Relativistic magnetotransport in graphene

Markus Müller

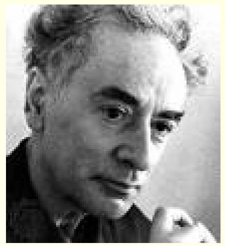
in collaboration with

Lars Fritz (Harvard)

Subir Sachdev (Harvard)

Jörg Schmalian (Iowa)

Landau Memorial Conference June 26, 2008



Outline

- Relativistic physics in graphene, quantum critical systems and conformal field theories

  → Relativistic signatures in magnetotransport:
  el.+th. conductivity, Peltier, Nernst effect etc.

- Hydrodynamic description

  → Collective, collision-broadened cyclotron resonance

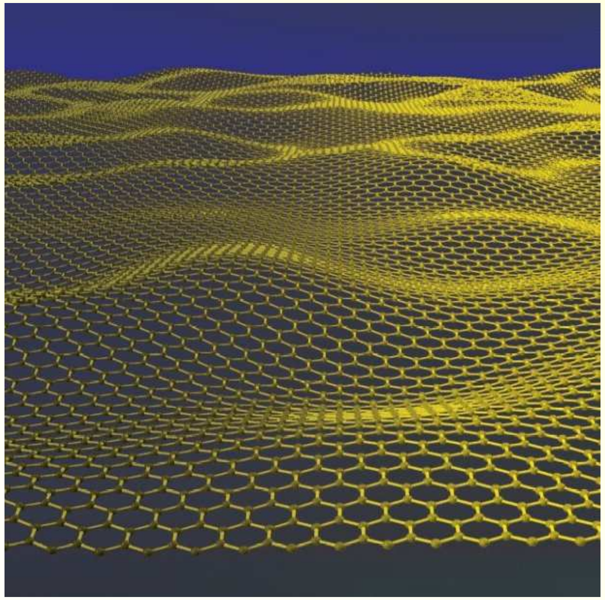
- Boltzmann equation

  → Recover and refine hydrodynamics with Boltzmann
  → Describe relativistic-to-Fermi liquid crossover
  → Go beyond hydrodynamics

Dirac fermions in graphene

(Semenoff '84, Haldane '88)

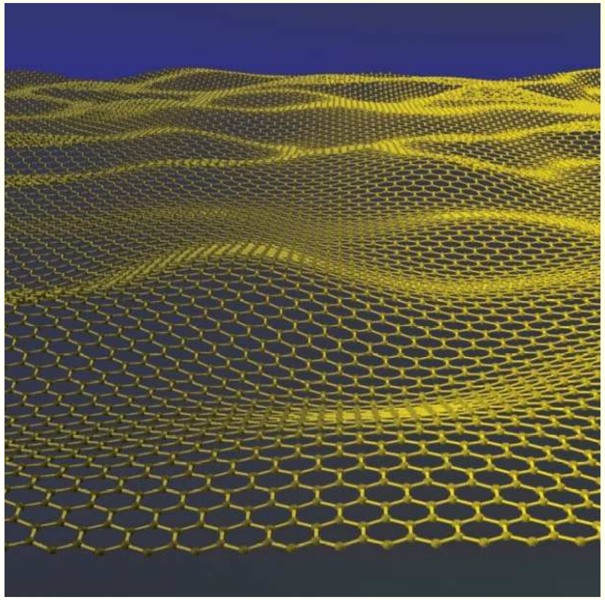
Honeycomb lattice of C atoms



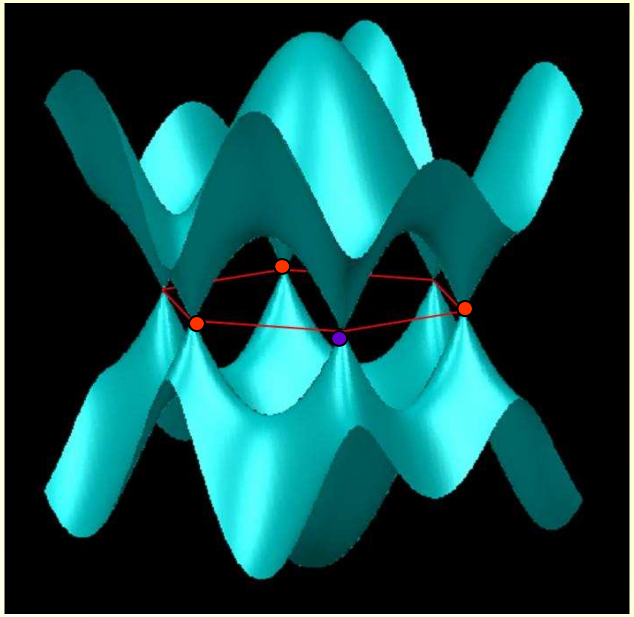
Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms



Tight binding dispersion



2 massless Dirac cones in the Brillouin zone: (Sublattice degree of freedom ↔ pseudospin)

Close to the two Fermi points **K**, **K'**:

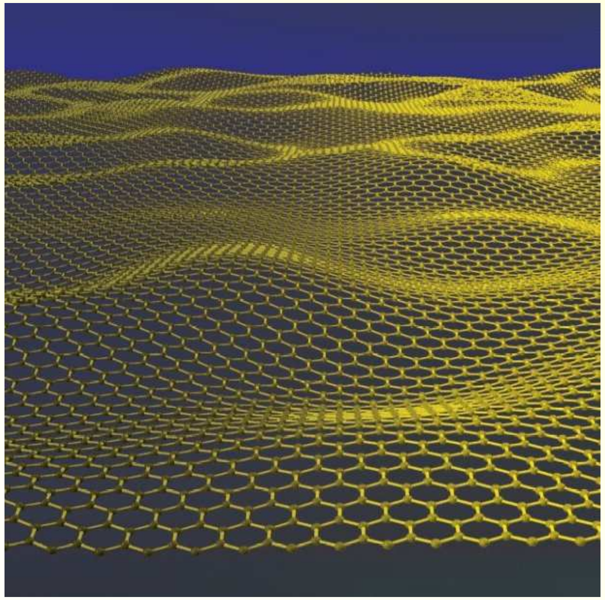
$$H \approx v_F \,(\mathbf{p}-\mathbf{K})\cdot \sigma_{\text{sublattice}}$$

$$\rightarrow \quad E_{\mathbf{k}} = v_F \left|\mathbf{k}-\mathbf{K}\right|$$

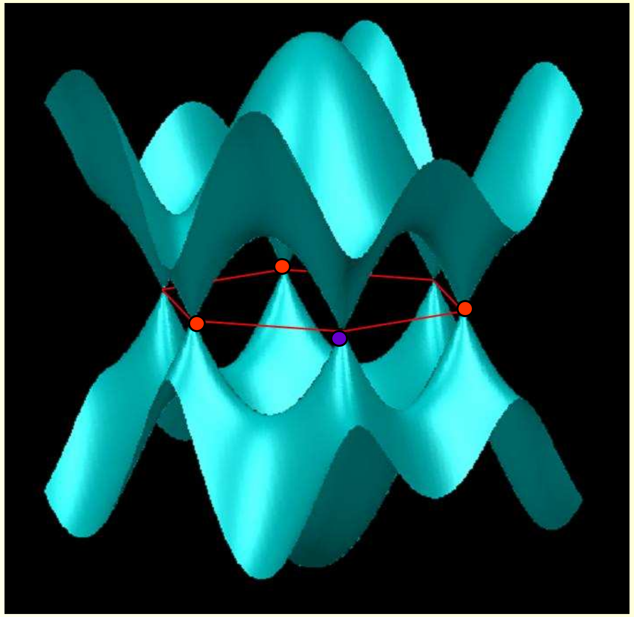
Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms



Tight binding dispersion



2 massless Dirac cones in the Brillouin zone: (Sublattice degree of freedom ↔ pseudospin)

Close to the two Fermi points **K**, **K'**:

$$H \approx v_F \, (\mathbf{p} - \mathbf{K}) \cdot \sigma_{\text{sublattice}}$$

$$\rightarrow \quad E_{\mathbf{k}} = v_F |\mathbf{k} - \mathbf{K}|$$

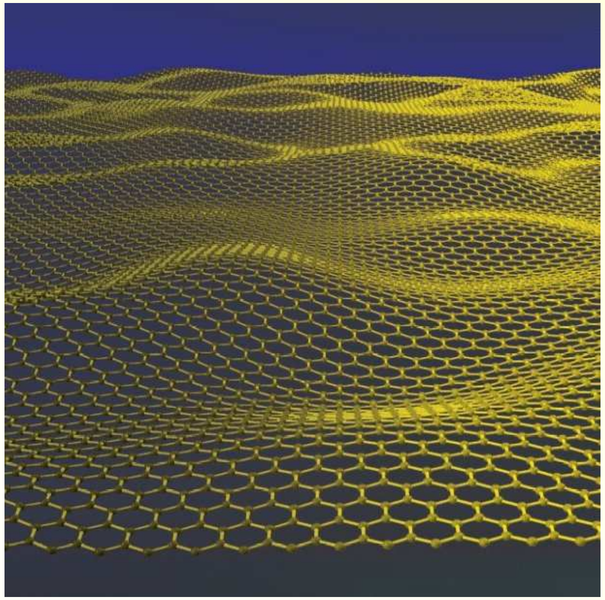
Fermi velocity (speed of light")

$$v_F \approx 1.1 \cdot 10^6 \text{ m/s} \approx \frac{c}{300}$$

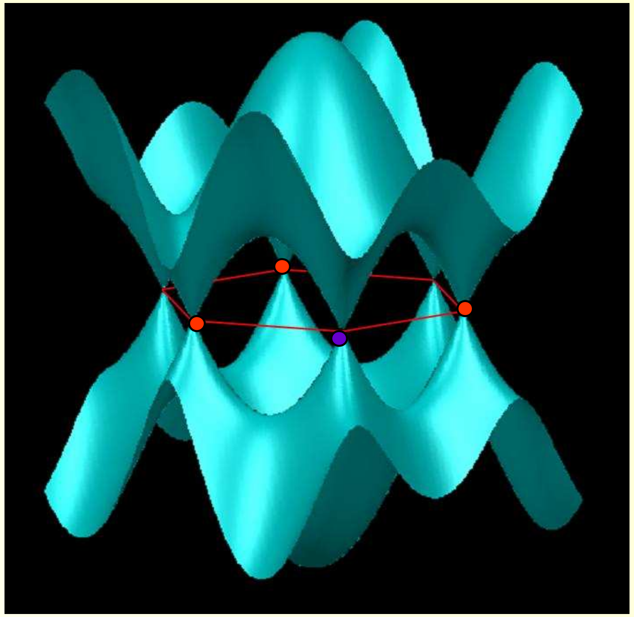
Dirac fermions in graphene

(Semenoff '84, Haldane '88)

Honeycomb lattice of C atoms



Tight binding dispersion



2 massless Dirac cones in the Brillouin zone: (Sublattice degree of freedom ↔ pseudospin)

Close to the two Fermi points **K**, **K'**:

$$H \approx \mathrm{v}_F \, (\mathbf{p} - \mathbf{K}) \cdot \sigma_{\text{sublattice}}$$
$$\rightarrow \quad E_{\mathbf{k}} = \mathrm{v}_F |\mathbf{k} - \mathbf{K}|$$

Fermi velocity (speed of light")

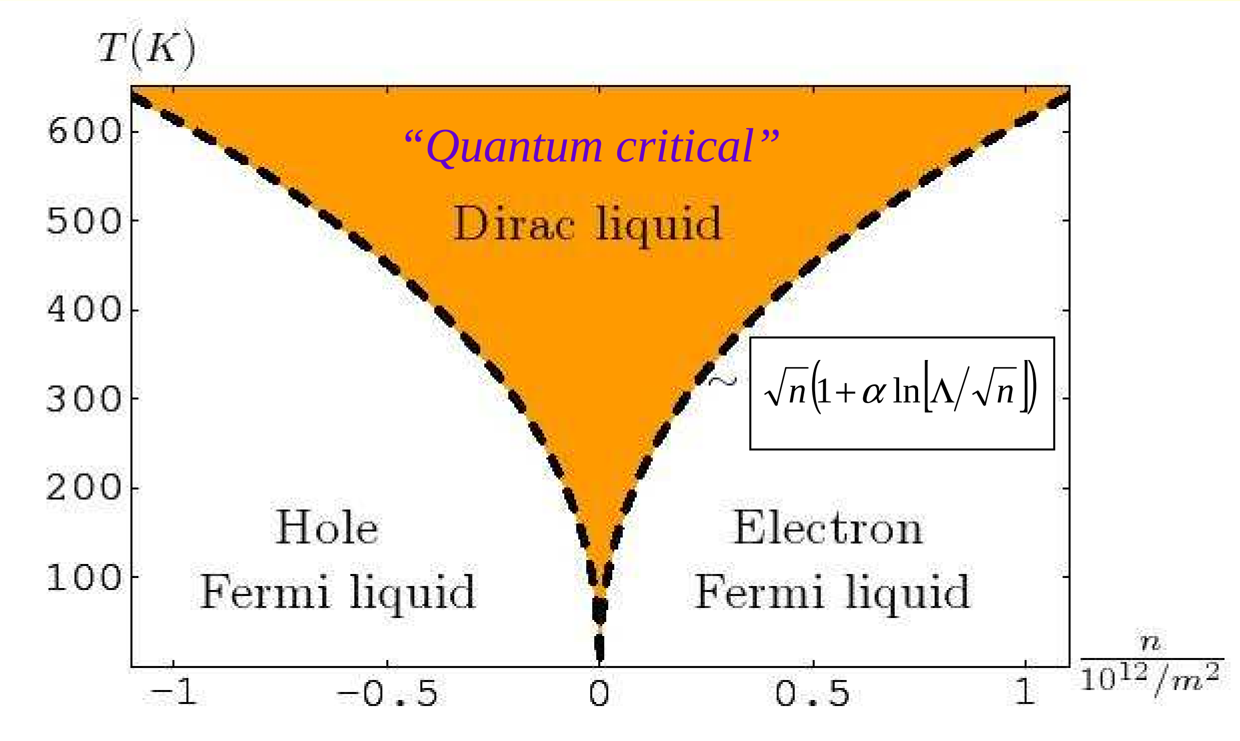
$$\mathrm{v}_F \approx 1.1 \cdot 10^6 \, \mathrm{m/s} \approx \frac{c}{300}$$

Coulomb interactions: Fine structure constant

$$\alpha \equiv \frac{e^2}{\varepsilon \hbar \mathrm{v}_F} = O(1)$$

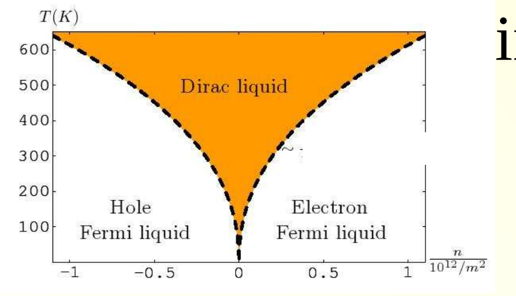
Relativistic fluid at the Dirac point

Expect relativistic plasma physics of interacting particles and holes!



