BRIEF COMMUNICATIONS

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An update on the energy dissipation rate in isotropic turbulence

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Direct numerical simulations of homogeneous turbulence in a periodic box are examined here to support the traditional expectation that the dissipation rate at high Reynolds numbers is independent of fluid viscosity, and is a constant of order unity when scaled on the integral scale and root-mean-square velocity. However, the numerical value of the constant appears to depend on details of forcing at low wavenumbers, or, perhaps, the structure of the large scale itself. © *1998 American Institute of Physics.* [S1070-6631(98)01002-2]

A basic premise in the phenomenology of turbulence is that the mean energy dissipation rate, $\langle\varepsilon\rangle$, is independent of the fluid viscosity, ν, as long as the latter is sufficiently small (or the Reynolds number is sufficiently high). It then follows from dimensional considerations that $D \equiv \langle\varepsilon\rangle L/u^3$ approaches a constant of order unity in the high-Reynolds-number limit. Here u and L are the velocity and length scales characteristic of the large scale motion of turbulence, such as the root-mean-square velocity and integral length scale. The significance of this premise has been emphasized at various times,[1–3] and attempts[4,5] have been made to test its tenability in experiments. The experimental situation is somewhat different in different classes of flows.[5] Here, we consider homogeneous and isotropic turbulence. In experimental realizations of this turbulence behind grids, $D \to D_\infty =$ constant at high Reynolds numbers. However, experiments also show (see Figs. 2 and 3 of Ref. 4) that D_∞ depends on some details of turbulence generation at the grid itself. The suggestion, then, is that the asymptotic value might depend on the nature of large-scale forcing, or, perhaps, on the structure of the large scale. With this in mind, data on $\langle\varepsilon\rangle L/u^3$, obtained by several authors from direct numerical simulations of turbulence in a periodic box, are collected and presented.

Table I lists all the relevant data. In each case, the originators of the data believe that the simulations have been carried out for ‘‘sufficiently long times.’’ The quantities $\langle\varepsilon\rangle$, L and u are defined as follows:

$$\frac{3}{2}u^2 = \int_0^\infty E(k)\,dk, \quad \langle\varepsilon\rangle = 2\nu\int_0^\infty k^2 E(k)\,dk,$$

$$L = \frac{\pi}{2u^2}\int_0^\infty \frac{E(k)}{k}\,dk.$$

Here $E(k)$ is the energy spectrum, and k is the wavenumber magnitude.

It is instructive to plot the data listed in Table I. Figure 1 shows that the data organize themselves into two groups, in each of which D decreases when R_λ is small, just as in experiments, and tends to asymptote to a constant value be-

TABLE I. $D \equiv \langle\varepsilon\rangle L/u^3$ as a function of the Taylor microscale Reynolds number $R_\lambda \equiv u\lambda/\nu$. The Taylor microscale $\lambda = (15\nu u^2/\langle\varepsilon\rangle)^{1/2}$. The numbers attributed to Yeung and Zhou are different by a few percent from the published data. The latter correspond to a slightly different definition of L. The revised data, consistent with the present definition of L, were kindly provided by P. K. Yeung.

Source	R_λ	D	Comments
Jimenez *et al.*[a]	35	1.09	forced
	61	0.82	
	94	0.70	
	168	0.69	
Wang *et al.*[b]	21	1.81	decaying
	68	0.86	
	132	0.62	
	100	0.49	forced
	151	0.43	
	195	0.49	
Yeung and Zhou[c]	38	0.69	forced
	90	0.46	
	140	0.44	
	180	0.41	
	240	0.40	
Cao *et al.*[d]	24	0.95	forced
	70	0.51	
	103	0.45	
	103	0.44	
	131	0.46	
	148	0.45	
	151	0.43	
	186	0.43	
	218	0.40	

[a]Reference 6.
[b]Reference 7.
[c]Reference 8.
[d]Reference 9.

