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Logarithmic scaling in the near-dissipation range of turbulence

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Abstract. A logarithmic scaling for structure functions, in the form $S_p \sim [\ln(r/\eta)]^{\zeta_p}$, where η is the Kolmogorov dissipation scale and ζ_p are the scaling exponents, is suggested for the statistical description of the near-dissipation range for which classical power-law scaling does not apply. From experimental data at moderate Reynolds numbers, it is shown that the logarithmic scaling, deduced from general considerations for the near-dissipation range, covers almost the entire range of scales (about two decades) of structure functions, for both velocity and passive scalar fields. This new scaling requires two empirical constants, just as the classical scaling does, and can be considered the basis for extended self-similarity.

Keywords. Logarithmic scaling; near-dissipation range; turbulence; extended self-similarity.

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The Kolmogorov approach to the phenomenological description of the inertial range of scales in turbulence requires that the scales $r \gg \eta$, where η is the Kolmogorov dissipation scale [1]. The dominant physical mechanism operating in the inertial range can be thought to be the Kolmogorov–Richardson cascade, and its application readily yields the well-known power-law scaling for structure functions [2,3]. On the other hand, the near-dissipation range of scales, for which $r > \eta$ but there exists no separation between the inertial and dissipation ranges, has a more complex dynamics arising from a strong competition between the cascade and dissipation mechanisms (see, for instance, refs [4–6] and the references cited there). Indeed, for small and moderate Reynolds numbers, the near-dissipation range can span most of the available range of scales. Analogous situation occurs also for the turbulent mixing of passive scalars.

One of the few properties known about the near-dissipation region is that it obeys the so-called extended self-similarity (ESS) [7], while the classical power-law scaling does not exist. In ESS the scaling relation between structure functions of different orders is given by

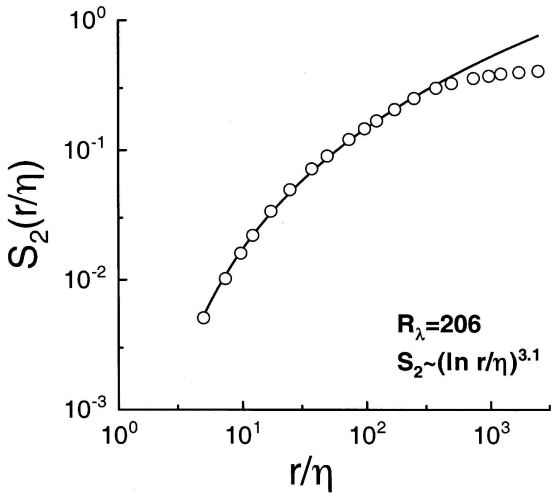

Figure 1. The second-order structure function $S_2(r/\eta)$ against r/η. The experimental data ($R_\lambda = 206$) are shown as circles. The solid curve is the best fit of (3) to the data, corresponding to the logarithmic scaling.

$$S_p(r) \sim S_q^{\beta_{p,q}}(r), \tag{1}$$

where $\beta_{p,q}$ is the exponent of the pth order structure function relative to that of the qth order. For the velocity field in the near-dissipation range, ESS implies that

$$S_p(r) = f(r/\eta)^{\zeta_p}, \tag{2}$$

where $f(x)$ is an unknown function different from a power law. Finding this function is crucial for the near-dissipation range. We show here, using experimental and numerical data, that the logarithmic function

$$S_p \sim [\ln(r/\eta)]^{\zeta_p} \tag{3}$$

can successfully replace the power law $S_p \sim r^{\zeta_p}$ in the near-dissipation range.

