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Rayleigh-number evolution of large-scale coherent motion in turbulent convection

J. J. Niemela and K. R. Sreenivasan
International Center for Theoretical Physics - Strada Costiera 11, 34014 Trieste, Italy

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Abstract. – At least up to Rayleigh numbers of the order 10^{13}, a feature of turbulent convection in confined containers is a self-organized and coherent large-scale motion ("mean wind"). For aspect ratio unity, the mean wind is comparable in scale to the container size. Its magnitude is measured here using short-time temperature correlations in a cylindrical container of aspect ratio unity; the working fluid is cryogenic helium and the Rayleigh numbers span from 10^6 to 10^{15}. The self-organizing advection of "plumes" by the mean wind leads to periodic temperature oscillations near the sidewall. Comparisons of the observed oscillation frequency to the rotational rate of the mean wind, however, have differed by a factor of 2 in the recent literature. It is argued here that this apparent discrepancy is the result of the evolution of the shape of the mean wind, from a tilted and nearly elliptical shape at low Rayleigh numbers to a squarish shape at high Rayleigh numbers, thereby altering the effective path length from which the rotational rate of the mean wind is deduced.

Introduction. – The large-scale circulation ("mean wind") in turbulent convection has been the subject of much interest [1–9] in recent years. The mean wind is a correlated motion over the entire container, and is intimately connected to the coherent release of plumes from the top and bottom boundary layers. The plumes both initiate the mean wind and are carried by it in a self-organizing process [8, 10]. However, this self-organization occurs only when the convection cycle time and the plume emission period are locked together. Unless sufficiently pinned in some manner (for example, by a small tilt of the apparatus), the wind is known to exhibit "sudden" and aperiodic reversals of direction [2, 4, 8]. We focus our discussion on containers of aspect ratio unity (the diameter D and height H are equal to each other).

Two methods of measurement are available on the magnitude of the mean wind. In the first method, by time shifting signals from two nearby temperature probes, separated by a known amount in the direction parallel to the presumed mean wind, and by obtaining the time delay needed to maximize the correlation between signals from the two probes, one can determine the magnitude of the mean wind. In the self-organized state, the inverse of the plume frequency gives the "cycle time" of the mean wind, assuming that the temperature perturbations are simply carried by it. By combining the two results, it has been estimated [1, 2, 4, 8] that the maximum path length traversed by the mean wind is close to $4L$, where L is twice the distance from the axis of the container to the position of the measurement probe. If the flow is sampled exactly at the sidewall, then $L = D$. It has been surmised therefore that the mean wind is in the form of a large-scale roll-type motion filling more or less the entire apparatus. This is consistent with correlations between two temperature probes located at opposite places on the sidewall [4]. In the second —and more direct— method, Qiu and Tong [7] used a laser Doppler anemometer to measure the magnitude of the mean wind in addition to the oscillating temperature field near the sidewalls. They show that the maximum path length for the mean wind is closer to $2L$, unmistakably different from $4L$.

The main purpose of this paper is to understand this seeming discrepancy. We do this by examining measurements spanning the Rayleigh number range between 10^6 and 10^{13}. Since the experimental apparatus has been discussed in refs. [3, 4], we cite here only a few features. The container is cylindrical in shape, 50 cm in height and 50 cm in diameter. The working fluid is low-temperature helium gas at various pressures and (cryogenic) temperatures, so a wide range of Rayleigh numbers, as well as a large value at the high end, can be achieved with relative ease. The mean wind is measured by the correlation of temperature signals from two sensors placed at the mid-height of the apparatus and at a distance of 4.4 cm from the sidewall. Thus, $L = D - 2 \times 4.4\,\text{cm} = 41.2\,\text{cm}$. The details of this measurement procedure are contained in ref. [4].

Results. – Let V_M be the speed of the mean wind, and f_P be the plume frequency (its inverse being the cycle time of the mean wind, as already remarked). Both are obtained from the same set of data, consisting of approximately a three-hour time series of temperature fluctuations measured from small sensors, which are neutron-transmutation-doped (NTD) germanium cubes, 250 micrometers on a side. The measurement of the mean-wind speed is robust: in 10 separate, but identical, runs at a given Ra, it is reproduced to within 4%. The time series of temperature from either sensor can be used to obtain f_P. The two sensors show no measurable difference between them: again, from 10 separate but nominally identical runs at the same Ra, the variation is observed to be about 2%. An exception occurs at the lowest Ra where the uncertainty is somewhat higher. At this Ra the emissions of plumes from the top and bottom boundary layers are not strongly correlated, and it becomes more difficult to assign a reliable value to f_P. It is indeed the case that, in this transitional regime, a different physical mechanism operates, as described by Howard [11] and others: plumes are emitted periodically from uncoupled boundary layers under conditions of marginal stability (see [12] for a discussion).