D. Sheehy, J. Schmalian, Phys. Rev. Lett. ***99****, 226803 (2007).*

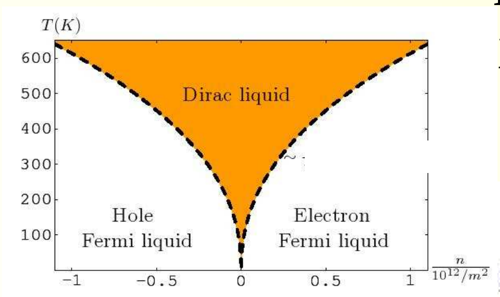
Transport and phase diagram

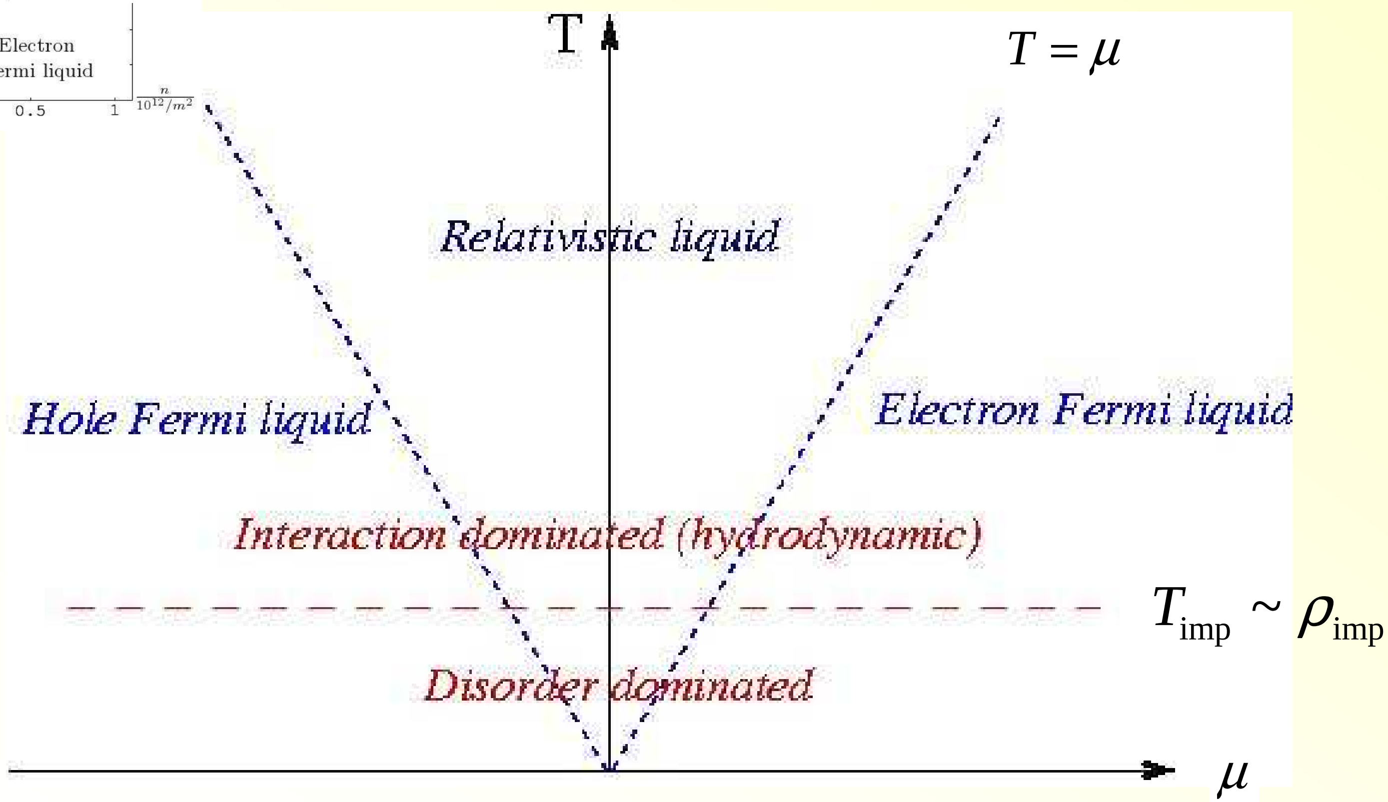


Expect relativistic plasma physics of interacting particles and holes!

Transport and phase diagram

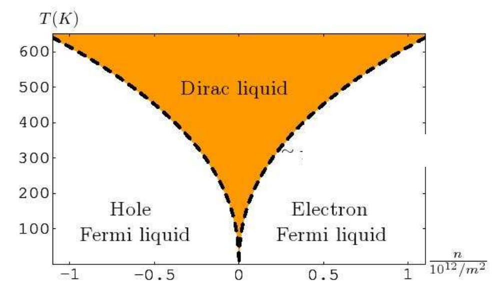
Expect relativistic plasma physics of interacting particles and holes!

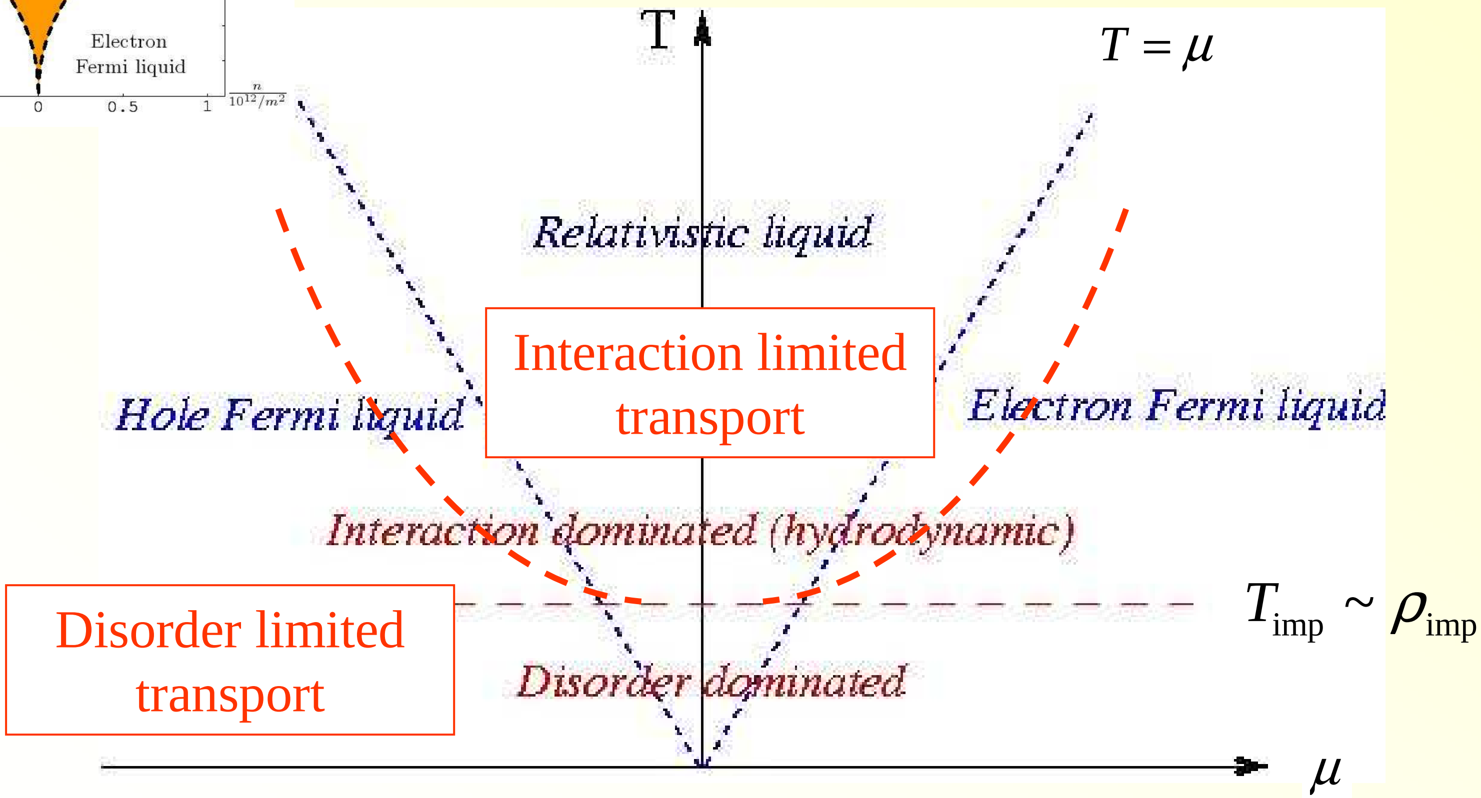




Transport and phase diagram

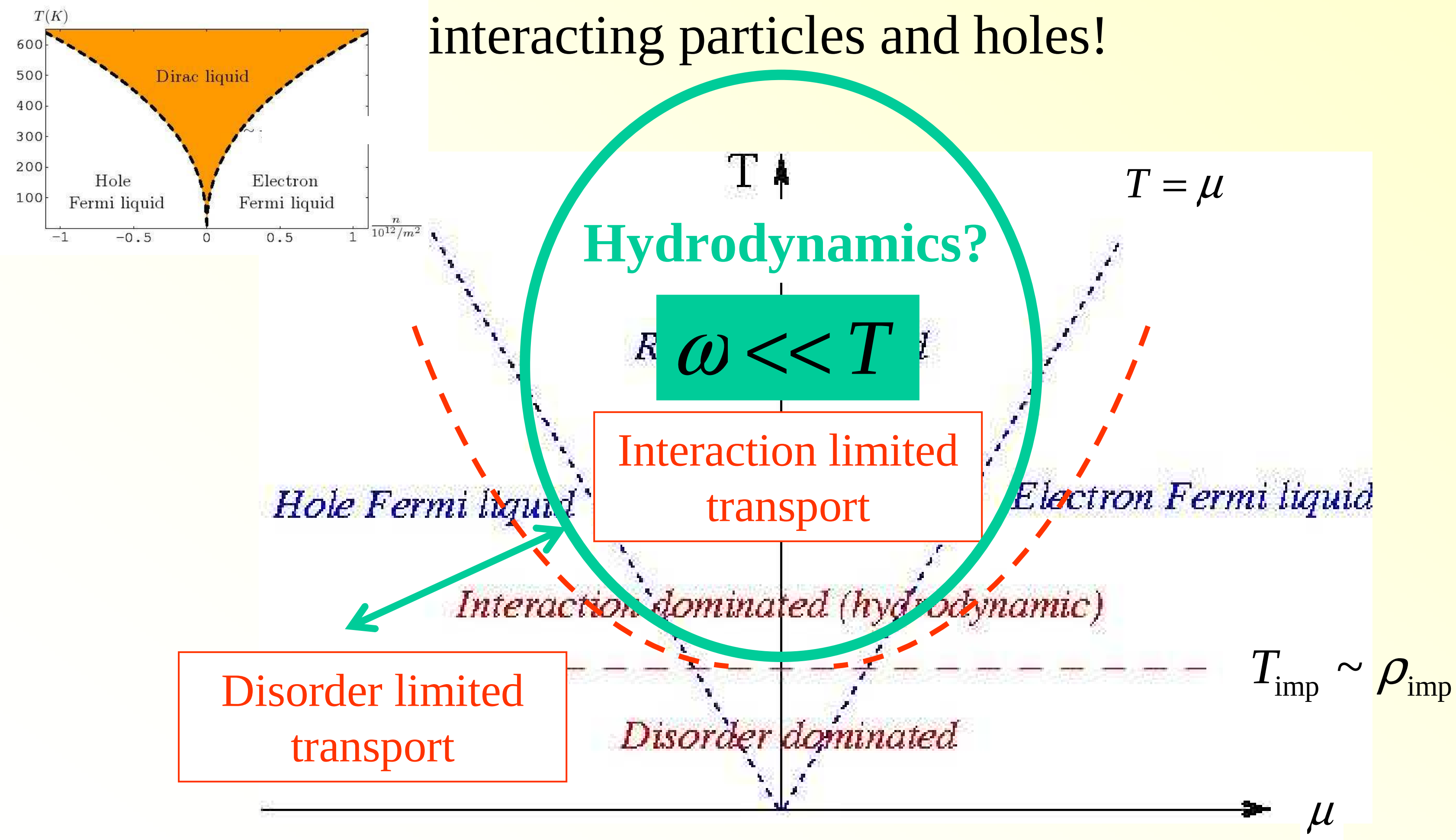
Expect relativistic plasma physics of interacting particles and holes!



