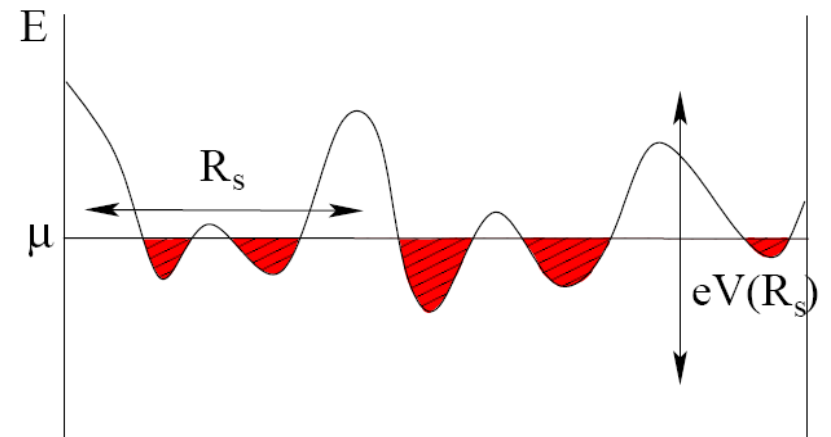
3. Chemical potential for bosons:  
Bose gas with scattering length  $\xi$

$$\mu(n) = \frac{\hbar^2}{2m} n \xi < E_F(n)!$$

4. Bose delocalization criterion:

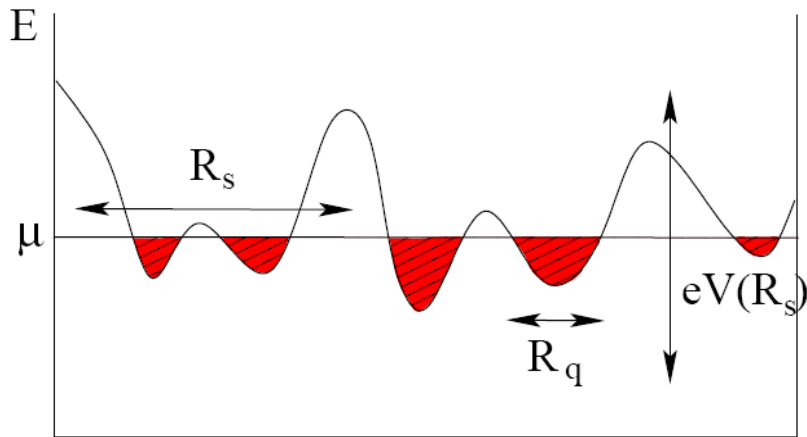
$$\mu(n) \sim eV_{\text{Cb}}(n)$$

$$\rightarrow n_{\text{SIT}}(N) = \frac{N^{1/2}}{(a\xi)^{3/4}}$$





# The Bose insulator



In the insulator:  $n < n_{\text{SIT}}$

Small droplets (boson lakes in the deepest wells) of size  $R_q < R_s$  !

Typical size of the droplets:

$$\mu(n = (N / R_q^3)^{1/2}) = eV_{\text{Cb}}(R_q)$$

BEC regime:  $\rightarrow R_q = (a\xi)^{1/2} < R_s(n)$

## Nature of the ground state

Level spacing in a droplet ( $\pm 1e$ ):  $\delta \sim (d\mu / dn R_q^3) = e^2 / R_q$

BEC:  $\xi < a$

Pair-breaking energy:

$$E_{\text{pair}} \sim \hbar^2 / m\xi^2$$

$$\rightarrow E_{\text{pair}} > \delta$$

$\rightarrow$  Breaking pairs is unfavorable, all electrons are paired!  
Single electron excitations are gapped!

