Available online at www.sciencedirect.com
SCIENCE DIRECT®
PHYSICS LETTERS A
Physics Letters A 319 (2003) 21–23
www.elsevier.com/locate/pla

Extended self-similarity of the small-scale cosmic microwave background anisotropy

A. Bershadskii [a,b,*], K.R. Sreenivasan [b]

[a] *ICAR, P.O. Box 31155, 91000 Jerusalem, Israel*
[b] *ICTP, Strada Costiera 11, I-34100 Trieste, Italy*

Received 25 May 2003; received in revised form 24 September 2003; accepted 25 September 2003

Communicated by V.M. Agranovich

Abstract

The extended self-similarity (ESS) of cosmic microwave background (CMB) radiation has been studied using recent data obtained by the space-craft based Wilkinson Microwave Anisotropy Probe. Using the ESS and the high angular scale resolution (arcminutes) of the data it is shown that the CMB temperature space *increments* exhibit considerable and systematic declination from Gaussianity for high order moments at the small angular scales. Moreover, the CMB space increment ESS exponents have remarkably close values to the ESS exponents observed in turbulence (in magnetohydrodynamic turbulence).

PACS: 98.70.Vc; 95.85.Bh; 98.80.Es

Keywords: Cosmic microwave background; Non-Gaussian multiscaling; Primordial turbulence

Interaction of the primordial magnetic filed with the baryon–photon fluid is widely discussed as a possible origin of the space anisotropies of the cosmic microwave background (CMB) radiation [1–6]. Recent analysis of the COBE data [6] showed that for very large cosmological scales (larger than 10°) the Anderson localization makes it impossible for the electromagnetic fields to propagate. However, for the angular scales less than 4° the electromagnetic fields can be actively involved into macroscopic dynamics (including turbulent motion of the baryon–photon fluid [1,7–10]). The release of the first results from the Wilkinson Microwave Anisotropy Probe (WMAP) can be very helpful for this problem. The high angular resolution (arcminutes) and space-craft origin of the WMAP data make the WMAP temperature maps very attractive.

In present Letter we will use a map made from the WMAP data [11]. The observations were made at frequencies 23, 33, 41, 61 and 94 GHz. The entire map consists of 3 145 728 pixels, in thermodynamic millikelvins. The map was cleaned from foreground contamination (due to point sources, galactic dust, free–free radiation, synchrotron radiation and so forth). Since some pixels are much noisier than others and there are noise correlations between pixels, Wiener filtered map is more useful than the raw one.

* Corresponding author.
E-mail address: ark2@eudoramail.com (A. Bershadskii).

0375-9601/$ – see front matter
doi:10.1016/j.physleta.2003.09.079

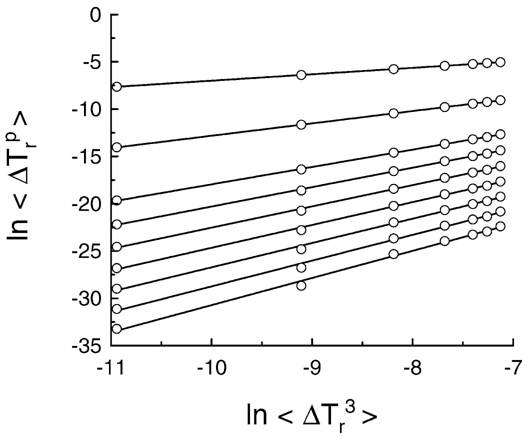

Fig. 1. Logarithm of moments of different orders $\langle|\Delta T_r|^p\rangle$ against logarithm of $\langle|\Delta T_r|^3\rangle$ for the cleaned and Wiener-filtered WMAP data. The straight lines (the best fit) are drawn to indicate the scaling (2).

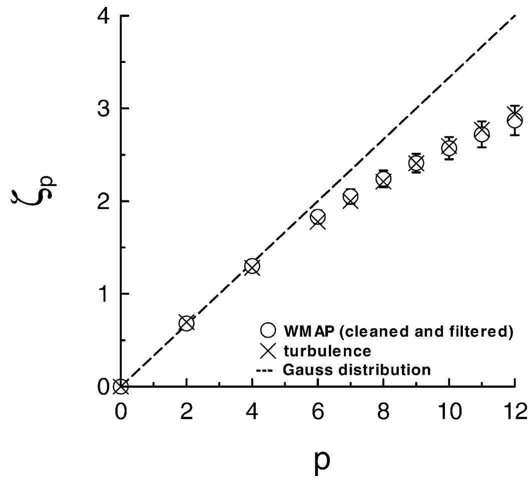

Fig. 2. The exponents ζ_p (open circles) extracted from Fig. 1 as slopes of the straight lines. The dashed straight line corresponds to the Gauss distribution ($\zeta_p = p/3$), while the crosses correspond to the turbulence ESS [12].

Wiener filtering suppresses the noisiest modes in a map and shows the signal that is statistically significant.