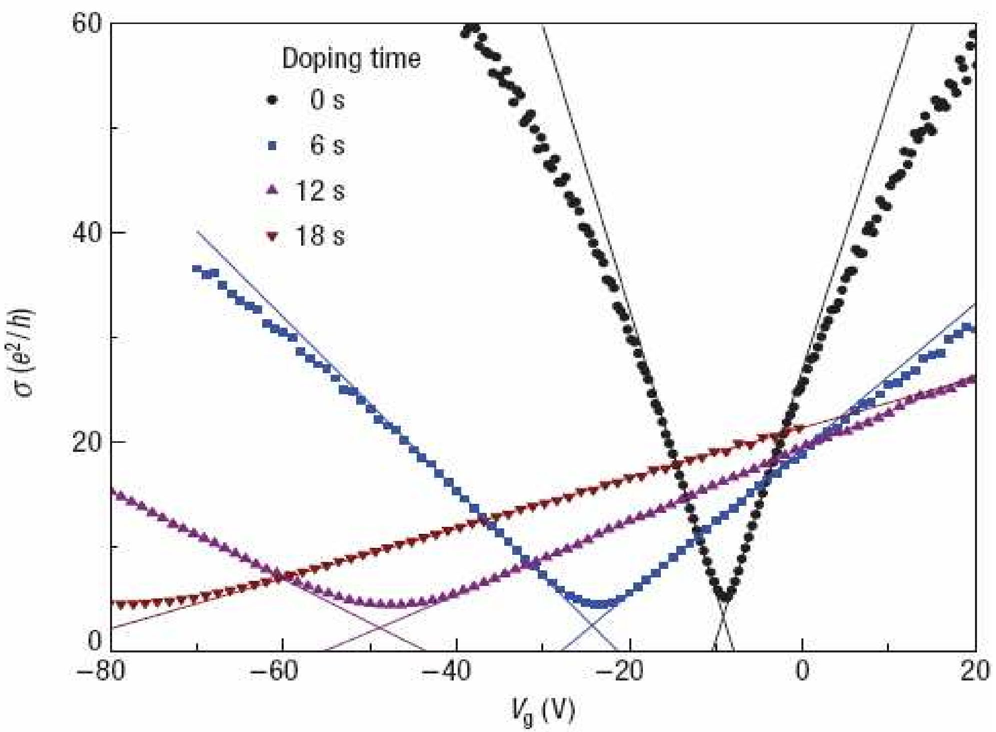


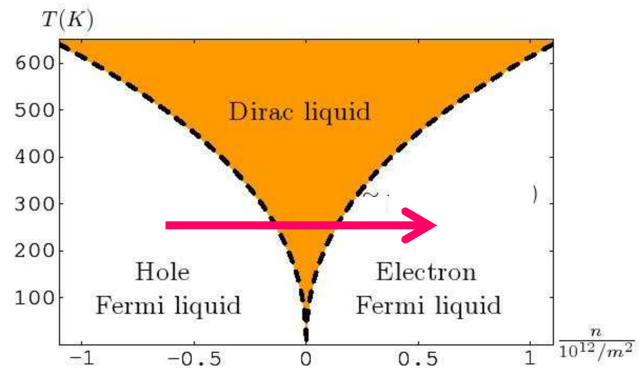
Transport and phase diagram

Expect relativistic plasma physics of interacting particles and holes!



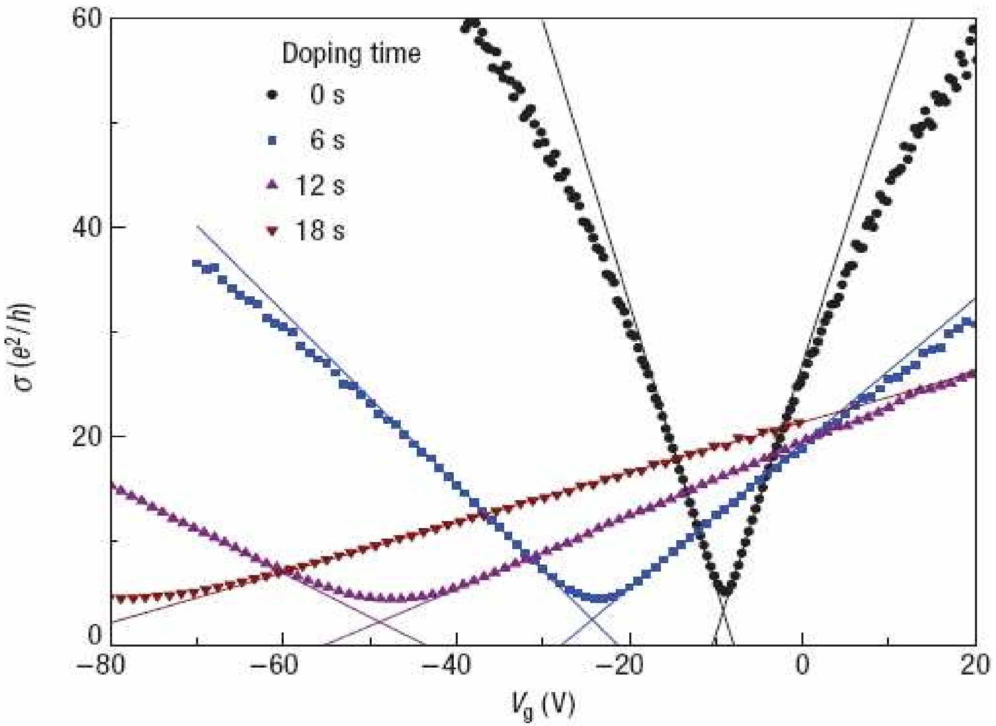
Conductivity in and across the relativistic regime?



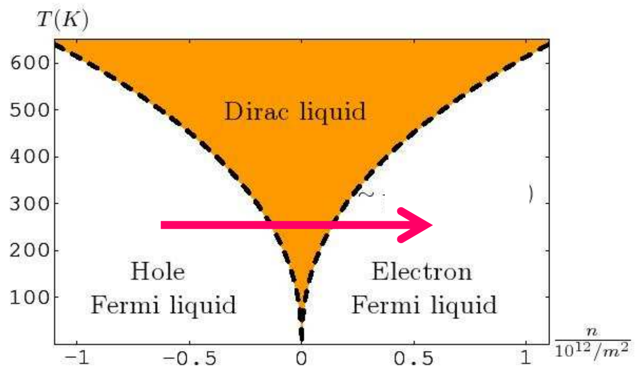


J.-H. Chen et al. Nat. Phys. ***4****, 377 (2008).*

Conductivity in and across the relativistic regime?



J.-H. Chen et al. Nat. Phys. ***4****, 377 (2008).*



+ Magnetotransport?
e.g., Hall, Nernst effect?

Other relativistic fluids:

- Bismuth (3d Dirac fermions with very small mass)
- Effective theories close to quantum phase transitions
- Conformal field theories
  E.g.: strongly coupled Non-Abelian gauge theories
  (QCD): tretament via AdS-CFT

Low energy effective theory at quantum phase transitions

Relativistic effective field theories $\leftrightarrow$ $z = 1$;
arise often due to particle-hole symmetry

Example: Superconductor-insulator transition (SIT)

Bhaseen, Green, Sondhi (PRL '07).
Hartnoll, Kovtun, MM, Sachdev (PRB '07)

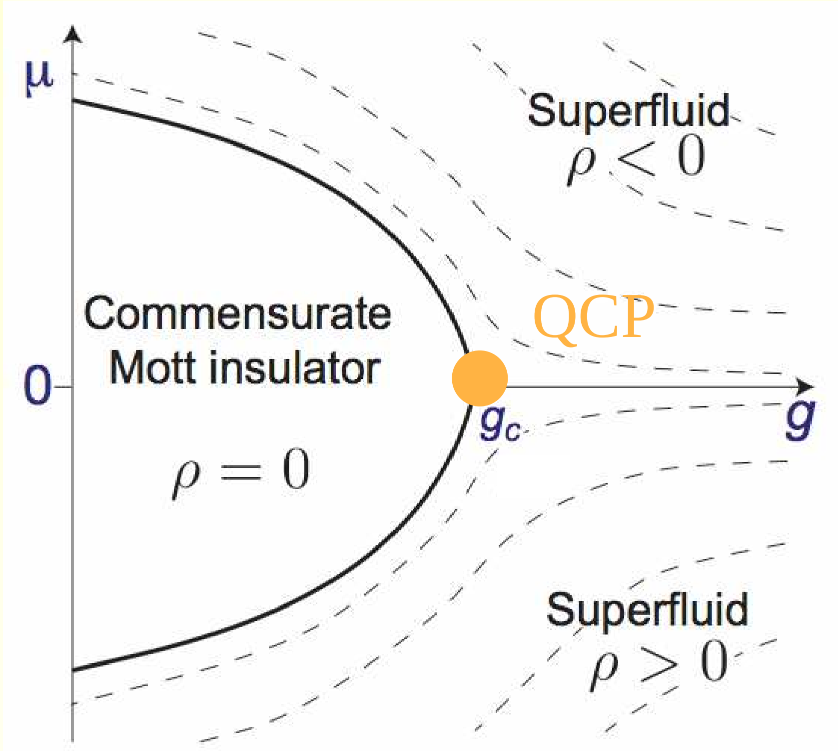
SI-transition: Bose Hubbard model

Bose-Hubbard model

$$H = -t\sum_{\langle ij\rangle} b_j^+ b_i + U\sum_i n_i^2 - \mu\sum_i n_i$$

Coupling

$$g \equiv \frac{t}{U}$$ tunes the SI-transition



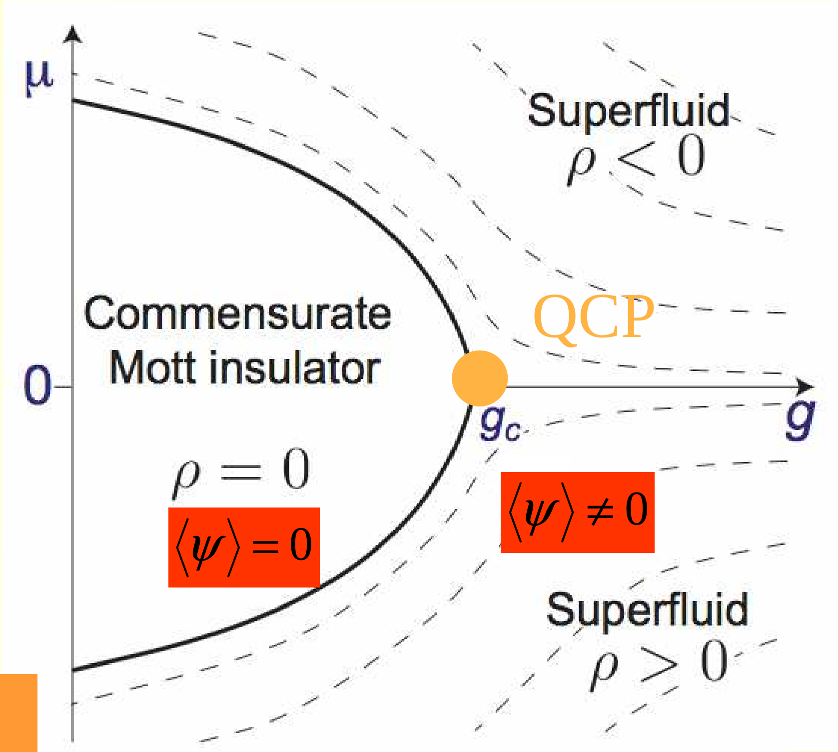
SI-transition: Bose Hubbard model

Bose-Hubbard model

$$H = -t\sum_{\langle ij \rangle} b_j^+ b_i + U\sum_i n_i^2 - \mu \sum_i n_i$$

Coupling

$g \equiv \dfrac{t}{U}$ tunes the SI-transition



Effective action around g_c ($\mu = 0$):

$$S = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 \left| \vec{\nabla}\psi \right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

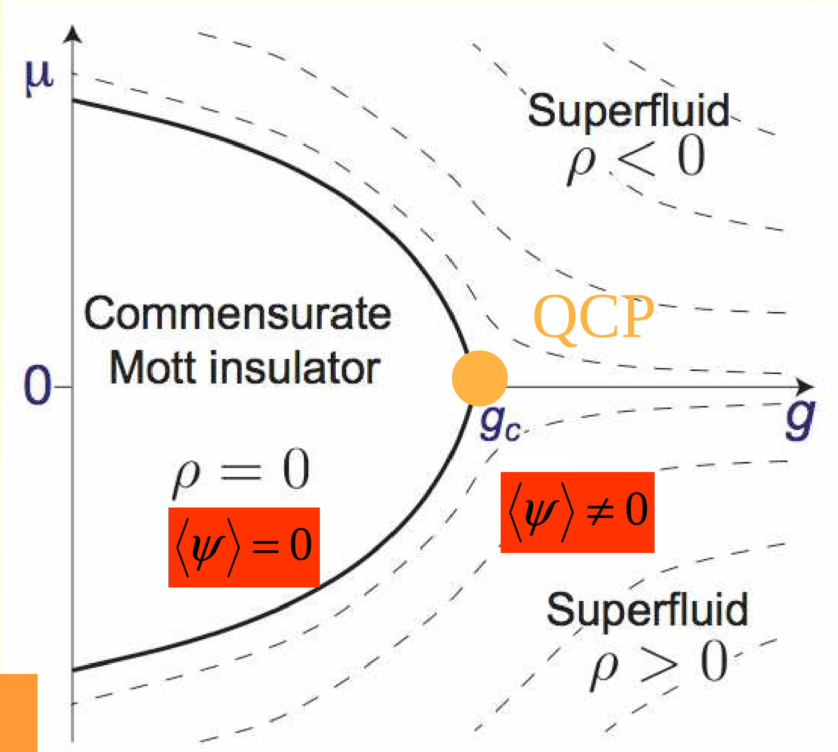
SI-transition: Bose Hubbard model

Bose-Hubbard model

$$H = -t\sum_{\langle ij\rangle} b_j^+ b_i + U\sum_i n_i^2 - \mu \sum_i n_i$$

Coupling

$g \equiv \dfrac{t}{U}$ tunes the SI-transition



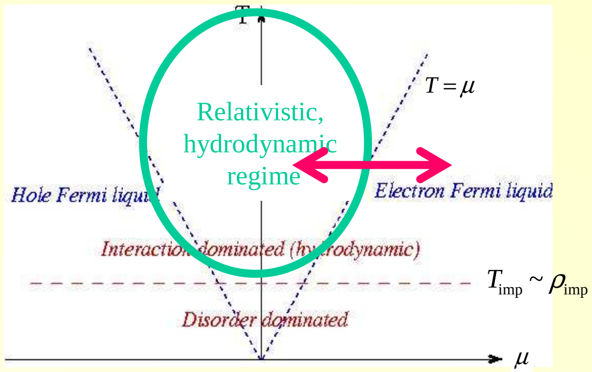
Effective action around g_c ($\mu = 0$):

$$S = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 \left|\vec{\nabla}\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

→ Relativistic field theory in d=2+1

Questions

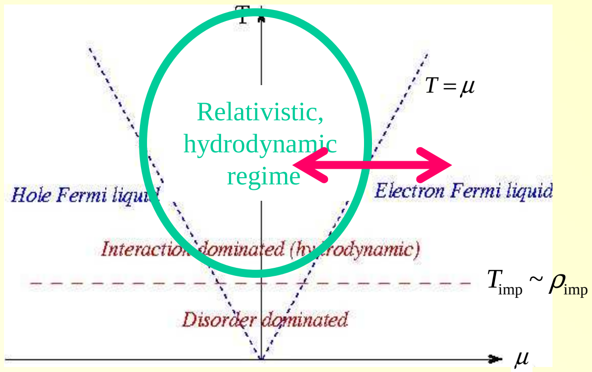
- Transport characteristics of the relativistic plasma in lightly doped graphene and close to quantum criticality?