# Properties of the insulator

## Tunneling

- Single particle gap
- SC spectrum of small droplets (and corresponding coherence peaks)

## Transport

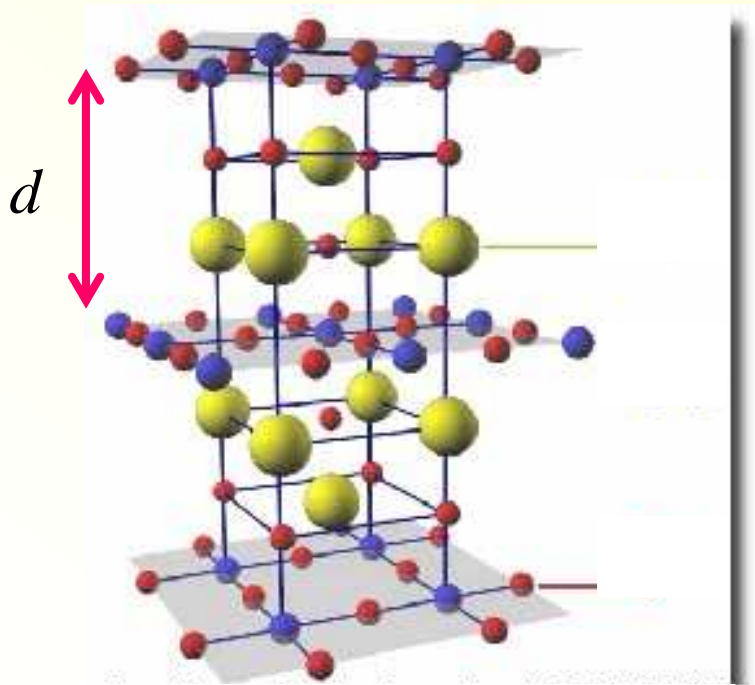
- At low T: Variable range hopping of pairs between droplets!
- In the BEC regime (strong coupling, small pairs) : always 2e-transport
- In the BCS regime (weak coupling): 1e-transport when  $\Delta < \delta$ .

$$\sigma = \sigma_0 \exp[-(T_{ES} / T)^{1/2}]$$

$$T_{ES} = 2.7 \cdot (2e)^2 / \kappa \xi_{2e}$$

# Layered superconductors

Examples: Cuprates (CuO compounds), pnictides (FeAs compounds)



$d$  : Distance between layers hosting the carriers

Differences 2d vs. 3d:

