Particle Dispersion in a Stratified Ekman Layer

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Abstract

The dispersion of massless and inertial particles in a stratified bottom Ekman layer is investigated. The study is carried out numerically, considering Lagrangian particles in a Large Eddy Simulation (LES) Eulerian field. The motion of the particles is studied in stable, neutral and unstable stratifications. The stratification affects the mean horizontal transport as well as the vertical diffusivity of the particulate. In stable conditions the swarm of particles remains segregated in the near bottom region and is strongly transported along the spanwise direction by the cross stream velocity. The opposite is true in convectively unstable conditions, where the high vertical diffusivity rapidly transports the particulate in the far field region where the cross-stream velocity is negligible.

1. Background and Problem Formulation

The problem of air pollution is nowadays a relevant issue. Consequently the comprehension of the dynamics of the motion of a dispersed phase in the atmospheric boundary layer (ABL) is of great importance. Despite the high degree of complexity and heterogeneity of the real-life ABL, from a very fundamental point of view, the ABL could be regarded as a bottom Ekman layer forced by free stream velocity (geostrophic wind) and subjected to earth's rotation around the vertical axis. During its cycle, the ABL is subjected to different conditions of thermal stratification (from convective to stably stratified) which dramatically influence the level of turbulent mixing in the air column. As regards the dispersed phase (*i.e.* the pollutant), a simplified model can neglect all the chemical and electrical interactions, and treat the pollutants usually present in the ABL only in a Lagrangian framework, *i.e.* as a cloud of particles. Although massive literature has been devoted to the analysis of the ABL (see Nieuwstadt (2005) for a review), in none of them the analysis of the dispersion of an ensemble of particles has been performed. On the other hand, in the literature, the study of dispersion of Lagrangian particles has been performed in very simple cases (see Yeung (2002) for a review), characterised by simple physics (isotropic and homogeneous, as well as simple wall turbulence). Particle dispersion in homogeneous stratified turbulence has been studied by Kimura and Herring (1996), whereas there are no studies on the effect of rotation in a $3D$ wall-bounded stratified turbulence.

In this paper, we study a problem which is archetypal of the dispersion of a pollutant in the low-atmosphere stratified boundary layer. The problem is sketched in Fig. 1. We consider a bottom Ekman layer subjected to different conditions of thermal stratification. In the different turbulent fields studied, we consider in the near wall region the release of different sets of Lagrangian particles, the difference being their density and size.

The study is carried out numerically, treating the dispersed phase as a swarm of Lagrangian particles moving in an Eulerian fluid. The flow field is governed by the Navier-Stokes equations subject to the Boussinesq approximation for the density field. The

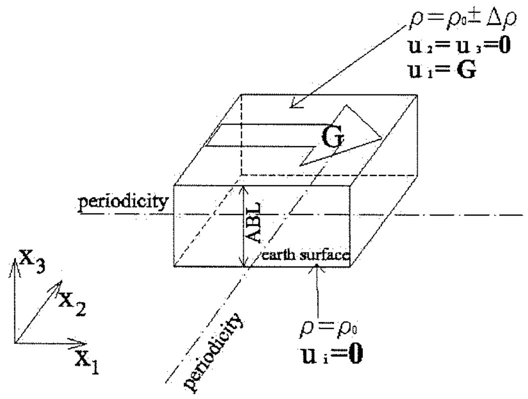

Figure 1: Schematic of the physical problem.

governing equations are solved using LES with a dynamic subgrid scale (SGS) model (the latter is described in Armenio and Sarkar (2002)). The position of the Lagrangian particles is advanced in time using the simplified form of the Maxey-Riley equation reported in Armenio and Fiorotto (2001). For now, the influence of gravity is neglected since the study is mostly focused on the understanding of the influence of inertia on the particle motion with respect to the turbulent structures present in the stratified field (the crossing trajectory problem). The interpolation of the Eulerian velocity field is performed using a $5th$-order Lagrangian polynomial in the horizontal planes of homogeneity and cubic splines in the inhomogeneous vertical direction. According to Armenio et al. (1999) the Lagrangian particles are moved using the filtered velocity field.

