# Nernst effect and quantum critical magnetotransport in superconductors and graphene

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in collaboration with

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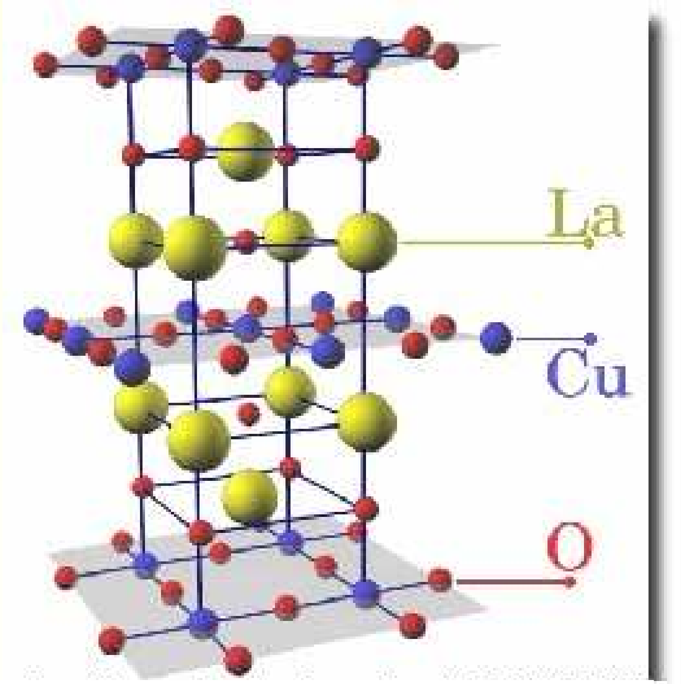
UCSB, 7th March, 2008

# Outline

- Nernst experiments in superconductors
- Hydrodynamic analysis of the thermo-electric response functions
- Applications to graphene: quantum critical transport and collective cyclotron motion
- Obtain hydrodynamic results *exactly* for a critical gauge theory via the AdS/CFT correspondence
- Comparison with experiments in high $T_c$'s

# $La_{2-x}Sr_xCuO_4$ (LSCO)



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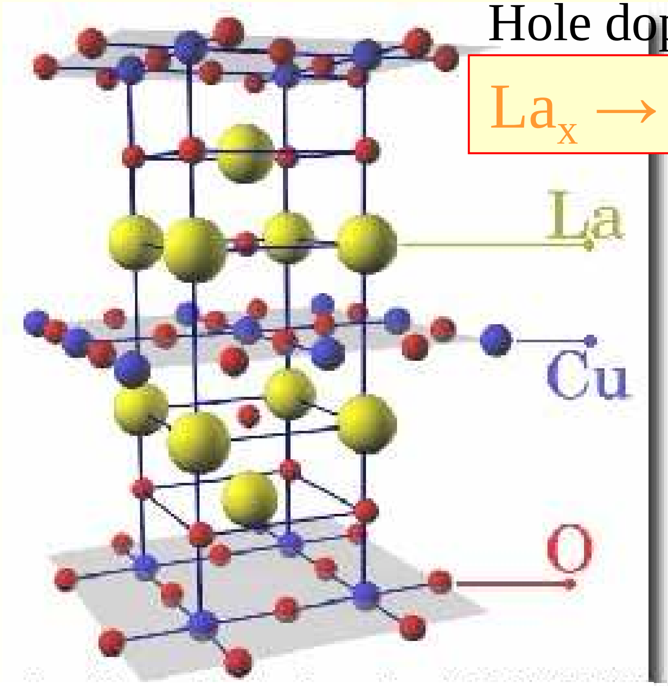


Hole doping:

$La_x \rightarrow Sr_x$

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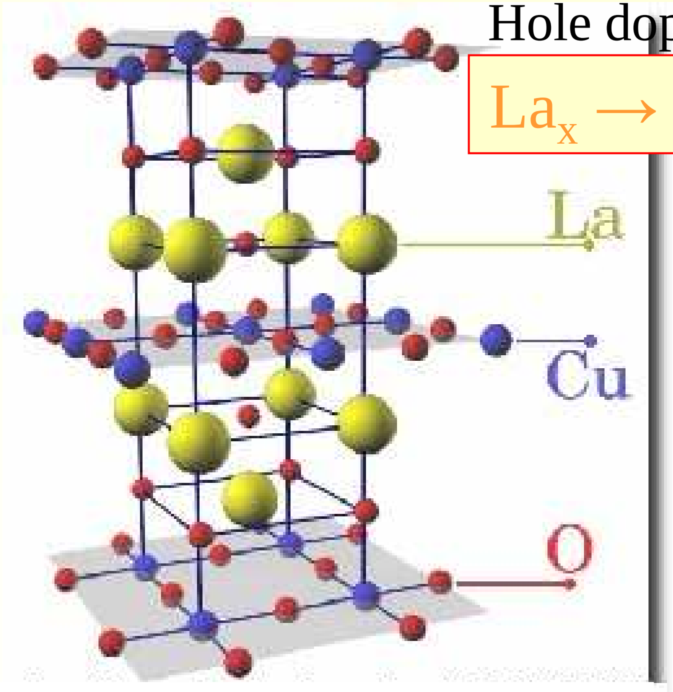
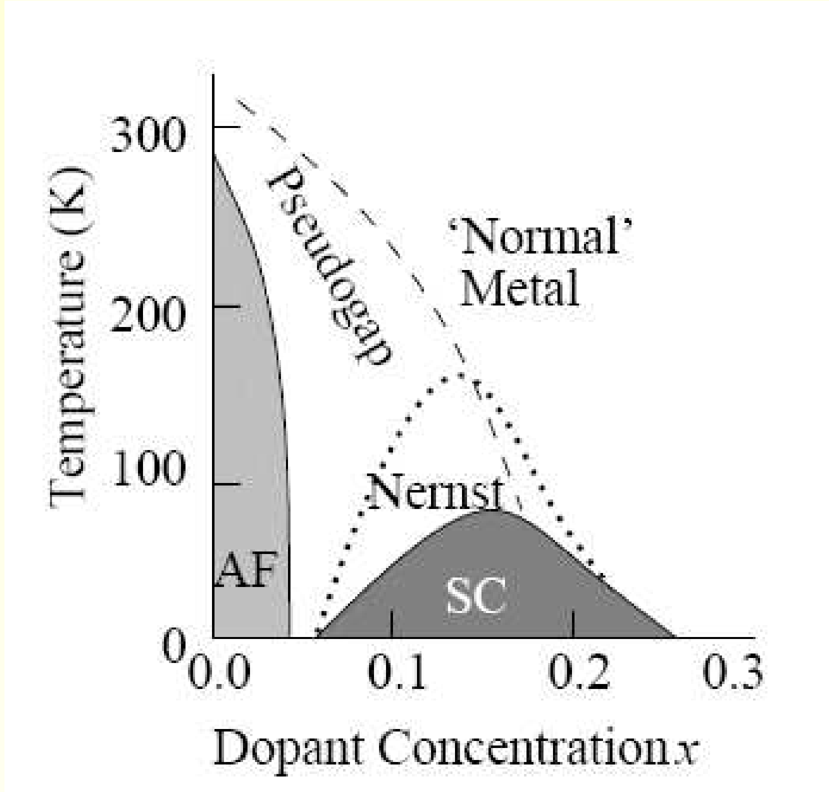


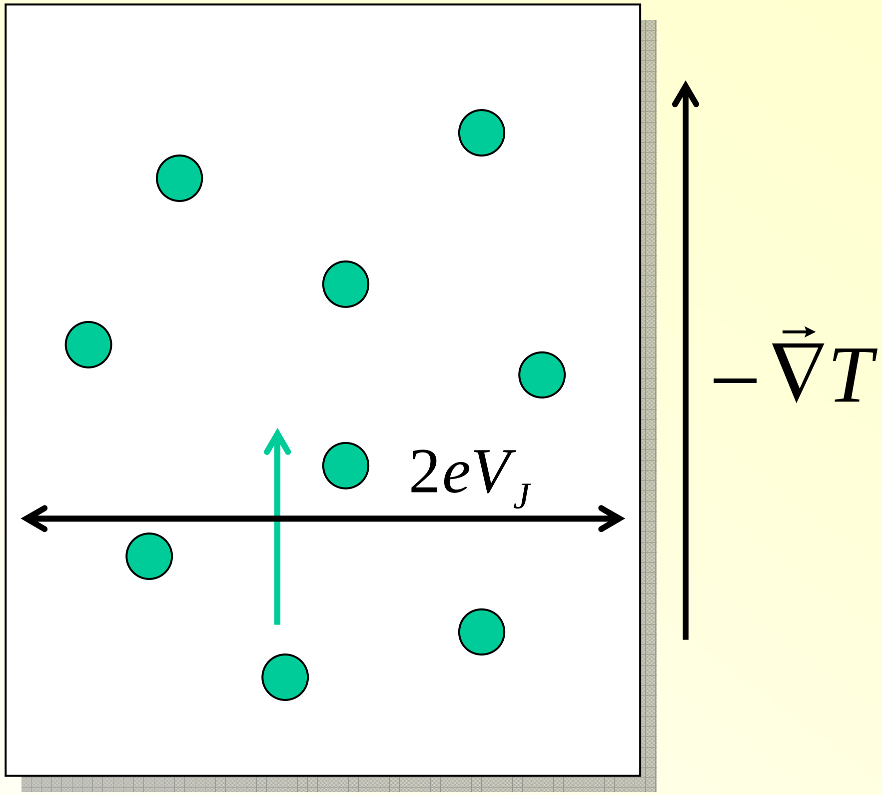




- Undoped $x=0$: antiferromagnetic Mott insulator
- Underdoped-optimally doped $0.05 < x < 0.17$:
  Strong Nernst signal up to $T=(2\text{-}3)T_c$
- Overdoped $0.17 < x$:
  BCS-like transition, very small Nernst signal above $T_c$

# Nernst effect ?

In the presence of a magnetic field: Transverse voltage due to a thermal gradient

(Hall effect: $-\vec{\nabla}T \rightarrow \vec{E}$ )




Nernst signal: $$e_N \equiv N = \frac{E_y}{-\vec{\nabla}_x T}$$

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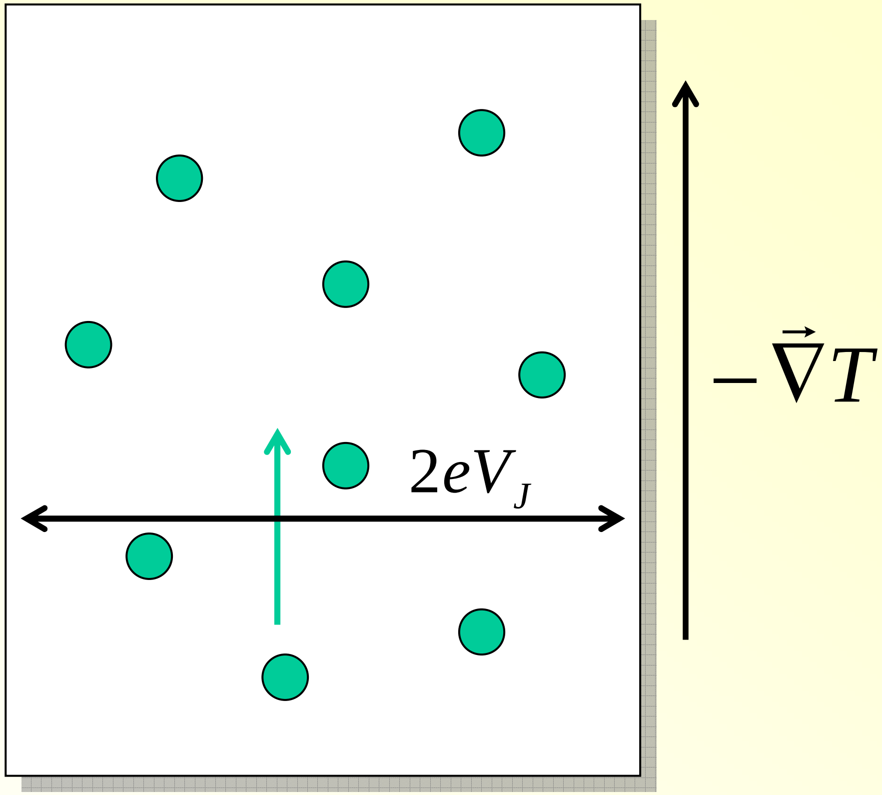
1. "Particle" view
2. "Vortex" view

