Cryogenic Buoyancy-Driven Turbulence

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Abstract. Fluid turbulence is of considerable importance both fundamentally, as a paradigm for all nonlinear systems with many degrees of freedom, and in applications. In recent years there has been considerable effort to take advantage of some unique properties of low temperature liquid and gaseous helium. In particular, studies of turbulent thermal convection in conventional fluids have been aided by the use of low temperature helium which principally allows the limit of large Reynolds and Rayleigh numbers to be attained under controlled conditions. We discuss some directions and recent progress in these studies.

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INTRODUCTION

Fluid turbulence is an important consideration in the context of industrial applications such as the dispersal of pollutants, mass and heat transfer, and flows around ships and aircraft. The problem is also a paradigm for strongly nonlinear systems, distinguished by the interaction of a large number of degrees of freedom. In either context there remain many open questions, which are of consequence to a number of closely related problems such as interstellar energy transport [1], weather prediction and planetary magnetic fields [2], and, more indirectly, perhaps even market fluctuations [3]. Because of the inherent complexity of these problems, progress depends on a substantial input from experiment. This has led to a search for optimal test fluids for laboratory work in fluid turbulence, which has in turn pointed to the use of low temperature helium. The adaptation of low temperature technology to the study of classical turbulence is not, however, without difficulty, but the benefits we believe outweigh the limitations, which are mostly related to the adaptation of measurement technology.

FLUID EQUATIONS

We briefly introduce here the fluid equations written for a coordinate system fixed in space, and assuming the applicability of continuum mechanics. Let us consider a volume of fluid of density ρ and subject to a velocity $\mathbf{u}$. Conservation of mass takes the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

which, in the case of constant density, reduces to

$$\nabla \cdot \mathbf{u} = 0. \quad (2)$$

For a Newtonian fluid (having a linear relation between stress and strain tensors), the momentum equation reduces to

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}_{\text{ext}}, \quad (3)$$

in terms of the pressure p, kinematic viscosity ν which is the ratio of the shear viscosity μ divided by the density ρ. For convenience of notation we have used the convective derivative $\frac{D}{Dt} = \frac{\partial}{\partial t} + \nabla \cdot \mathbf{u}$. An unspecified external body force term is represented by $\mathbf{F}_{\text{ext}}$.

For thermally driven flows, we have the buoyancy force of magnitude $F_{ext} = g\alpha\Delta T$ in the direction of the gravitational acceleration g, where α is the coefficient of thermal expansion and ΔT is the temperature difference across a layer of fluid in the direction of gravity. With the exception of variations in density that produce the buoyancy force, a simplifying Boussinesq approximation assumes that all other fluid property variations occurring as a result of the imposed thermal gradient can be neglected. The equation for energy conservation in the Boussinesq approximation is

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T - \mathbf{u} \cdot \nabla T. \quad (4)$$

Here κ is the thermal diffusivity of the fluid at temperature T. Additionally to these partial differential equations we must also supply the appropriate initial and boundary conditions; e.g., rigid or "stress-free" surfaces, conducting or insulating boundaries, rough or smooth surfaces, etc.

To motivate the principal dynamical control parameter for turbulent flows– the Reynolds number Re– we briefly consider isothermal flows driven solely by pressure gradients. Rescaling all velocities by some characteristic velocity $\mathbf{U}$, all lengths by some characteristic length L, times by L/U and normalizing the pressure by

a factor ρU^2 the momentum equation (3) without the external force term can be re-written as

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u}, \qquad (5)$$

where the Reynolds number is identified with $Re = UL/\nu$. It may be thought that when Re is very large (i.e. for turbulent flow) the viscous term in the momentum equation (the term $\nu\nabla^2\mathbf{u}$ in equation 3) becomes unimportant compared to the inertial term (the term $\mathbf{u} \cdot \nabla\mathbf{u}$). The Reynolds number can be defined of course on any scale, not just the largest. If we consider that at some scale the locally defined Re becomes of order unity, allowing for the domination of viscosity, we can understand conceptually the existence of a smallest scale in a turbulent flow.

It is evident from equation 5 that we do not have to take into account particular dimensioned values of the viscosities, lengths, velocities, etc. This is what allows us to connect in a general way diverse phenomena ranging from the flow of hydrogen and helium gas at astrophysical scales to, say, the the flow of blood in tiny capillaries. In the case where the geometrical factors are the same, Re becomes a dynamical similarity parameter and its equivalence between flows with otherwise different length and velocity scales, or different viscosities, is the principle behind wind-tunnel model testing.