The LES equations are solved under the following boundary conditions (see Fig. 1). A constant velocity G is imposed on the top surface of the domain while on the bottom surface the velocity is set to zero; since the flow field is homogeneous in the horizontal directions, periodicity was set in the directions x_1 and x_2 for the velocity, pressure and density fields. We considered the cases of stable and unstable stratification by imposing a negative or a positive fixed density gap between the top and the bottom surfaces of the domain. Our simulations are representative of a 90°-latitude case, *i.e.* only the vertical component Ω_3 of the rotation vector is considered. In the problem under investigation, the velocity shear is aligned with the rotation vector and with the density gradient. The molecular Prandtl number is chosen to be 0.71, which corresponds to thermally stratified air. The Reynolds number of the simulation $Re = GD/\nu$ is set equal to 400 as in the simulations of Coleman et al. (1990), with $D = \sqrt{2\nu/f}$ the viscous Ekman depth, f the Coriolis parameter and ν the kinematic viscosity of the fluid.

Our grid has $N_1 = 32$, $N_2 = 128$ and $N_3 = 64$ cells, respectively, in the x_1, x_2 and x_3 directions. The domain is $L_1 = L_2 = 5.8\delta$ in the geostrophic wind direction and in the cross stream direction, respectively, whereas its height is $L_3 = 1.8\delta$ where $\delta = u^*/f$, $u^* = \sqrt{\tau_{wall}/\rho_0}$, ρ_0 is the reference density of the fluid and τ_{wall} is the wall shear stress. The grid spacing is uniform in the horizontal directions while the cells are clustered near the bottom surface along the vertical direction. The grid spacing made nondimensional with the wall unit ν/u^* is: $\Delta x_1^+ = 59$, $\Delta x_2^+ = 15$ and $\Delta x^+_{3\ min;max} = 0.40\ ;\ 20$.

The model was validated against the DNS results of the 90°-latitude case (A90) of Cole-

Table 1: Particle parameters.

	N_p	d_p/δ	ρ_P/ρ_0	$St = \tau_p/\tau_f$
tracers	15,000	3.8e-5	1	
P1	15,000	3.8e-5	1217	1.026
P2	15,000	3.8e-5	2435	2.053
P3	15,000	3.8e-5	4869	4.106

man et al. (1990). Our first (see Figs. 2a,b) and second order statistics compared very well with the reference data.

We consider three cases of stratification, namely the neutral case (CN), a stably stratified case (CS) and an unstable (convective) case (CU). The overall level of stratification is quantified by the Richardson number $Ri = -(g\,\Delta\rho\, D^2)/(\rho_0\, G^2\, L_3)$ where g is the gravitational acceleration, $\Delta\rho$ is the density gap between the bottom wall and the free-stream region. In the CS case we set $Ri = 0.001$, whereas $Ri = -0.00001$ was used in the CU case. The choice of a smaller value of Ri in the convective case was due to the limited height of the domain; however, even in this case, we will show that unstable stratification appreciably affects particle dispersion.

The parameters of the particles considered here are listed in Table 1: N_p is the number of released particles, d_p and ρ_p are respectively the diameter and the density of the particles, $\tau_p = \rho_p d_p^2/18\mu$ is the particle characteristic time, μ is the dynamic viscosity of the fluid and $\tau_f = \nu/u^{*2}$ is a turbulent time scale. The choice of the particle Stokes numbers St is such to span a wide range of values of ρ_p/ρ_0, which are representative of solid particles in air. The particles are released over six horizontal planes equispaced within 30 wall units. The initial velocity of the particles is set equal to that of the fluid field at the particle injection. Finally, their motion is followed for a nondimensional time $t\,f = 9$.

2. Results and Discussion

Figure 2 shows relevant first-order and second-order statistics for the cases of stratification investigated: the statistics are reported as a function of the domain height nondimensionalized by δ. As in Coleman et al. (1992) all quantities denoted with $< \dot{} >$ are averaged over the horizontal planes, and also over time, for a period of at least $tf = 2\pi$. For the values of Ri considered, the mean streamwise velocity of Fig. 2a is weakly affected by stratification, apart from the case CS where an increase of the mean gradient in the near wall region is observed. Similarly, the mean spanwise velocity profile of the CS case, Fig. 2b, exhibits larger gradients in a thinner region, thus indicating that the main effect of stable stratification is the reduction of the turbulent layer. The increase of the mean gradients of the velocity components in the CS case, as discussed by Armenio and Sarkar (2002) for a plane channel flow case, is due to the decrease of the turbulent shear stresses (Figs. 2c,d) and to the consequent increase of the molecular shear stresses. The opposite is true in the CU case, although due to the small value of Ri considered here, small differences are observable when comparing CU to the neutral case.