The effective path length S of the mean wind can be written as, say,

$$S = \frac{V_\text{M}}{f_\text{P}} = \alpha L. \tag{1}$$

For motion along a perfect square of side L, we have $\alpha = 4$. In our helium experiments, we have measured both V_M and f_P as functions of Ra up to about $Ra \sim 10^{13}$; for higher Ra, the flow becomes sufficiently disorganized so that one cannot confidently assign a magnitude to it. In fig. 1, we plot the measured α as a function of Ra. It is clear that α starts out near a value of 2 at the lowest Ra. This data point is well within the transition region discussed above, where the wind is not fully developed. According to [7], the flow becomes fully developed only for $Ra > 5 \times 10^7$. All other data above the transition point correspond to the self-sustained state in which the wind prevails; the α values above the transition monotonically increase from about 2.8 (as inferred from the interpolation between the first two data points) to about 4 asymptotically. The rest of the paper is related to the interpretation of this main result.

We now note that the limiting values of α in fig. 1 are consistent with the shapes of the

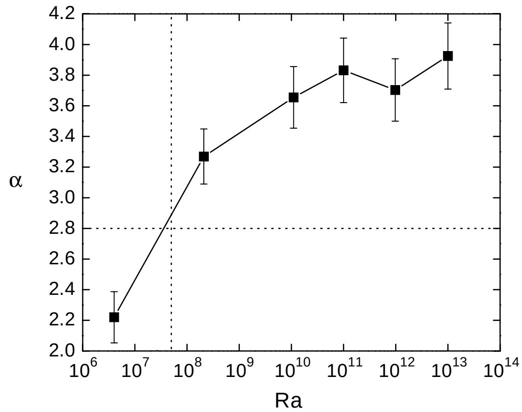

Fig. 1 – The parameter α (see eq. (1)), evaluated from the present measurements, plotted as a function of Ra. In this case, the relevant distance $L = 50\,\text{cm} - (2 \times 4.4\,\text{cm}) = 41.2\,\text{cm}$, reflecting the fact that the probes are sampling the velocity at a distance of 4.4 cm from the sidewall. The vertical dotted line denotes the position of the minimum Ra for which the re-circulating flow is fully developed, for which we expect α to be equal to or greater than about 2.8, corresponding to the horizontal dotted line (see text). The error bars reflect the combined uncertainties in the determination of V_{M} and f_{P}.

mean wind shown in fig. 2. The ellipse shown by the dotted line is close to the shape deduced by direct flow visualization and velocity measurements of ref. [6]; for that shape, one expects an α of about 2.8 ($\approx 2\sqrt{2}$), as fig. 1 indeed confirms. (The data below the transition to the fully developed recirculation flow might qualitatively be expected to give a smaller "effective" α than 2.8; this is also in concurrence with fig. 1.) The dashed line is closer to the situation at large Ra, as supported by multiprobe measurements at high Ra [4]. The intermediate variation of α represents the gradual transformation of one shape of the mean wind into another.

What causes the tilt at low Rayleigh numbers? Observations suggest that the largest

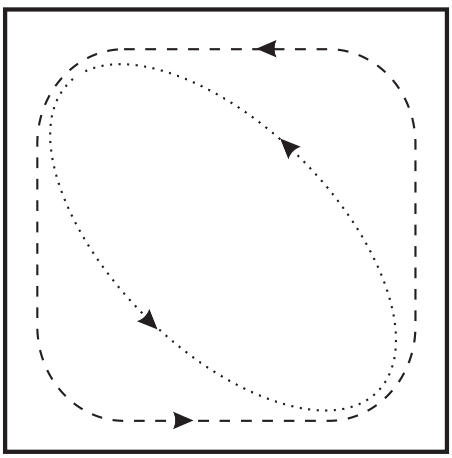

Fig. 2 – Two schematic configurations of the mean wind in turbulent convection in a container of aspect ratio unity. For lower Ra, the wind is roughly elliptical in form, illustrated by the dotted line; as Ra increases, it becomes increasingly squarish (dashed line).

