Description of the ICTP AGCM (SPEEDY) Version 41

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This description is organized as follows: In A, the model dynamical core is briefly discused and an overview of the physical parameterizations is given. In B, the formulation of the physical parameterizations is presented in more detail.

New features with respect to version 40 are that version 41 is very modular, making the coupling to ocean and land surface models easier. Furthermore, the model has now a spatially varying depth for the optional ocean mixed-layer and a stability-dependent stratocumulus cloud parameterization. The models boundary conditions are derived from the ERA interim re-analysis for the 30-year period 1979 to 2008. Additionally, a filter (RAW) is used in the time integration which makes the scheme 3rd order accurate.

A Model formulation.

A.1 The dynamical core.

The ICTP AGCM (nicknamed SPEEDY, for "Simplified Parameterizations, privitivE-Equation DYnamics") is based on a spectral dynamical core developed at the Geophysical Fluid Dynamics Laboratory (see Held and Suarez 1994). It is a hydrostatic, σ-coordinate, spectral-transform model in the vorticity-divergence form described by Bourke (1974), with semi-implicit treatment of gravity waves. The basic prognostic variables are vorticity (Vor), divergence (Div), absolute temperature (T) and the logarithm of surface pressure ($log(p_s)$); the code also computes the evolution of a number of additional variables which are simply advected by the dynamical core (with sources and sinks specified by the physical parametrizations). In SPEEDY, the only additional variable currently used is specific humidity (Q).

The time stepping uses a leapfrog scheme, with a time filter (Robert 1966) to suppress the computational mode. It also uses an additional filter (RAW) introduced by Amezcua et al. (2011), which makes the time integration scheme 3rd order accurate (instead of the standard 1st order accuracy in the leapfrog scheme). Horizontal hyper-diffusion of Vor, Div, T and Q has the form of the fourth-power of the Laplacian, applied on σ surfaces; a corrective term which simulates diffusion on pressure surfaces is used for T and Q, to avoid spurious diffusion over topography.

The standard horizontal resolution corresponds to a triangular spectral truncation at total wavenumber 30 (T30), with a standard gaussian grid of 96 by 48 points. In the current formulation of SPEEDY, eight vertical layers are used, with boundaries (half-levels) at σ values of 0, 0.05, 0.14, 0.26, 0.42, 0.60, 0.77, 0.90 and 1.

The prognostic variables (apart from $log(p_s)$) are specified at σ levels intermediate between the upper and lower boundaries (the so-called full levels), namely at σ 0.025, 0.095, 0.20, 0.34, 0.51, 0.685, 0.835 and 0.95. In practice, the top two

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and the bottom layer provide a "bulk" representation of the stratosphere and the planetary boundary layer (PBL) respectively. The output data are post-processed on pressure levels at 30, 100, 200, 300, 500, 700, 850 and 925 hPa.

A.2 Overview of physical parameterizations.

A set of physical parametrization schemes has been developed starting from basic principles used in more complex GCMs, with a number of simplifying assumptions which are suited to a model with a coarse vertical resolution. A brief description of processes represented by the schemes is given here; a more detailed and quantitative description can be found in B.

Convection - A simplified mass-flux scheme is activated where conditional instability is present (namely, where saturation moist static energy decreases with height between the lowest layer (PBL) and the upper-tropospheric layers), and where humidity in the PBL exceeds a prescribed threshold. The cloud-base mass flux (at the top of the PBL) is such that the PBL humidity is relaxed towards the threshold value on a time-scale of 6 hours. Detrainment occurs only at the cloud-top level (determined by the conditional instability criterion), while entrainment occurs in the lower half of the troposphere. The air in the updrafts is assumed to be saturated.

Large-scale condensation - When relative humidity exceeds a σ-dependent threshold, specific humidity is relaxed towards the corresponding threshold value on a time-scale of 4 hours, and the latent heat content removed from the atmosphere is converted into dry static energy.

Clouds - Cloud cover and thickness are defined diagnostically from the values of relative humidity in an air column including all tropospheric layers except the PBL and the amount of total precipitation. Stratocumulus clouds are treated separately based on the static stability in the PBL.

Short-wave radiation. - The shortwave radiation schemes uses two spectral bands, one of which represents the near-infrared portion of the spectrum. Radiation is reflected at cloud top and at the surface; the cloud albedo is proportional to the total cloud cover. The shortwave transmissivities of the model layers are functions of layer mass, specific humidity and cloud cover.

Long-wave radiation - The longwave radiation schemes uses four spectral bands, one for the atmospheric "window" and the other ones for the spectral regions of absorption by water vapour and carbon dioxide. For each layer, transmissivities in the four bands are defined as a function of layer mass and humidity. The effect of clouds is represented as a descrease in the transmissivity of the "window" band, as a function of cloud cover. The downward emission from each layer is computed as a weighted function of the temperature at the full level and of the (interpolated) temperature at the half-level below. For the upward emission, the temperature at the full level and at the half-level above is used.

Surface fluxes of momentum and energy - Surface fluxes are defined by bulk aerodynamic formulas with different exchange coefficients between land and sea. Coefficients for (sensible and latent) heat fluxes also depend on a simple stability index, while the coefficient for the momentum flux over land is a function of topographic height. A skin temperature over land is defined from the surface energy balance.

Vertical diffusion - Vertical diffusion is composed of three terms: a redistribution of dry static energy and moisture between the two lowest model layers, which simulates shallow convection in regions of conditional instability; a diffusion of water vapour in stable conditions which acts in the lower troposphere, depending on the vertical profile of relative humidity; and a diffusion of dry static energy in case the lapse rate approaches (or exceeds) the dry-adiabatic limit.

A.3 Boundary conditions.

As any atmospheric model, SPEEDY requires appropriate boundary conditions to determine the fluxes of momentum, heat and moisture at the surface, and the flux of incoming solar radiation at the top of the atmosphere.