Figures 1 and 2 show the longitudinal second-order structure function of the velocity field calculated using the data obtained in a wind tunnel at $R_\lambda = 206$ and 487, respectively. Here R_λ is the so-called Taylor microscale Reynolds number which varies as the square root of the large-scale Reynolds number. The flow was a combination of the wake and homogeneous turbulence behind a grid and is more fully described in ref. [8]. Following convention, we invoke Taylor's hypothesis [1] to equate temporal statistics to spatial statistics. For convenience, we normalize the scale r by the Kolmogorov scale η. The solid curves in these figures are the best fit by eq. (3) for $\zeta_2 \simeq 3.1$ and 3.3, respectively. We consider in figure 3 the quantity $S_2(r)$ calculated using data from a high-resolution direct numerical simulation of homogeneous steady three-dimensional turbulence [9], corresponding to 1024^3 grid points and $R_\lambda = 460$. The solid curve in this figure also corresponds to the best fit by eq. (3), with $\zeta_2 \simeq 3.2$. The logarithmic scaling applies well to the near-dissipation region and, although we do not particularly expect it, to a considerable part of the inertial range as well, overlapping with the 2/3rds form of Kolmogorov.

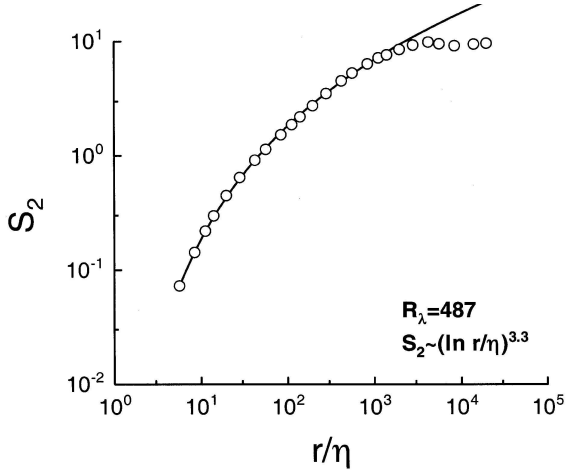

Figure 2. Same as in figure 1, but for $R_\lambda = 487$.

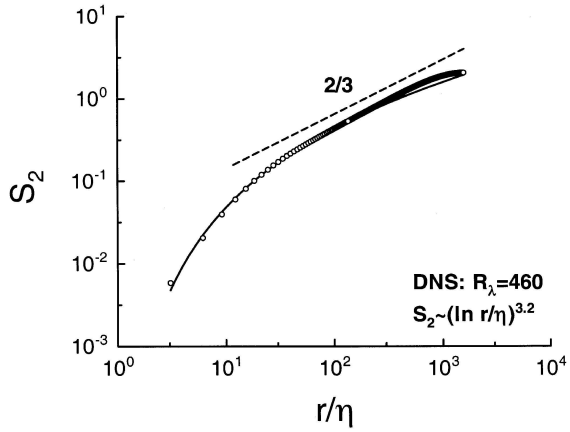

Figure 3. The second-order structure function $S_2(r/\eta)$ against r/η. The DNS data [9] ($R_\lambda = 460$) are shown as circles. The solid curve is the best fit of (3) corresponding to the logarithmic scaling. The dashed straight line is the Kolmogorov's 2/3rds scaling form, nominally thought to be applicable in the inertial range (ignoring effects of small-scale intermittency).

Figure 4 shows the structure functions of different orders for wind-tunnel data ($R_\lambda = 487$) and the logarithmic scaling (3) is shown in the figure as solid curves. The exponents ζ_p normalized by the exponent ζ_3 are shown in figure 5 as circles. For comparison we show by crosses in this figure the relative exponents $\zeta_{p,3}$ obtained using ESS in the atmospheric turbulence for large $R_\lambda = 10{,}340$ [10]. As described at some length in [10], the ESS exponents are very close to those determined directly.

The fact that the normalized logarithmic exponents are also the same as the power-law exponents clearly shows why the ESS works well at low Reynolds numbers. We understand that this observation merely shifts emphasis on explaining