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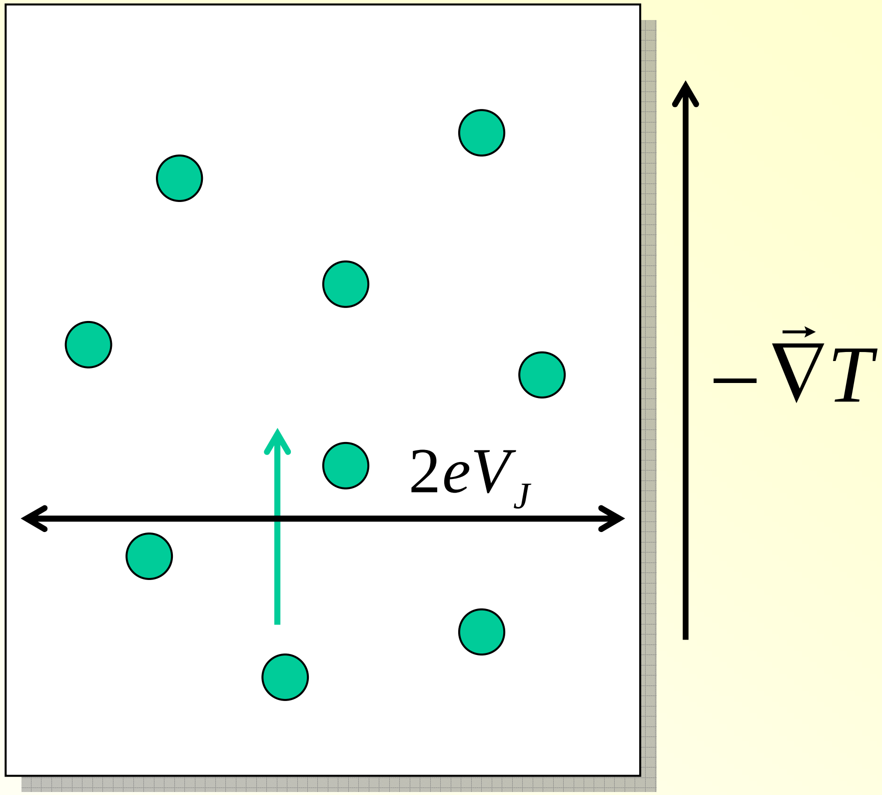
**2. "Vortex" view**

$$2eV_J = \hbar \partial_t \varphi$$
$$= 2\pi \hbar \partial_t n_V$$

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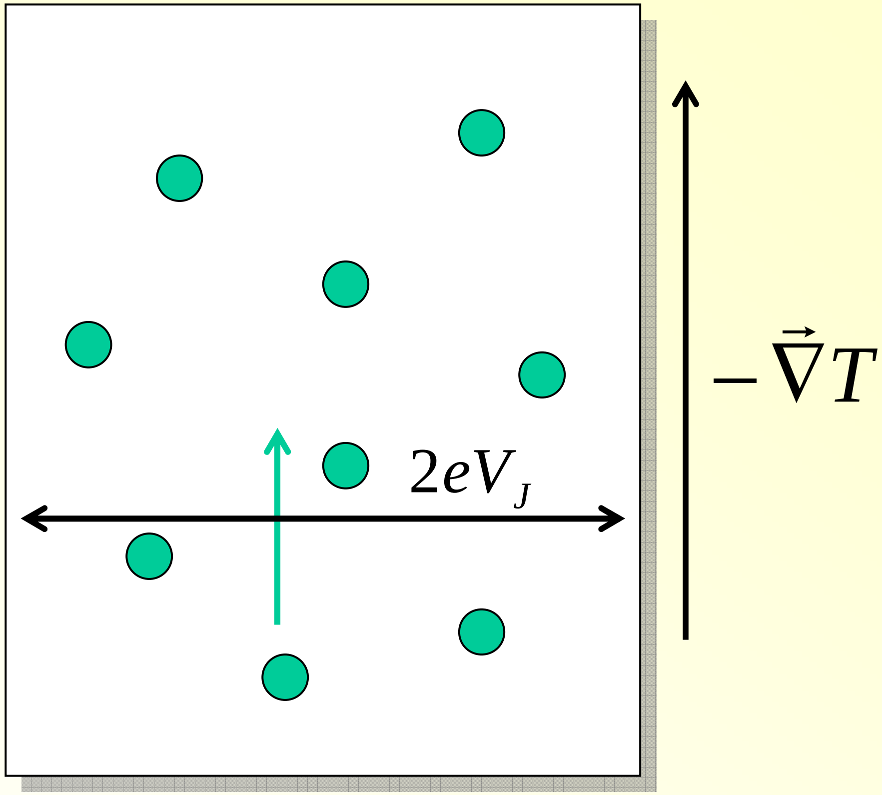


Nernst signal: $$e_N \equiv N = \frac{E_y}{-\vec{\nabla}_x T}$$

In Fermi liquids: $e_N$ very small

→

Big Nernst signal above $T_c$ ↔ Evidence for a "vortex liquid"?

# Vortex liquid?

Two scenarii for superconducting transition:

$$\Psi = |\Psi| e^{i\varphi}$$

1) BCS-type: Amplitude vanishes at $T_c$

$$\langle |\Psi|^2 \rangle \to 0$$

2) Phase fluctuations kill long range order:
(in purely 2d: Kosterlitz-Thouless)

$$\langle e^{i\varphi} \rangle \to 0$$

while a “vortex (Cooper pair) liquid” with local pairing amplitude $|\Psi|^2 > 0$ survives.
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Probe with Nernst effect!

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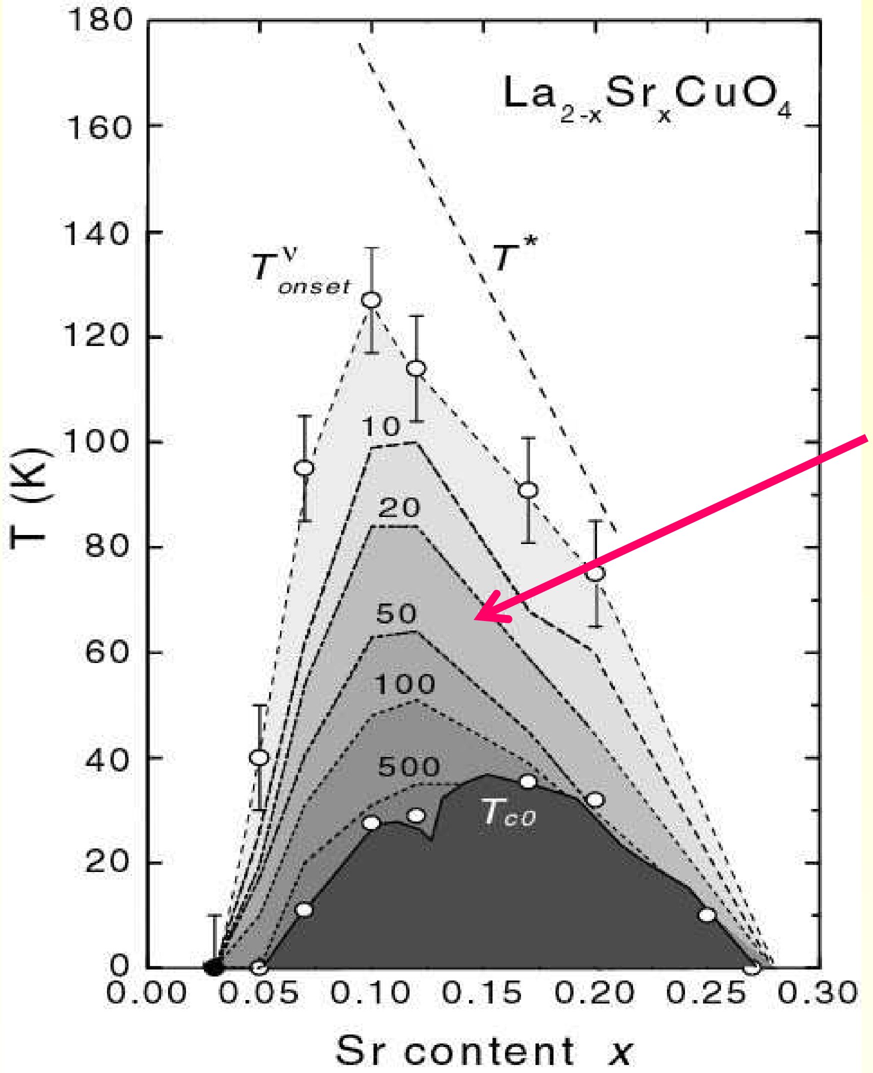
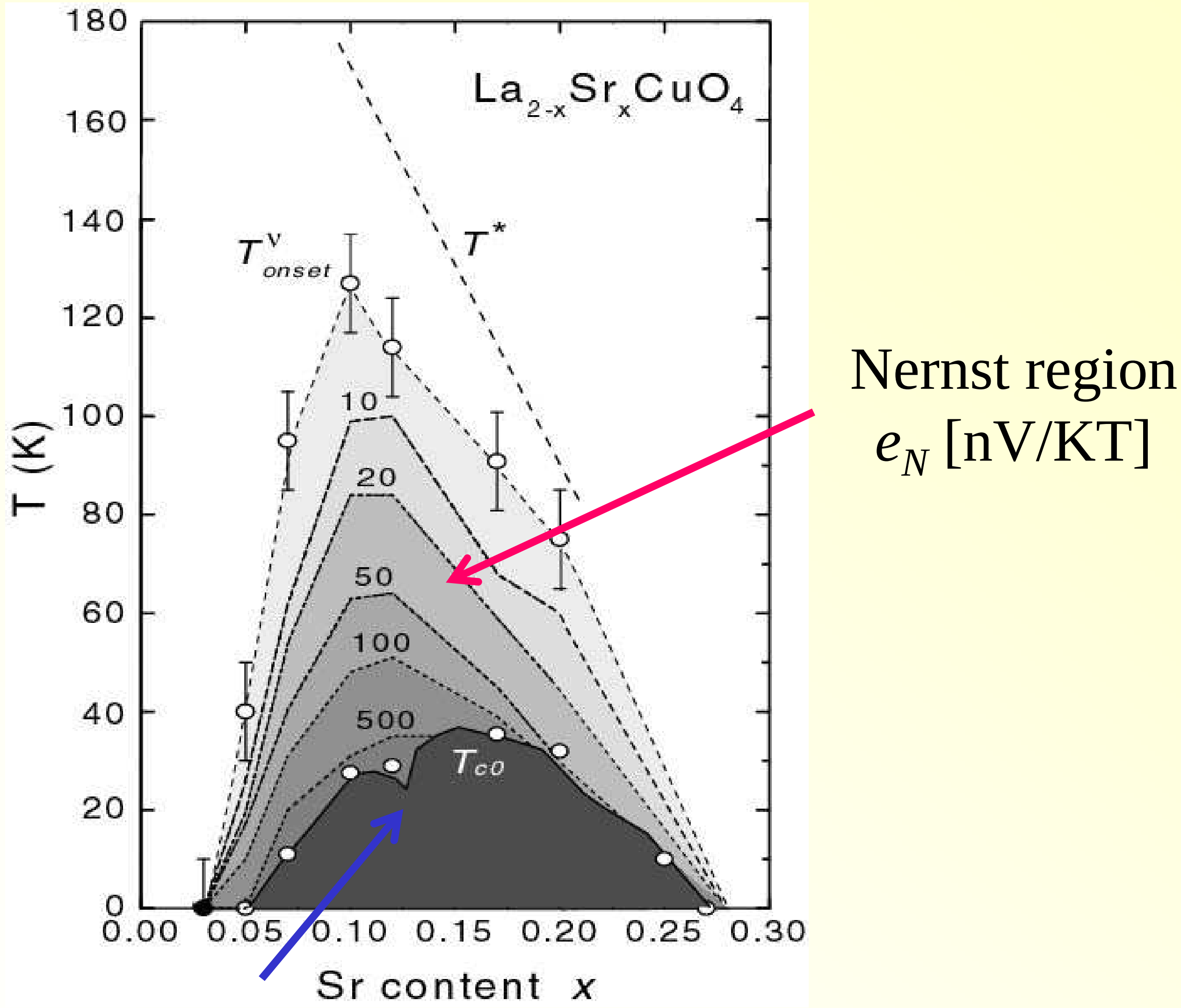
# LSCO Phase diagram



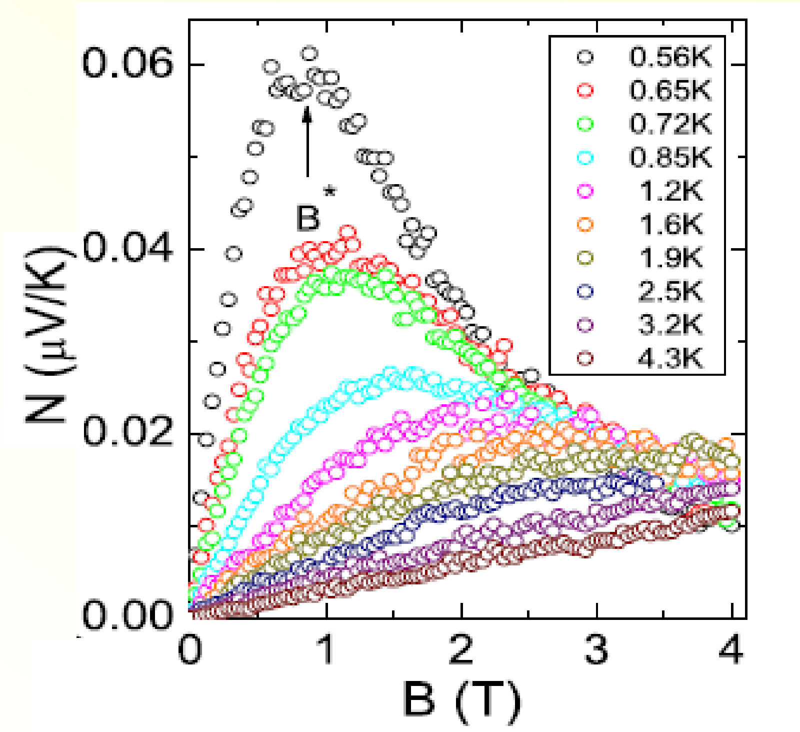

Nernst region
$e_N$ [nV/KT]