At the surface, in addition to topographic height and (fractional) land-sea mask, the model requires climatological fields of the following variables:

- sea surface temperature (SST);
- sea ice fraction;
- soil temperature in the deep soil layer (about 1 m);
- moisture in the top soil layer and the root-zone layer;
- snow depth;
- bare-surface albedo (in the absence of snow or sea ice);
- fraction of land-surface covered by vegetation.

For the last two fields, annual-mean values are used, while all other fields are specified as monthly means and are linearly interpolated to get daily-updated values.

The bare-surface albedo is linearly combined with assigned values of sea-ice and snow albedo to get a net surface albedo, using weights which are linearly dependent on sea-ice fraction and snow cover respectively. Similarly, the soil moisture in the top soil layer and in the root zone are linearly combined (using the vegetation fraction) to define a soil moisture availability index, which is used to compute evaporation over land. (See B.6 for more details).

All climatological fields have been computed by averaging the corresponding data from the European Centre for Medium-Range Weather Forecasts' re-analysis (ERA Interim; see Dee et al. 2011) in the period 1979-2008.

For soil and sea ice, the temperature the upper layer (1m for soil and 1.8 m for sea ice) are estimated using an extended force-restore type approach.

The model allows a time-varying SST anomaly to be superimposed to the climatological SST; the anomaly can be either specified from an input file or computed by a coupled ocean model. As an option, speedy employs a slab Ocean model, with a depth that varies between 60 m in the extratropical and 40 m in the tropical regions. Ocean temperature anomalies are calculated due to net heatflux anomalies at the ocean surface. In order to determine the heatflux anomalies, SPEEDY has to be run previously using prescribed SSTs.

At the top of the atmosphere, the incoming flux of solar radiation is computed daily from astronomical formulae (the model has no daily cycle). Empirical, seasonally varying functions are used to define the absorption of solar radiation by ozone in the stratosphere, and the latitudinal variations of the optical depth for solar radiation depending on the daily-averaged zenith angle. The use of a proper ozone climatology is planned for future model versions.

B Formulation of physical parametrizations.

B.1 General definitions.

Physical parametrization tendencies at time step t are computed from (time-filtered) model variables at step *t-1*, and integrated forward to step *t+1*. All parametrization schemes described below are run as independent modules, except for computation of surface fluxes, which uses the downward radiative flux at the surface as an input to define a (diagnostic) skin temperature over land, and returns an upward flux of longwave radiation to the radiation code.

The parametrization driver first computes grid-point values of the 'primary' variables (U and V wind components, temperature T, specific humidity Q, geopotential Φ, surface pressure p_s) from their spectral representations, then defines a number of additional diagnostic variables:

- saturation specific humidity: Q^{sat} ;
- relative humidity: RH ;
- dry static energy: $SE = c_p T + \Phi$;
- moist static energy: $MSE = SE + L_c Q$;
- saturation moist static energy: $MSS = SE + L_c Q^{sat}$;

where c_p and L_c are the specific heat at constant pressure and the latent heat of condensation respectively.

All variables are defined at full levels; however, values of some variables are also needed at half-levels (layer boundaries) to define fluxes. Let A_k be the value of a generic variable A at full level k (k=1,...,N, with k increasing with σ, i.e from top to bottom). Defining h=1/2 layer, let A_{k+h} be the value at the half-level representing the lower boundary of layer k, and A_{k-h} the value at its upper boundary. Unless stated otherwise, half-level values used by parametrization schemes are obtained by linear interpolation in log(σ):

$$A_{k+h} = A_k + (A_{k+1} - A_k)\,\frac{log(\sigma_{k+h}) - log(\sigma_k)}{log(\sigma_{k+1}) - log(\sigma_k)} \qquad (1)$$

Most parametrization schemes work by defining upward and downward fluxes of momentum, energy or moisture at layer boundaries. For a generic variable A, upward and downward fluxes at half-level $k \pm h$ will be indicated by ${}^uF^A_{k\pm h}$ and ${}^dF^A_{k\pm h}$ respectively. The flux *absorbed* in layer k is:

$$\Delta F^A_k = (\ {}^uF^A_{k+h} - {}^uF^A_{k-h}\ ) + (\ {}^dF^A_{k-h} - {}^dF^A_{k+h}\ ) \qquad (2)$$

Fluxes of momentum and moisture are converted into wind and moisture tendencies by:

$$\frac{\partial A_k}{\partial t} = \frac{g\ \Delta F^A_k}{\Delta p_k} \qquad (3)$$

where $A = U,\ V$, or Q, and:

$$\Delta p_k = p_s\ (\ \sigma_{k+h} - \sigma_{k-h}\ ) \qquad (4)$$

while energy fluxes are converted into temperature tendencies by:

$$\frac{\partial T_k}{\partial t} = \frac{g\ \Delta F_k^{EN}}{c_p\ \Delta p_k} \tag{5}$$

where EN = SE, SR (shortwave radiation), or LR (longwave radiation).

B.2 Convection.

The convection scheme is a simplified version of the mass-flux scheme developed by Tiedke (1993). It represents the updraft of saturated air from the PBL to a suitably defined "top-of-convection" (TCN) level in the middle or upper troposphere, and the compensating large-scale descending motion. Entrainment into the updraft occurs in the lower troposphere above the PBL, while detrainment is only allowed in the TCN layer (which greatly simplifies the computation of latent heat release). The scheme also represents a "secondary" exchange of moisture between the PBL and the layers below the TCN level.

The convection scheme is activated in conditionally unstable regions where MSS decreases with height, and where humidity in both ,the PBL and one level above, (level N and N-1) exceeds a prescribed threshold. Specifically, convection occurs at a given grid-point if: A tropospheric level exists (with $1 < k < N - 1$), such that:

$$MSS_N > MSS_{k+h} \tag{6}$$

If this criterion if fullfilled convection is activated where the actual moist static energy at N-1 is larger than that at k+h

$$MSS_{N-1} > MSS_{k+h} \tag{7}$$

or where the humidity in both ,the PBL and one level above, (level N and N-1) exceeds a prescribed threshold

$$Q_N > Q_{thr1} = RH_{cnv}\ Q_N^{sat} \quad ; Q_{N-1} > Q_{thr2} = RH_{cnv}\ Q_{N-1}^{sat} \quad . \tag{8}$$

The TCN level is defined as the highest tropospheric level for which Eq. 6 is satisfied.