Questions

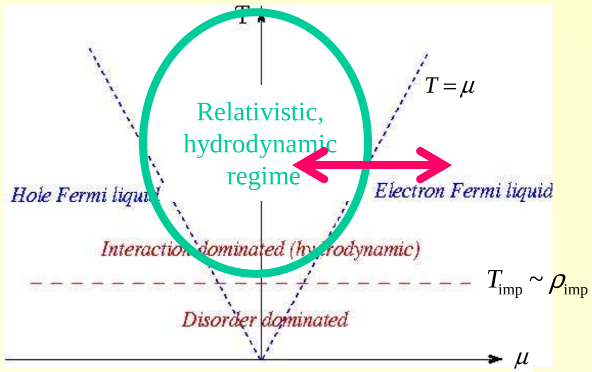
- Transport characteristics of the relativistic plasma in lightly doped graphene and close to quantum criticality?

- How does the relativistic regime connect to Fermi liquid behavior at large doping?



Questions

- Transport characteristics of the relativistic plasma in lightly doped graphene and close to quantum criticality?
- How does the relativistic regime connect to Fermi liquid behavior at large doping?
- What is the range of validity of relativistic magneto-hydrodynamics?
- Beyond hydrodynamics?



Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma_{\lambda a}^\dagger(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

B-field: $i\vec{\nabla} \rightarrow i\vec{\nabla} - e\vec{A}/c$

Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma_{\lambda a}^\dagger(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

B-field: $i\vec{\nabla} \to i\vec{\nabla} - e\vec{A}/c$

Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma_{\lambda a}^\dagger(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

Coulomb interactions

$$H_1 = \frac{1}{2} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Psi_a^\dagger(\mathbf{k}_2 - \mathbf{q}) \Psi_a(\mathbf{k}_2) V(\mathbf{q}) \Psi_b^\dagger(\mathbf{k}_1 + \mathbf{q}) \Psi_b(\mathbf{k}_1)$$

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}$$

Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

B-field: $i\vec{\nabla} \to i\vec{\nabla} - e\vec{A}/c$

Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma^\dagger_{\lambda a}(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

Coulomb interactions

$$H_1 = \frac{1}{2} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Psi^\dagger_a(\mathbf{k}_2 - \mathbf{q}) \Psi_a(\mathbf{k}_2) V(\mathbf{q}) \Psi^\dagger_b(\mathbf{k}_1 + \mathbf{q}) \Psi_b(\mathbf{k}_1)$$

Coulomb marginally irrelevant!

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}$$

$$\alpha \equiv \frac{e^2}{\varepsilon_r \hbar v_F} = O(1)$$

RG:

$$\frac{d\alpha}{d\ell} = -\frac{\alpha^2}{4} + \mathcal{O}(\alpha^3)$$

$$\alpha(T) = \frac{\alpha^0}{1 + (\alpha^0/4)\ln(\Lambda/T)} \overset{T \to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

B-field: $i\vec{\nabla} \to i\vec{\nabla} - e\vec{A}/c$

Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma_{\lambda a}^\dagger(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

Coulomb interactions

$$H_1 = \frac{1}{2} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Psi_a^\dagger(\mathbf{k}_2 - \mathbf{q}) \Psi_a(\mathbf{k}_2) V(\mathbf{q}) \Psi_b^\dagger(\mathbf{k}_1 + \mathbf{q}) \Psi_b(\mathbf{k}_1)$$

Coulomb marginally irrelevant!

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}$$

Screening neglected (down by factor α)

Model of graphene

Graphene with Coulomb interactions and disorder

$$H = H_0 + H_1 + H_{\text{dis}}$$

B-field: $i\vec{\nabla} \to i\vec{\nabla} - e\vec{A}/c$

Tight binding kinetic energy

$$H_0 = -\sum_{a=1}^{N} \int d\mathbf{x} \left[\Psi_a^\dagger \left(i v_F \vec{\sigma} \cdot \vec{\nabla} + \mu \right) \Psi_a \right]$$

$$H_0 = \sum_{\lambda=\pm} \sum_{a=1}^{N} \int \frac{d^2k}{(2\pi)^2} \lambda v_F k \, \gamma_{\lambda a}^\dagger(\mathbf{k}) \gamma_{\lambda a}(\mathbf{k})$$

Coulomb interactions

$$H_1 = \frac{1}{2} \int \frac{d^2k_1}{(2\pi)^2} \frac{d^2k_2}{(2\pi)^2} \frac{d^2q}{(2\pi)^2} \Psi_a^\dagger(\mathbf{k}_2 - \mathbf{q}) \Psi_a(\mathbf{k}_2) V(\mathbf{q}) \Psi_b^\dagger(\mathbf{k}_1 + \mathbf{q}) \Psi_b(\mathbf{k}_1)$$

Coulomb marginally irrelevant!

$$V(\mathbf{q}) = \frac{2\pi e^2}{\varepsilon |\mathbf{q}|}$$

Screening neglected (down by factor α)

Disorder: charged impurities

$$H_{\text{dis}} = \int d\mathbf{x} V_{\text{dis}}(\mathbf{x}) \Psi_a^\dagger(\mathbf{x}) \Psi_a(\mathbf{x})$$

$$V_{\text{dis}}(\mathbf{x}) = \sum_i \delta(\mathbf{x} - \mathbf{x}_i) \frac{Ze^2}{\varepsilon |\mathbf{x} - \mathbf{x}_i|}.$$

Time scales

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Inelastic scattering rate
(Electron-electron interactions)

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar} \frac{1}{\max[1, \mu/T]}$$

Relativistic regime (μ < T):
Relaxation rate set by temperature,
like in quantum critical systems!

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar}$$

Time scales

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Inelastic scattering rate
(Electron-electron interactions)

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar} \frac{1}{\max\left[1, \mu/T\right]}$$

Relativistic regime ($\mu < T$):
Relaxation rate set by temperature,
like in quantum critical systems!

$$\tau_{\mathrm{ee}}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar}$$

2. Elastic scattering rate
(Scattering from charged impurities)
Subdominant at high T

$$\tau_{\mathrm{imp}}^{-1} \sim \frac{\left(Ze^2/\varepsilon\right)^2 \rho_{\mathrm{imp}}}{\hbar} \frac{1}{\max\left[T, \mu\right]}$$

Time scales

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Inelastic scattering rate
(Electron-electron interactions)

$$\tau_{\rm ee}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar} \frac{1}{\max[1, \mu/T]}$$

Relativistic regime ($\mu < T$):
Relaxation rate set by temperature,
like in quantum critical systems!

$$\tau_{\rm ee}^{-1} \sim \alpha^2 \frac{k_B T}{\hbar}$$

2. Elastic scattering rate
(Scattering from charged impurities)
Subdominant at high T

$$\tau_{\rm imp}^{-1} \sim \frac{(Ze^2/\varepsilon)^2 \rho_{\rm imp}}{\hbar} \frac{1}{\max[T, \mu]}$$

3. Deflection rate due to magnetic field
(Cyclotron frequency of non-interacting particles with typical thermal energy)

$$\tau_{\rm B}^{-1} \sim \omega_c^{\rm typ} \sim \frac{eB{\rm v}_F^2}{\max[T, \mu]}$$

Regimes

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Hydrodynamic regime:
(collision-dominated)

$$\tau_{\text{ee}}^{-1} >> \tau_{\text{imp}}^{-1}, \tau_{\text{B}}^{-1}, \omega$$

Regimes

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Hydrodynamic regime:
(collision-dominated)

$$\tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \tau_{\mathrm{B}}^{-1}, \omega$$

2. Ballistic magnetotransport
(large field limit)

$$\tau_{\mathrm{B}}^{-1} > \tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \omega$$

Regimes

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

1. Hydrodynamic regime:
(collision-dominated)

$$\tau_{ee}^{-1} >> \tau_{imp}^{-1}, \tau_{B}^{-1}, \omega$$

2. Ballistic magnetotransport
(large field limit)

$$\tau_{B}^{-1} > \tau_{ee}^{-1} >> \tau_{imp}^{-1}, \omega$$

3. Disorder limited transport
(inelastic scattering ineffective due to nearly conserved momentum)

$$\mu >> T \qquad \tau_{ee}^{-1} \gtrless \tau_{imp}^{-1}$$

Hydrodynamic Approach

Hydrodynamics

Hydrodynamic collision-dominated regime

$$\tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{imp}}^{-1}, \tau_{\mathrm{B}}^{-1}, \omega$$

Long times,
Large scales

$$t >> \tau_{\mathrm{ee}}$$

Hydrodynamics

Hydrodynamic collision-dominated regime $\tau_{ee}^{-1} >> \tau_{imp}^{-1}, \tau_{B}^{-1}, \omega$

Long times,
Large scales $t >> \tau_{ee}$

- Local equilibrium established: $T_{loc}(r)\, ,\, \mu_{loc}(r)\, ;\; \vec{u}_{loc}(r)$
- Study relaxation towards global equilibrium
- Slow modes: Diffusion of the density of conserved quantities:
  - Charge
  - Momentum
  - Energy

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Energy-momentum tensor $$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$ $$\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

Current 3-vector $$J^\mu = \rho u^\mu + \nu^\mu$$ $$\begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \end{pmatrix}$$

u^μ : Energy velocity: $u^\mu = (1,0,0) \rightarrow$ No energy current

ν^μ : Dissipative current ("heat current")

$\tau^{\mu\nu}$: Viscous stress tensor (Reynold's tensor)

Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Energy-momentum tensor $T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$ $\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$

Current 3-vector $J^\mu = \rho u^\mu + \nu^\mu$ $\begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \end{pmatrix}$

u^μ : Energy velocity: $u^\mu = (1,0,0) \rightarrow$ No energy current

ν^μ : Dissipative current ("heat current")

$\tau^{\mu\nu}$: Viscous stress tensor (Reynold's tensor)

+ Thermodynamic relations

$$\varepsilon + P = Ts + \mu\rho, \quad d\varepsilon = Tds + \mu d\rho,$$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

$$J^\mu = \rho u^\mu + \nu^\mu \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

Conservation laws (equations of motion):

$$\partial_\mu J^\mu = 0$$ Charge conservation

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

$$J^\mu = \rho u^\mu + \nu^\mu \qquad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

Conservation laws (equations of motion):

$$\partial_\mu J^\mu = 0$$ Charge conservation

Energy/momentum conservation

$$\partial_\nu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} T^{0\nu} \delta_{\mu 0}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y \\ -E_x & 0 & B \\ -E_y & -B & 0 \end{pmatrix}$$

$$\vec{E} = -i\vec{k} \frac{2\pi}{|k|} \rho_{\vec{k}}$$ Coulomb interaction

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

$$J^{\mu} = \rho u^{\mu} + \nu^{\mu}$$

$$T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} + Pg^{\mu\nu} + \tau^{\mu\nu}$$

Conservation laws (equations of motion):

$$\partial_{\mu} J^{\mu} = 0$$ Charge conservation

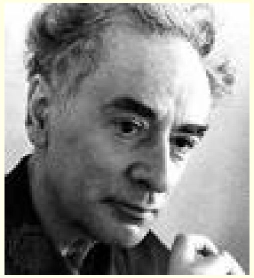
Energy/momentum conservation

$$\partial_{\nu} T^{\mu\nu} = F^{\mu\nu} J_{\nu} + \frac{1}{\tau_{\text{imp}}} T^{0\nu} \delta_{\mu 0}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y \\ -E_x & 0 & B \\ -E_y & -B & 0 \end{pmatrix}$$

$$\vec{E} = -i\vec{k} \frac{2\pi}{|k|} \rho_{\vec{k}}$$ Coulomb interaction

Heat current and viscous tensor?



Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$

→ Entropy current $S^{\mu} = Q^{\mu} / T$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$

$\rightarrow$ Entropy current $S^{\mu} = Q^{\mu} / T$

Positivity of entropy production (Second law):

$$\partial_{\mu} S^{\mu} \equiv A_{\alpha}(\partial T, \partial\mu, F^{\mu\nu})\nu^{\alpha} + B_{\alpha\beta}(\partial T, \partial\mu, F^{\mu\nu})\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^{\mu} = \text{const.} \times A^{\mu}(\partial T, \partial\mu, \partial u; F^{\mu\nu})$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B_{\alpha}^{\alpha}$$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $Q^\mu = (\varepsilon + P)u^\mu - \mu J^\mu$

$\rightarrow$ Entropy current $S^\mu = Q^\mu / T$

Positivity of entropy production (Second law):

$$\partial_\mu S^\mu \equiv A_\alpha(\partial T, \partial\mu, F^{\mu\nu})\nu^\alpha + B_{\alpha\beta}(\partial T, \partial\mu, F^{\mu\nu})\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^\mu = \text{const.} \times A^\mu(\partial T, \partial\mu, \partial u; F^{\mu\nu})$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^\alpha_\alpha$$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$

$\rightarrow$ Entropy current $S^{\mu} = Q^{\mu} / T$

Positivity of entropy production (Second law):

$$\partial_{\mu} S^{\mu} \equiv A_{\alpha}\left(\partial T, \partial \mu, F^{\mu\nu}\right) \nu^{\alpha} + B_{\alpha\beta}\left(\partial T, \partial \mu, F^{\mu\nu}\right) \tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^{\mu} = \text{const.} \times A^{\mu}\left(\partial T, \partial \mu, \partial u; F^{\mu\nu}\right)$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^{\alpha}_{\alpha}$$

$\rightarrow$

$$\nu^{\mu} = \sigma_Q (g^{\mu\nu} + u^{\mu} u^{\nu}) \left[(-\partial_{\nu}\mu + F_{\nu\lambda} u^{\lambda}) + \mu \frac{\partial_{\mu} T}{T} \right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^{\mu} u^{\lambda}) [\eta (\partial_{\lambda} u^{\nu} + \partial^{\nu} u_{\lambda}) + (\zeta - \eta) \delta^{\nu}_{\lambda} \partial_{\alpha} u^{\alpha}]$$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $$Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$$

$\rightarrow$ Entropy current $$S^{\mu} = Q^{\mu}/T$$

Positivity of entropy production (Second law):

$$\partial_{\mu} S^{\mu} \equiv A_{\alpha}(\partial T, \partial\mu, F^{\mu\nu})\nu^{\alpha} + B_{\alpha\beta}(\partial T, \partial\mu, F^{\mu\nu})\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^{\mu} = \text{const.} \times A^{\mu}(\partial T, \partial\mu, \partial u; F^{\mu\nu})$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^{\alpha}_{\alpha}$$

$\rightarrow$

$$\nu^{\mu} = \sigma_Q (g^{\mu\nu} + u^{\mu}u^{\nu}) \left[(-\partial_{\nu}\mu + F_{\nu\lambda}u^{\lambda}) + \mu \frac{\partial_{\mu} T}{T} \right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^{\mu}u^{\lambda})[\eta(\partial_{\lambda}u^{\nu} + \partial^{\nu}u_{\lambda}) + (\zeta - \eta)\delta^{\nu}_{\lambda}\partial_{\alpha}u^{\alpha}]$$

B small!