The mean density profiles as well as the buoyancy flux $< \rho' u_3' >$, and consequently the

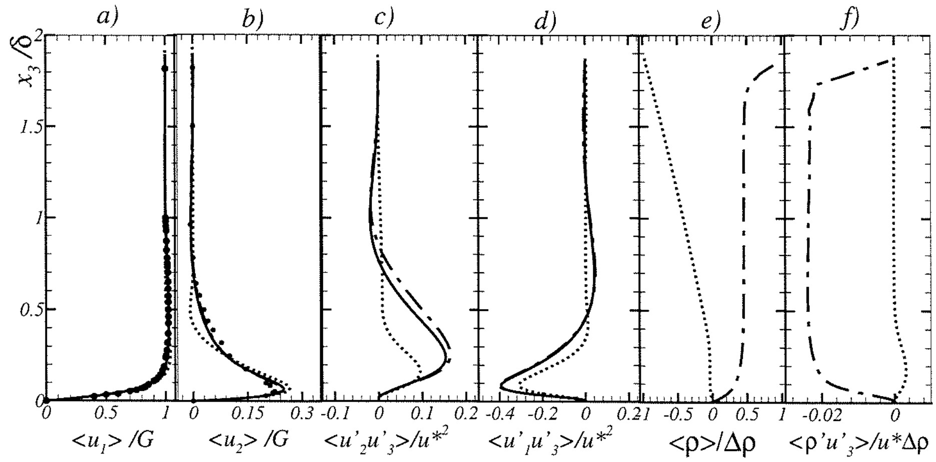

Figure 2: Vertical profiles of the following nondimensional quantities: a) Mean velocity in the geostrophic wind (streamwise) direction; b) Mean transversal velocity ; c) Transversal-vertical Reynolds shear stress ; d) Streamwise-vertical Reynolds shear stress; e) Mean density; f) Buoyancy flux. CS, dotted lines; CN solid lines; CU dash-dotted lines. The dots in a) and b) are the DNS case A90 data of Coleman et al. (1990).

vertical transport of mass, are dramatically affected by stratification. Figure 2e shows that, in CS, the mean density profile is well mixed in a very thin region above the solid wall; in the CU case, the mean density exhibits an almost flat profile up to 1.7δ, and a sharp gradient is observable at the top of the domain due to the imposition of a constant value of density at the top boundary. In CS, the buoyancy flux is very small and confined within a thin layer approximately equal to 0.4δ, whereas large convective buoyancy fluxes are observable along almost the whole fluid column in the CU case (Fig. 2f). Note that, even though the Ri is two orders of magnitude smaller in CU than in CS, the nondimensional buoyancy flux appears much larger in CU than in CS. The different behavior observed in the two cases has been attributed to the fact that (a detailed discussion is given in Stocca (2006)) the density field in CS exhibits a wave-like behavior above the mixed region, whereas, in CU, the presence of convective cells is observed, with vertical extension of the order of the domain height.

Figure 3 shows the vertical mean displacement ($D_3(t) = 1/N_p \sum_{j=1}^{N_p}[x_{3,j}(t) - x_{3,j}(t_0)]$, with $x_{3,j}(t)$ indicating the vertical position of the j-particle at the time t and t_0 the initial time of the simulation) and the vertical particle diffusivity ($kp_3(t) = 1/2[dxp_3^2(t)/dt]$, where $xp_3^2(t) = 1/N_p \sum_{j=1}^{N_P}[x_{3,j}(t) - x_{3,j}(t_0)]^2$ is the vertical particle dispersion). Only the statistics relative to fluid tracers and to particles P3 are shown since, in the range of St numbers considered, the behavior of particles with an increasing inertia has shown to vary monotonically.

The effect of inertia is to reduce the vertical displacement of the cloud (Figs. 3a,b,c) since, as is well known, inertia acts as a low-pass filter and makes the particulate matter less sensitive to a wide range of turbulent scales. Stratification greatly affects the vertical displacement of the particulate. As regards the tracers, in CS, the vertical motion is suppressed by about 40% with respect to the neutral case, whereas an increase of D_3 of the