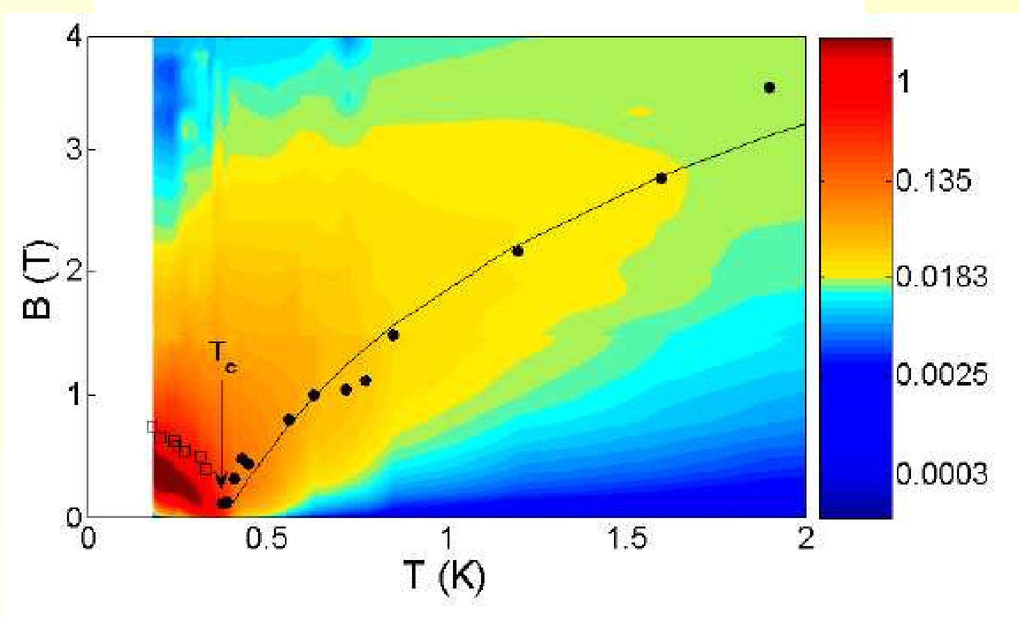
# LSCO Phase diagram




Dip in $T_c$ near $x$=1/8 indicates proximity of insulator

# Nernst effect in $Nb_{0.15}Si_{0.18}$




*(A. Pourret, H. Aubin, J. Lesueur, C. A. Marrache-Kikuchi, L. Bergé, L. Dumoulin, K. Behnia, arxiv:0701376 (2007))*

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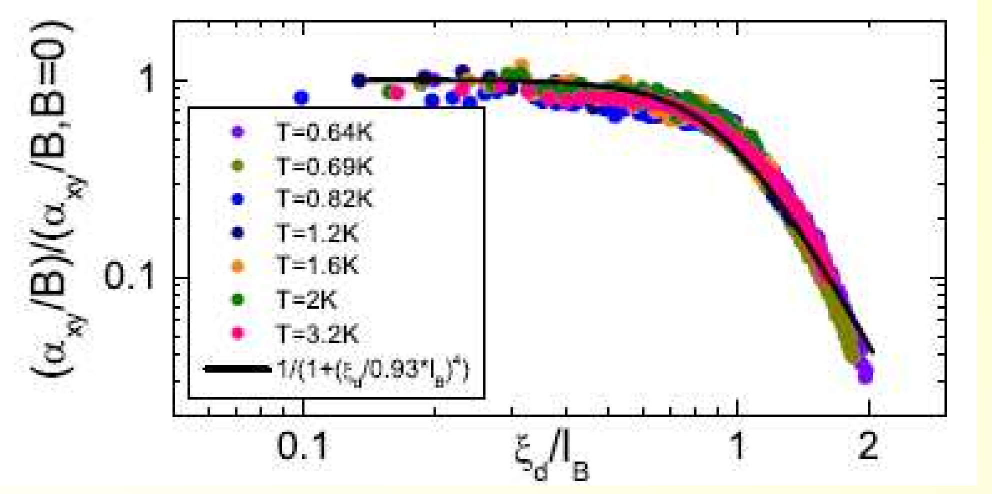


$$\frac{\alpha_{xy}}{B} = \frac{C}{1+\left(\xi_d/\ell_B\right)^4} = \frac{C}{1+\left(B/B_0\right)^2}$$

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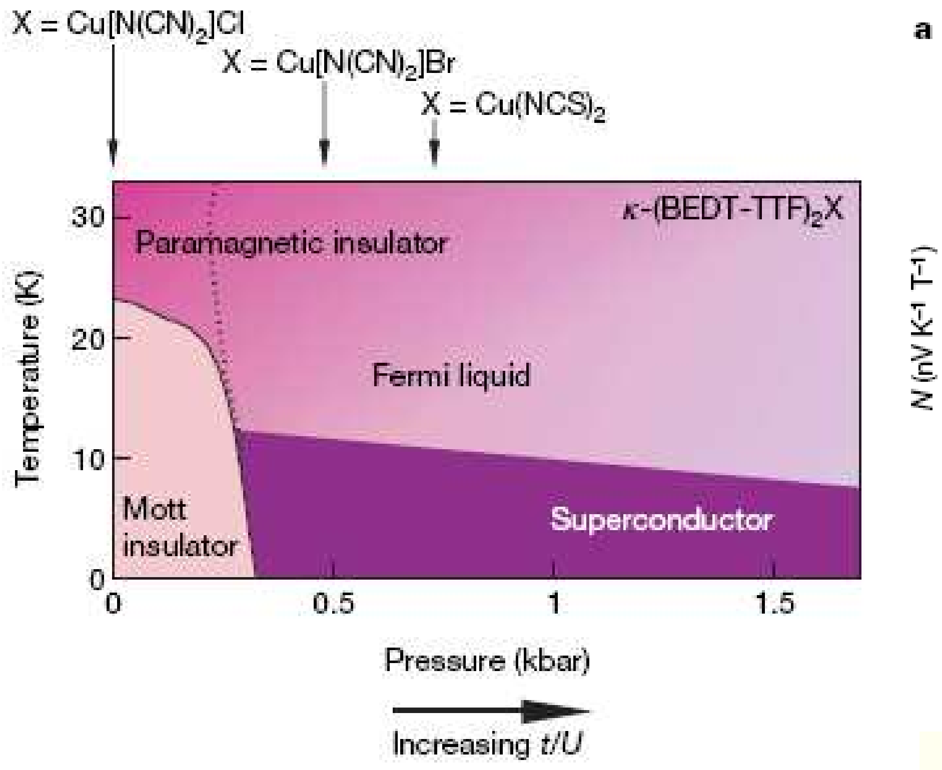
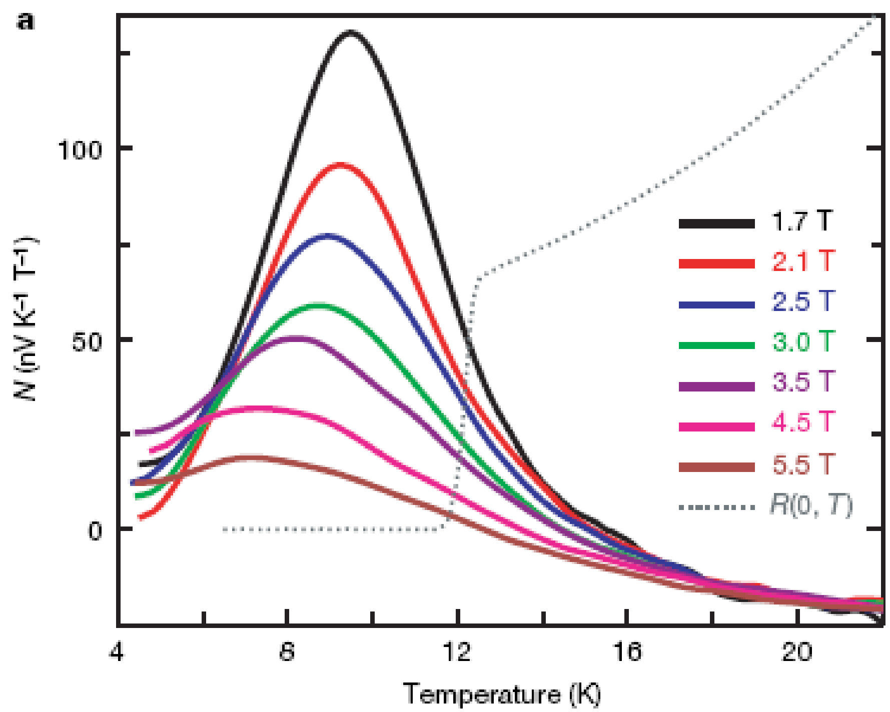
# Organic superconductors

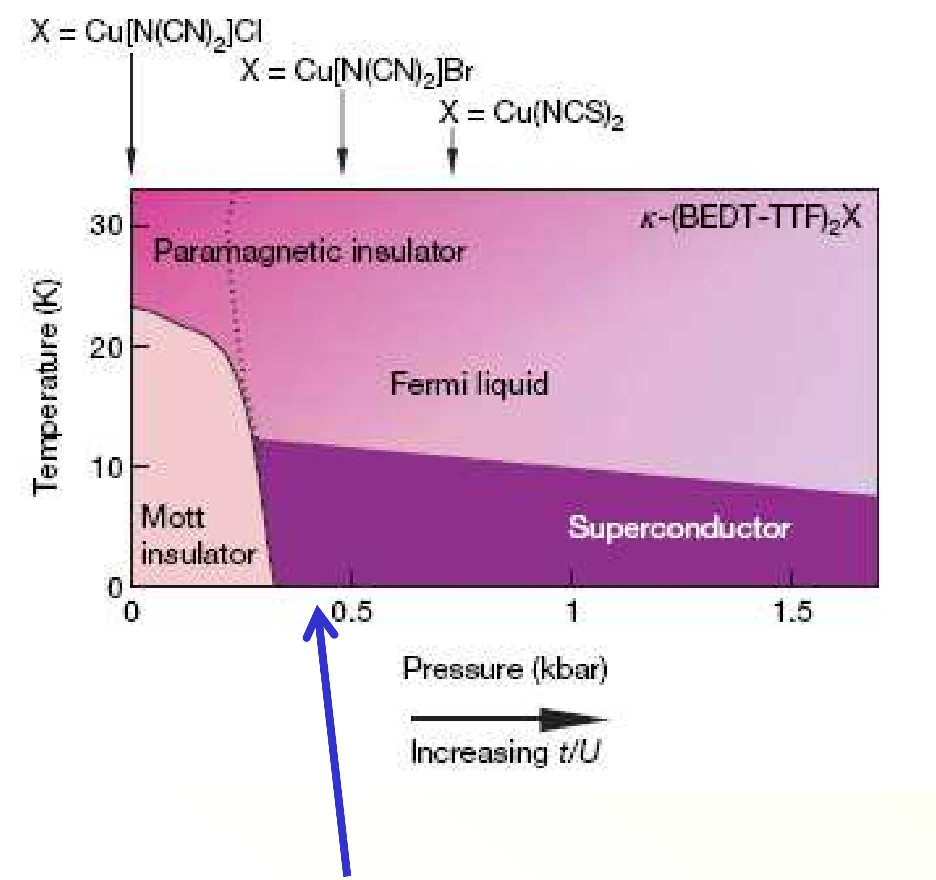


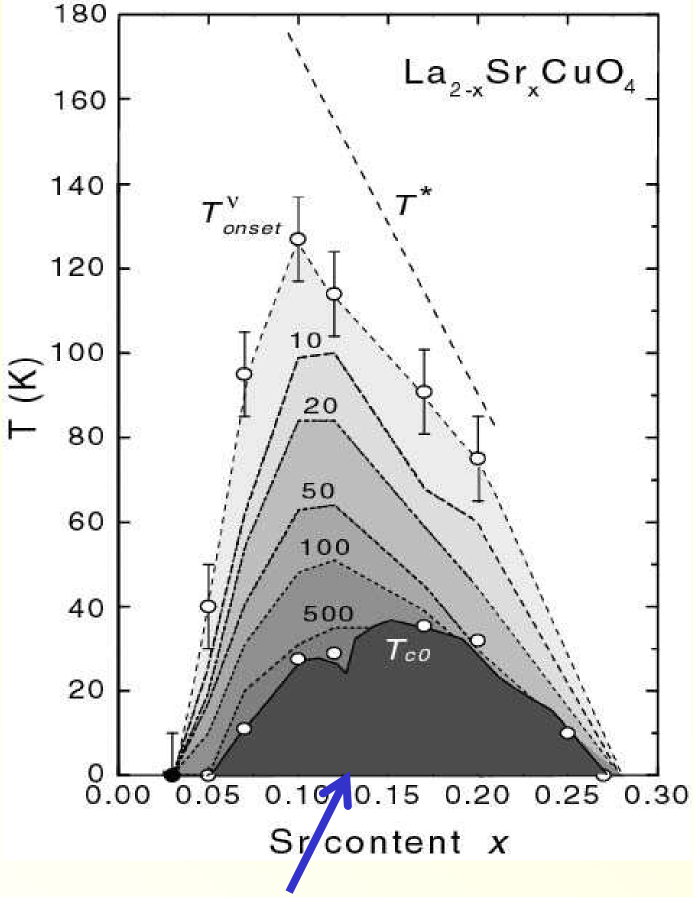




*M. Nam, A. Ardavan, S. J. Blundell, and J. A. Schlueter, Nature* ***449****, 584 (2007).*