Relativistic Hydrodynamics

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$

$\rightarrow$ Entropy current $S^{\mu} = Q^{\mu} / T$

Positivity of entropy production (Second law):

$$\partial_{\mu} S^{\mu} \equiv A_{\alpha}(\partial T, \partial \mu, F^{\mu\nu})\nu^{\alpha} + B_{\alpha\beta}(\partial T, \partial \mu, F^{\mu\nu})\tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^{\mu} = \text{const.} \times A^{\mu}(\partial T, \partial \mu, \partial u; F^{\mu\nu})$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^{\alpha}_{\alpha}$$

$\rightarrow$

$$\nu^{\mu} = \sigma_Q (g^{\mu\nu} + u^{\mu}u^{\nu}) \left[(-\partial_{\nu}\mu + F_{\nu\lambda}u^{\lambda}) + \mu \frac{\partial_{\mu} T}{T} \right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^{\mu}u^{\lambda})[\eta(\partial_{\lambda}u^{\nu} + \partial^{\nu}u_{\lambda}) + (\zeta - \eta)\delta^{\nu}_{\lambda}\partial_{\alpha}u^{\alpha}]$$

Irrelevant for response at $k \rightarrow 0$

Relativistic Hydrodynamics

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Landau-Lifschitz,
Relat. plasma physics

Heat current and viscous tensor?

Heat current $Q^{\mu} = (\varepsilon + P)u^{\mu} - \mu J^{\mu}$

$\rightarrow$ Entropy current $S^{\mu} = Q^{\mu} / T$

Positivity of entropy production (Second law):

$$\partial_{\mu} S^{\mu} \equiv A_{\alpha}(\partial T, \partial \mu, F^{\mu\nu}) \nu^{\alpha} + B_{\alpha\beta}(\partial T, \partial \mu, F^{\mu\nu}) \tau^{\alpha\beta} \geq 0$$

$$\Rightarrow \nu^{\mu} = \text{const.} \times A^{\mu}(\partial T, \partial \mu, \partial u; F^{\mu\nu})$$

$$\tau^{\mu\nu} = \text{const.} \times B^{\mu\nu} + \text{const.} \times \delta^{\mu\nu} B^{\alpha}_{\alpha}$$

$\rightarrow$

$$\nu^{\mu} = \sigma_Q (g^{\mu\nu} + u^{\mu} u^{\nu}) \left[(-\partial_{\nu}\mu + F_{\nu\lambda} u^{\lambda}) + \mu \frac{\partial_{\mu} T}{T} \right]$$

$$\tau^{\mu\nu} = -(g^{\mu\lambda} + u^{\mu} u^{\lambda}) [\eta(\partial_{\lambda} u^{\nu} + \partial^{\nu} u_{\lambda}) + (\zeta - \eta) \delta^{\nu}_{\lambda} \partial_{\alpha} u^{\alpha}]$$

Irrelevant for response at k $\rightarrow$ 0

One single transport coefficient (instead of two)!

Meaning of σ_Q ?

- Dimension of electrical conductivity

- At zero doping (particle-hole symmetry):

$$\sigma_Q = \sigma_{xx}(\rho_{\text{imp}} = 0)$$

= Universal d.c. conductivity of the pure system

Why is $\sigma_{xx}(\rho_{\text{imp}} = 0)$ finite ??

Universal conductivity σ_Q

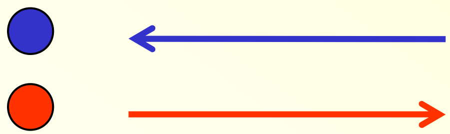
K. Damle, S. Sachdev, (1996).

Particle-hole symmetry ($\rho = 0$)

- Key: Charge current without momentum (energy current)!

(particle)

(hole)



$$\vec{J} \neq 0, \quad \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite "quantum critical" conductivity!

Universal conductivity σ_Q

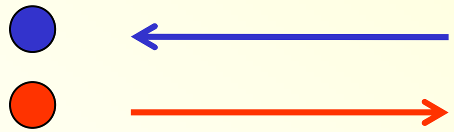
K. Damle, S. Sachdev, (1996).

Quantum critical situation: Particle-hole symmetry ($\rho = 0$)

- Key: Charge current without momentum (energy current)

(particle)

(hole)

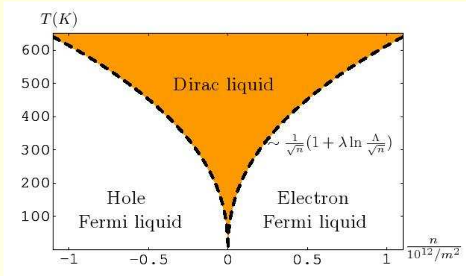


$$\vec{J} \neq 0, \quad \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite "quantum critical" conductivity!
- As in quantum criticality:
  Relaxation time set by temperature alone

$$\tau_{ee} \approx \frac{\hbar}{\alpha^2 k_B T}$$



Universal conductivity σ_Q

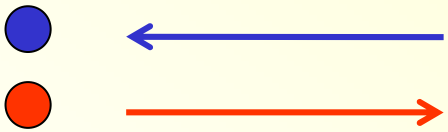
K. Damle, S. Sachdev, (1996).

Quantum critical situation: Particle-hole symmetry ($\rho = 0$)

- Key: Charge current without momentum (energy current)

(particle)

(hole)

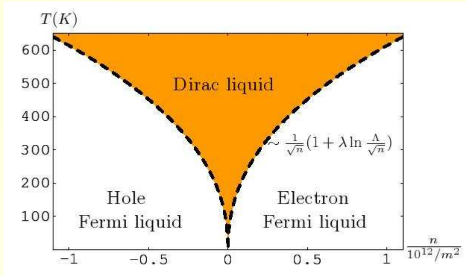


$$\vec{J} \neq 0, \quad \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite “quantum critical” conductivity!
- As in quantum criticality:
  Relaxation time set by temperature alone

$$\tau_{ee} \approx \frac{\hbar}{\alpha^2 k_B T}$$



→ Universal conductivity

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma_Q \sim \frac{e}{k_B T / \mathrm{v}^2}\left(e\frac{(k_B T)^2}{(\hbar \mathrm{v})^2} \right)\frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

Universal conductivity σ_Q

K. Damle, S. Sachdev, (1996).

Quantum critical situation: Particle-hole symmetry ($\rho = 0$)

- Key: Charge current without momentum (energy current)

(particle)

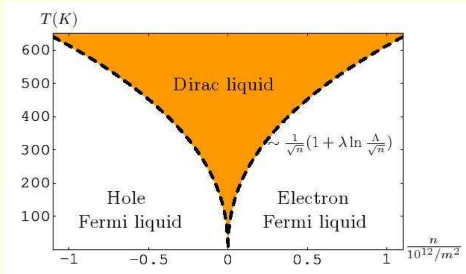
(hole)

$$\vec{J} \neq 0, \quad \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite "quantum critical" conductivity!
- As in quantum criticality: Relaxation time set by temperature alone

$$\tau_{ee} \approx \frac{\hbar}{\alpha^2 k_B T}$$



→ Universal conductivity

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma_Q \sim \frac{e}{k_B T/\mathrm{v}^2}\left(e\frac{(k_B T)^2}{(\hbar \mathrm{v})^2}\right)\frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

Exact (Boltzmann)

$$\sigma_Q(\mu = 0) = \frac{0.76}{\alpha^2}\frac{e^2}{h}$$

Universal conductivity σ_Q

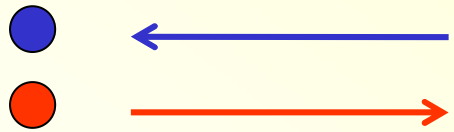
K. Damle, S. Sachdev, (1996).

Quantum critical situation: Particle-hole symmetry (ρ = 0)

- Key: Charge current without momentum (energy current)

(particle)

(hole)

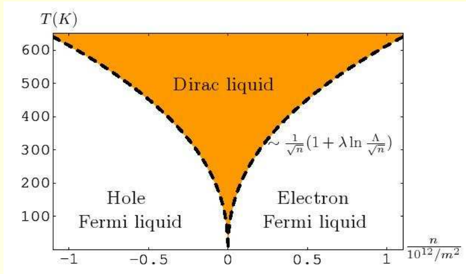


$$\vec{J} \neq 0, \quad \vec{P} = 0$$

Pair creation/annihilation leads to current decay

- Finite "quantum critical" conductivity!
- As in quantum criticality: Relaxation time set by temperature alone

$$\tau_{ee} \approx \frac{\hbar}{\alpha^2 k_B T}$$



→ Universal conductivity

$$\sigma_{Drude} = \frac{e}{m}\rho\tau \rightarrow \sigma_Q \sim \frac{e}{k_B T / v^2}\left(e\frac{(k_B T)^2}{(\hbar v)^2}\right)\frac{\hbar}{\alpha^2 k_B T} \sim \frac{1}{\alpha^2}\frac{e^2}{h}$$

Exact (Boltzmann)

$$\sigma_Q(\mu = 0) = \frac{0.76}{\alpha^2}\frac{e^2}{h}$$

Marginal irrelevance of Coulomb:

$$\alpha \approx \frac{4}{\log(\Lambda/T)}$$

Thermoelectric response

S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B ***76****, 144502 (2007).*

Charge and heat current:

$$J^{\mu} = \rho u^{\mu} - \nu^{\mu}$$

$$Q^{\mu} = (\varepsilon + P) u^{\mu} - \mu J^{\mu}$$

Thermo-electric response

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix}$$

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

etc.

Thermoelectric response

*S. Hartnoll, P. Kovton, MM, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).*

Charge and heat current:

$$J^{\mu} = \rho u^{\mu} - \nu^{\mu}$$

$$Q^{\mu} = (\varepsilon + P) u^{\mu} - \mu J^{\mu}$$

Thermo-electric response

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \hat{\sigma} & \hat{\alpha} \\ T\hat{\alpha} & \hat{\bar{\kappa}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix}$$

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

etc.

i) Solve linearized hydrodynamic equations
ii) Read off the response functions *(Kadanoff & Martin 1960)*

Results from Hydrodynamics

Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

$$\sigma_{xx}(\omega, k; B = 0) = \left(\sigma_Q + \frac{\rho^2}{P + \varepsilon}\frac{\tau}{1 - i\omega\tau}\right)$$

Universal conductivity at the quantum critical point $\rho = 0$

Drude-like conductivity, divergent for $\tau \to \infty, \omega \to 0, \rho \neq 0$

Momentum conservation ($\rho \neq 0$)!

Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

Coulomb correction $\left(g = 2\pi e^2\right)$

$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)\left[1 - \frac{igk}{\omega}\left(\sigma_Q + \frac{\tau}{1-i\omega\tau}\frac{\rho^2}{P+\varepsilon}\right)\right] + O\left(k^2\right)$$

Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

Coulomb correction $\left(g = 2\pi e^2\right)$

$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)\left[1 - \frac{igk}{\omega}\left(\sigma_Q + \frac{\tau}{1-i\omega\tau}\frac{\rho^2}{P+\varepsilon}\right)\right] + O\left(k^2\right)$$

Thermal conductivity:

$$\kappa_{xx}(\omega, k; B=0) = \sigma_Q \frac{\mu^2}{T} + \frac{s^2 T}{P+\varepsilon}\frac{\tau}{1-i\omega\tau} + \mathcal{O}(k^2).$$

Relativistic Wiedemann-Franz-like relations between σ and κ in the quantum critical window!

Response functions at B=0

Symmetry z ⟶ -z : $\sigma_{xy} = \alpha_{xy} = \kappa_{xy} = 0$

Longitudinal conductivity:

Coulomb correction $\left(g = 2\pi e^2\right)$

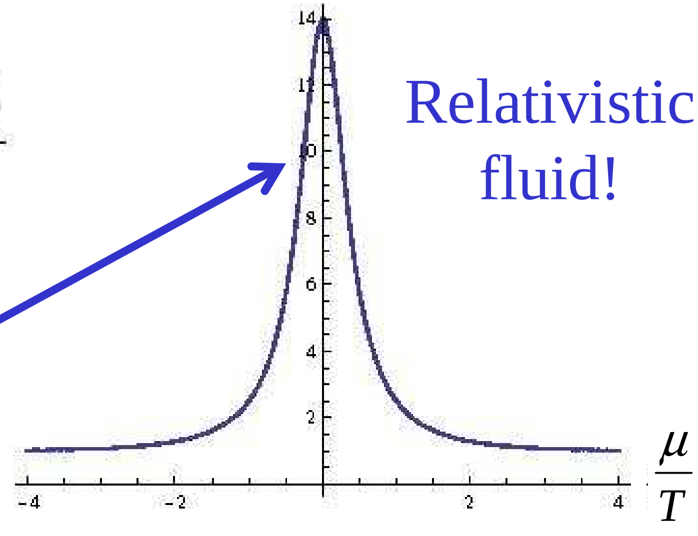
$$\sigma_{xx}(\omega, k; B=0) = \left(\sigma_Q + \frac{\rho^2}{P+\varepsilon}\frac{\tau}{1-i\omega\tau}\right)\left[1 - \frac{igk}{\omega}\left(\sigma_Q + \frac{\tau}{1-i\omega\tau}\frac{\rho^2}{P+\varepsilon}\right)\right] + O\left(k^2\right)$$

Thermopower:

$$\alpha_{xx}(\mu, \omega=0) = -\frac{\pi^2}{3e}k_B^2 T \frac{d\sigma(\mu, \omega=0)}{d\mu}$$

Only valid in the Fermi liquid regime,
but violated in the relativistic window.

$$-\frac{3e}{\pi^2}\frac{1}{k_B^2 T}\frac{\alpha_{xx}}{d\sigma_{xx}/d\mu}$$



Relativistic fluid!

