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Multiscale SOC in turbulent convection

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Abstract

Using data obtained in a laboratory thermal convection experiment at high Rayleigh numbers, it is shown that the multiscaling properties of the observed mean wind reversals are quantitatively consistent with analogous multiscaling properties of the Bak–Tang–Wiesenfeld prototype model of self-organized criticality in two dimensions.

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Keywords: Convection; Multiscaling; SOC; Turbulence

1. Introduction

The existence of well-organized large-scale motion in turbulent thermal convection is an intriguing recent discovery. It has been under active investigation for some time, both theoretically and experimentally (see [1–15] and references therein). At least in a convection apparatus whose aspect ratio (=the ratio of the diameter to the height) is unity, the large-scale motion is in the form of a persistent circulation (on the average), its physical extent being of the order of the size of the apparatus itself. This is often called the "mean wind". The mean wind evolves with the Rayleigh number (which can be regarded as an external tunable parameter in the flow), and seems to asymptote to a well-defined shape only beyond Rayleigh numbers of the order of 10^{10}. However, beginning more or less at this Rayleigh number, at least for three or so further decades,

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the mean wind undergoes abrupt and apparently irregular reversals of direction. These reversals are the object of the study here.

The measurements to be analyzed were obtained at a Rayleigh number of 1.5×10^{11} in thermal convection occurring in a closed container of circular cross-section of 50 cm diameter and aspect ratio unity. The working fluid was cryogenic helium gas. The same data have been analyzed in the past [12], where the fluid dynamical origin of reversals was proposed to be the imbalance between buoyancy and friction (with inertia playing a secondary role). It was further suggested that a stochastic "change of stability" occurs between two metastable states corresponding to the opposite direction of the wind. The main physical quantity analyzed in Ref. [12] was the interval τ (the life-time of the metastable states, or the interval between reversals). The time of actual switching between the two states was short on the scale of some average measure of τ, so the wind reversals could be regarded as abrupt. The last property (perhaps also the precise details of reversals themselves) is most likely dependent on the specific boundary conditions of the experiment, but it is thought that the statistical properties of the duration times τ are insensitive to such details. The analysis of Ref. [12] indicated a general dynamic mechanism similar to self-organized criticality (SOC) [16–23]. The particular emphasis of this paper is the elaboration of this analogy.

Self-organized criticality occurs through a nonlinear feedback mechanism. There could be many possible scenarios of SOC in our system where numerous plumes and jets are developed as a result of boundary layer and thermal instabilities, all of which are embedded in a background of strong turbulent fluctuations prevalent in the bulk of the apparatus. It was suggested recently for plasma turbulence (see Refs. [24,25]) that instabilities governed by a threshold may lead to self-organized criticality by producing transport events at all scales (avalanches). These avalanches are due to local accumulation of energy, leading to an increasing gradient. Once the gradient exceeds an appropriate threshold, a burst of activity, which expels the accumulated energy, ensues. This process can be renewed, much like a domino effect, leading to a large transport event. Specific conditions of the thermal convection in the container make these states metastable and produce random reversals.

2. Multiscaling properties of the mean wind and SOC

The probability density function (PDF) of the life-times of the metastable states, τ, observed in the laboratory data on the wind, exhibits the characteristic power-law

$$p(\tau) \sim \tau^{-1} , \tag{1}$$

as shown in Fig. 1.

Different physical mechanisms can lead to this power law. It could be the "exchange of stability" under turbulent noise effect [12], or it could be the SOC as mentioned in the introduction. These mechanisms differ substantially. The differences can be ascribed in part, almost trivially, to the lack of long time correlations for the avalanches in the "monoscale" SOC models, leading to the inability of SOC to incorporate the far-from-equilibrium characteristics of hydrodynamic systems such as turbulent