# Quantum criticality

Proximity to transition: Superconductor ↔ Mott insulator




Proximity to an insulator near $x$=1/8

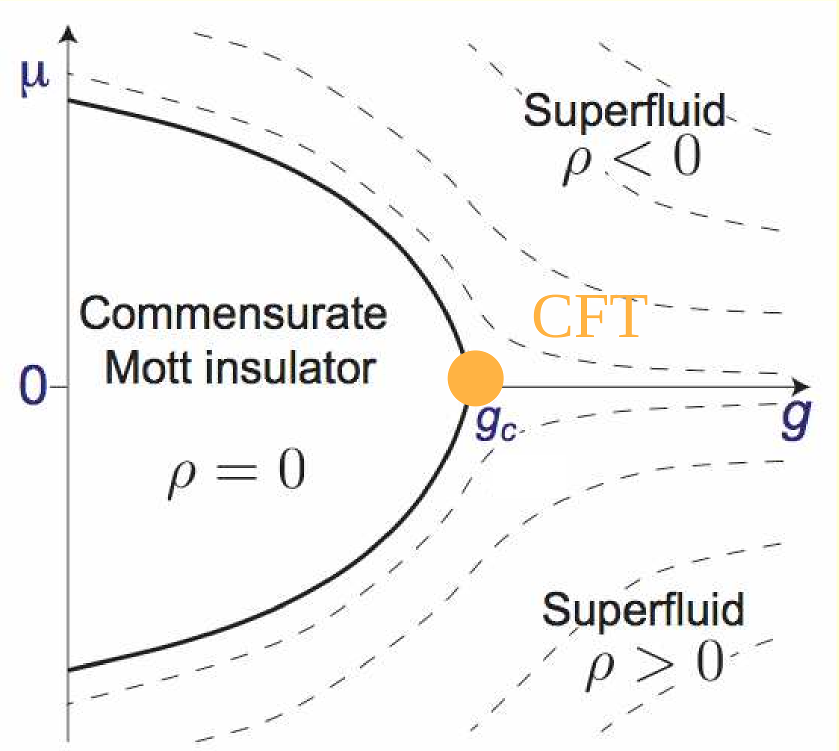
# SI-transition: Bose Hubbard model

Bose-Hubbard model

$$H = -t\sum_{\langle ij\rangle} b_j^+ b_i + U\sum_i n_i^2 - \mu\sum_i n_i$$

Coupling

$$g \equiv \frac{t}{U}$$ tunes the SI-transition



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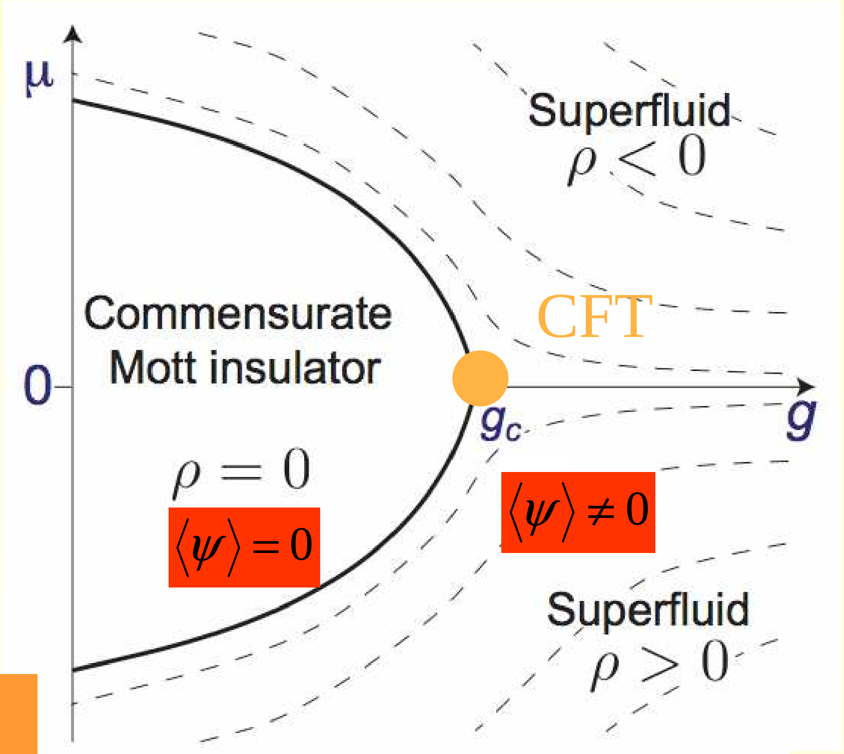


Effective action around $g_c$ ($\mu = 0$):

$$S = \int d^2r d\tau \left[ |\partial_\tau \psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

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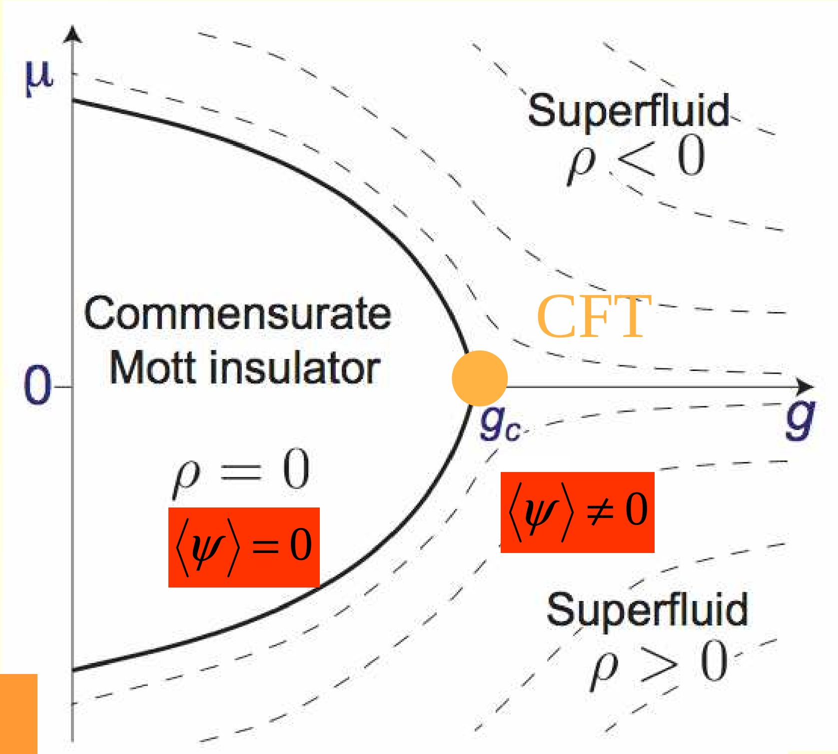


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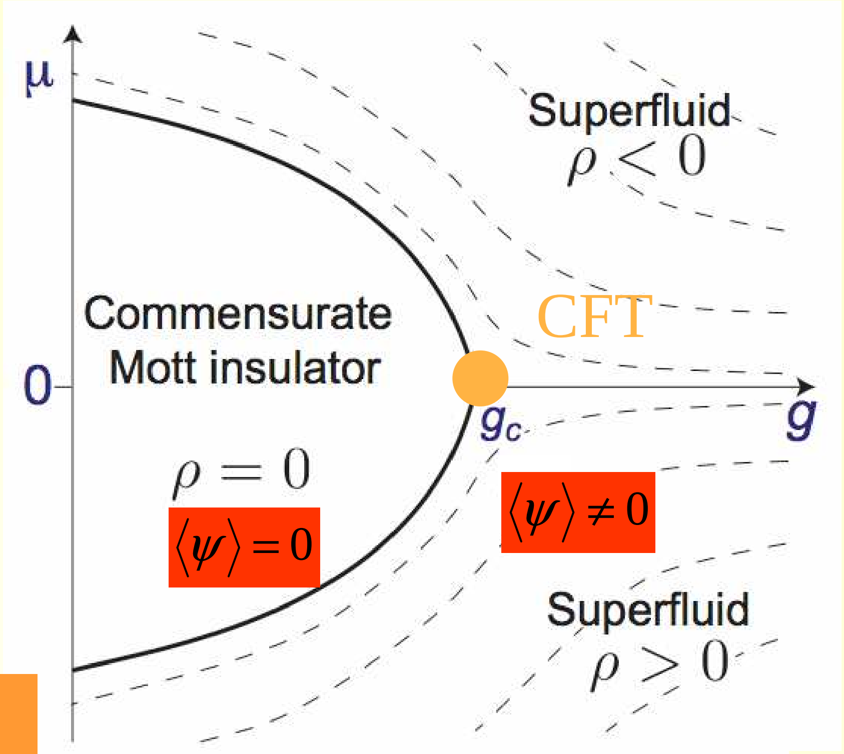


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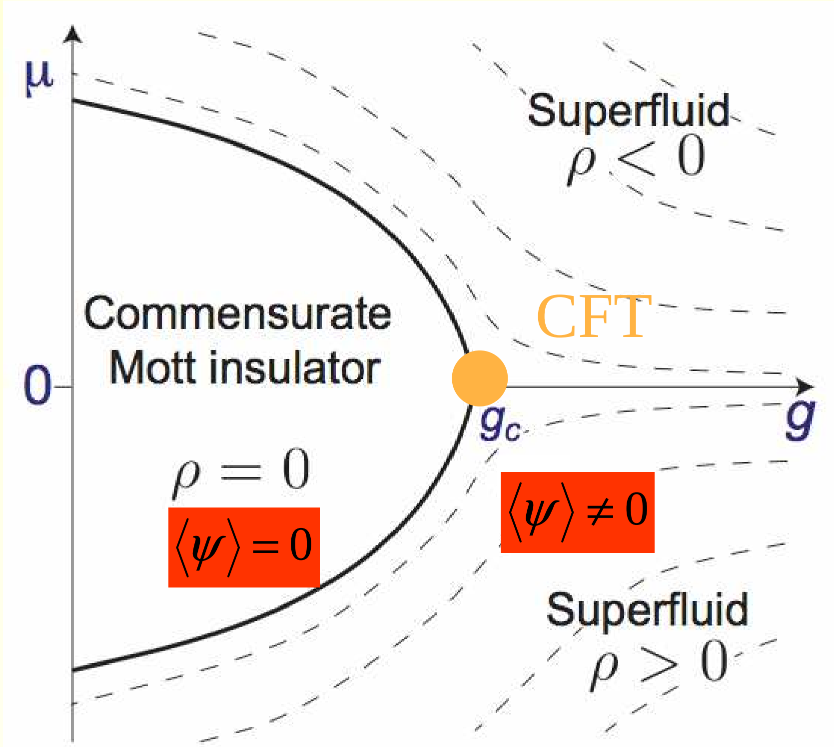

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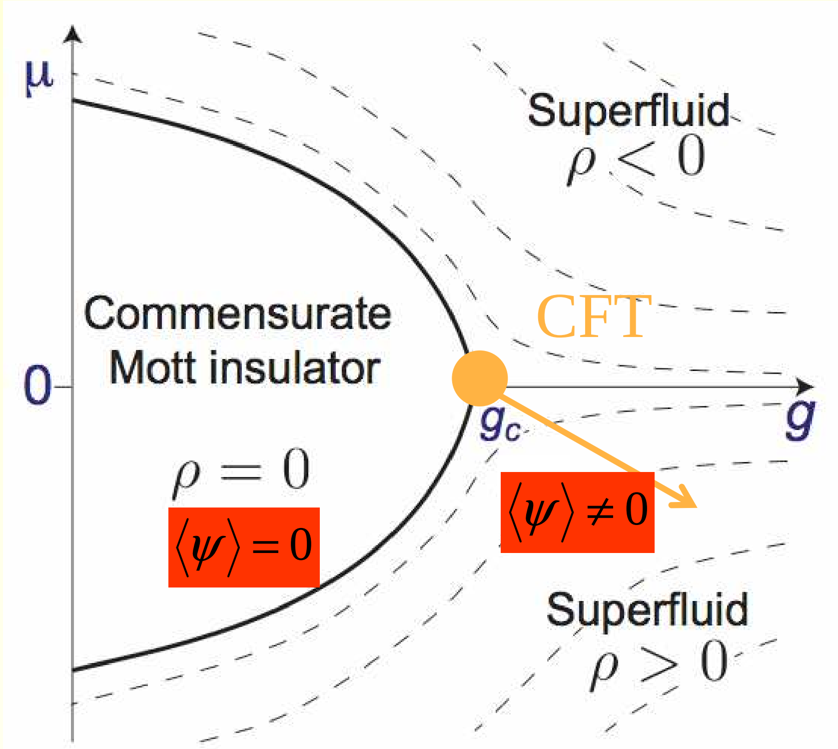
Perturb the CFT with

- a chemical potential $\mu$
- a magnetic field $B$

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$$\nabla \times \vec{A} = B$$