Over the selected grid points, the scheme first defines the fluxes of mass *(m)*, humidity and dry static energy at the top of the PBL:

$${}^{u}F^{m}_{N-h} = {}^{d}F^{m}_{N-h} = F^* \tag{9}$$

$${}^{u}F^{Q}_{N-h} = F^* \cdot Q_N^{sat} \quad ; \quad {}^{d}F^{Q}_{N-h} = F^* \cdot Q_{N-h} \tag{10}$$

$${}^{u}F^{SE}_{N-h} = F^* \cdot SE_N \quad ; \quad {}^{d}F^{SE}_{N-h} = F^* \cdot SE_{N-h} \tag{11}$$

The closure of the scheme (i.e. the determination of F^*) is obtained by requiring the convective moisture tendency in the PBL to be equivalent to a relaxation of humidity towards the threshold specified by Q_{thr} with relaxation time τ_{cnv}:

$$\left(\frac{\partial Q_N}{\partial t}\right)_{cnv} = -\ \frac{g\ F^*\ (\ Q_N^{sat} - Q_{N-h}\ )}{\Delta p_N} = -\ \frac{Q_N - Q_{thr}}{\tau_{cnv}} \tag{12}$$

Solving Eq. 12 for F^* gives:

$$F^* = \frac{\Delta p_N}{g\ \tau_{cnv}}\ \frac{Q_N - Q_{thr}}{Q_N^{sat} - Q_{N-h}} \tag{13}$$

In the 'intermediate' layers (between the PBL and the TCN layer), the upward fluxes are increased by the contribution due to entrainment (E^m), while the downward fluxes are computed from the modified mass flux and the half-level values of humidity and dry static energy (since the upward and downward mass fluxes have the same magnitude, just one value needs to be defined):

$$E_k^m = \epsilon(\sigma_k)\ F_{k+h}^m \tag{14}$$

$$F_{k-h}^m = F_{k+h}^m\ +\ E_k^m \tag{15}$$

$${}^{u}F_{k-h}^{Q} =\ {}^{u}F_{k+h}^{Q}\ +\ E_k^m\ Q_k\ \ ;\ \ {}^{d}F_{k-h}^{Q} = F_{k-h}^m\ Q_{k-h} \tag{16}$$

$${}^{u}F_{k-h}^{SE} =\ {}^{u}F_{k+h}^{SE}\ +\ E_k^m\ SE_k\ \ ;\ \ {}^{d}F_{k-h}^{SE} = F_{k-h}^m\ SE_{k-h} \tag{17}$$

where the entrainment coefficient $\epsilon(\sigma_k)$ varies linearly with σ, from 0 at $\sigma = 0.5$ to a maximum value at σ_{N-1}.

It should be noted that, since no detrainment occurs in 'intermediate' layers, there is no need to compute the condensation in the updraft within these layers, since the energy released by such a process is simply transported upwards and does not modify the local temperature tendency. Therefore, condensation is only computed in the TCN layer, where the fluxes at the upper boundary are set to zero. A part of the upward moisture flux at the TCN lower boundary in converted into convective precipitation:

$$P_{cnv} =\ {}^{u}F_{k0+h}^{Q}\ - F_{k0+h}^m\ Q_{k0+h}^{sat} \tag{18}$$

where $k0 = k(TCN)$. Then, the net fluxes of moisture and energy into the TCN layer are modified as follows:

$$\Delta F_{k0}^{Q} =\ {}^{u}F_{k0+h}^{Q}\ -\ {}^{d}F_{k0+h}^{Q}\ -\ P_{cnv} \tag{19}$$

$$\Delta F_{k0}^{SE} =\ {}^{u}F_{k0+h}^{SE}\ -\ {}^{d}F_{k0+h}^{SE}\ +\ L_c\ P_{cnv} \tag{20}$$

From Eqs. 8 to 19, net fluxes of moisture and dry static energy can be computed for all tropospheric layers. These represent the effects of deep convective systems extending up to the TCN layer. The effects of shallower, non-precipitating convective systems are crudely represented by an additional diffusion of moisture between the PBL and the 'intermediate' layers, which acts wherever the relative humidity in the the latter layers falls below a threshold $RH'_{cnv} < RH_{cnv}$. In such a case, the net moisture flux in the *k-th* intermediate layer is increased by a quantity

$$\Delta' F_k^Q = \epsilon'\ F^*\ (\ RH'_{cnv}\ Q_k^{sat}\ -\ Q_k\ ) \tag{21}$$

while the net moisture flux in the PBL is decreased by a corresponding amount.

B.3 Large-scale condensation.

Large-scale condensation is modelled as a relaxation of humidity towards a reference value, which occurs in tropospheric layers whenever relative humidity exceeds a σ-dependent threshold RH_{lsc}. In such a case, a humidity tendency is defined as:

$$\left(\frac{\partial Q_k}{\partial t}\right)_{lsc} = - \frac{Q_k \; - \; RH(\sigma_k)_{lsc} \, Q_k^{sat}}{\tau_{lsc}} \tag{22}$$

and the corresponding temperature tendency as:

$$\left(\frac{\partial T_k}{\partial t}\right)_{lsc} = - \frac{L_c}{c_p} \left(\frac{\partial Q_k}{\partial t}\right)_{lsc} \tag{23}$$

where the dependence of RH_{lsc} on σ is as follows:

$$RH(\sigma_k)_{lsc} = RH^1_{lsc} \; + \; \Delta RH_{lsc} \, (\sigma_k^2 - 1) \tag{24}$$

The precipitation flux due to large-scale condensation is given by (the opposite of) the vertical integral of humidity tendencies in the troposphere:

$$P_{\;lsc} = - \frac{1}{g} \sum_{k=2}^{N} \Delta p_k \left(\frac{\partial Q_k}{\partial t}\right)_{lsc} \tag{25}$$

B.4 Clouds and shortwave radiation.