In those flows where buoyancy derived from thermal gradients plays a role–which evidently occurs in natural gradients more often than not, and in which we are mostly interested here–another control parameter, namely the Rayleigh number

$$Ra = g\alpha\Delta T L^3/\nu\kappa, \qquad (6)$$

emerges in the fluid equations. Physically, the Rayleigh number measures the ratio of the rate of potential energy release due to buoyancy with the rate of its dissipation due to thermal and viscous diffusion. Additionally, a parameter composed only of fluid properties, the Prandtl number,

$$Pr = \nu/\kappa, \qquad (7)$$

appears. Physically, Pr is a measure of the ratio of time scales due to thermal diffusion ($\tau_\theta = L^2/\kappa$) and momentum diffusion ($\tau_\nu = L^2/\nu$).

WHY HELIUM?

Turbulent flows occur in the limit of large Re and Ra, although there does not always exist any sharp boundary beyond which a flow becomes turbulent as one of these parameters is increased. Considering that Re or Ra in fluid turbulence are measured on a logarithmic scale,

the attainment of the highest possible control parameter values is a useful target for experimental research. If suitably high Re or Ra were attainable using common fluids such as air or water, there would be little motivation for pursuing the difficult task of pushing low-temperature technology. However, this does not appear to be the case. The principal advantage of helium, as advertised numerous times (see for example ref. [4]) is its very small kinematic viscosity, even in the gas phase, leading to large Re without the need for large apparatus dimensions or of large velocities. Of course, it can be made to have a rather large viscosity as well in the gas phase (for very low densities). Similarly the ratio of fluid properties appearing in the Rayleigh number (see equation 6) can be orders of magnitude larger in cryogenic helium than for conventional fluids, and, conversely, can also be made rather small (also in the gas phase for very low densities). We have emphasized the capacity of low temperature helium gas in particular to produce a wide variation in the fluid properties (hence also Re, Ra) because we are often seeking to accurately measure scaling relations of some property of the turbulent flow, assumed invariant in some asymptotic limit of high Re or Ra. For this we need some decades of the control parameter in the turbulent regime, which are better obtained in a single apparatus having consistent boundary conditions, protocol, etc.

RAYLEIGH-BÈNARD CONVECTION

Turning our attention now to buoyancy-driven flows, we note at the outset that it is not realistic to suppose that we can solve all real problems, which may have complicated boundary conditions, multiple phases and chemical species, magnetic fields, and so on. Consequently, most work focuses on a simpler problem for which reasonable progress can be made, but one that still contains the essential physics of the real problem in which we are ultimately interested. In the case of thermally-driven turbulence, this is the so-called Rayleigh-Bènard convection (RBC). In standard RBC, a thin, laterally infinite fluid layer is contained between two surfaces (either rigid or "stress-free") held at constant temperature. Usually the expansion coefficient is positive and so, when the fluid is heated from below (temperature decreasing from bottom to top), a mechanically unstable density gradient forms. The applied stress is measured in terms of the Rayleigh number, Ra, defined above. With increasing Ra the dynamical state goes from a uniform and parallel roll pattern at the onset of convection ($Ra \sim 10^3$) to turbulent flow at $Ra \sim 10^7 - 10^8$. For turbulent flows, the Prandtl number Pr determines the nature and relative sizes of the viscous and thermal boundary layers that are established on the solid surfaces.

Threlfall [6] is usually credited as being the first to recognize the advantages of using low temperature helium gas to investigate the thermal turbulence problem in the laboratory. Later low temperature work by Libchaber and co-workers [7] considerably broadened the awareness of the problem.

EXPERIMENTAL FEATURES

We briefly note some salient features of the low temperature experiments, which do not vary in substantial respects from one experiment to the next (the reader is invited to look in the various original publications [7, 8, 9, 10] for more detailed information). To approximate the constant temperature top and bottom boundaries, annealed OFHC copper is used (thermal conductivity near 1 kW/m-K at helium temperatures). Suitable heaters are attached to both plates which vary in design from experiment to experiment. On the bottom plate a constant heat current is applied while the heater at the top plate (in contact with a helium bath) is used to regulate its temperature. Typically, small (nominally 200-300 micrometer on a side) semiconductor crystals, either doped germanium or silicon, are placed in the flow to measure temperature fluctuations within the gas which is confined laterally by thin wall stainless steel walls. The sample cell is surrounded by thermal shields at various graded temperatures.