B > 0 : Cyclotron resonance

E.g.: Longitudinal conductivity

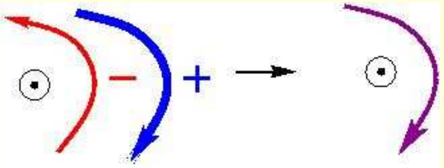
$$\sigma_{xx}(\omega) = \sigma_Q \frac{\omega\left(\omega + i\gamma + i\omega_c^2/\gamma\right)}{\left(\omega + i\gamma\right)^2 - \omega_c^2}$$

Poles in the response

$$\omega = \pm\omega_c^{\mathrm{QC}} - i\gamma - i/\tau$$

Collective cyclotron frequency of the relativistic plasma

$$\omega_c^{\mathrm{QC}} = \frac{\rho B}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2} \leftrightarrow \omega_c^{\mathrm{FL}} = \frac{eB}{m}$$



B > 0 : Cyclotron resonance

E.g.: Longitudinal conductivity

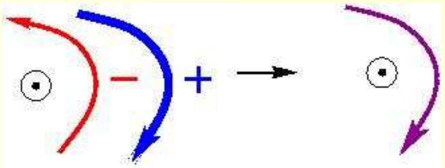
$$\sigma_{xx}(\omega) = \sigma_Q \frac{\omega\left(\omega + i\gamma + i\omega_c^2/\gamma\right)}{\left(\omega + i\gamma\right)^2 - \omega_c^2}$$

Poles in the response

$$\omega = \pm\omega_c^{\mathrm{QC}} - i\gamma - i/\tau$$

Collective cyclotron frequency of the relativistic plasma

$$\omega_c^{\mathrm{QC}} = \frac{\rho B}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2} \leftrightarrow \omega_c^{\mathrm{FL}} = \frac{e B}{m}$$



Intrinsic, interaction-induced broadening

(↔ Galilean invariant systems:
No broadening due to Kohn's theorem)

$$\gamma = \sigma_{\mathrm{Q}} \frac{B^2}{(\varepsilon + P)/\mathrm{v}_{\mathrm{F}}^2}$$

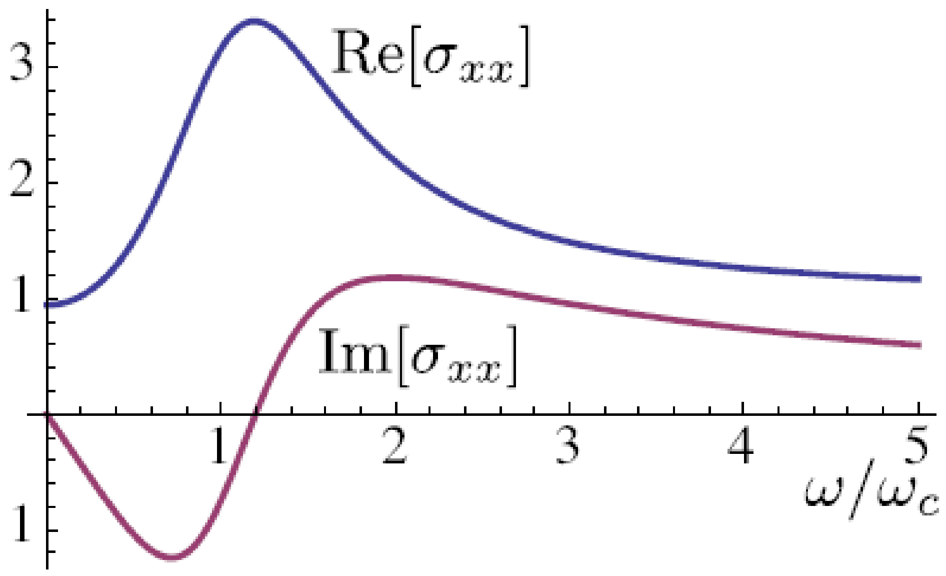
B > 0 : Cyclotron resonance

Longitudinal conductivity

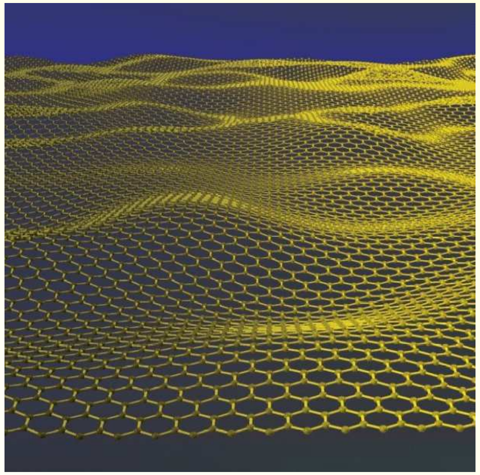
$$\sigma_{xx}(\omega, k) = \sigma_Q \frac{(\omega + i/\tau)(\omega + i/\tau + i\gamma + i\omega_c^2/\gamma)}{(\omega + i/\tau + i\gamma)^2 - \omega_c^2}$$

Poles in the response

$$\omega = \pm\omega_c^{\mathrm{QC}} - i\gamma - i/\tau$$

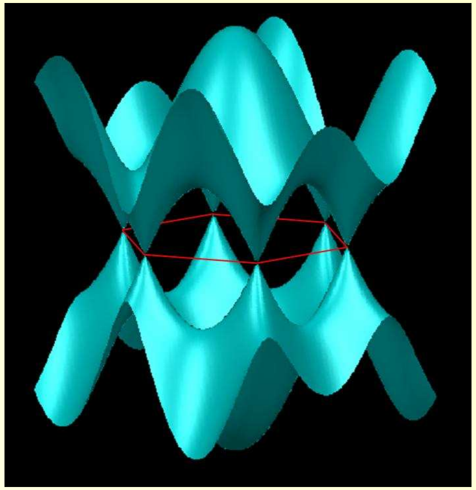


Can the resonance be observed?



$$\omega = \pm\omega_c - i\gamma - i/\tau$$

$$v_F = 1.1\cdot 10^6\, m/s$$
$$\approx c/300$$



Conditions to observe collective cyclotron resonance

Collison-dominated regime	$\hbar\omega_c << \alpha^2 k_B T$
Small broadening	$\gamma, \tau^{-1} < \omega_c$
Quantum critical regime	$\rho \le \rho_{th} = \dfrac{(k_B T)^2}{(\hbar v_F)^2}$
High T: no Landau quantization	$E_{LL} = \hbar v_F \sqrt{\dfrac{2eB}{\hbar c}} << k_B T$

Parameters:

$T \approx 300K$

$B \approx 0.1T$

$\rho \approx 10^{11} cm^{-2}$

$\omega_c \approx 10^{13} s^{-1}$

Does relativistic hydrodynamics apply?

- Do T and μ not break relativistic invariance?
- Validity at large chemical potential?
- Beyond linearization in magnetic field?
- Treatment of disorder?

Boltzmann Approach

MM, L. Fritz, and S. Sachdev, cond-mat 0805.1413.

→ Recover and refine the hydrodynamic description

→ Describe relativistic-to-Fermi-liquid crossover

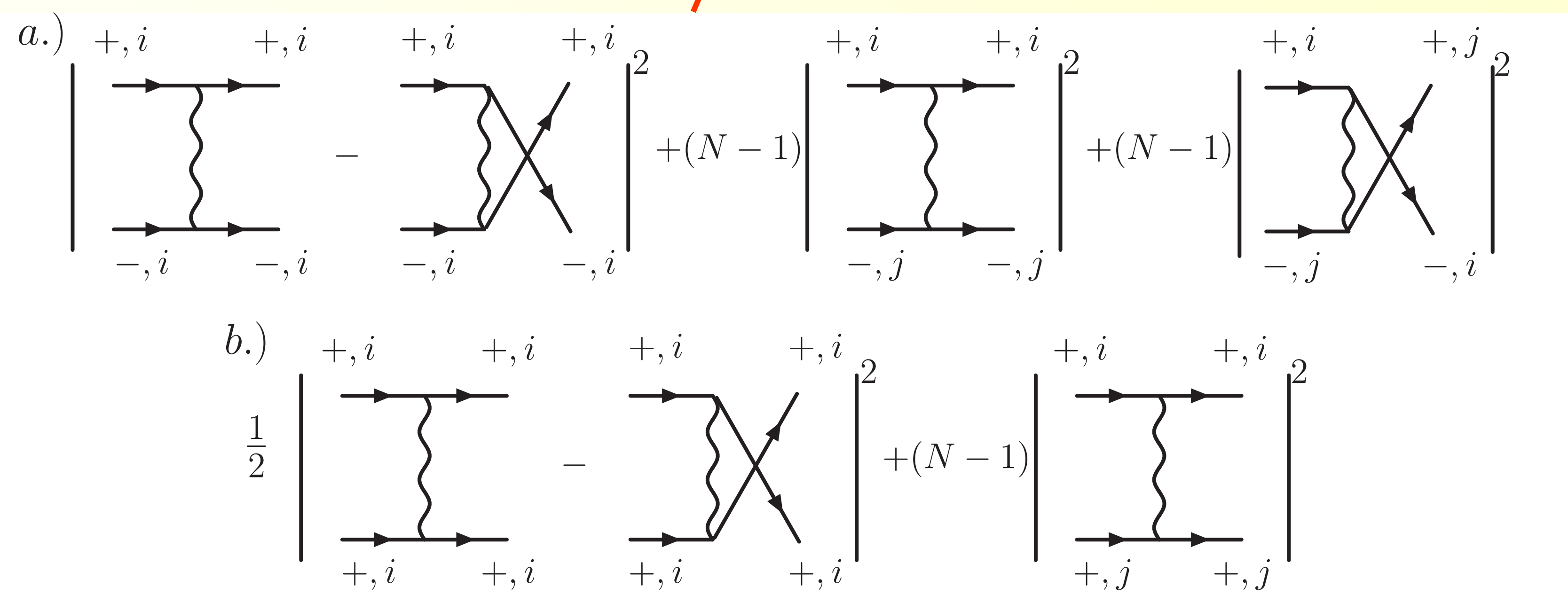
→ Go beyond hydrodynamics

σ_Q from Boltzmann

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_{\pm}(\mathbf{k},t) = \alpha^2 I^{Cb}_{\text{coll}}\left[\mathbf{k},t \mid \{f_{\pm}(\mathbf{k}',t)\}\right] + \Delta\, I^{dis}_{\text{coll}}\left[\mathbf{k},t \mid \{f_{\pm}(\mathbf{k}',t)\}\right]$$



σ_Q from Boltzmann

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = \alpha^2 I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \left\{f_\pm(\mathbf{k}',t)\right\}\right] + \Delta\, I_{\text{coll}}^{dis}\left[\mathbf{k},t \mid \left\{f_\pm(\mathbf{k}',t)\right\}\right]$$

Linearization: $$f_\pm(\mathbf{k},t) = f_\pm^{eq}(\mathbf{k},t) + \delta f_\pm(\mathbf{k},t)$$

σ_Q from Boltzmann

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = \alpha^2 I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \left\{f_\pm(\mathbf{k}',t)\right\}\right] + \Delta\, I_{\text{coll}}^{dis}\left[\mathbf{k},t \mid \left\{f_\pm(\mathbf{k}',t)\right\}\right]$$

Linearization: $f_\pm(\mathbf{k},t) = f_\pm^{eq}(\mathbf{k},t) + \delta f_\pm(\mathbf{k},t)$

Great simplification: Divergence of forward scattering amplitude in 2d

Amp [] $\to\infty$

$\to$ Equilibration along unidimensional spatial directions

At p-h symmetry: $f_\pm(\mathbf{k},t) = f_\pm^{eq}\left(\mathbf{k}, \mu \to \mu_{eq} + \delta\mu(t)\right)\ ;\ \delta\mu = C(t)\frac{\mathbf{E}\cdot\mathbf{k}}{k}$

σ_Q from Boltzmann

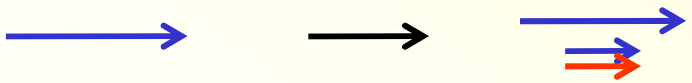
L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = \alpha^2 I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right] + \Delta\, I_{\text{coll}}^{dis}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right]$$

Linearization: $\boxed{f_\pm(\mathbf{k},t) = f_\pm^{eq}(\mathbf{k},t) + \delta f_\pm(\mathbf{k},t)}$

Great simplification: Divergence of forward scattering amplitude in 2d

Amp [] $\rightarrow\infty$

$\rightarrow$ Equilibration along unidimensional spatial directions

At p-h symmetry: $\boxed{f_\pm(\mathbf{k},t) = f_\pm^{eq}\left(\mathbf{k},\mu \rightarrow \mu_{eq} + \delta\mu(t)\right)\ ;\ \delta\mu = C(t)\frac{\mathbf{E}\cdot\mathbf{k}}{k}}$

$\longrightarrow$ $\boxed{\sigma_Q(\mu = 0) \approx \frac{0.76}{\alpha^2}\frac{e^2}{h}}$

σ_Q from Boltzmann

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = \alpha^2 I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right] + \Delta\, I_{\text{coll}}^{dis}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right]$$

General analysis in linear response:

$$\begin{aligned} f_\lambda(\mathbf{r},\mathbf{k},\omega) &= 2\pi\delta(\omega) f_\lambda^0(k,T(\mathbf{r})) \\ &+ f_{\lambda k}^0[1-f_{\lambda k}^0]\frac{v_F}{T^2}\mathbf{e}_\mathbf{k}\cdot\left[e\mathbf{E}(\omega)g_{\parallel,\lambda}^{(E)}\left(\frac{v_F k}{T},\omega\right) + \nabla T(\omega) g_{\parallel,\lambda}^{(T)}\left(\frac{v_F k}{T},\omega\right)\right] \\ &+ f_{\lambda k}^0[1-f_{\lambda k}^0]\frac{v_F}{T^2}(\mathbf{e}_\mathbf{k}\times\mathbf{e}_z)\cdot\left[\mathbf{E}(\omega)g_{\perp,\lambda}^{(E)}\left(\frac{v_F k}{T},\omega\right) + \nabla T(\omega) g_{\perp,\lambda}^{(T)}\left(\frac{v_F k}{T},\omega\right)\right] \end{aligned}$$

σ_Q from Boltzmann

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = \alpha^2 I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right] + \Delta\, I_{\text{coll}}^{dis}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right]$$