Although the cloud and radiation schemes used in SPEEDY are much simpler than the schemes used in state-of-the art GCMs, still they represent the most complex part of the parametrization package, and that which depends on the largest number of parameters. For reasons of brevity, only the basic principles of the schemes will be described here. A more detailed description can be found in Corbetta (1999).

Clouds properties are defined diagnostically from values of relative in the tropospheric air column and the amount of total precipitation. Clouds are assumed to have their base at the interface between the lowest two model layers, and their top at the upper boundary of highest layer in which the following conditions are both satisfied:

$$RH_k \;>\; RH_{cl} \tag{26}$$

$$Q_k \;>\; Q_{cl} \quad . \tag{27}$$

The cloud-top layer will be denoted by k_{cltop}.

With the conditions 26 and 27 satisfied, the cloud cover is determined by the sum of terms proportional to the square root of total precipitation and the square of $RH_k - RH_{cl}$:

$$CLC = min \left[1., wp_{cl} \sqrt{min[pm_{cl}, P_{\;lsc} + P_{\;cnv}]} + min \left[1., \frac{RH_k - RH_{cl}}{RH'_{cl} - RH_{cl}}) \right]^2 \right] \tag{28}$$

If the top layer of precipitation is higher then the cloud top layer, $k_{prtop} < k_{cltop}$, then k_{cltop} is set to k_{prtop}.

Stratocumulus clouds at the top of the boundary layer are parameterized depending on the static stability (derived from the gradient in dry static energy in the boundary layer, GSE_N), and exist if GSE_N exceeds the threshold $GSES0$. The statocumulus cover CLS over oceans then varies between 0 and the maximum value $CLSMAX$, depending on the convective and large-scale cloud cover

$$CLS = FST\ max(CLSMAX - CLF\ CLC, 0)\ , \tag{29}$$

where

$$FST = max\left(0., min\left[1., \frac{GSE_N - GSES0}{GSES1 - GSES0}\right]\right) \tag{30}$$

Over land, the statocumulus cover CLS is further modified to be proportional to the surface relative humudity $CLSL = CLS\ RH_N$.

Once cloud properties are defined, the shortwave radiation (SR) starts by computing the incoming flux of solar radiation ${}^dF_0^{sol}$ at the top of the atmosphere from astronomical formulae. Since the model has just two layers above the troposphere, which is intended as a "dynamical boundary" for the tropospheric motion rather than as a representation of the vertically-averaged upper atmosphere, the code is designed to reproduce lower-stratosperic temperatures in the first two (highest) model layers. The absorption by ozone in the lower stratosphere is defined by an idealised function of latitude; the corresponding flux (ΔF_{lst}^{ozone}) is considered as absorbed by the first model layer. In addition, as a crude representation of the radiative effects of the *unresolved* upper stratosphere, a small latitudinally-dependent fraction (ΔF_{ust}^{ozone}) of the incoming solar radiation is turned directly into outgoing longwave radiation.

After subtracting the ozone absorption, the residual downward flux into the first model layer:

$${}^dF_h^{SR} = {}^dF_0^{sol} - \Delta F_{ust}^{ozone} - \Delta F_{lst}^{ozone} \tag{31}$$

is partitioned into two bands, one corresponding to the visible part of the spectrum, the second to the near-infrared. For each band and for each tropospheric layer, a transmissivity τ_k^{SR} is defined as a function of the daily-averaged zenith angle (again a prescribed function of latitude), layer depth, specific humidity and cloud properties, and the downward propagation of SW radiation is simply modelled (for each band) by:

$${}^dF_{k+h}^{SR} = {}^dF_{k-h}^{SR}\ \ \tau_k^{SR} \tag{32}$$

In the cloud-top layer, the radiation entering the upper boundary is first modified by subtracting the flux reflected by clouds, so that for $k = k_{cltop} < N$ Eq. (29) becomes:

$${}^dF_{k+h}^{SR} = {}^dF_{k-h}^{SR}\ (1 - A_{cl}\ CLC)\ \ \tau_k^{SR} \tag{33}$$

where A_{cl} is the cloud albedo. (For simplicity, cloud effects are neglected in the near-infrared band). At the surface (i.e. level $N{+}h$), we have reflection due to the stratocumulus clouds

$${}^dF_{N+h}^{SR} = {}^dF_{N-h}^{SR}\ (1 - A_{cls}\ CLS)\ \ \tau_N^{SR} \tag{34}$$

where A_{cls} is the stratocumulus cloud albedo.

Furthermore, at the surface (i.e. level $N+h$), a climatological albedo A_s is defined as a function of seasonally varying fields of sea ice and snow depth. The upward flux at the surface is defined as:

$$ {}^{u}F_s^{SR} \; = \; {}^{d}F_s^{SR} \; A_s \tag{35} $$

and the upward propagation of shortwave radiation is modelled by:

$$ {}^{u}F_{k-h}^{SR} \; = \; {}^{u}F_{k+h}^{SR} \;\; \tau_k^{SR} \tag{36} $$

with the flux reflected by clouds added at the upper boundary of the cloud-top layer. Since practically the whole flux in the near-infrared band is absorbed in the downward propagation, the upward part is only modelled for the visible band.

B.5 Longwave radiation.

In the parametrization scheme for longwave radiation (LR), the infrared spectrum ($5\mu m \leq \lambda \leq 50\mu m$) is partitioned into four regions (again referred to as "bands", although two of them include different spectral intervals with similar optical properties):

- the so-called "infrared window" between 8.5 and 11 μm (band 1);
- the band of strong absorption by CO_2 around 15 μm (band 2);
- the aggregation of regions with weak or moderate absorption by water vapour (band 3);
- the aggregation of regions with strong absorption by water vapour (band 4).