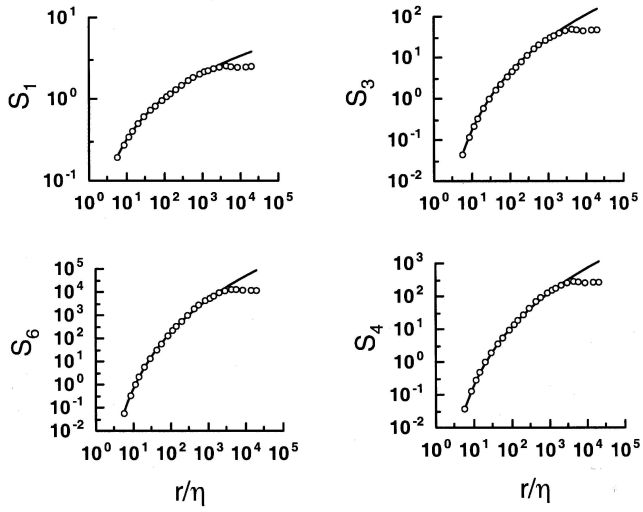

Figure 4. Structure functions of different orders ($p = 1, 3, 4$ and 6) for $R_\lambda = 487$ (cf. fiigure 2). The solid curves are drawn to indicate the logarithmic scaling (3).

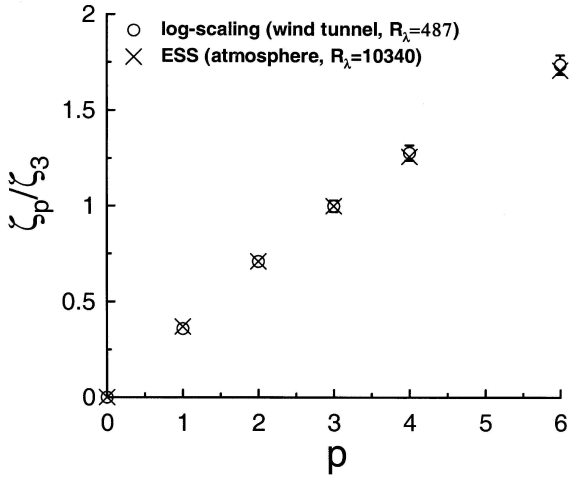

Figure 5. Normalized exponents ζ_p/ζ_3 against p for $R_\lambda = 487$ (circles). Crosses are the ESS exponents obtained for the atmospheric turbulence data at $R_\lambda = 10{,}340$ [10].

why these two sets of exponents should be identical, but leave this question for the future.

For the passive scalar we use the data acquired with a cold-wire probe in the wake of a heated cylinder in a wind tunnel (see ref. [11] for details of the experiment). Temperature can be considered a passive scalar for the conditions of the experiment. We used measurements on the centerline of the wake. Figures 6 and 7 show the

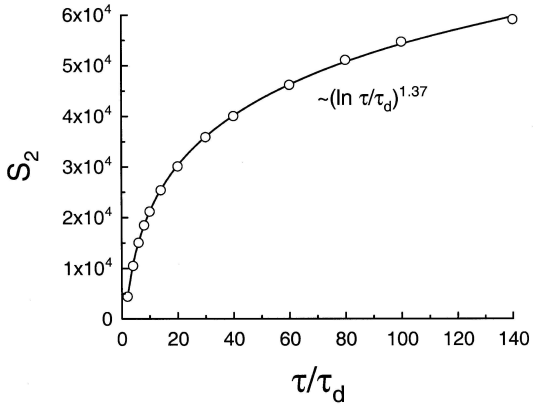

Figure 6. Second-order structure function of the temperature fluctuations for the heated wake.

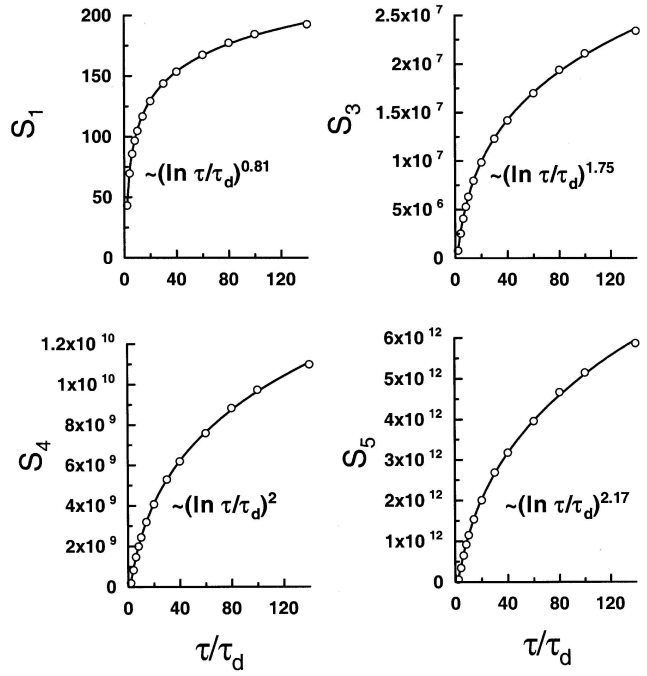

Figure 7. As in figure 3, but for different orders of the structure function.