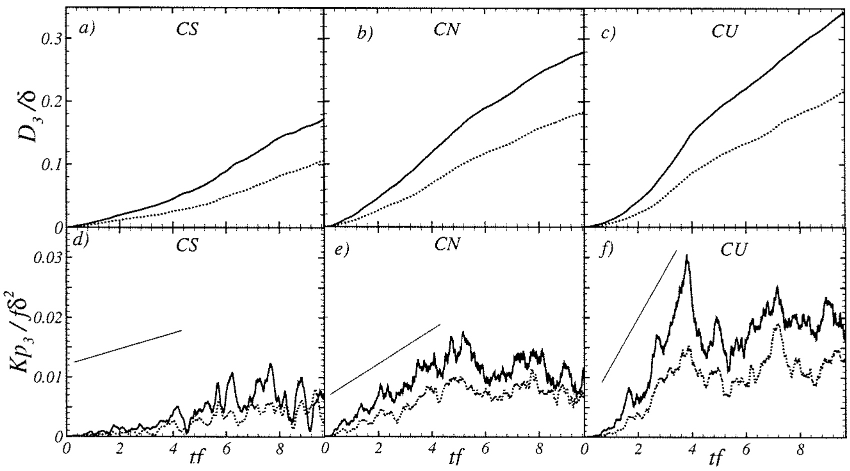

Figure 3: Lagrangian statistics for tracers (solid lines) and P3 particles (dotted lines). Vertical displacement of the particle cloud in the a) CS case; b) CN case; c) CU case. Vertical particle diffusivity in the d) CS case; e) CN case; f) CU case. The straight lines in d) ,e) ,f) indicate the mean slopes of the diffusivity in the ballistic regime.

order of 10% is observed in the convective case. These differences hold for inertial particles, although in this case the vertical displacement appears reduced. The time evolution of the vertical particle diffusivity is shown in Figs. 3d,e,f for the cases and the kinds of particles discussed above. In all cases we observe the presence of two regimes, namely the ballistic one where dispersion increases with t^2 ($kp_3 \sim t$) and the Brownian one where dispersion increases with t and $kp_3 \sim const.$ This behavior is related to the shape of the Lagrangian auto-correlation function (see Taylor (1921) for details). As is well-known, stable stratification destroys large scales and anisotropic pancake-like scales populate the flow field, whereas convective conditions increase the presence of large turbulent scales in the flow field. This effect has its own counterpart on the ballistic regime of dispersion: specifically, stable stratification reduces the autocorrelation function and thus the slope of the diffusivity kp_3 with respect to the neutral case, whereas the opposite is true for the convective case. As a result, vertical particle diffusivity appears greatly enhanced going from CS to CU. Figures 3d,e,f also show that inertia acts toward the reduction of vertical particle diffusivity in all cases of stratification. This has to be attributed to the fact that inertia makes particles less sensitive to a wide range of turbulent scales (crossing-trajectory effect).

Finally, in Fig. 4, we show a top view of the tracers at the final time of simulation for the three cases of stratification. We observe that the initial deviation of the swarm, with respect to the geostrophic wind direction increases from CU to CS, accordingly to whether the velocity vector deviates more in the stable case than in the unstable one. Moreover Fig. 4a shows that, in CS, a larger number of particles deviates from the streamwise direction and is transported along the spanwise one. This has to be attributed to the fact that stable stratification reduces the vertical displacement of the particles and consequently confines them in the near wall layer where large values of the spanwise velocity component are present. Conversely, in CU, (Fig. 4c) the particulate is rapidly transported in the

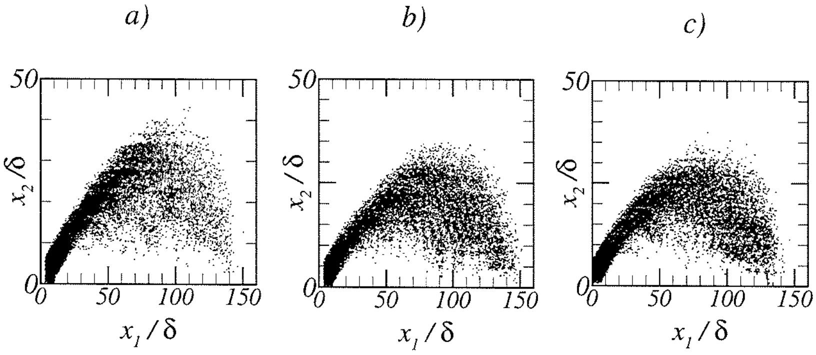

Figure 4: Top view of the position of tracers at $tf = 9$: a) CS case; b) CN case; c) CU case.

far field where the spanwise velocity component is very small and thus the transport is mainly in the geostrophic wind direction. Similar behavior (not shown) is observed for inertial particles. In this case, since inertia reduces the vertical transport, the deviation effect discussed above is more pronounced in the three cases of stratification.

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