As in the SR code, a transmissivity is computed for each band and model layer as a function of layer depth, humidity and cloud properties. The effect of clouds is modelled as a (strong) decrease in the transmissimity in the "window" band. So, if

$$ \Delta p'_k \; = \; \frac{p_s}{p_0} \, (\sigma_{k+h} - \sigma_{k-h}) \tag{37} $$

is the normalised layer depth (where $p_0 = 10^5$ hPa), the transmissivity in the "window" band is given by

$$ \tau_{k,1}^{LR} \; = \; exp \, [\, - \, (\, \alpha_{win}^{LR} + \alpha_{cl}^{LR} \; CLC) \; \Delta p'_k \;] \tag{38} $$

in cloudy regions/layers (i.e. where $CLC > 0$ and $k_{cltop} \leq k \leq N - 1$), by

$$ \tau_{k,1}^{LR} \; = \; exp \; (\, - \, \alpha_{win}^{LR} \; \Delta p'_k \;) \tag{39} $$

otherwise. For the CO_2 and water-vapour bands, transmissivities are given respectively by:

$$ \tau_{k,2}^{LR} \; = \; exp \; (\, - \, \alpha_{CO_2}^{LR} \; \Delta p'_k \;) \tag{40} $$

$$ \tau_{k,3}^{LR} \; = \; exp \; (\, - \, \alpha_{wv1}^{LR} \; Q_k \; \Delta p'_k \;) \tag{41} $$

$$ \tau_{k,4}^{LR} \; = \; exp \; (\, - \, \alpha_{wv2}^{LR} \; Q_k \; \Delta p'_k \;) \tag{42} $$

where the α coefficients are constant absorptivity parameters. Typically, in the middle and lower troposphere (in clear-sky conditions),

$$\tau_{k,4}^{LR} \ < \ \tau_{k,2}^{LR} \ < \ \tau_{k,3}^{LR} \ < \ \tau_{k,1}^{LR} \tag{43}$$

except in the tropics, where $\tau_{k,3}^{LR} < \tau_{k,2}^{LR}$ close to the surface. In the upper troposphere and in the stratosphere, the CO_2 band has the lowest clear-sky transimissivity.

After setting the downward flux at the upper boundary to zero, the LR code proceeds by computing the downward transmission and emission in each band:

$${}^{d}F_{k+h}^{LR} \ = \ {}^{d}F_{k-h}^{LR} \ \tau_k^{LR} \ + \ (\ 1 - \tau_k^{LR}\ )\ {}^{d}B_k \tag{44}$$

In the equation above (where the band index is omitted for simplicity), ${}^{d}B_k$ represents the downward blackbody emission in a given spectral band. This is computed as a weighted function of temperature at the centre and at the lower boundary of the layer:

$${}^{d}B_k \ = \ f_b(T_k)\ \sigma_{SB}\ [\ T_k^4 \ + \ w_k^{LR}\ (\ T_{k+h}^4 - T_k^4\ )\ ] \tag{45}$$

where σ_{SB} is the Stefan-Boltzmann constant, f_b gives the fraction of energy emitted in each band as a function of temperature, and the weight w_k^{LR} depends on the layer transmissivity in a given band ($w_k^{LR} = 0$ when $\tau_k^{LR} = 1$).

At the lower boundary, the blackbody emission from the surface ${}^{u}B_s$ (multiplied by a constant surface emissivity ϵ_s) is partitioned among the four bands, and a small contribution from LR reflection is added:

$${}^{u}F_s^{LR} \ = \ \epsilon_s \ \ f_b(T_s)\ {}^{u}B_s \ + \ (1-\epsilon_s)\ {}^{d}F_s^{LR} \tag{46}$$

Then, for each band the upward propagation is modelled by:

$${}^{u}F_{k-h}^{LR} \ = \ {}^{u}F_{k+h}^{LR} \ \tau_k^{LR} \ + \ (\ 1 - \tau_k^{LR}\ )\ {}^{u}B_k \tag{47}$$

$${}^{u}B_k \ = \ f_b(T_k)\ \sigma_{SB}\ [\ T_k^4 \ + \ w_k^{LR}\ (\ T_{k-h}^4 - T_k^4\ )\ ] \tag{48}$$

where the upward emission is computed from the temperature at the centre and the upper boundary of each layer. Eqs. (41) and (44) imply that, when the layer transmissivity is close to zero, the upward and downward emissions are dependent on the temperatures at the respective boundaries, while they approach each other (as functions of mid-layer temperature) when transmissivity is high.

Finally, since in the (dry) stratospheric layers neither the water-vapour nor the ozone emission/absorption are explicitely modelled, a seasonally-varying, zonally-symmetric correction term is added to the LR flux emitted by such a layer.

B.6 Surface fluxes.

In SPEEDY, surface fluxes are modelled using rather standard aerodynamic formulas (see e.g. chapter 4 in Hartmann 1994). However, since the PBL is represented by just one layer, one cannot use variables at the lowest model level as approximations of near-surface variables. Also, vertical gradients between two model levels cannot be used to estimate PBL stability properties which may affect the definition of exchange coefficients.

Therefore, the first step in the estimation of surface fluxes is the definition of near-surface atmospheric values of wind, temperature and humidity (U_{sa}, V_{sa}, T_{sa}, Q_{sa}, RH_{sa}) through a suitable extrapolation procedure.

Near-surface wind is simply assumed to be proportional to the wind at the lowest full level:

$$U_{sa} = f_{wind}\, U_N \quad ; \quad V_{sa} = f_{wind}\, V_N \tag{49}$$

For temperature and moisture, two options are available. The simplest method, normally used in GCMs with multi-level PBL, assumes that near-surface values of potential temperature and specific humidity are the same as at the lowest model level. In the second method (used in the integrations described below), temperature is extrapolated at σ=1 using values at the two lowest levels (T_N, T_{N-1}) and assuming a linear profile in $\log(\sigma)$ (although T_{sa} is not allowed to be lower than T_N). Then, relative humidity is set equal to the lowest-level value ($RH_{sa} = RH_N$), and Q_{sa} is recomputed from T_{sa} and RH_{sa}.