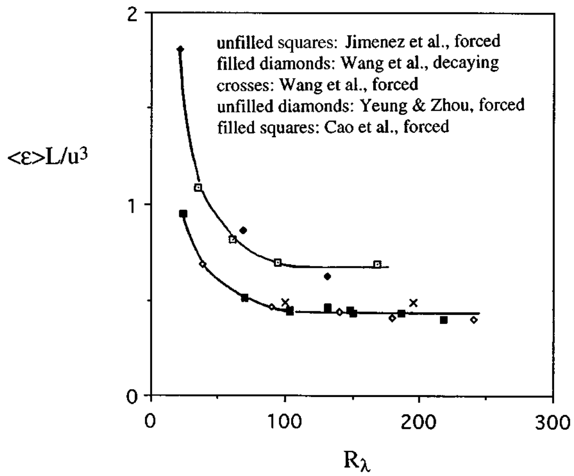

FIG. 1. The variation of the quantity $\langle\varepsilon\rangle L/u^3$ with the Taylor microscale Reynolds number, R_λ, in simulations of homogeneous and isotropic turbulence in periodic box. The symbols, described on the figure, correspond to different sources of data noted in Table I.

yond some R_λ. However, the numerical value of D_∞ is not the same in the two groups. To compare them meaningfully with experiments,[4] the scales L and u used there have to be redefined slightly. The redefinition leads to $D_\infty \approx 0.73$ for square grids of round bars, and is in rough agreement with the D_∞ for the upper curve in Fig. 1. It was noted in Ref. 4 that D_∞ assumes different values for grids of different configurations, especially for the active grids of Gad-el-Hak and Corrsin.[10]

Yeung and Zhou used a stochastic forcing confined to the lowest two or three wavenumber shells, while Wang *et al.* and Cao *et al.* maintain the energy of a few lowest modes according to the $k^{-5/3}$ energy spectrum. It is heartening to note that the forced data of Wang *et al.* and of Yeung and Zhou agree with each other, but one cannot dismiss the fact that they both differ from the forced calculations of Jimenez *et al.* and the decay data of Wang *et al.* The former maintained the energy peak essentially at $k=1$, and introduced negative viscosity for $k<3$ in order to compensate for the energy decay. In all the forced cases, it might be said that the resolution of the large-scale is a major factor: there is no perceptible gap between the large-scale and the box-size. The energy in the decay data of Wang *et al.* did not peak at the lowest wavenumber but was shifted to the right, suggesting that the large-scale resolution might be better. Yet, the decay data agree with one set of forced data—though it should be said that there are only three R_λ values for the former, and that they do not totally preclude the possibility of further decrease with increasing R_λ— but not with the other two. It is not clear why this is so.

Despite this lack of clarity, the principal message of Fig. 1 is that D asymptotes to a constant value, but that D_∞ can perhaps be manipulated moderately—even in isotropic turbulence—by adjusting in some manner the forcing scheme or the large structure. Some preliminary calculations of Juneja (private communication) suggest that the same degree of manipulation might also be possible by varying the initial conditions. At present, we do not know enough to say precisely how this can be done in a controlled way. To resolve this issue, one ought to implement systematic changes in the forcing scheme, the large-scale structure, and initial conditions. That the large structure does influence the constant D_∞ is clear from experiments in homogeneously sheared flows; in Ref. 5, it is shown that $D_\infty = D_\infty(S)$, S being a non-dimensional shear parameter.

One is now left with the question as to whether the nature of forcing at the large scale, and the resulting differences in the structure of the large scale, affect other aspects of turbulence as well. We have examined various small-scale statistics from the sources cited here. There seems to be no perceptible difference in this regard. But the scaling range—as determined, for example, by Kolmogorov's 4/5-ths law[11]—does depend on the nature of forcing: it can be extended or contracted depending on how one deals with the energy level of the lowest few wavenumbers.

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