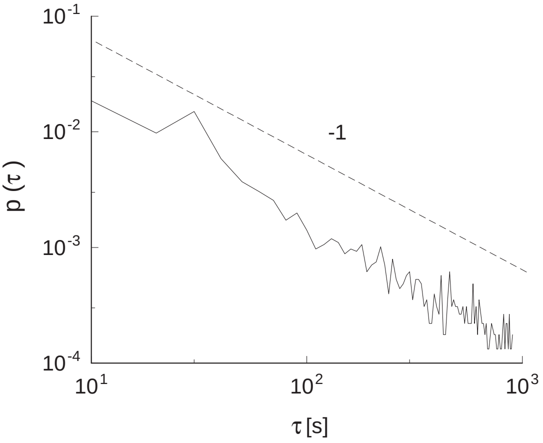

Fig. 1. The probability density function of the life-times of the metastable states of the wind (in log–log scales). The dashed line indicates the power law (1).

convection. This trivial distinction between the two mechanisms disappears if we consider the two-dimensional Bak–Tang–Wiesenfeld (BTW) model [18,19], which obeys a specific form of multiscaling for the PDF of several avalanche measures [20,22,23]. This model has bursts with long-time correlation, and other turbulence-like intermittent properties, if studied on the time scales of its waves (see below). The question now is this: Can one still distinguish between turbulence and the multiscaling SOC in this situation? Or, are there still significant differences between the model and the specific physical phenomenon? We shall address this issue briefly in this paper, using as example the life-time τ of the metastable states of the mean wind.

The BTW prototype model of self-organized criticality is defined on a square lattice [19]. The number of "grains" stacked on a given site i is denoted by $z_i = 0, 1, 2, \ldots$. If $z_i < 4$, $\forall i$, the configuration is stable by construction. A random site k is selected and a grain is added to it, thus increasing z_k to $z_k + 1$. If, in the process, $z_k \geqslant 4$, immediately z_k is reduced to $z_k - 4$. This is called "toppling". The expelled four grains are received one each by the four nearest neighbor site of k. The grains disappear if k is a boundary site. It is clear that the toppling at site k may lead to topplings at the next microscopic time step, and so on. Thus, an avalanche made by a total number $s \geqslant 0$ of topplings occurs before a new stable configuration is reached, and a new grain is added to a random site. After many additions the system reaches a stationary critical state in which the properties of the avalanches are sampled.

A key notion of the BTW model is that of wave decomposition of avalanches [26]. The first wave is obtained as the set of all topplings which can take place as long as the site of addition is prevented from a possible second toppling. The second wave is constituted by the topplings occurring after the second toppling of the additional site takes place and before a third one is allowed, and so on. The total number of topplings in an avalanche is the sum of those of all its waves. The wave size, s, has the power-law PDF

$$p(s) \sim s^{-1} . \tag{2}$$

Returning now to convection, as mentioned in the introduction, the times taken to switch from one metastable state to the other are very short in comparison with the interval between switchings; in addition, the magnitude of the mean thermal wind velocity is approximately *constant* during the metastable events. Therefore, the size of the events in the experiment (in SOC terms) is proportional to their duration; that is, $s \sim \tau$ (compare Eqs. (1) and (2), and Ref. [27]).

With this identification, one can take the analogy a step further. A characteristic similar to local dissipation rate in turbulence,

$$\varepsilon_t = \sum_{k=1}^{t} (s_{k+1} - s_k)^2/t , \tag{3}$$

was introduced in the Ref. [23] in order to characterize multiscaling properties of SOC. The multiscaling (if it exists) has the form

$$\frac{\langle \varepsilon_t^p \rangle}{\langle \varepsilon_t \rangle^p} \sim t^{-\mu_p} . \tag{4}$$

Such multiscaling was observed in Ref. [23] for the two-dimensional BTW prototype model of the SOC. Since $s \sim \tau$ in our case (see above and Ref. [27]), we calculated the analogous "local dissipation rate" for the duration τ obtained in our experiment. The multiscaling and corresponding scaling exponents μ_p, given by Eq. (4), are shown in Figs. 2 and 3, respectively. Circles in Fig. 3 correspond to the exponents computed for our data whereas the triangles correspond to computations performed in Ref. [23] for the two-dimensional BTW model of SOC.