From the variables defined above, surface air density (ρ_{sa}) is also computed, and an "effective" surface wind speed is defined as:

$$|V_0| = \left(U_{sa}^2 + V_{sa}^2 + V_{gust}^2 \right)^{1/2} \tag{50}$$

where the V_{gust} constant represents the contribution of unresolved wind variability.

As far as soil moisture is concerned, the information regarding the water content in the top soil layer (W_{top}) and in the root layer below (W_{root}) is condensed into a climatological soil-water availability index α_{sw}. This non-dimensional ratio depends on vegetation fraction (f_{veg}), and on soil moisture values at field capacity (W_{cap}) and wilting point (W_{wil}) (see e.g. Viterbo and Beljiars 1995 for the definition of these parameters and their use in more complex schemes):

$$\alpha_{sw} = \frac{D_{top}\, W_{top} \; + \; f_{veg}\, D_{root} \;\; max\; (\, W_{root} - W_{wil},\; 0\,)}{D_{top}\, W_{cap} \; + \; D_{root}\; (\, W_{cap} - W_{wil}\,)} \tag{51}$$

where D_{top} and D_{root} are the layer depths.

Once the appropriate variables are defined, surface stresses, sensible heat flux and evaporation are computed over land surface as:

$${}^{u}F_{ls}^{U} \; = \; \tau_{ls}^{U} \; = \; -\rho_{sa}\; C_l^D\; |V_0|\; U_{sa} \tag{52}$$

$${}^{u}F_{ls}^{V} \; = \; \tau_{ls}^{V} \; = \; -\rho_{sa}\; C_l^D\; |V_0|\; V_{sa} \tag{53}$$

$${}^{u}F_{ls}^{SE} \; = \; SHF_{ls} \; = \; \rho_{sa}\; C_l^H\; |V_0|\; c_p\; (T_{skin} - T_{sa}) \tag{54}$$

$${}^{u}F_{ls}^{Q} \; = \; E_{ls} \; = \; \rho_{sa}\; C_l^H\; |V_0| \;\; max\; [\; \alpha_{sw} Q^{sat}(T_{skin}, p_s) - Q_{sa}\;,\; 0\;] \tag{55}$$

where the drag coefficient C_l^D is an increasing function of topographic height, and the heat exchange coefficient C_l^H is dependent on the difference of potential temperature between the surface and the lowest model layer. The skin temperature definition is given below in section B.6.1.

Over sea surface, surface stresses have the same formulation as over land, but with a constant drag coefficient C_s^D, while sensible heat and moisture fluxes:

$${}^{u}F_{ss}^{SE} \; = \; SHF_{ss} \; = \; \rho_{sa}\; C_s^H\; |V_0|\; c_p\; (T_{sea} - T_{sa}) \tag{56}$$

$${}^{u}F_{ss}^{Q} \; = \; E_{ss} \; = \; \rho_{sa}\; C_s^H\; |V_0| \;\; max\; [\; Q^{sat}(T_{sea}, p_s) - Q_{sa}\;,\; 0\;] \tag{57}$$

use a heat exchange coefficient C_s^H with a different mean value but the same dependence on potential temperature differences as for the land fluxes.

Differently from most GCMs, SPEEDY uses a fractional land-sea mask rather than a binary one. This feature allows a more accurate interpolation of surface fluxes over sea in view of future coupling with ocean models of different resolution and domain. So, in all grid points where the land-fraction is between 0.1 and 0.9, both land and sea surface fields are defined, and grid-point-average surface fluxes are defined as weighted averages of land and sea fluxes. This approach is also used to compute the emission of longwave radiation from the surface from the Stefan-Boltzmann formula.

B.6.1 Skin Temperature

Over land surfaces, a skin temperature (T_{skin}) is derived from the surface energy balance assuming a skin layer with zero heat capacity (see e.g., Viterbo and Beljaars 1995):

$$F_s^{SR} + F_s^{LR} + SHF_{ls} + E_{ls} - GHF_{ls} = 0 \quad , \tag{58}$$

where the ground heat flux, $GHF_{ls} = \lambda(T_{skin} - T_s)$, is specified on the basis of the soil temperature (see section B.7). Since Eq. (58) is a nonlinear function of T_{skin}, an exact solution would require an iterative method. In order to avoid such a potentially slow solution, Eq. (58) is solved approximately by linearization assuming constant stability coefficients C_l^H and small skin temperature adjustments with respect to the soil temperature $\delta T_{skin} = T_{skin} - T_s$. Therefore all fluxes in Eq. 58 are estimated on the basis of $T_{skin} = T_s$. Then the skin temperature is perturbed and the linear version of Eq. 58 is solved for the perturbation δT_{skin}, which is added to T_s. Subsequently, all fluxes are corrected on basis of the skin temperature adjustment. The error due to linearization is small (order of 0.1 K).

B.7 Soil and sea ice temperatures

For a soil layer of the thickness d_s the temperature is calculated according to an extended force-restore method (Hirota et al. 2002):

$$d_s c_s \frac{\partial T_s}{\partial t} = GHF_{ls} - d_s c_s \tau_s^{-1}(T_s - T_{clims}) \quad , \tag{59}$$

where GHF_{ls} represents the net heatflux into the ground defined in Eq. 58, c_s heat capacity of soil, τ_s is a damping timescale for T_s-anomalies and T_{clims} a climatological soil temperature with a seasonal cycle that is prescribed.

Over sea ice an equation similar to Eq. 59 is applied

$$d_i c_i \frac{\partial T_i}{\partial t} = GHF_{is} - d_i c_i \tau_i^{-1}(T_i - T_{climi}) \quad . \tag{60}$$

The net heatflux into the sea ice surface is GHF_{is} evaluated from

$$GHF_{is} = F_s^{SR} + F_s^{LR} + SHF_{ss} + E_{ss} \quad , \tag{61}$$

by taking into account only point with sea ice content larger than zero. d_i is the depth of the ice and varies according to

$$d_i = d_{imax} + (d_{imin} - d_{imax})\cos(lat)^2 \quad , \tag{62}$$

c_i heat capacity of ice, τ_i is a damping timescale for T_i-anomalies and T_{climi} a climatological ice temperature with a seasonal cycle the is prescribed.