In laboratory experiments there is an additional length scale imposed by the need to laterally contain the fluid, the width-to-height aspect ratio $\Gamma = D/H$, where here D is the diameter for the more typical case of cylindrical containers. As the dynamics of thermal convection depend explicitly on H, they are significantly affected only in the limit in which the ratio of width to height is small. Having both Γ and Ra large, however, is difficult (note that H appears in the numerator of one and the denominator of the other). For absolute heights large enough to obtain high Ra, making D much larger would involve a technical and economic challenge. An exception is with the use of cryogenic helium gas, where a diameter of reasonable dimension can be coupled to a much smaller height without sacrificing too much of the upper limit of attainable Ra (recalling its H^3 dependence). Such an experiment has recently been performed [11] in aspect ratio 4. Note that while this does not represent any record for large Γ, it does represent the first attempt to obtain very high Ra in a moderately large Γ experiment. We will briefly discuss some of the preliminary observations in the context of turbulent heat transfer.

HEAT TRANSFER AND SCALING

The benefits of low temperature experiments to obtain scaling relations has been clearly evident in the context of dimensionless heat transfer, represented by the Nusselt number Nu defined as

$$Nu = \frac{q}{q_{cond}} = \frac{qH}{\lambda\Delta T}, \qquad (8)$$

where q is the total heat flux, q_{cond} is the value the heat flux would have in the absence of convection, and λ is the thermal conductivity of the fluid. Nu represents the ratio of the effective turbulent thermal conductivity of the fluid to its molecular value and can reach values of over 10^4 in helium experiments [8] thus demonstrating the enormous enhancement of thermalization (one of the motivations for Threlfall's pioneering work in cryogenic turbulent convection–see ref. [6]).

THE CLASSICAL RESULTS

To partially motivate the push to larger Γ it is useful to briefly consider the expected scaling between Nu and Ra. The "classical" prediction (see, e.g., refs. [12, 13]) is a power law relation $Nu \sim Ra^\beta$ with $\beta = 1/3$. This value for the exponent can be easily motivated by considering that the thermal gradients occur only in very thin diffusive boundary layers near each horizontal heated surface, and that the intervening fluid, being fully turbulent, acts more or less like a thermal short circuit (the reader might be convinced of this scaling by taking the general power law relation above and assuming the physical heat flux Q appearing in Nu to have no implicit height dependence). In the limit of infinite Pr at least, where the problem becomes more tractable, the exponent $\beta = 1/3$ has recently been rigorously established [14]. The other so-called "classical" relation is a power law (albeit with logarithmic corrections) with $\beta = 1/2$, mostly due to the work of Kraichnan [15]. A modern theory [16] presents a $Ra - Pr$ phase portrait with different power law relations in neighboring regions combining to influence Nu, including separate relations with the two values of β given above.

SCALING RESULTS FROM EXPERIMENTS

Experiments have in fact generally measured exponents less than 1/3; for instance, Libchaber and co-workers championed the exponent 2/7 and subsequent theory which seemed to pin it down [7], although this is no longer considered the correct asymptotic limit [16]. Others have observed larger scaling exponents closer to 1/2

at high Ra [9], albeit for operating points quite close to the critical point of the fluid. The highest Ra obtained to date was in an experiment of Niemela *et al.* [8], in which, to lowest order, an exponent of 0.31 was observed over 10 decades of Ra up to $Ra \sim 10^{17}$. The main experimental results of each of these groups were obtained in $\Gamma = 1/2$ containers, which more or less represents an historically accepted minimum. However, Γ may play a significant role, at least for small values of it, through the action of a robust and organized "wind" which sweeps through the entire container [17, 18, 19]. Low temperature experiments have subsequently been performed [10] in a container of $\Gamma = 1$ for Rayleigh numbers up to $Ra \sim 10^{15}$. One of the interesting results at $\Gamma = 1$ was that at high enough Ra for the mean wind to have become significantly disordered, but low enough for the Boussinesq approximation to remain valid, roughly a decade of power-law scaling with $\beta = 1/3$ emerged.