General analysis in linear response:

$$\begin{aligned} f_\lambda(\mathbf{r},\mathbf{k},\omega) &= 2\pi\delta(\omega) f_\lambda^0(k,T(\mathbf{r})) \\ &+ f_{\lambda k}^0[1-f_{\lambda k}^0]\frac{v_F}{T^2}\mathbf{e}_\mathbf{k}\cdot\left[e\mathbf{E}(\omega) g_{\parallel,\lambda}^{(E)}\left(\frac{v_F k}{T},\omega\right) + \nabla T(\omega) g_{\parallel,\lambda}^{(T)}\left(\frac{v_F k}{T},\omega\right)\right] \\ &+ f_{\lambda k}^0[1-f_{\lambda k}^0]\frac{v_F}{T^2}(\mathbf{e}_\mathbf{k}\times\mathbf{e}_z)\cdot\left[\mathbf{E}(\omega) g_{\perp,\lambda}^{(E)}\left(\frac{v_F k}{T},\omega\right) + \nabla T(\omega) g_{\perp,\lambda}^{(T)}\left(\frac{v_F k}{T},\omega\right)\right] \end{aligned}$$

Central element of analysis: Choose appropriate basis $g_{\lambda=\pm}(k,t) = \sum_n a_n \phi_n(\lambda,k)$

$\phi_0(\lambda,k) = k,$	Momentum or energy-current mode
$\phi_1(\lambda,k) = \lambda.$	Charge current mode

σ_Q from Boltzmann

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

Boltzmann equation in Born approximation

$$\left(\partial_t + e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial}{\partial\mathbf{k}}\right) f_\pm(\mathbf{k},t) = \alpha^2 I_{\text{coll}}^{Cb}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right] + \Delta\, I_{\text{coll}}^{dis}\left[\mathbf{k},t \mid \{f_\pm(\mathbf{k}',t)\}\right]$$

General analysis in linear response:

$$\begin{aligned} f_\lambda(\mathbf{r},\mathbf{k},\omega) &= 2\pi\delta(\omega) f_\lambda^0(k, T(\mathbf{r})) \\ &\quad + f_{\lambda k}^0[1 - f_{\lambda k}^0]\frac{v_F}{T^2}\mathbf{e}_\mathbf{k}\cdot\left[e\mathbf{E}(\omega) g_{\parallel,\lambda}^{(E)}\left(\frac{v_F k}{T},\omega\right) + \nabla T(\omega) g_{\parallel,\lambda}^{(T)}\left(\frac{v_F k}{T},\omega\right)\right] \\ &\quad + f_{\lambda k}^0[1 - f_{\lambda k}^0]\frac{v_F}{T^2}(\mathbf{e}_\mathbf{k}\times\mathbf{e}_z)\cdot\left[\mathbf{E}(\omega) g_{\perp,\lambda}^{(E)}\left(\frac{v_F k}{T},\omega\right) + \nabla T(\omega) g_{\perp,\lambda}^{(T)}\left(\frac{v_F k}{T},\omega\right)\right] \end{aligned}$$

Central element of analysis: Choose appropriate basis $g_{\lambda=\pm}(k,t) = \sum_n a_n \phi_n(\lambda,k)$

$$\phi_0(\lambda,k) = k, \qquad \text{Momentum or energy-current mode}$$
$$\phi_1(\lambda,k) = \lambda. \qquad \text{Charge current mode}$$

$$\sum_\lambda \int d^2k\, f_{\lambda k}^0(1 - f_{\lambda k}^0)\phi_{n\geq 2}(\lambda,k)\phi_{0,1}(\lambda,k) = 0$$

Relativistic dispersion ensures that φ_0 only couples to φ_1 for clean systems!

Conductivity: σ_Q

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

General doping:

Clean system:

$$\sigma_{xx}(\omega;\mu,\Delta=0) = e^2 \frac{\rho^2 v_F^2}{\varepsilon + P} \frac{1}{(-i\omega)} + \sigma_Q$$

Precise expression for σ_Q!

$$\sigma_Q(\mu,\omega) = \frac{e^2}{\hbar}\frac{1}{\alpha^2}\frac{2\hat{g}_1}{N}\left[I_+^{(1)} - \frac{\rho^2(\hbar v)^2}{(\varepsilon+P)T}\right]^2 \frac{1}{1-i\omega\tau_{ee}}$$

Conductivity: σ_Q

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

General doping:

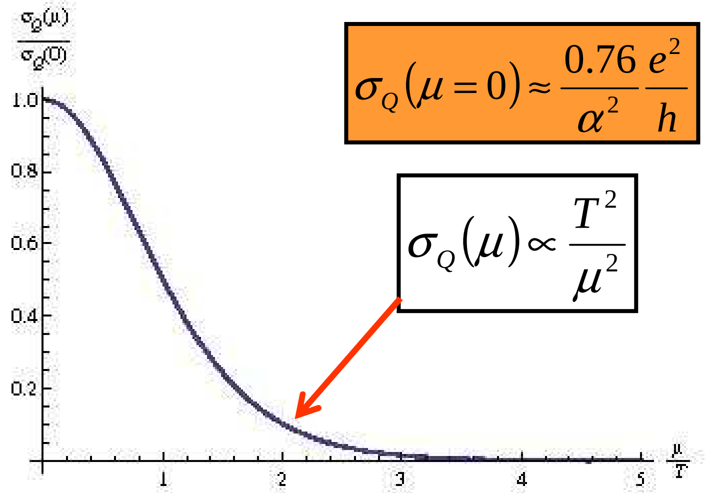
Clean system:

$$\sigma_{xx}(\omega;\mu,\Delta=0) = e^2\frac{\rho^2 v_F^2}{\varepsilon+P}\frac{1}{(-i\omega)} + \sigma_Q$$

Precise expression for σ_Q!

$$\sigma_Q(\mu,\omega) = \frac{e^2}{\hbar}\frac{1}{\alpha^2}\frac{2\hat{g}_1}{N}\left[I_+^{(1)} - \frac{\rho^2(\hbar v)^2}{(\varepsilon+P)T}\right]^2\frac{1}{1-i\omega\tau_{ee}}$$

Gradual disappearance of relativistic physics



$$\sigma_Q(\mu=0) \approx \frac{0.76}{\alpha^2}\frac{e^2}{h}$$

$$\sigma_Q(\mu) \propto \frac{T^2}{\mu^2}$$

Will appear in all Boltzmann formulae below!

Conductivity: crossover

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

General doping:

Lightly disordered system: $$\sigma_{xx}(\omega;\mu,\Delta) = \frac{e^2}{\tau_{\rm imp}^{-1} - i\omega}\frac{\rho^2 v_F^2}{\varepsilon + P} + \sigma_Q + \delta\sigma(\Delta,\omega,\mu)$$

$$\delta\sigma(\Delta,\omega,\mu) = \mathcal{O}(\Delta/\alpha^2)$$ ← Correction to hydrodynamics

Conductivity: crossover

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

General doping:

Lightly disordered system:

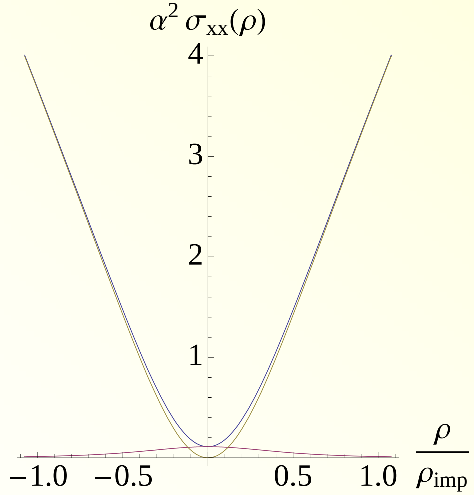
$$\sigma_{xx}(\omega;\mu,\Delta) = \frac{e^2}{\tau_{\rm imp}^{-1} - i\omega}\frac{\rho^2 v_F^2}{\varepsilon + P} + \sigma_Q + \delta\sigma(\Delta,\omega,\mu)$$

$$\delta\sigma(\Delta,\omega,\mu) = \mathcal{O}(\Delta/\alpha^2)$$

← Correction to hydrodynamics

Fermi liquid regime:

$$\sigma_{xx}(\omega = 0; \mu \gg T) \approx \frac{e^2 \rho^2 v_F^2 \tau_{\rm imp}}{\varepsilon + P} = \frac{2}{\pi}\frac{1}{(Z\alpha)^2}\frac{e^2}{h}\frac{\rho}{\rho_{\rm imp}}$$



Conductivity: crossover

L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

General doping:

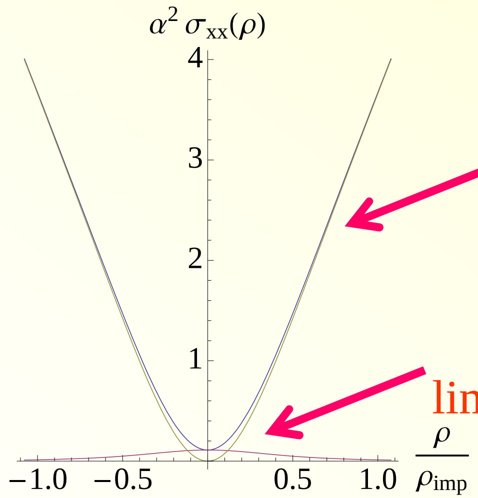
Lightly disordered system:

$$\sigma_{xx}(\omega;\mu,\Delta) = \frac{e^2}{\tau_{\rm imp}^{-1} - i\omega}\frac{\rho^2 v_F^2}{\varepsilon + P} + \sigma_Q + \delta\sigma(\Delta,\omega,\mu)$$

$$\delta\sigma(\Delta,\omega,\mu) = \mathcal{O}(\Delta/\alpha^2)$$ ← Correction to hydrodynamics

Fermi liquid regime:

$$\sigma_{xx}(\omega = 0; \mu \gg T) \approx \frac{e^2\rho^2 v_F^2 \tau_{\rm imp}}{\varepsilon + P} = \frac{2}{\pi}\frac{1}{(Z\alpha)^2}\frac{e^2}{h}\frac{\rho}{\rho_{\rm imp}}$$



Impurity limited conductivity

Universal Cb limited conductivity

Conductivity: crossover

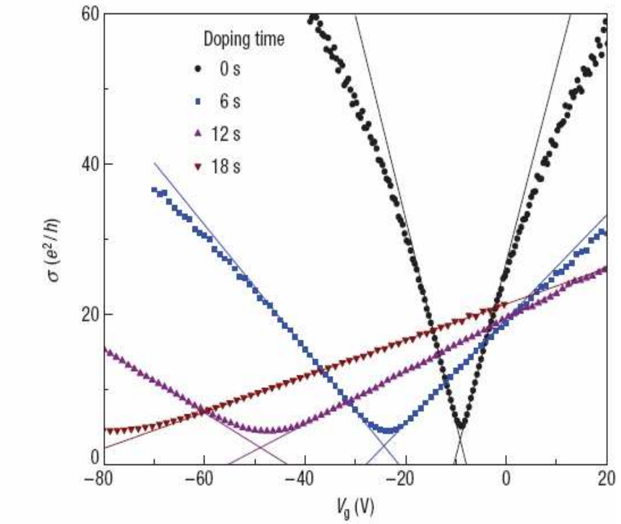
L. Fritz, J. Schmalian, MM, and S. Sachdev, condmat 0802.4289

General doping:

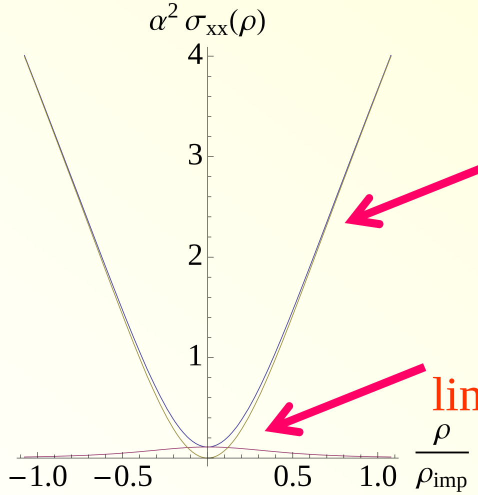
Lightly disordered system:
$$\sigma_{xx}(\omega;\mu,\Delta) = \frac{e^2}{\tau_{\rm imp}^{-1} - i\omega}\frac{\rho^2 v_F^2}{\varepsilon + P} + \sigma_Q + \delta\sigma(\Delta,\omega,\mu)$$
$$\delta\sigma(\Delta,\omega,\mu) = \mathcal{O}(\Delta/\alpha^2)$$
← Correction to hydrodynamics

Fermi liquid regime:
$$\sigma_{xx}(\omega = 0; \mu \gg T) \approx \frac{e^2\rho^2 v_F^2 \tau_{\rm imp}}{\varepsilon + P} = \frac{2}{\pi}\frac{1}{(Z\alpha)^2}\frac{e^2}{h}\frac{\rho}{\rho_{\rm imp}}$$

*J.-H. Chen et al. Nat. Phys. **4**, 377 (2008).*



↔



Impurity limited conductivity

Universal Cb limited conductivity

Magnetotransport

- Strategy: describe the slow dynamics of the momentum mode φ_0 in very weak disorder and moderate magnetic field

Magnetotransport

• Strategy: describe the slow dynamics of the momentum mode φ_0 in very weak disorder and moderate magnetic field

Result: Full thermoelectric response (for general B) obtained in terms of thermodynamic quantities + only 2 independent transport coefficients (collision matrix elements)!

Magnetotransport

• Strategy: describe the slow dynamics of the momentum mode φ_0 in very weak disorder and moderate magnetic field

Result: Full thermoelectric response (for general B) obtained in terms of thermodynamic quantities + only 2 independent transport coefficients (collision matrix elements)!