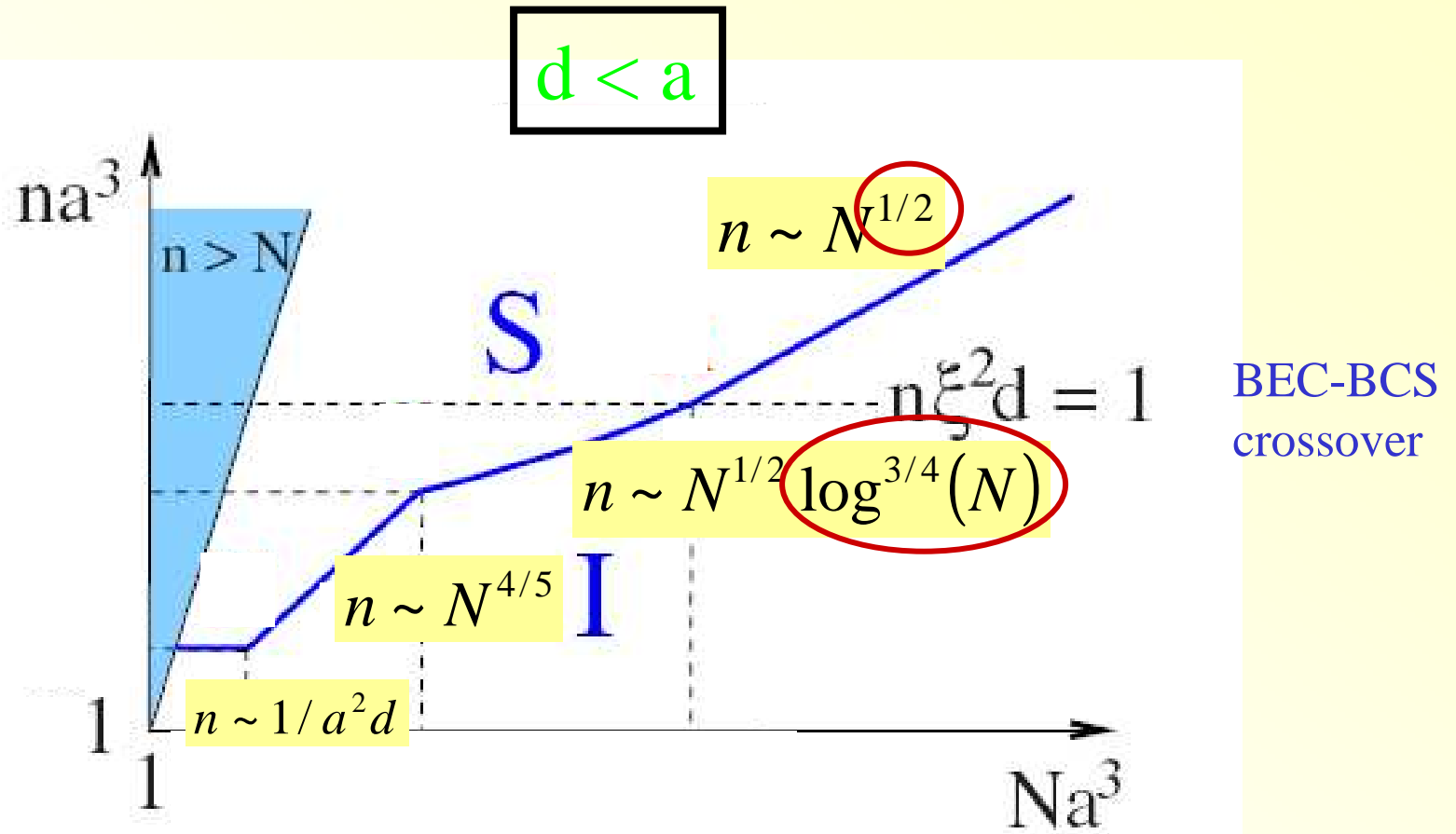
1. Nonlinear screening is **modified** when  $d > R_s$ ; need to account for **anisotropic dielectric constant**