Eqs. 59 and 60 are solved by discretization in time and as time-step one day is used. This means that the net surface fluxes (GHF_{ls}, GHF_{is}) are averaged over one day.

The solution method implemented in SPEEDY for 59 and 60 is such that the time-discretized equations (implicit for $\Delta T_s, \Delta T_i$) are re-written in anomaly form (omitting the subscripts for simplicity):

$$\Delta T(t+1) = \frac{\tau}{\tau + \delta t}\Delta T(t) + \frac{\delta t \; GHF(t)}{d \; c} - (T_{clim}(t+1) - T_{clim}(t)) \quad , \tag{63}$$

where $\Delta T = T - T_{clim}$ is the anomaly from climatology and $\delta t = 86400s = 1$ day is the timestep. After calculating 63, the anomaly is added to the climatological temperature T_{clim} to determine the temperature T.

B.8 Vertical diffusion.

Three different processes are modelled by the vertical diffusion scheme:

- a shallow convection, which redistributes moisture and dry static energy between the two lowest layers in cases of conditional instability;
- a (slow) diffusion of moisture in stable conditions;
- a (fast) redistribution of dry static energy occuring when the lapse rate is close to the dry-adiabatic limit.

For shallow convection (acting between levels N and $N-1$), the condition of occurrence is:

$$MSE_N > MSS_{N-1} \tag{64}$$

where the use of MSE instead of MSS at level N prevents the process from occurring in dry regions. If the above condition is satisfied, net upward fluxes of dry static energy and moisture are defined at the interface between the two layers:

$$^uF^{SE}_{N-h} \;=\; F^*_{shc} \; (MSE_N - MSS_{N-1}) \tag{65}$$

$$^uF^{Q}_{N-h} \;=\; F^*_{shc} \; Q^{sat}_N \;\; max \; (RH_N - RH_{N-1} \; , \; 0 \;) \tag{66}$$

where

$$F^*_{shc} \;=\; \frac{\Delta p_N}{g \; \tau_{shc}} \tag{67}$$

In stable conditions, or at higher levels, vertical diffusion is activated wherever the vertical gradient of a (scalar) variable is outside some reference bounds. Let Γ^A be the reference gradient of variable A with respect to a generic vertical coordinate Z. The difference between the values of A at adjacent levels is checked against Γ^A, and if:

$$A_{k+1} - A_k \;>\; \Gamma^A \; (\; Z_{k+1} - Z_k \;) \tag{68}$$

then a net upward flux of A is defined at the interface between the two layers:

$$^uF^A_{k+h} \;=\; F^*_{vdf} \; (\; A^*_k - A_k \;) \tag{69}$$

where

$$F^*_{vdf} \; = \; \frac{C_0 \; p_s}{g \; \tau_{vdf}} \tag{70}$$

and A^*_k is defined below.

If the adimensional coefficient C_0 is equal to the average σ-depth of model layers, then the tendencies implied by Eq. (60) are (approximately) equivalent to a relaxation of A_k towards A^*_k with time scale τ_{vdf}. Different time scales may be used for different variables and/or stability conditions.

For moisture diffusion, a reference gradient of relative humidity (Γ^{RH}) with respect to σ is specified, so that Eqs. (59-60) become:

$$RH_{k+1} - RH_k \; > \; \Gamma^{RH} \; (\; \sigma_{k+1} - \sigma_k \;) \tag{71}$$

$${}^u F^Q_{k+h} \; = \; F^*_{vdf} \; Q^{sat}_k \; (\; RH_{k+1} - RH_k \;) \tag{72}$$

For dry static energy, the gradient Γ^{SE} with respect to geopotential is used in Eq. (59); this parameter is linearly related to the ratio between the actual and the dry-adiabatic lapse rate:

$$\Gamma^{SE} \; = \; \frac{\partial SE}{\partial \Phi} \; = \; 1 + \frac{c_p}{g} \, \frac{\partial T}{\partial z} \tag{73}$$

Diffusion occurs when $SE_k < SE^*_k$, where:

$$SE^*_k \; = \; SE_{k+1} \; + \; \Gamma^{SE} \; (\; \Phi_k - \Phi_{k+1} \;) \tag{74}$$

B.9 Ocean mixed-layer

As an optional facility, a simple slab ocean mixed layer of constant depth has been implemented. The application of the mixed-layer requires however, that a heat-flux climatology is calculated from a previous integration with identical parameter settings, but prescribed SSTs. The formulation is similar to e.g. Kiel et al. (2002):

$$d_o c_o \frac{\partial T_o}{\partial t} = F_{net} + Q - d_o c_o \tau_o^{-1} (T_o - T_{climo}) \quad , \tag{75}$$

where F_{net} represents the net heatflux into the Ocean

$$F_{net} = F^{SR}_s + F^{LR}_s + SHF_{ss} + E_{ss} \quad , \tag{76}$$

Q represents the impact of deep water heat exchange and ocean transport on the SSTs and $d_o c_o \tau_o^{-1}(T_o - T_{climo})$ is an additonal damping term towards climatological SSTs. The mixed-layer depth d_o varies between 40 m (d_{omin} in the tropical and 60 m (d_{omax}) in extratropical regions

$$d_0 = d_{omax} + (d_{omin} - d_{omax}) \cos(lat)^3 \tag{77}$$

Eq. 75 can be re-written in anomaly form by introducing an equation for the climatological ocean temperature, assuming that the ocean deep water exchange and ocean transport infuence only the climatological SSTs:

$$d_o c_o \frac{\partial T_{climo}}{\partial t} = \overline{F}_{net} + Q \quad , \tag{78}$$

which enables to write Eq. 75 as

$$d_o c_o \frac{\partial \Delta T_o}{\partial t} = \Delta F_{net} - d_o c_o \tau_o^{-1} \Delta T_o \quad , \tag{79}$$

where $\Delta T_o = T_o - T_{climo}$ and $\Delta F_{net} = F_{net} - \overline{F}_{net}$. The climatological net heatflux $\overline{F}_{net}$ must be calculated by an integration with SPEEDY using prescribed SSTs. As Eqs. 59 and 60, Eq. 79 can be formulated, after discretization in time (implicit for ΔT_o), in the formulation that is implemented in SPEEDY

$$\Delta T_o(t+1) = \frac{\tau_o}{\tau_o + \delta t} \Delta T_o(t) + \frac{\delta t \ \Delta F(t)}{d_o \ c_o} \quad . \tag{80}$$

$\delta t = 86400s = 1$ day is the timestep. The ocean mixed-layer temperature is determined by adding the anomaly ΔT_o to T_{climo}.