• At small *B*, one transport coefficient is subdominant
→ Relativistic hydrodynamics with only one transport coefficient σ_Q is recovered!

$$\tau_{\mathrm{ee}}^{-1} >> \tau_{\mathrm{B}}^{-1}$$

$$\sigma_{xx}(\omega, B) = \sigma_{xx}^{\mathrm{MHD}}(\omega, B) + \mathcal{O}(b/\alpha^2, \omega/\alpha^2)$$

Corrections to hydrodynamics

Magnetotransport

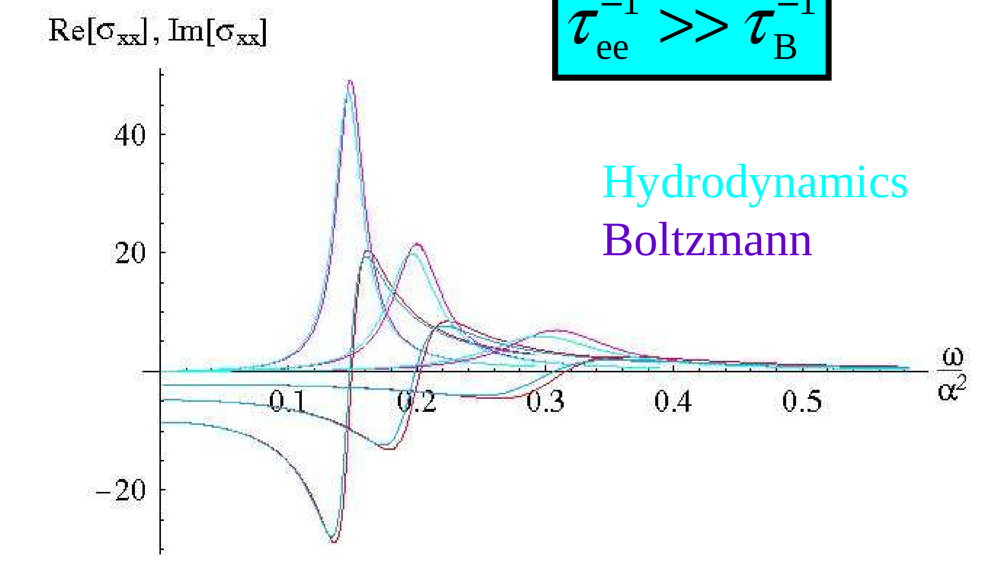
• Strategy: describe the slow dynamics of the momentum mode φ_0 in very weak disorder and moderate magnetic field

Result: Full thermoelectric response (for general B) obtained in terms of thermodynamic quantities + only 2 independent transport coefficients (collision matrix elements)!

• At small *B*, one transport coefficient is subdominant
→ Relativistic hydrodynamics with only one transport coefficient σ_Q is recovered!

$$\tau_{ee}^{-1} >> \tau_B^{-1}$$

Cyclotron resonance:



Cyclotron resonance revisited

Crossover to Fermi liquid regime:

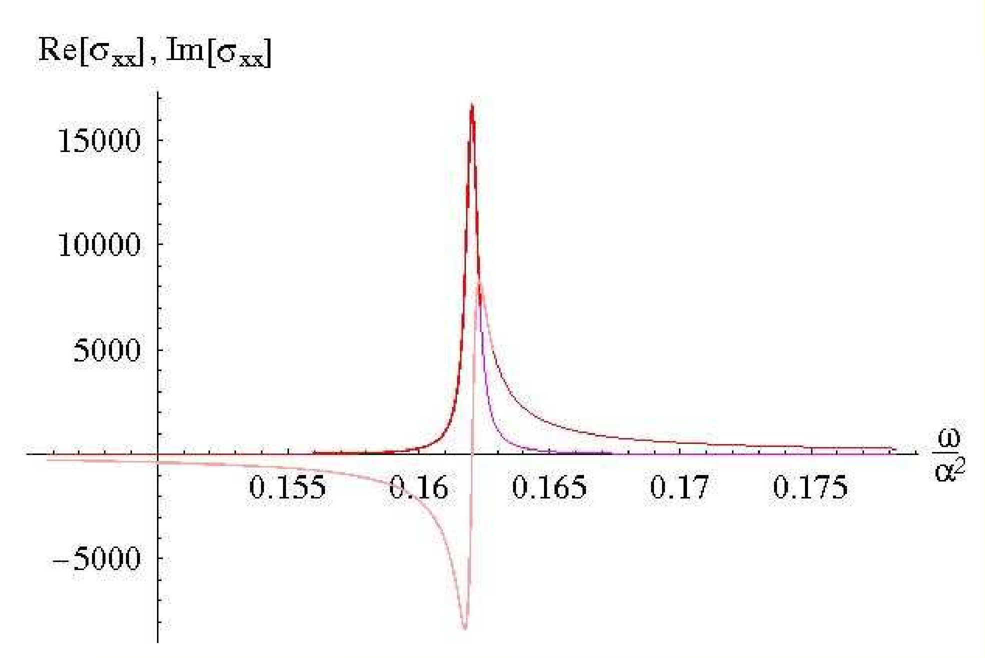
- Semiclassical ω_c recovered at $\mu >> T$

$$\omega_c^{(0)} = \frac{\rho B}{\varepsilon + P} \rightarrow \frac{eB}{\mu / v_F^2} = \frac{eB}{\hbar k_F / v_F}$$

- Broadening goes to zero - Kohn's theorem recovered: Non-broadening of the resonance for a single parabolic band.

$$\gamma \equiv \frac{\sigma_Q B^2 v_F^2}{(\varepsilon + P)}$$

$$\gamma \propto \sigma_Q(\mu) \overset{\mu >> T}{\longrightarrow} 0$$

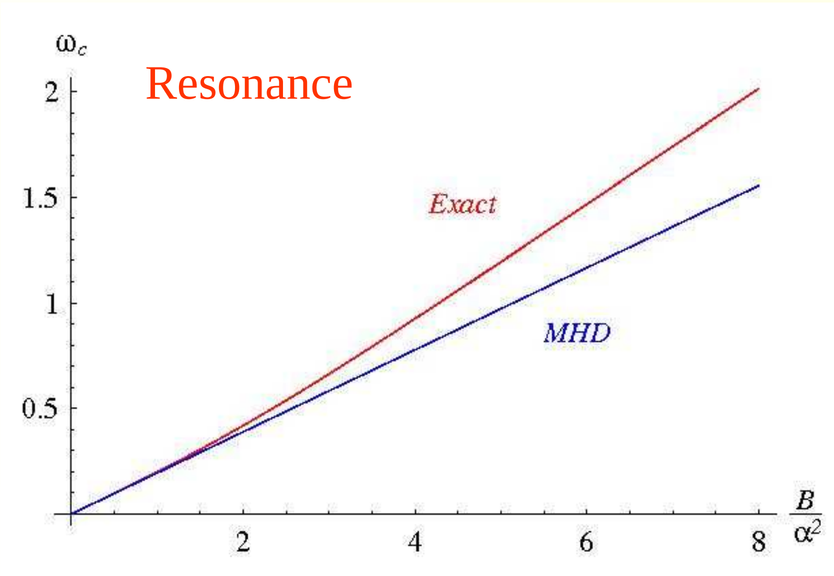


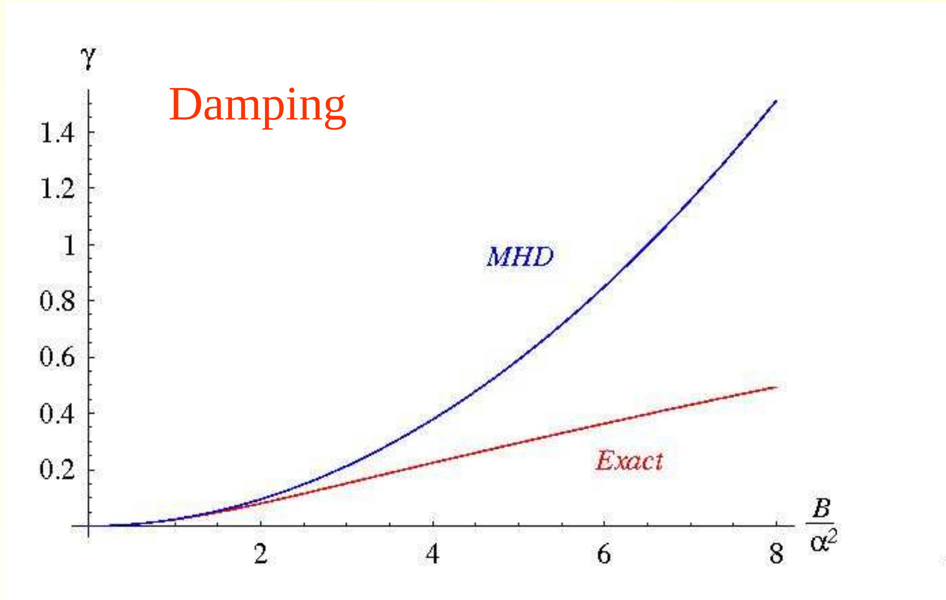
Cyclotron resonance revisited

Beyond hydrodynamics: Towards ballistic magnetotransport

$$\mu = T$$

Large fields $$\tau_{B}^{-1} > \tau_{ee}^{-1} >> \tau_{imp}^{-1}, \omega$$





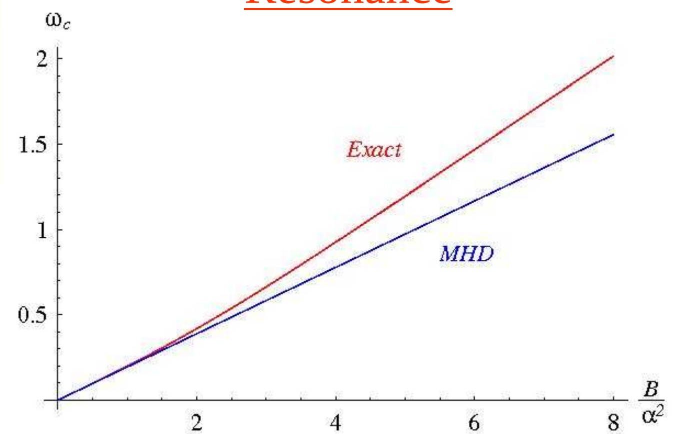
Strongly coupled liquids

Same trends as in exact (AdS-CFT) results for strongly coupled relativistic fluids!

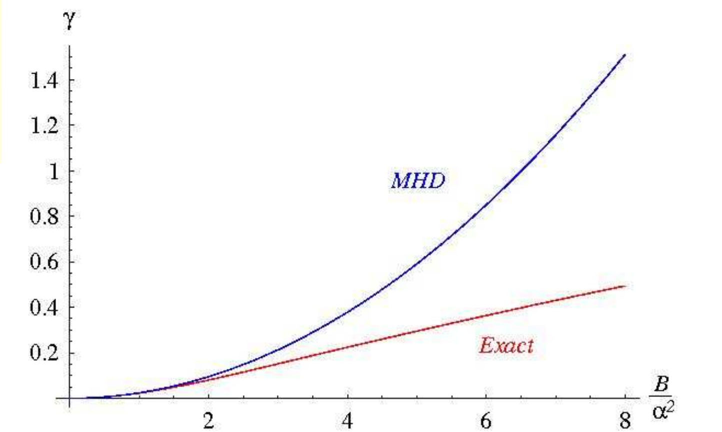
S. Hartnoll, C. Herzog (2007)

Graphene

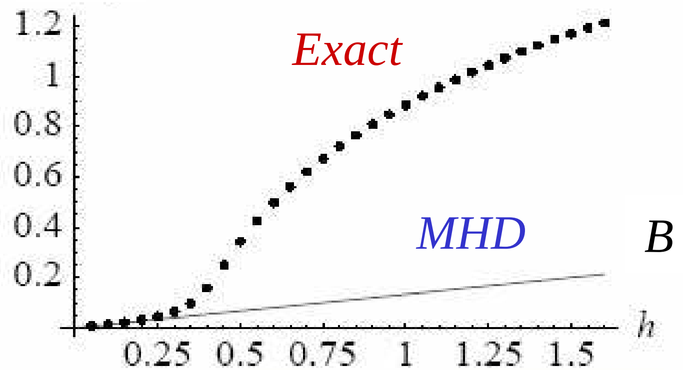
Resonance

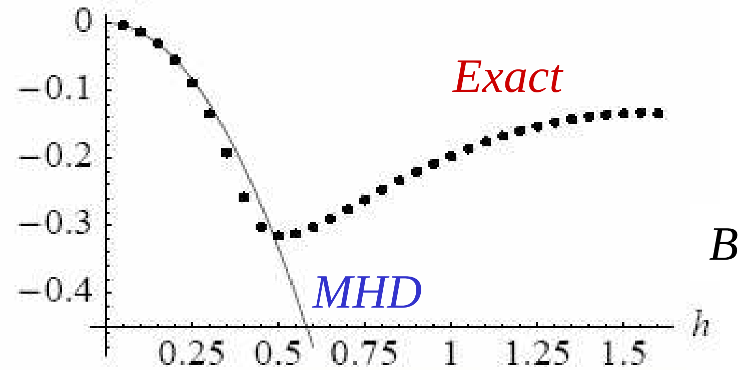


Damping

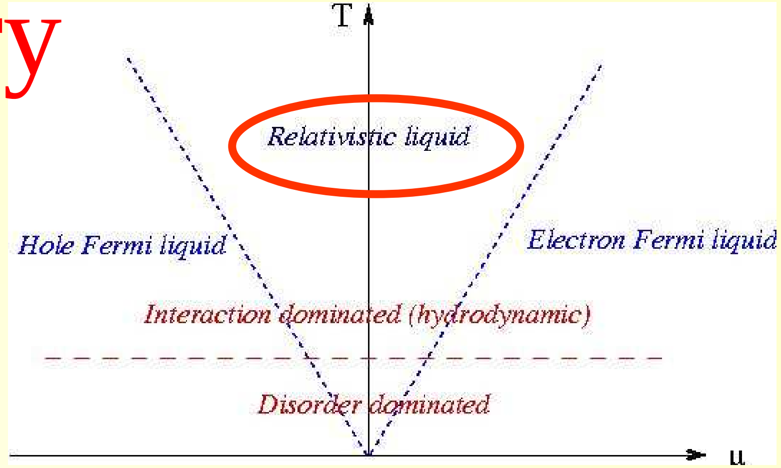


$\mathcal{N}$= 4 SUSY SU(N) gauge theory [flows to CFT at low energy]





Summary



- Relativistic physics in graphene and quantum critical systems

- Hydrodynamic description:

→ collective cyclotron resonance in the relativistic regime
→ covariance: 6 frequency dependent response functions given by thermodynamics and *only one* parameter σ_Q.

- Boltzmann approach

→ Confirmed and refined hydrodynamic description
→ Understood relativistic-to-Fermi liquid crossover:
  - From universal Coulomb-limited to disorder-limited linear conductivity in graphene
  - From collective-broadened to semiclasscial sharp cyclotron resonance

→ Beyond hydrodynamics: describe large fields and disorder