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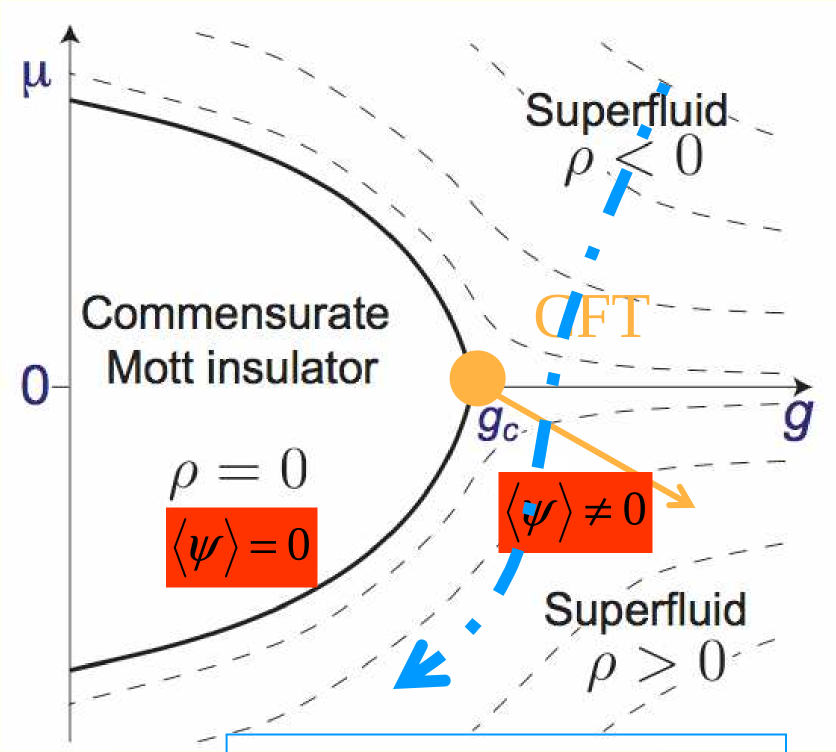


Doping route: μ(x), g(x)

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# Hydrodynamic Approach

# Fluid Dynamics

Two transport regimes:

I. Ballistic regime (collisonless)

Short times,
Small scales

$t << \tau_{rel}$

II. Hydrodynamic regime (collision-dominated)

Long times
Large scales

$t >> \tau_{rel}$

# Recall: Hydrodynamics

II. Hydrodynamic regime (collisiondominated)

Short times,
Large scales

$t >> \tau_{rel}$

# Recall: Hydrodynamics

## II. Hydrodynamic regime (collisiondominated)

Short times,
Large scales

$t >> \tau_{rel}$

- Local equilibrium established: $T_{loc}(r), \mu_{loc}(r); \vec{u}_{loc}(r)$

- Study relaxation towards global equilibrium

- Slow modes: Diffusion of the density of conserved quantities:
  - Charge
  - Momentum
  - Energy

# Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B* ***76****, 144502 (2007).*

Energy-momentum tensor $$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

Current 3-vector $$J^\mu = \rho u^\mu + \nu^\mu$$

$u^\mu$ : Energy velocity: $u^\mu = (1,0,0) \rightarrow$ No energy current

$\nu^\mu$ : Dissipative current (“heat curreny”)

$\tau^{\mu\nu}$ : Viscous stress tensor (Reynold’s tensor)

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+ Thermodynamic relations

$$\varepsilon + P = Ts + \mu\rho, \quad d\varepsilon = T ds + \mu d\rho,$$

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$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu$ Energy/momentum conservation

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y \\ -E_x & 0 & B \\ -E_y & -B & 0 \end{pmatrix}$$

$$\vec{E} = -i\vec{k}\frac{2\pi}{|k|}\rho_{\vec{k}}$$

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Q: **How to determine the dissipative terms $\nu^\mu$, $\tau^{\mu\nu}$?**

*(Landau-Lifschitz)*

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Positivity of entropy production: →

$$\partial_\mu\left(\frac{Q^\mu}{T}\right) = a_{1\mu}\partial^\mu T + a_{2\mu}\partial^\mu \mu + a_{3\mu}F^{\mu\nu}u_\nu + b_{\mu\nu}\partial^\mu u^\nu \geq 0$$

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Irrelevant for response at k → 0

# Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

$$J^\mu = \rho u^\mu + \nu^\mu \qquad T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

Conservation laws (equations of motion):

$$\partial_\mu J^\mu = 0$$ Charge conservation

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y \\ -E_x & 0 & B \\ -E_y & -B & 0 \end{pmatrix}$$

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu$$ Energy/momentum conservation

$$\vec{E} = -i\vec{k}\,\frac{2\pi}{|k|}\rho_{\vec{k}}$$

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} T^{0\nu}\delta_{\mu 0}$$ Momentum relaxation

Positivity of entropy production: →

$$\nu^\mu = \sigma_Q (g^{\mu\nu} + u^\mu u^\nu)\left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T}\right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^\mu u^\lambda)\left[\eta(\partial_\lambda u^\nu + \partial^\nu u_\lambda) + (\zeta - \eta)\delta^\nu_\lambda \partial_\alpha u^\alpha\right]$$

Irrelevant for response at k → 0

**One single** transport coefficient (instead of two)!

# Relativistic hydrodynamics

• at the S-I transition

• in graphene! *MM, and S. Sachdev, cond-mat 0801.2970.*

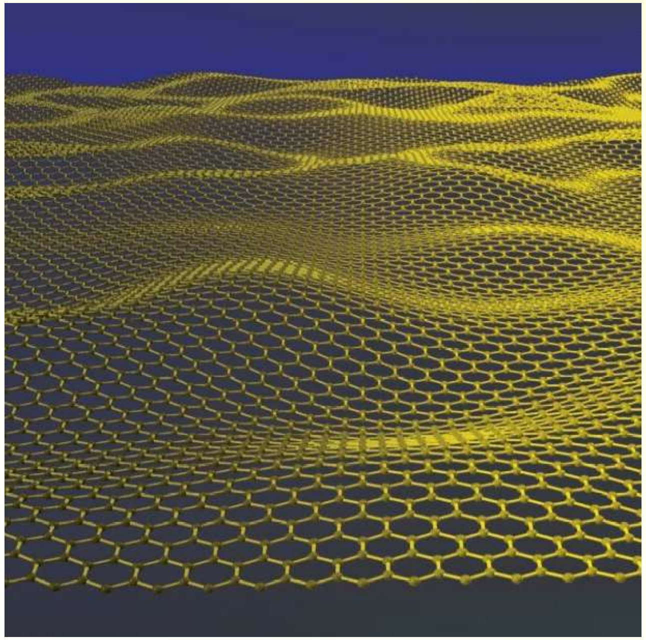
→ Get a feel for the “quantum critical” $\sigma_Q$ in graphene:
Calculation from a quantum Boltzmann equation

*L. Fritz, J. Schmalian, MM, and S. Sachdev, cond-mat 0802.4289.*

# Relativistic plasma in graphene

*MM, and S. Sachdev, cond-mat 0801.2970.*

Honeycomb lattice of C atoms



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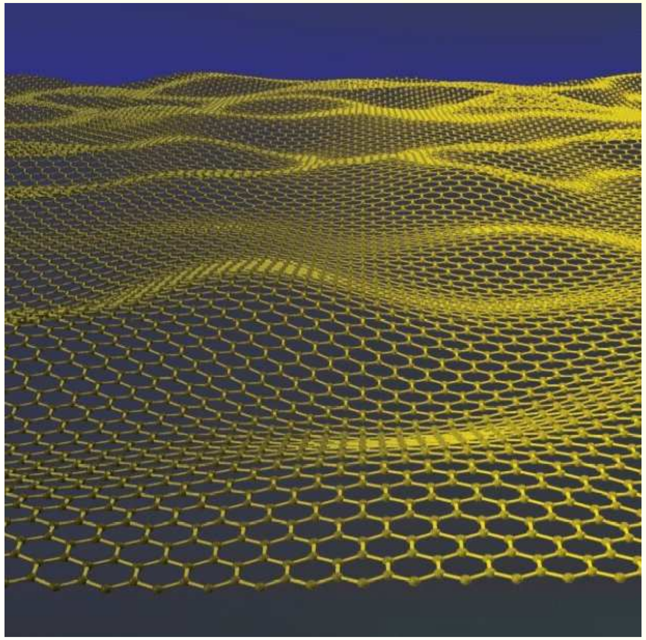
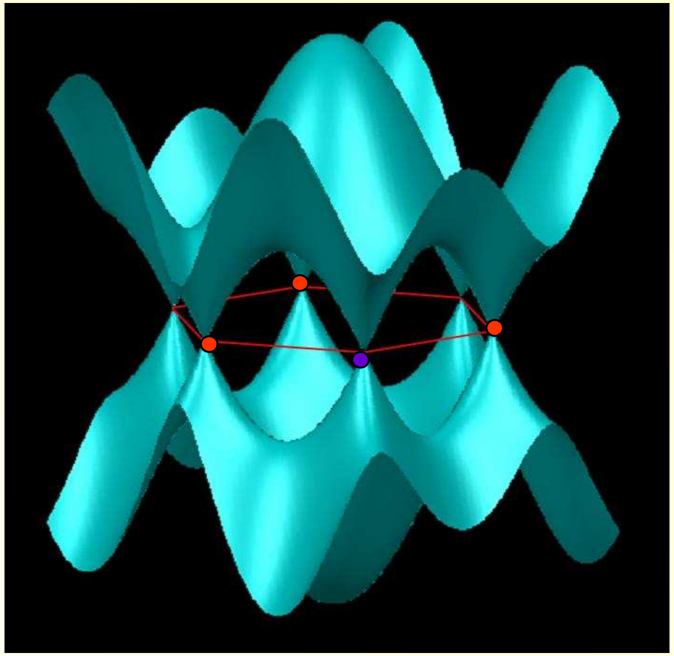


Tight binding dispersion



Close to the two
Fermi points **K**, **K'**:

$$H \approx v_F \left(\mathbf{p} - \mathbf{K}\right) \cdot \boldsymbol{\sigma}_{\text{sublattice}}$$

$$\rightarrow \quad E_{\mathbf{k}} = v_F \left|\mathbf{k} - \mathbf{K}\right|$$

Relativistic (Dirac) cones

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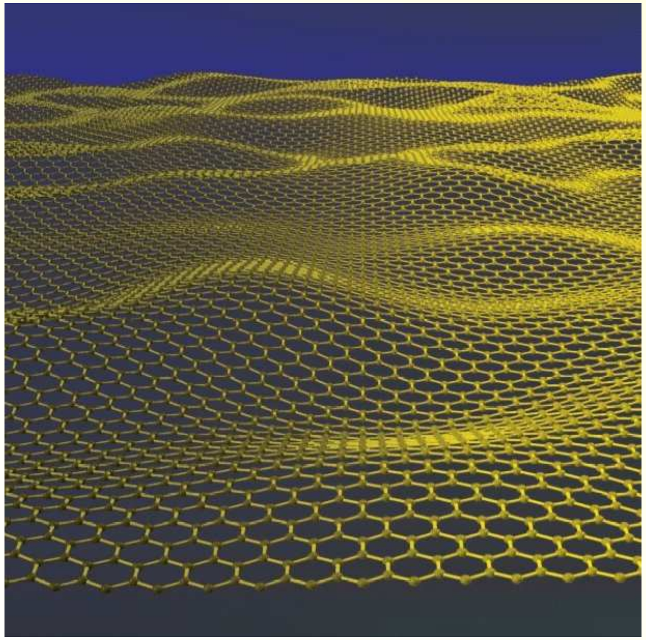

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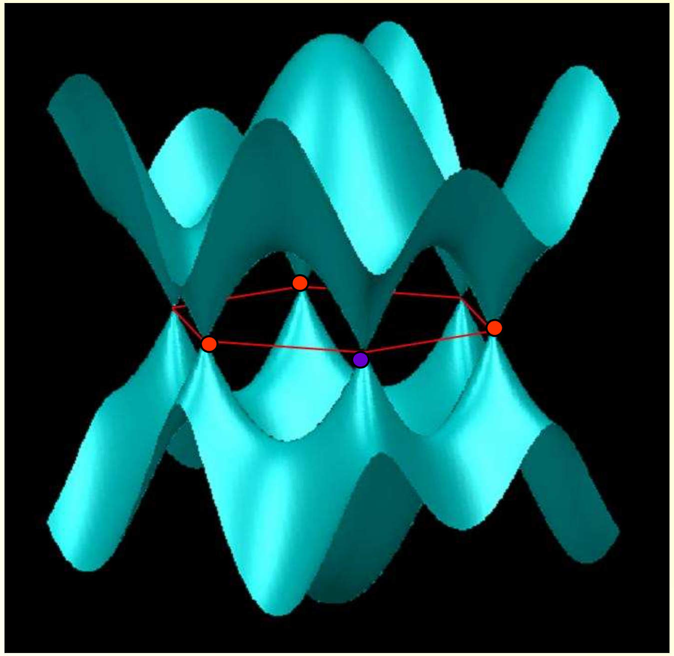

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Relativistic (Dirac) cones

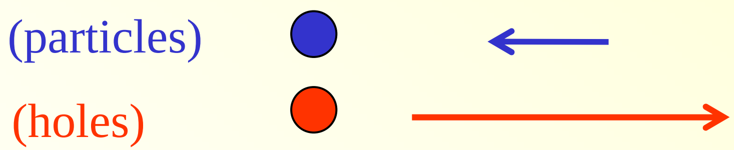
$$\mathrm{v} \equiv \mathrm{v}_F \approx 10^6 \,\mathrm{m/s} \approx \frac{c}{300}$$
$$\sim 500\ \mathrm{v}_{\text{high } T_c}\ !$$

# Universal conductivity $\sigma_Q$

Standard situation: No particle-hole symmetry ($\rho \neq 0$)

- Current is carried predominantly by majority carriers

- Finite current implies finite momentum:




$$\vec{J} \neq 0, \;\; \vec{J}^{E} \neq 0$$

- In the absence of impurities:
  Momentum conservation implies infinite conductivity!