The main tool for extracting cosmological information from the CMB maps is the angular power spectra. However, such spectra use only a fraction of the information on hand. An additional tool is the probability density function (PDF) of the CMB temperature *fluctuations*, or deviations from a mean value. The WMAP temperature fluctuations have been confirmed to be Gaussian. Now, taking advantage of the good angular resolution of this map, we can study statistical properties of angular *increments* of CMB temperature for different values of angular separation. The increments are defined as

$$\Delta T_r = \big(T(\mathbf{R}+\mathbf{r}) - T(\mathbf{R})\big), \tag{1}$$

where $\mathbf{r}$ is dimensionless vector connecting two pixels of the map separated by a distance r, and the structure functions of order p as $\langle|\Delta T_r|^p\rangle$ where $\langle\cdot\rangle$ means a statistical average over the map.

It is well known that for complex stochastic (turbulent) systems the temperature fluctuations $\delta T = T - \langle T\rangle$ themselves can exhibit rather good Gaussianity, while the space increments (1) are significantly non-Gaussian. So-called extended self-similarity is an useful tool to check non-Gaussian properties of the data. The ESS is defined as the generalized scaling (see, for instance, Ref. [12] for application of the ESS to different turbulent processes):

$$\big\langle|\Delta T_r|^p\big\rangle \sim \big\langle|\Delta T_r|^3\big\rangle^{\zeta_p}. \tag{2}$$

For any Gaussian process the exponents ζ_p obey to simple equation

$$\zeta_p = \frac{p}{3}. \tag{3}$$

Therefore, systematic deviation from the simple law (3) can be interpreted as deviation from Gaussianity. An additional remarkable property of the ESS is that the ESS holds rather well even in situations when the ordinary scaling does not exit, or cannot be detected due to small scaling range (that takes place in our case) [12].

Fig. 1 shows logarithm of moments of different orders $\langle|\Delta T_r|^p\rangle$ against logarithm of $\langle|\Delta T_r|^3\rangle$ for the cleaned and Wiener-filtered WMAP data. The straight lines (the best fit) are drawn to indicate the ESS (2).

Fig. 2 shows the exponents ζ_p (open circles) extracted from Fig. 1 as slopes of the straight lines. We also show (dashed straight line) in Fig. 2 dependence of ζ_p on p for the Gaussian distributions (3).

One can see that starting from $p \simeq 6$ the data decline systematically from the Gaussian straight line and follow, in this declination, quite close to the ESS exponents observed for turbulence [12] (indicated in the figure by the crosses).

Since the quoted "turbulent" values of the exponent ζ_p are practically the same both for classic Kolmogorov fluid turbulence and for the Alfven-wave dominated magnetohydrodynamic (MHD) turbulence [12] it is impossible, using the presented data, decide—what type of turbulence (Kolmogorov's or MHD) could be considered as the CMB modulation origin in this case. According to the up to date knowledge of the processes in the cosmic baryon–photon fluid the last (MHD) type of turbulence seems to be more plausible [1–5]. It is know now that the Alfven waves with small enough wavelength, which can oscillate appreciably before recombination, become overdamped (the photon mean-free-path becomes large enough for dissipative effects to overcome the oscillations). In this case, the longest wavelength which suffers appreciable damping by photon viscosity, has a scale $k^{-1} \sim V_A L_S^C$, where L_S^C is the usual comoving Silk (or photon viscosity) scale and V_A is the Alfven speed. Since the Alfven speed is $V_A \sim 3.8 \times 10^{-4} B_{\mathrm{pr}} \ll 1$ (B_{pr} is the present-day magnetic field in units 10^{-9} Gauss), only comoving wavelengths smaller than $10^{23} B_{\mathrm{pr}}$ cm suffer appreciable damping. Therefore, it seems plausible that the arcminute WMAP data could be modulated by the Alfven-wave dominated turbulence, that results in the evident systematic declination of the small-scale WMAP temperature space increments from the Gaussianity (Fig. 2).

Acknowledgements

The authors are grateful to C.H. Gibson for discussions, to M. Tegmark group for providing the data, and to the Referee for suggestion.

References

[1] A. Brandenburg, K. Enqvist, P. Olesen, Phys. Rev. D 54 (1996) 1291.
[2] K. Subramanian, J.D. Barrow, Phys. Rev. Lett. 81 (1998) 3575.
[3] K. Jedamzik, V. Katalinic, A.V. Olinto, Phys. Rev. D 57 (1998) 3264.
[4] K. Subramanian, J.D. Barrow, Phys. Rev. D 58 (1998) 083502.
[5] R. Durrer, T. Kahniashvili, A. Yates, Phys. Rev. D 58 (1998) 123004.
[6] A. Bershadskii, Phys. Lett. B 559 (2003) 107.
[7] R. Beck, A. Bradebburg, D. Moss, A. Shukurov, D. Sokoloff, Annu. Rev. Astron. Astrophys. 34 (1996) 153.
[8] A.D. Dolgov, D. Grasso, A. Nicolis, Phys. Rev. D 66 (2002) 103505.
[9] A. Kosowsky, A. Mack, T. Kahniashvili, Phys. Rev. D 66 (2002) 024030.
[10] A. Bershadskii, K.R. Sreenivasan, Phys. Lett. A 299 (2002) 149.
[11] M. Tegmark, A. de Oliveira-Costa, A. Hamilton, astro-ph/0302496.
[12] R. Benzi, L. Biferale, S. Ciliberto, M.V. Struglia, R. Tripiccione, Physica D 96 (1996) 162.