The furthest step in the direction of fully turbulent RBC in a laterally extended system has only been done recently [11], in which a $\Gamma = 4$ container was used to obtain Ra up to 10^{13} (note that the highest Ra is sacrificed with increase in aspect ratio as the latter is practically effected by lowering the height while maintaining the diameter fixed). In this $\Gamma = 4$ experiment a mean wind also existed but was considerably less robust than in the smaller aspect ratio containers. A scaling exponent of approximately 0.31 was observed at low Ra (albeit in the turbulent regime) as in the case of Niemela *et al.* [8], but was followed by a transition to almost 2 decades of scaling consistent with $\beta = 1/3$ over the ultimate range of Ra for which the Boussinesq approximation could be assumed valid. Corresponding to this transition in scaling, the long-time correlation between temperature probes separated by a diameter vanished. Thus it appears as though the $\Gamma = 4$ experiments may approach the kind of conditions postulated for observing the $\beta = 1/3$ regime; namely, a more disordered bulk flow that separates the action of the two thermal boundary layers. Presumably, the heat transfer is influenced less by the mean wind, which otherwise sweeps more robustly through the entire cell in small-Γ containers, affecting Nu by its coupling to gradients in the imperfectly insulating sidewalls [20, 21, 10, 22]. It should also be mentioned that the exponent $\beta = 1/3$ is consistent with the corresponding region of the $Ra - Pr$ phase space according to ref. [16].

DISCUSSION

The $\Gamma = 4$ experiment in turbulent convection discussed above represents a different direction from that pursued in the past for helium experiments; namely the simultaneous attainment of both large Ra and large Γ, thus ex-

amining more accurately the problem of turbulent RBC. It should be possible to expand even further in this direction.

It is worth pointing out, however, that recent advances in numerical modelling have enabled the effects of finite Γ to be properly accounted for at high Ra. We note that there continues to be steady and substantial progress in this direction (R. Verzicco, personal communication).

REFERENCES

1. Spiegel, E.M. *Ann. Rev. Astron. Astrophys.* **9**, 323-352 (1971).
2. Glatzmaier, G.A., Coe, R.C., Hongre, L. & Roberts, P.H. *Nature* **401**, 885-890 (1999).
3. Mandelbrot, B.B. A Multifractal Walk Down Wall Street *Scientific American* February, 77-73 (1999).
4. Niemela, J.J. "Thermal turbulence in cryogenic helium gas" *Physica B* **329-333**, 429-430 (2003).
5. Frisch, U. *Turbulence, the Legacy of A. N. Kolmogorov*(Cambridge University Press, Cambridge, England) (1995).
6. Threlfall, D.C. *J. Fluid. Mech.* **67** 17-28 (1975).
7. Castaing, B., Gunaratne, G., Heslot, F., Kadanoff, L., Libchaber, A., Thomae, S., Wu, X.Z., Zaleski, A. & Zanetti, G. *J. Fluid Mech.* **204**, 1-30 (1989).
8. Niemela, J.J., Skrbek, L., Sreenivasan, K.R. & Donnelly, R.J. *Nature* **404**, 837-840 (2000).
9. Chavanne, X., Chilla, F., Chabaud, B., Castaing, B. & Hebral, B. *Phys. Fluids* **13**, 1300-1320 (2001).
10. Niemela J.J. & Sreenivasan K.R. *J.Fluid Mech.* **481** 355 (2003) .
11. Niemela, J.J. & Sreenivasan K.R. *to appear in J. Fluid Mech.* (2005).
12. Malkus, W.V.R. *Proc. R. Soc. London* **A 225**, 196-212 (1954).
13. Howard, L.N. *Proc. Eleventh International Congress on Applied Mechanics* (Ed. H. Görtler) Berlin: Springer, pp. 1109-1115 (1966).
14. Constantin, P. & Doering, C. R., *J. Stat. Phys.* **94**, 159Ǔ172 (1999).
15. Kraichnan, R.H. *Phys. Fluids* **5**, 1374-1389 (1962).
16. Grossmann, S. & Lohse, D. *Phys. Rev. E.* **66**, 016305 (2002).
17. Niemela, J.J., Skrbek, L., Sreenivasan, K.R. & Donnelly, R.J. *J. Fluid Mech.* **449**, 169-178 (2001).
18. K. R. Sreenivasan, A. Bershadskii and J. J. Niemela, Phys. Rev. E **65**, 056306 (2002).
19. Verzicco R. & Camussi, R. *J. Fluid Mech.* **477**, 19 (2003).
20. Ahlers, G. *Phys. Rev. E.* **63**, art. no. 015303 (2001).
21. Roche, P., Castaing, B., Chabaud, B., Hebral, B. & Sommeria, J. *Euro. Phys. J.* **24**, 405-408 (2001).
22. Verzicco, R. *J. Fluid Mech.* **473**, 201 (2002).