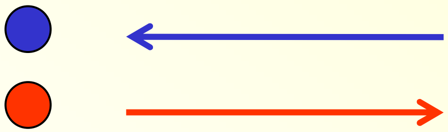
# Universal conductivity $\sigma_Q$

Quantum critical situation: Particle-hole symmetry ($\rho = 0$)

- Charge current without momentum (energy current)

(particle)

(hole)



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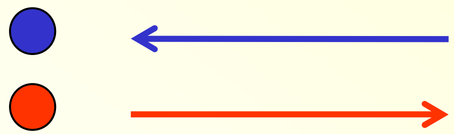

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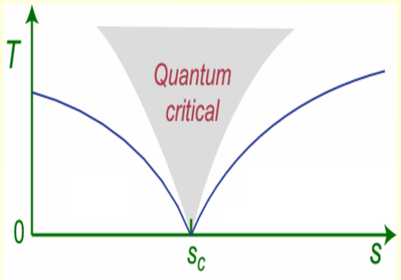
- Quantum criticality:
  Relaxation time set by temperature alone
  (interaction strength: $\alpha = e^2 / h\mathrm{v}$ )

$$\tau_{rel} \approx \frac{\hbar}{\alpha^2 k_B T}$$



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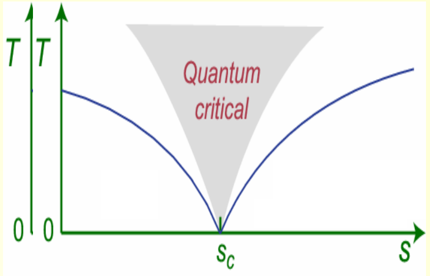



→ Universal quantum critical conductivity

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma_Q \sim \frac{e}{k_B T / \text{v}^2}\left( e \frac{(k_B T)^2}{(\hbar \text{v})^2} \right) \frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

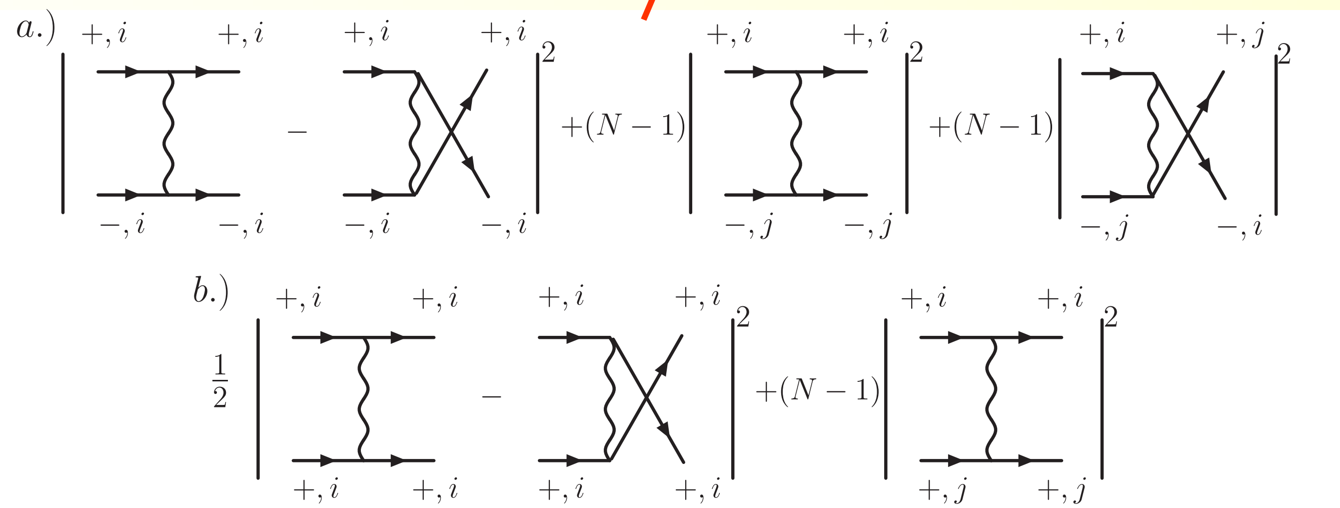
# Universal conductivity $\sigma_Q$ : graphene

*L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289*

Quantum critical situation: Particle-hole symmetry ($\rho = 0$), no impurities

Quantum Boltzmann equation

$$\left(\partial_t + e\mathbf{E}\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = I_{\text{collision}}\left[\left\{f_\pm(\mathbf{k}',t)\right\}\right] \quad \propto \alpha^2$$



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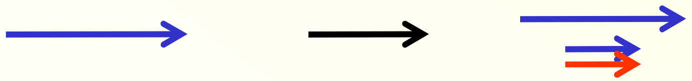
Great simplification: Divergence of collinear scattering amplitude

Amp [  ] $\to \infty$

$\rightarrow$ Equilibration along unidimensional spatial directions

$$f_\pm(\mathbf{k},t) = f_\pm^{eq}\left(\mathbf{k}, \mu \to \mu_{eq} + \delta\mu(t)\right) \ ; \ \delta\mu = C(t)\frac{\mathbf{E}\cdot\mathbf{k}}{k}$$

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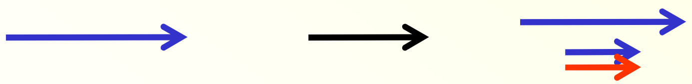
Amp [  ] $\rightarrow\infty$

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→ $$\sigma(\omega = 0) \approx \frac{0.76}{\alpha^2}\frac{e^2}{h}$$

# Thermoelectric response

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Charge and heat current: $J^\mu = \rho u^\mu + \nu^\mu$ $\quad Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$

Thermo-electric response in the particle picture

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix} \qquad \hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

etc.

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Nernst signal

$$e_N \equiv \vartheta_{yx}$$

Nernst coefficient

$$\nu = e_N / B$$

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Task: i) Solve linearized hydrodynamic equations;
ii) Read off the response functions (Kadanoff & Martin 1960)

# Results

# Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)$$

Universal conductivity at the quantum critical point $\rho = 0$

Drude-like conductivity, divergent for $\tau \to \infty, \omega \to 0, \rho \neq 0$

Momentum conservation ($\rho \neq 0$)!

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Thermal conductivity:

$$\kappa_{xx}(\omega, k; B=0) = \sigma_Q\frac{\mu^2}{T} + \frac{s^2 T}{P+\varepsilon}\frac{\tau}{1-i\omega\tau} + \mathcal{O}(k^2).$$

Relativistic Wiedemann-Frantz-like relations between σ and κ!

# B > 0 : Cyclotron resonance

E.g.: Hall conductivity

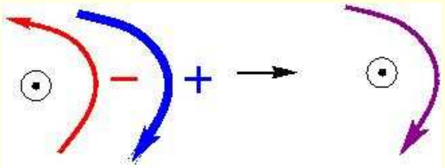
$$\sigma_{xy}(\omega,k) = -\frac{\rho}{B}\frac{\omega_c^2+\gamma^2+2\gamma(1/\tau-i\omega)}{(\omega+i/\tau+i\gamma)^2-\omega_c^2}$$

Poles in the response

$$\omega = \pm\omega_c^{rel} - i\gamma - i/\tau$$

Collective cyclotron frequency of the relativistic plasma

$$\omega_c^{rel} = \frac{v^2}{c^2}\frac{2e\,B}{(\varepsilon+P)/\rho c} \leftrightarrow \omega_c^{nonrel} = \frac{2e\,B}{mc}$$



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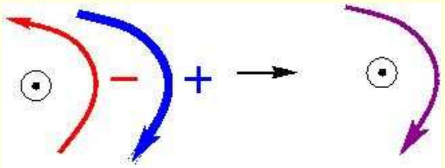


Intrinsic, interaction-induced broadening
(↔ Galilean invariant systems:
No broadening due to Kohn's theorem)

$$\gamma = \sigma_Q \frac{v^2}{c^2}\frac{B^2}{\varepsilon+P}$$

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Longitudinal conductivity

$$\sigma_{xx}(\omega, k) \;=\; \sigma_Q \frac{(\omega + i/\tau)\,(\omega + i/\tau + i\gamma + i\omega_c^2/\gamma)}{(\omega + i/\tau + i\gamma)^2 - \omega_c^2}$$

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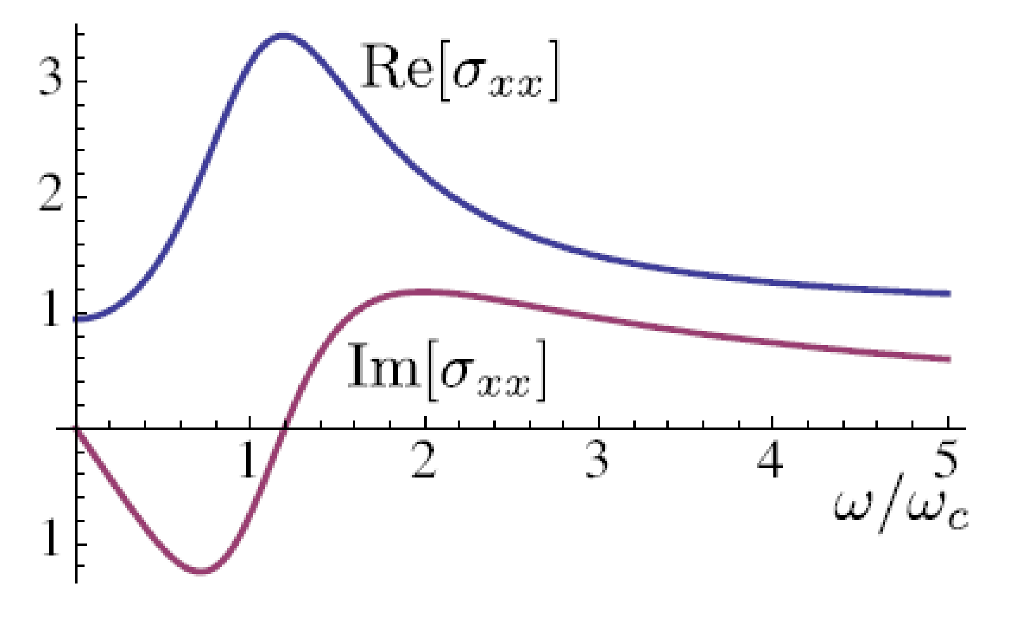

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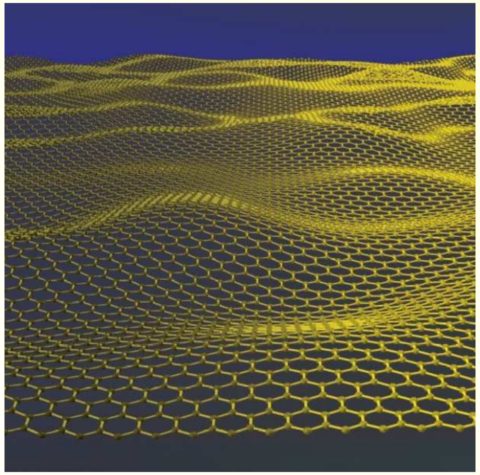
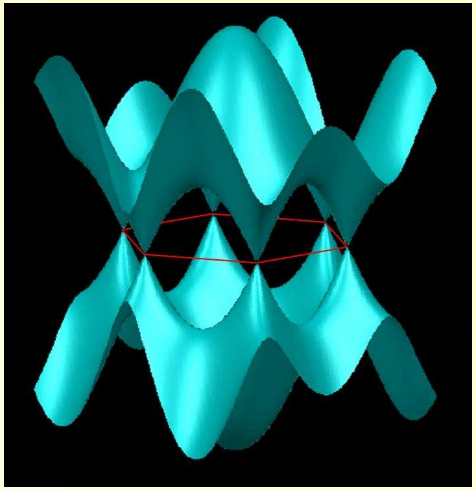


$$\omega = \pm\omega_c^{rel} - i\gamma - i/\tau$$

$$v = 1.1\cdot 10^6\, m/s$$
$$\approx c/300$$



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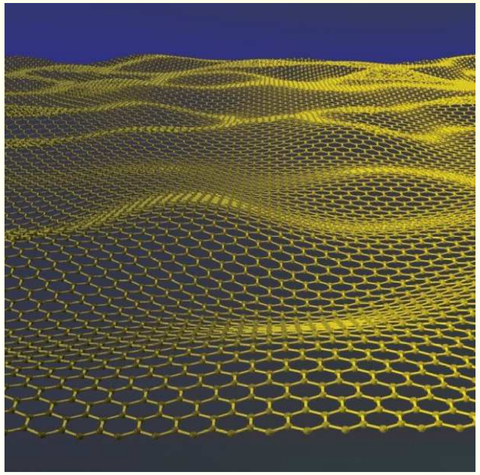

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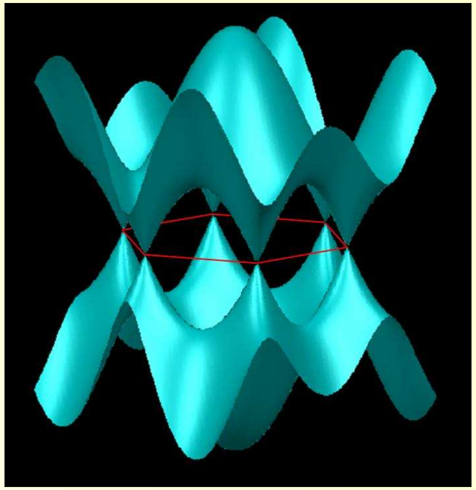

## Conditions to observe resonance

| | |
|---|---|
| Negligible Landau quantization | $E_{LL} = \hbar v\sqrt{\frac{2eB}{\hbar c}} << k_B T$ |
| Hydrodynamic, collison-dominated regime | $\hbar\omega_c^{rel} << k_B T$ |
| Negligible broadening | $\gamma, \tau^{-1} < \omega_c^{rel}$ |
| Relativistic, quantum critical regime | $\rho \le \rho_{th} = \frac{(k_B T)^2}{(\hbar v)^2}$ |

$T \approx 300K$

$B \approx 0.1T$

$\rho \approx 10^{11} cm^{-2}$

$\omega_c \approx 10^{13} s^{-1}$

AdS/CFT correspondence:

Recover magnetohydrodynamics from String theory techniques

# AdS/CFT

The AdS/CFT correspondence (Maldacena, Polyakov) relates CFTs to the quantum gravity theory of a black hole in Anti-de Sitter (AdS) space.