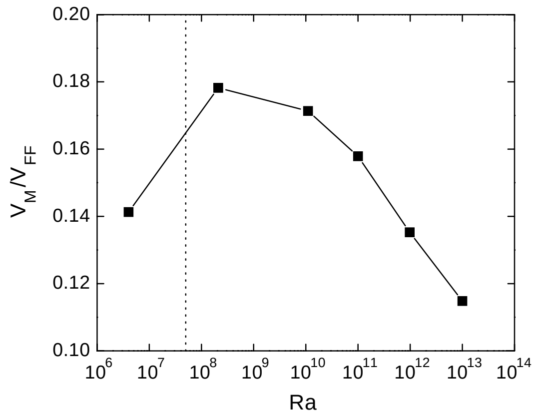

Fig. 3 – The ratio of the speed of the mean wind, V_{M}, to the freefall velocity, $V_{\mathrm{FF}} \equiv \sqrt{g\alpha\Delta T H}$, as a function of Ra. V_{FF} is the maximum velocity due to a conversion of the buoyant potential energy of a fluid parcel as it traverses a height H in an adverse temperature difference ΔT. As in fig. 1, the dotted line denotes the minimum Ra for which the recirculating flow is fully developed.

plumes are created as a rule close to the wall. We may imagine that when these large plumes detach from the top and bottom plates, they create an unbalanced suction leading to horizontal pressure gradients. They also lower the local buoyancy directly adjacent to the sidewalls, so that the subsequent plume will be weaker and hence more strongly subject to the pressure gradients. It follows that the rising plumes are also displaced horizontally. Thus, in steady state, this effect is likely to lead to a tilted shape for the resultant motion. Conversely, the large horizontal flow (which is a part and parcel of the mean wind) has opposite signs near the top and bottom walls, and so produces a "torque" to nonlinearly distort itself in shape and orientation. As this horizontal flow decreases, the torque and the resulting tilt will decrease. For consistency with figs. 1 and 2, we should thus expect that the torque, or the horizontal flow, must diminish with increasing Ra. This is indeed the general trend of the mean wind relative to the scale velocity of buoyancy, the so-called "free-fall" velocity (see fig. 3). The free-fall velocity is the maximum velocity acquired by a fluid parcel by releasing its gravitational potential energy against the mean adverse density gradient, and is given by $V_{\mathrm{FF}} \equiv \sqrt{\alpha g \Delta T H}$, where ΔT is the temperature difference, g is the acceleration due to gravity and α is the thermal expansion coefficient of the gas.

The trend shown in fig. 3 might seem counterintuitive at first, since we know that the associated Reynolds number *increases* according to $Ra^{0.47}$ [4] or something close; however, this increase occurs from the operational fact that, as Ra increases, the decrease in wind speed is more than compensated by the decrease in the kinematic viscosity of the working fluid. As noted with respect to fig. 2, the velocity corresponding to the lowest measured Ra does not correspond to the self-organized recirculating flow, which occurs only for $Ra > 5 \times 10^7$; the consequent change in the nature of the mean wind is clearly evident. Interestingly, a consequence of the weaker recirculating flow at very high Ra might be the uncoupling of the top and bottom boundary layers and the possibility of observing the "classical" heat transfer scaling: $Nu \sim Ra^{1/3}$. Indeed, such a scaling regime has recently been inferred in ref. [12].

It must be said that the shorter path length observed at low Rayleigh numbers could perhaps also be explained as follows (see [6]). A strong plume, rising along one part of the sidewall, triggers a pressure perturbation in the top-wall boundary layer when it "hits" it. This pressure perturbation instantaneously travels along the top-wall boundary layer and triggers the release of a plume. This plume sinks along the opposite part of the sidewall. If it is assumed that the cycle repeats itself, it is not hard to argue that the effective path length per cycle will be $2L$. However, this scenario requires that two different mechanisms for a coherent, recirculating flow operate at low and high Rayleigh numbers, and is thus less attractive to us.

Conclusions. – In summary, we have argued that the seemingly disparate results connected to the path length of the mean wind are related to, and can be explained by, the change in its shape with Rayleigh number. (Note that the path length is estimated by measurements of the frequency of temperature oscillations and the rotational rate of the mean wind.) We were able to deduce this result only because a large range of Ra could be attained in the same apparatus by using cryogenic helium at varying pressures. By examining smaller ranges of Ra, one could easily, perhaps erroneously, favor one or the other of the configurations shown in fig. 1.

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