3. Concluding remarks

The fluid flow here has many features such as jets, boundary layers (both attached and detached), plumes, and so forth. These are hydrodynamical entities which, in the end, may have to be explained through their hydrodynamic origin. Amidst this complexity, however, the comparison of the multiscaling exponents shown in Fig. 3 suggests that the multiscale SOC is a possible model for the observed reversal of the thermal mean wind. We might ask: Is the mechanism in the flow indeed the same as that of two-dimensional BTW, or does the agreement of multiscaling exponents μ_p shown in Fig. 3 characterize some class of the multiscale SOC universality? In some sense, the first question looks for more detailed dynamical understanding, while the latter looks for a statistical analogy. We cannot answer the first question with any certainty yet, but the answer to the second question seems to be in the affirmative.

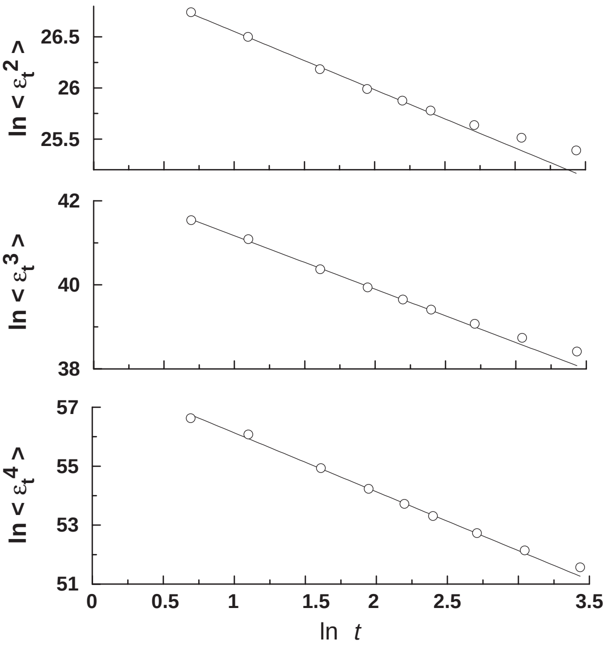

Fig. 2. Moments of the "local dissipation rate" of the duration times of the metastable state (or, in this case, their sizes). The straight lines (the best fit) are drawn in order to indicate (in log–log scales) the multiscaling given by Eq. (4).

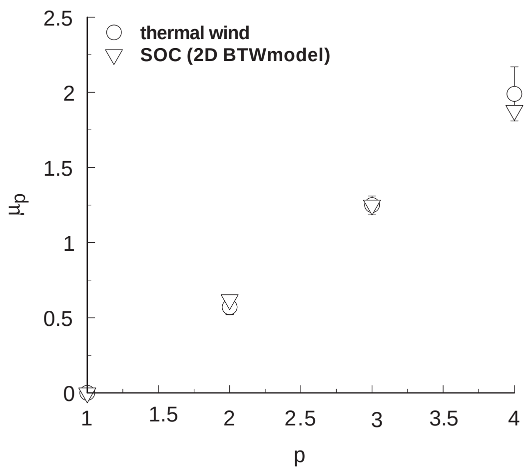

Fig. 3. Multiscaling exponents from Eq. (4) for the mean wind (circles) and for the two-dimensional BTW prototype model of SOC (triangles).

Acknowledgements

For sometime soon after SOC was invented, Per Bak was actively proposing SOC as the leading mechanism to explain turbulence. By the time he wrote his book [16], however, Per seemed to have withdrawn that emphasis. One imagines that this may have resulted at least partly from the cautionary notes he constantly received from the likes of one of us (KRS). It is therefore especially fitting to point out a quantitative, albeit small, feature of turbulence that bears similarity to SOC, even if the implications of this observation are not yet fully clear. We appreciate the efforts of the organizers in holding the meeting and arranging this publication honoring the ebullient character of Per Bak. We are pleased to be a part of it, and are sad at the same time that his life was terminated prematurely.

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