The ocean mixed-layer may be run over selected domains that can be specified when running the model.

B.10 Table of constants

Symbol	Equation	Process/Definition	Value
		Convection	
RH_{cnv}	8	*Relative humidity threshold in PBL*	0.9
τ_{cnv}	12, 13	*Relaxation time for PBL humidity*	6 *hours*
RH'_{cnv}	21	*Relative humidity threshold in intermediate layers*	0.7
ϵ'	21	*ratio between secondary and primary mass flux at cloud − base Reduction*	0.8
		Large − scale condensation	
τ_{lsc}	22	*Relaxation time for humidity*	4 *hours*
RH^{1}_{lsc}	24	*Relative humidity threshold at* $\sigma = 1$	0.9
ΔRH_{lsc}	24	*Vertical range of relative humidity threshold*	0.1
		Clouds and shortwave radiation	
RH_{cl}	26	*Relative humidity limit corresp. to cloud cover* $= 0$	0.30
Q_{cl}	27	*Absolute humidity threshold for cloud cover*	0.2 g/kg
RH'_{cl}	28	*Relative humidity limit corresp. to cloud cover* $= 1$	1.
wp_{cl}	28	*cloud c. weight for the sq. root of precip.*	0.2
pm_{cl}	28	*max. prec. (mm/day) contributing to cloud cover*	10.
$CLSMAX$	29	*Maximum stratocumulus cloud cover*	0.60
CLF	29	*Rescaling factor in Stratocumulus calculation*	1.2
$GSES0$	30	*Static stability threshold for Stratocumulus clouds*	0.25
$GSES1$	30	*Upper static stability threshold for Stratocumulus clouds*	0.40
A_{cl}	33	*Cloud albedo (for cloud cover* $= 1$*)*	0.43
A_{cls}	34	*StratocumulusCloud albedo (for cloud cover* $= 1$*)*	0.50
		Longwave radiation	
α^{LR}_{win}	38	*Absorptivity coef. for dry air in window band*	0.3
α^{LR}_{cl}	38	*Absorptivity coef. for clouds in window band*	12.0
$\alpha^{LR}_{CO_2}$	40	*Absorptivity coef. in* CO_2 *band*	6.0
α^{LR}_{wv1}	41	*Absorptivity coef. in weak − abs. water vapour band*	$0.7\ (g/kg)^{-1}$
α^{LR}_{wv2}	42	*Absorptivity coef. in strong − abs. water vapour band*	$50.0\ (g/kg)^{-1}$
ϵ_s	46	*Surface longwave emissivity*	0.98
		Surface fluxes	
f_{wind}	49	*Ratio of near − surf. wind to lowest − level wind*	0.95
V_{gust}	50	*Wind speed of sub − grid scale gusts*	$5\ m/s$
D_{top}	51	*Depth of top soil layer*	$7\ cm$
D_{root}	51	*Depth of root layer*	$21\ cm$
W_{cap}	51	*Soil wetness at field capacity (volume fraction)*	0.30
W_{wil}	51	*Soil wetness at wilting point (volume fraction)*	0.17
C^{D}_{l}	52, 53	*Drag coefficient over land*	$2.4 \cdot 10^{-3}$ (1)
C^{D}_{s}	(*text*)	*Drag coefficient over sea*	$1.8 \cdot 10^{-3}$
C^{H}_{l}	54, 55	*Heat exchange coefficient over land*	$1.2 \cdot 10^{-3}$ (2)
C^{H}_{s}	56, 57	*Heat exchange coefficient over sea*	$0.9 \cdot 10^{-3}$ (2)
λ	58	*Coef. for ground heatflux*	$7\ W\ m^{-2} K^{-1}$

		Soil and sea ice temperatures	
d_s	59	*depth of interactive soil layer*	$1\ m$
c_s	59	*heat capacity of organic soil*	$2.5 \cdot 10^6\ Jk^{-1}m{-}1$
τ_s	59	*damping time scale for soil temperature anomalies*	$40 \cdot 86400s$
c_i	60	*heat capacity of ice*	$1.93 \cdot 10^6\ Jk^{-1}m{-}1$
τ_i	60	*damping time scale for seaice temperature anomalies*	$30 \cdot 86400s$
d_{imin}	62	*minimumdepth of seaice*	$1.5\ m$
d_{imax}	62	*maximumdepth of seaice*	$2.5\ m$
		Vertical diffusion	
τ_{shc}	67	*Relaxation time for shallow convection*	$6\ hours$
τ_{vdf}	70	*Relaxation time for vertical diffusion*	$24\ hours\ (humidity)$
			$6\ hours\ (dry\ st.\ en.)$
Γ_{RH}	71	*Reference gradient of relative humidity (w.r.t. σ)*	0.5
Γ_{SE}	73, 74	*Reference gradient of dry static energy (w.r.t. geopot.)*	0.1
		Slab Ocean tempeartures	
c_o	75	*heat capacity of slab*	$4.18 \cdot 10^6\ Jk^{-1}m{-}1$
τ_o	75	*damping time scale for ocean temperature anomalies*	$90 \cdot 86400s$
d_{omin}	77	*minimumdepth of slab*	$40\ m$
d_{omax}	77	*maximumdepth of slab*	$60\ m$

(1) Value for topographic height = 0 . (2) Value for neutral stability conditions.

B.11 References

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