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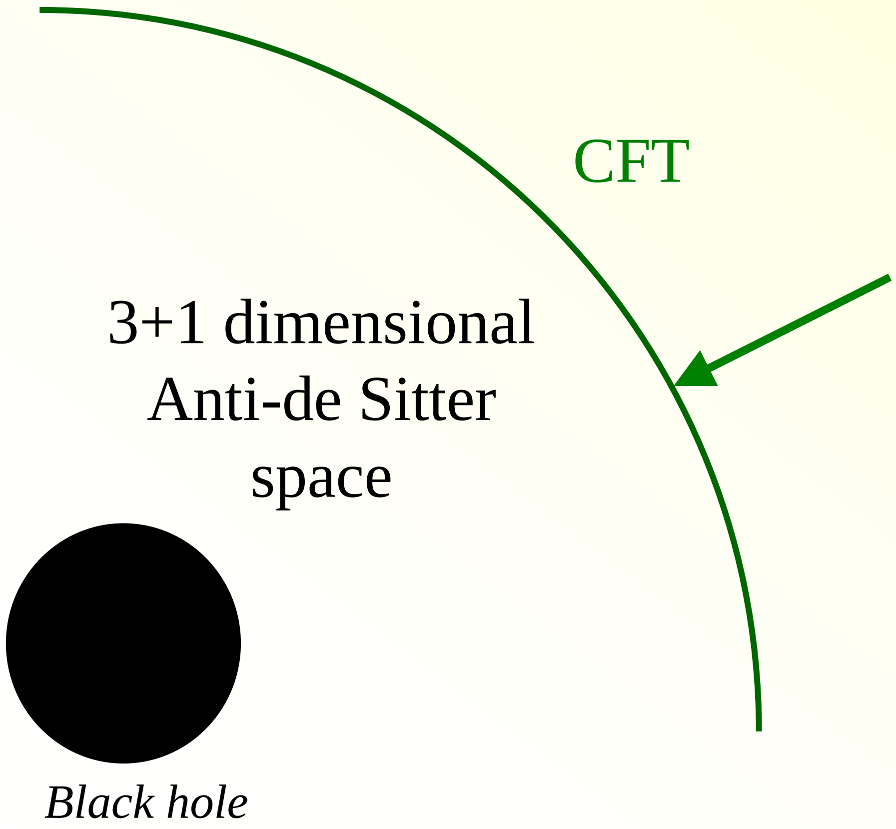


- 2+1 dimensional CFT holographically represents the black hole physics, the CFT living on the boundary of $AdS_{3+1}$ space
- The temperature of the CFT equals the Hawking temperature of the black hole.

# AdS/CFT

Goal:

- Solve exactly a conformal field theory (CFT), obtain $\sigma_Q$
- Soluble theories:
  Supersymmetric Yang-Mills theory, perturbed by

  - a chemical potential
  - a magnetic field

# AdS/CFT

Simplest gravitational dual to $\mathrm{CFT}_{2+1}$: Einstein-Maxwell theory

$$I = \frac{1}{g^2}\int d^4x\sqrt{-g}\left[-\frac{1}{4}R + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\right].$$

(embedded in M theory as $AdS_4 \times S^7$: $1/g^2 \sim N^{3/2}$)

It has a black hole solution (with electric and magnetic charge):

$$ds^2 = \frac{\alpha^2}{z^2}\left[-f(z)dt^2 + dx^2 + dy^2\right] + \frac{1}{z^2}\frac{dz^2}{f(z)},$$

$$F = h\alpha^2 dx \wedge dy + q\alpha dz \wedge dt,$$

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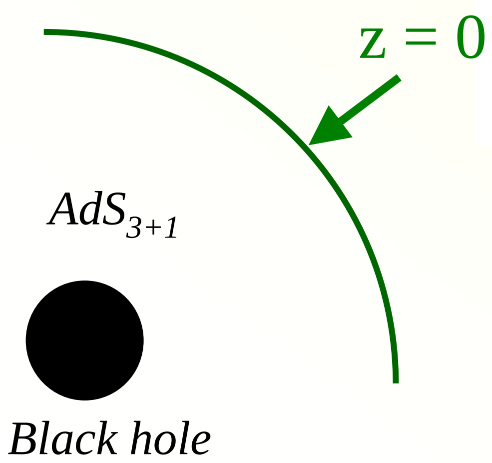


Electric charge $q$ and magnetic charge, $h$

$\leftrightarrow$ $\mu$ and $B$ for the CFT

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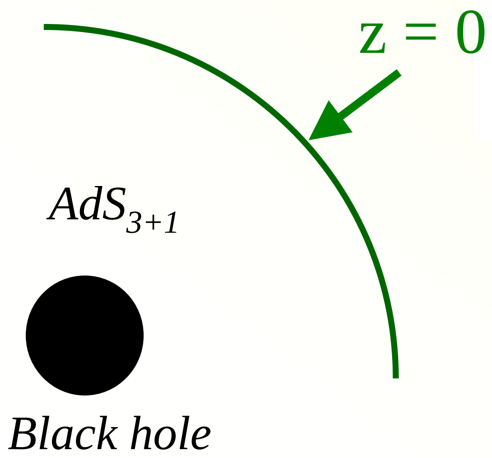
$$F = h\alpha^2 dx \wedge dy + q\alpha dz \wedge dt ,$$

$$f(z) = 1 + (h^2 + q^2)z^4 - (1 + h^2 + q^2)z^3 .$$




Background $\leftrightarrow$ Equilibrium

Transport $\leftrightarrow$ Perturbations in $g_{tx,ty}, A_{x,y}$ .

Response via Kubo formula from $\delta^2 I/\delta(g,A)^2$.

# AdS/CFT

## Main results

- Precise agreement with MHD, *without* imposing the principle of positivity of entropy production!

- Exact value for $\sigma_Q$.

- Proven potential to go beyond MHD

  *S. Hartnoll+Ch. Herzog: beyond small B, calculation of $\tau_{imp}(\rho,B)$.*

# Comparison of hydrodynamics with experiments in high $T_c$'s

# Nernst signal (B > 0)

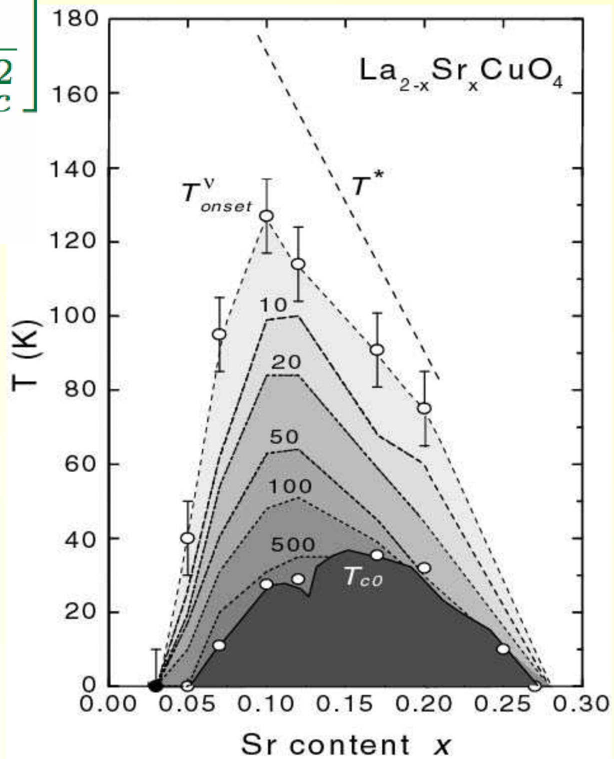
$$e_N \equiv N = \frac{E_y}{-\vec{\nabla}_x T} \qquad (\vec{J} = 0)$$

Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\text{imp}}}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]$$

Quantum unit of the Nernst signal

$$\frac{k_B}{2e} = 43.086\ \mu\text{V/K}$$



# Comparison with experiment: Peltier coefficient

$$\alpha_{xy} = \left(\frac{2ek_B}{h}\right)\left(\frac{s/k_B}{B/\phi_0}\right)\left[\frac{\gamma^2 + \omega_c^2 + \gamma/\tau_{\rm imp}\{1 - \mu\rho/(Ts)\}}{(\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

Quantum critical scaling: $\varepsilon, P = \# T^3 \ ; \ s = \# T^2 \ ; \ \sigma_{\rm Q} = \#$

$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \# T^3\right)}{T^6 + \# B^2\rho^2\tau_{imp}^2}$$

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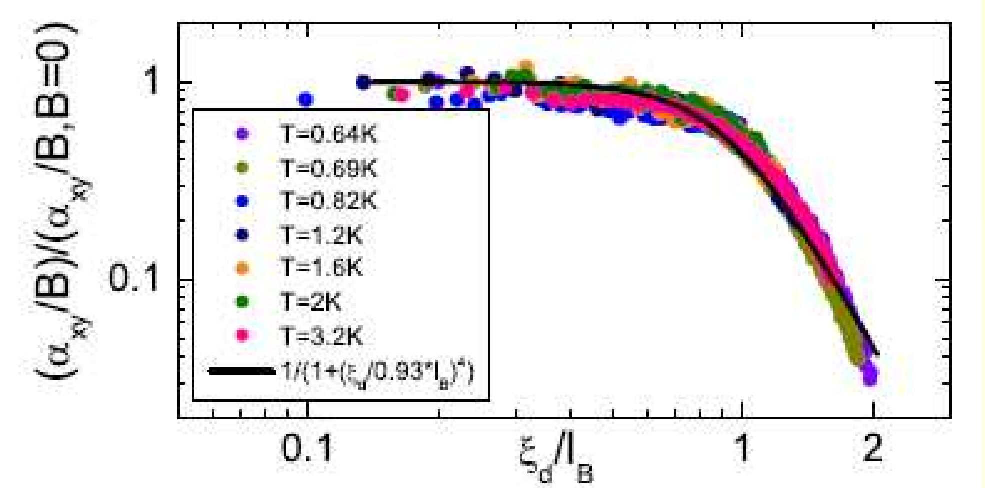
$$\frac{\alpha_{xy}}{B} = \frac{C}{1+\left(\xi_d/\ell_B\right)^4} = \frac{C}{1+\left(B/B_0\right)^2}$$