2. Delocalization criterion:

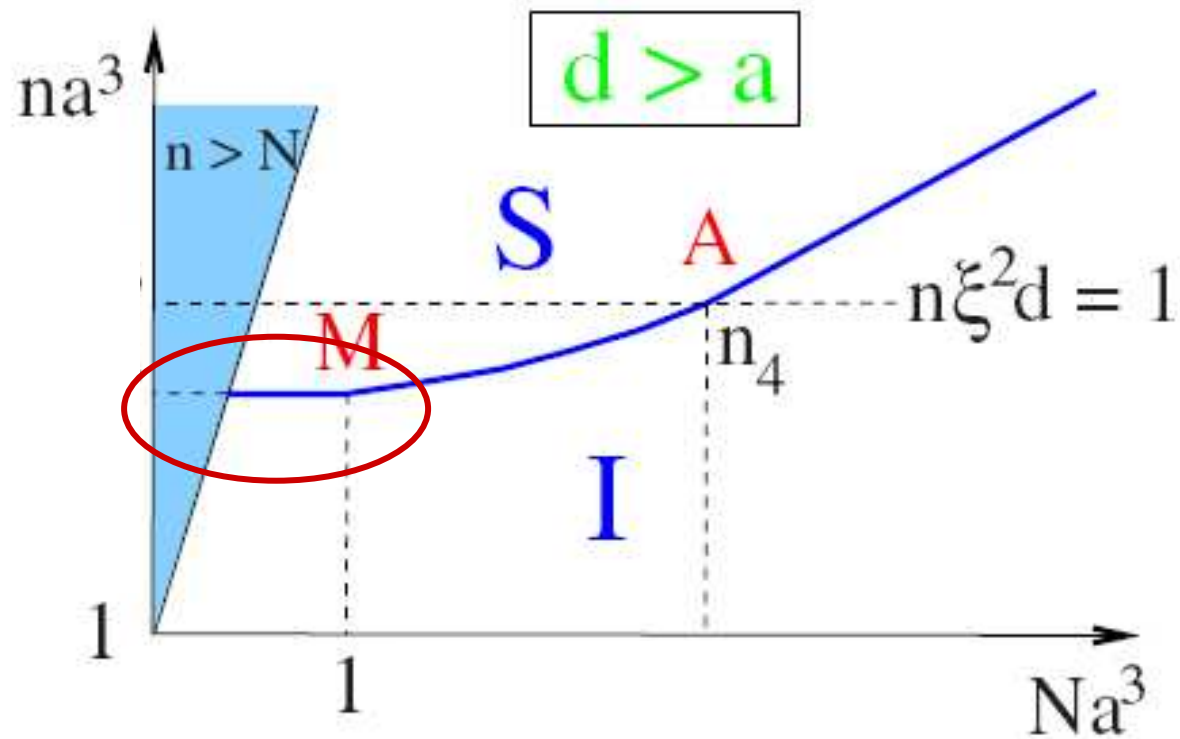
$$\mu_{2D}(n) = eV_{Cb}(n, N)$$

Only extra log at the BCS-BEC crossover! 
$$\mu_{2D, BEC}(n) = \frac{\hbar^2 nd}{m \log(1/\xi^2 nd)} = \frac{E_F(n)}{\log(1/\xi^2 nd)}$$

# SIT Phase diagram – 2D

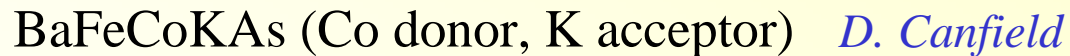
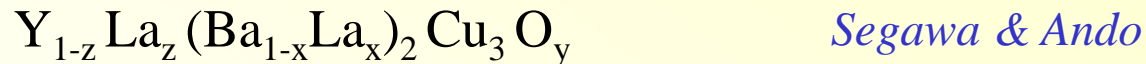


# SIT Phase diagram – 2D



# Applicability to high Tc ?

Candidate systems:



Condition for the SI transition to occur in the BEC regime:

$$n_{\text{SIT}} \xi^3 < 1 \quad \leftrightarrow \quad \xi < a$$

Parameters for typical underdoped high Tc's:

$$a = \frac{\hbar^2 \kappa}{m_{\text{pair}} (2e)^2}$$

BSCCO:  $a \sim 4-5\text{\AA}$

$$\xi_{\text{typ}} \approx 1-2\text{nm}$$



At the border of  
BEC-BCS  
crossover

# BEC-regime in bipolarons

Condition for the SI transition to occur in the BEC regime:

$$n_{\text{SIT}} \xi^3 < 1 \quad \leftrightarrow \quad \xi < a$$

Two independent parameters:

- Electron-phonon coupling:  $\alpha > \alpha_c = 2.9$  (in 2d)
- Ratio between electronic and static dielectric constant:  
 $\eta = \kappa_e / \kappa$  needs to be  $\eta \gg 1$  :

$$\frac{\xi}{a} \sim \alpha^2 \eta < 1$$

# Summary

- SIT in the presence of strong Coulomb disorder:  
Delocalization of preformed pairs in a self-consistently screened disorder potential
- Non-linear screening is less efficient with bosons  
(exclusion principle less effective in BEC regime)

$$n_{SIT}^{(bosons)} > n_{SIT}^{(fermions)}$$

- Low T transport in the insulator in the BEC regime is always dominated by pairs