temperature structure functions of different orders and the solid curves are the best fits drawn to indicate agreement with eq. (3) (i.e., logarithmic scaling). Values of ζ_p/ζ_2 calculated from figures 6 and 7 are shown in figure 8 as circles. We also show by triangles in this figure the ESS exponents obtained for passive scalar in fully developed atmospheric turbulence using *ordinary* scaling in the *inertial* range of scales [12].

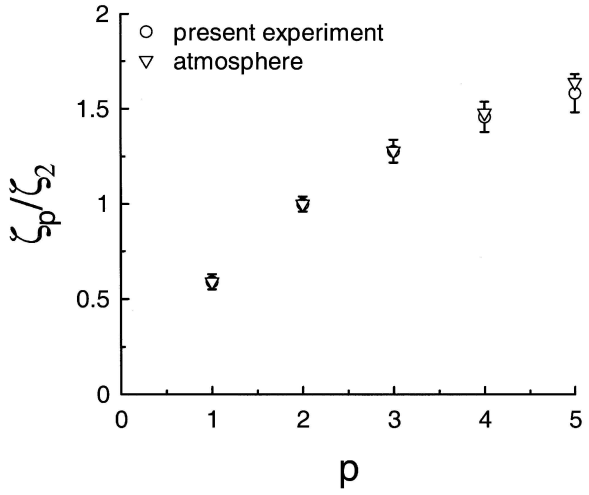

Figure 8. Exponents for the passive scalar extracted from figures 6 and 7 (circles) and for the fully developed atmospheric turbulence (ordinary scaling in the inertial range, triangles [12]).

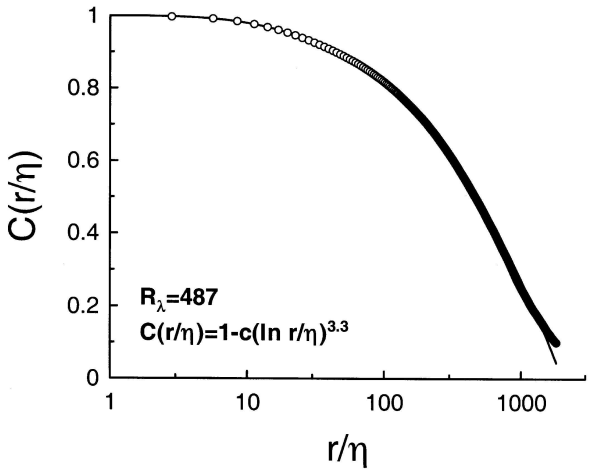

Figure 9. Correlation function for velocity fluctuations against r/η for $R_\lambda = 487$ (circles). The solid curve is drawn in the figure to indicate agreement with the logarithmic scaling shown in figure 2.

It should be pointed out that the logarithmic scaling requires two fitting constants just as the ordinary scaling does, and, in the examples discussed above, the range of scales covered by this scaling is about two decades. Application of these results to correlation functions and spectra is in good agreement with the structure functions analysis (see, for instance, figure 9) as well as with other available data sets.

Thus, we have clear experimental indication that the logarithmic scaling is an appropriate tool for the description of data in the near-dissipation range. This

scaling applies to nearly the entire range of scales in flows of low and modest Reynolds numbers. At high Reynolds numbers, it overlaps with conventional power laws. The normalized logarithmic exponents are the same as the classical ones, thus demonstrating why ESS works so well at low and moderate Reynolds numbers. These comments apply to both velocity and scalar fields. The theory concerning these observations will be discussed elsewhere [13].

Acknowledgement

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