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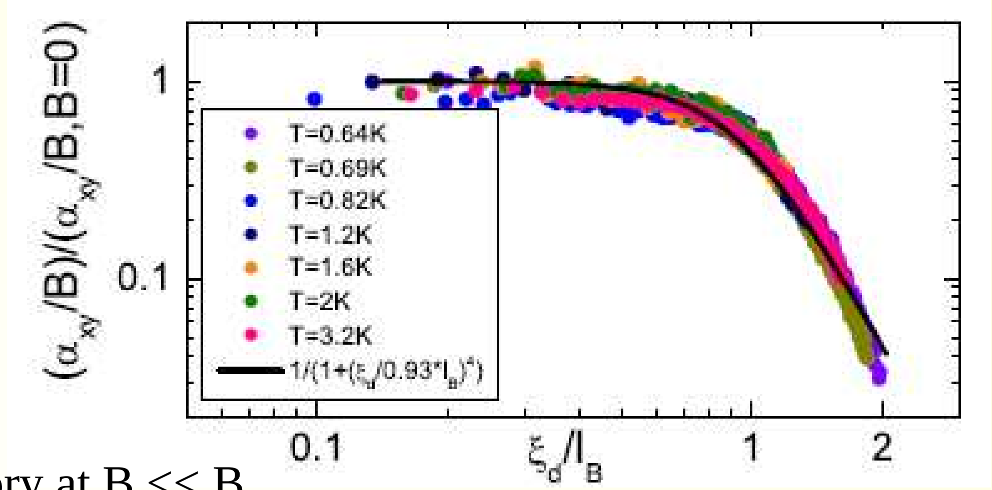
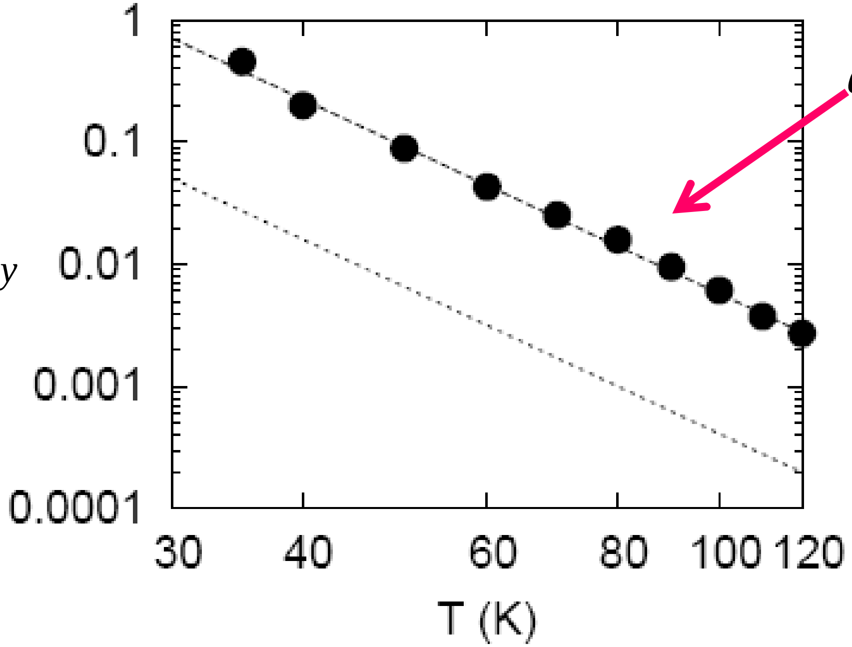
Matches with Gaussian fluctuation theory at B << $B_0$



# LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$




$\alpha_{xy}$

$$\alpha_{xy} \propto \frac{1}{T^4}$$

$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \#T^3\right)}{T^6 + \#B^2\rho^2\tau_{imp}^2}$$

(T not too large)

$$\frac{\alpha_{xy}}{B}(B \to 0) \approx \left(\frac{2ek_B}{h\phi_0}\right)\frac{\Phi_s}{\Phi_{\varepsilon+P}^2}\left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2\frac{\rho^2(\hbar v)^6}{(k_B T)^4}$$

*Y. Wang et al., Phys. Rev. B* ***73****, 024510 (2006).*

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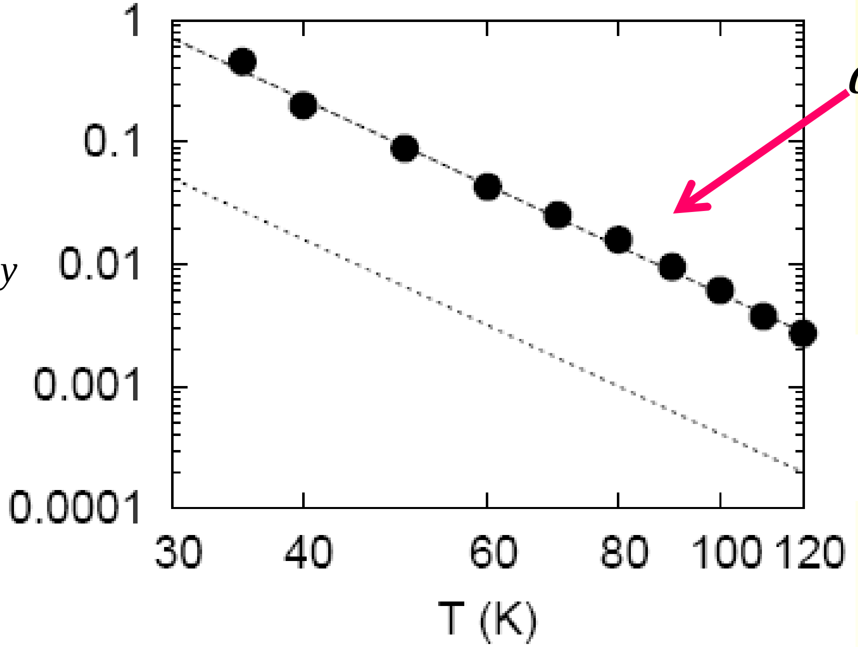


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$\hbar \mathrm{v} \approx 47 \mathrm{meV}\,\mathring{A}$

$\mathrm{v} \approx 2.5 \cdot 10^{-5} c$ $\quad\quad \tau_{imp} \approx 10^{-12}\,\mathrm{s}$

$\approx 10^{-2}\,\mathrm{v}_{Graphene}$

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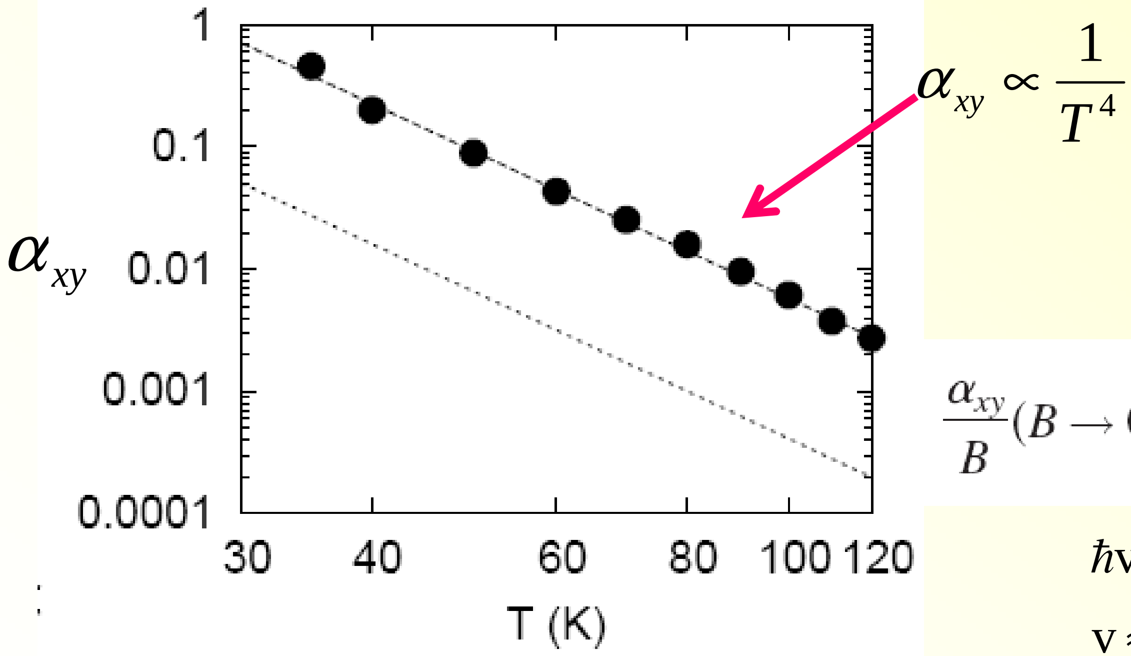
Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$




*Y. Wang et al., Phys. Rev. B* ***73****, 024510 (2006).*

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$\hbar v \approx 47 \text{meV Å}$

$v \approx 2.5 \cdot 10^{-5} c \qquad \tau_{imp} \approx 10^{-12}\,\text{s}$

$\approx 10^{-2}\, v_{Graphene}$

**$\rightarrow$ Prediction for $\omega_c$:**

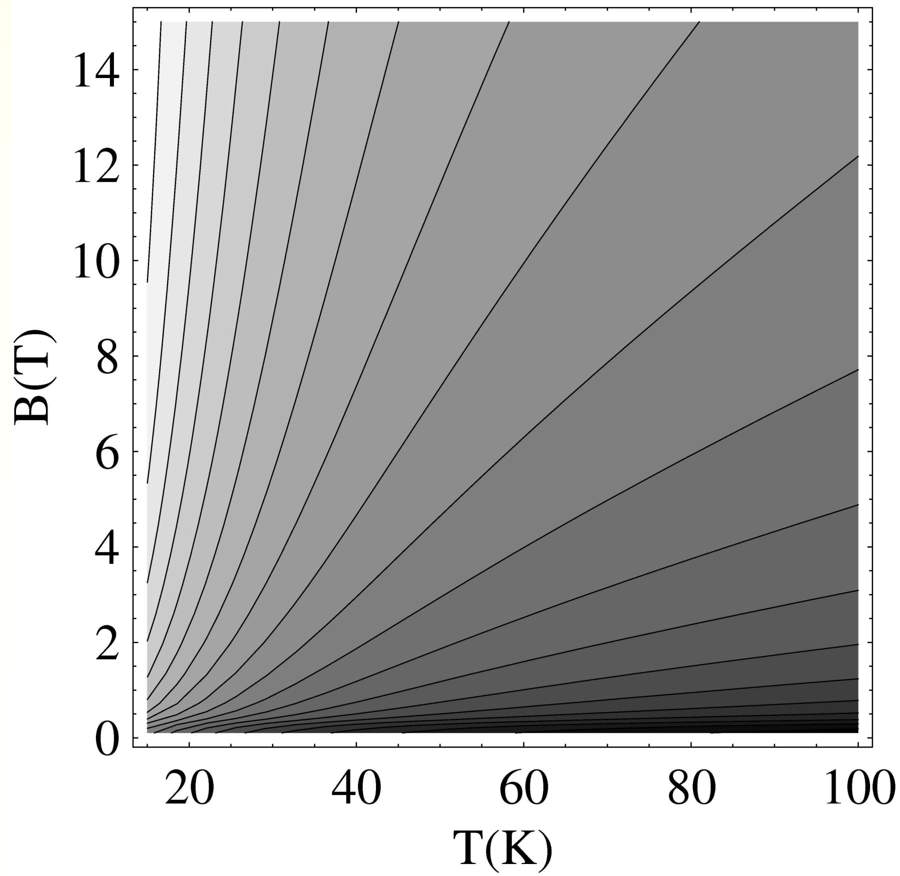
$$\omega_c = 6.2\ \text{GHz}\frac{B}{1\ \text{T}}\left(\frac{35\ \text{K}}{T}\right)^3$$

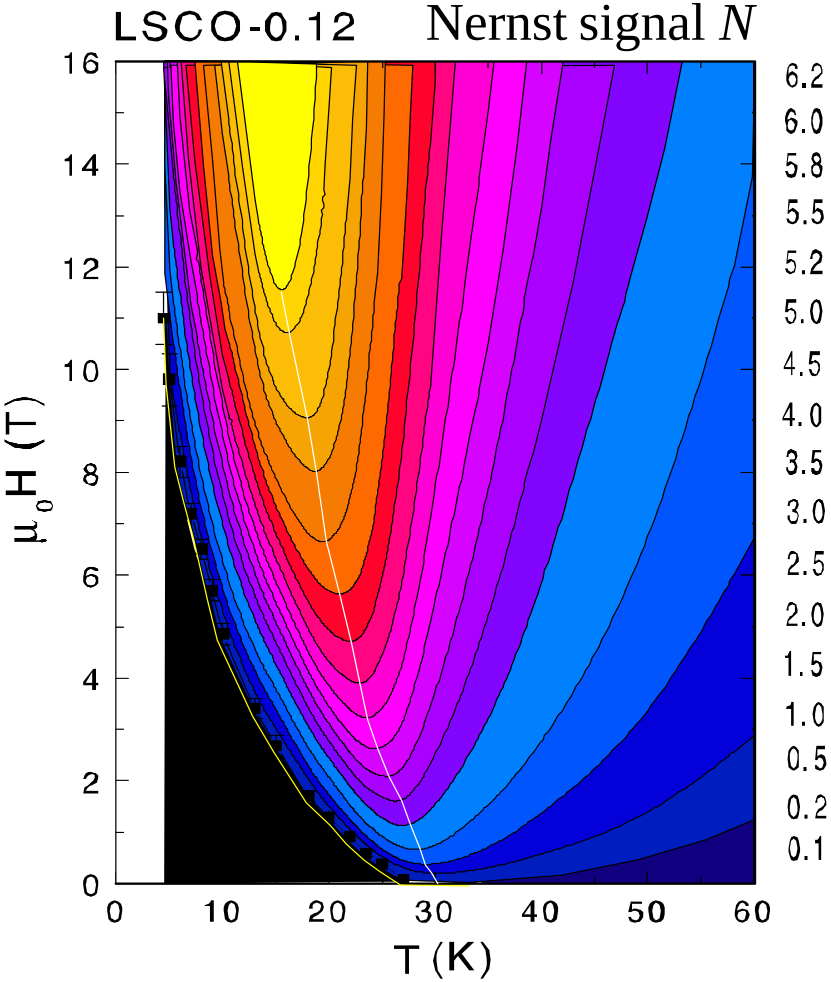
- 2 ordes of magnitude smaller than the cyclotron frequency of free electrons
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$.

# LSCO Experiments

*B*,*T* -dependence

Theory for $\alpha_{xy} \approx \sigma_{xx} N$




*Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B* ***73****, 024510 (2006).*

# Conclusions

- General theory of transport in a weakly disordered “vortex liquid’’ state close to a QCP.
- Simplest model reproduces many trends of the Nernst measurements in cuprates.
- Collective cyclotron resonance observable in graphene
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.