On the universality of the Kolmogorov constant

Katepalli R. Sreenivasan
Mason Laboratory, Yale University, New Haven, Connecticut 06520-8286

(Received 11 May 1995; accepted 13 July 1995)

All known data are collected on the Kolmogorov constant in one-dimensional spectral formula for the inertial range. For large enough microscale Reynolds numbers, the data (despite much scatter) support the notion of a "universal" constant that is independent of the flow as well as the Reynolds number, with a numerical value of about 0.5. In particular, it is difficult to discern support for a recent claim that the constant is Reynolds number dependent even at high Reynolds numbers. © *1995 American Institute of Physics.*

I. INTRODUCTION

The classical formalism of Kolmogorov[1] for three-dimensional turbulence at high Reynolds numbers leads to explicit expressions for various inertial range quantities. In particular, for the so-called one-dimensional longitudinal spectral density defined as

$$\langle u_1^2 \rangle = \int_0^\infty \phi_1(k_1)\,dk_1, \tag{1}$$

where $\langle u_1^2 \rangle$ and k_1 are the mean-square fluctuation velocity and the wavenumber component in the "longitudinal" direction x_1, one obtains the relation[2]

$$\phi_1(k_1) = C_K \langle \epsilon \rangle^{2/3} k_1^{-5/3}; \tag{2}$$

where $\langle \epsilon \rangle$ is the mean value of the energy dissipation rate, and the constant C_K, named after Kolmogorov, is presumed to be "universal." The subject of this paper is the nature and numerical value of this Kolmogorov constant.

Often in turbulence literature, "Kolmogorov constant" denotes the prefactor in three-dimensional spectrum, as well as that in second-order structure function for longitudinal velocity increments. Clearly, local isotropy—which is a forerunner of universality in Kolmogorov's formalism—implies that those two constants are, respectively, $\frac{55}{18}C_K$ and about $4.02C_K$ (see, for example, Ref. 3).

Since several past attempts (see, for example, Refs. 3–6) have been made to collect available data, and the conclusions reached there were not very different from those of the present work, one might wonder whether this paper is needed at all. First, much more data have now become available in the 25 or so years since these past efforts, and it is now possible to make a more complete examination of all the data. Second, in contrast to the general belief held until recently, it has been claimed[7] that C_K possesses an explicit Reynolds number dependence. Even a weak dependence could lead to far-reaching conclusions,[8–10] and is of crucial importance to some recent theoretical considerations.[11] Together, these reasons prompted us to take a thorough look at the experimental situation.

We have examined more than 100 spectra obtained in various flows, and a few remarks should be made about the procedure used for determining C_K. First, all the data examined here are from single-point measurements in which Taylor's hypothesis has been invoked to relate frequency spectrum to wavenumber spectrum. The effect of this plausible approximation is not known precisely—despite several laudable efforts[12–14] to quantify them—and no further comments will be made on this matter. Second, the analysis here will be based on the longitudinal spectra, although a comment will be made in Sec. V on the effect of using spectra of transverse velocity components. Third, all dissipation measurements in shear flows have been made by assuming local isotropy; we shall briefly remark on the effect of this approximation.

A few additional remarks seem worthwhile. First, in many cases examined here (especially where the authors did not specify the Kolmogorov constant themselves), we have plotted the compensated spectral quantity $\psi(k_1) = k_1^{5/3}\phi_1(k_1)/\langle \epsilon \rangle^{2/3}$, and determined the constant from the wavenumber region in which ψ was reasonably constant. This scheme is better than fitting straight-lines to data in log-log plots, but is not foolproof (unless the scaling regime is several decades in extent). The principal difficulty is the slight but inevitable curvature of the region expected to be flat. The situation can be improved, as pointed out by Kraichnan,[15] by using some model for dissipative and large scale regions, thus producing sharper cut-offs and extending the flat region. Unfortunately, this cannot be done without introducing *ad hoc* models, and so will not be attempted here. Different methods of estimation using the *same* data could result in an uncertainty of the order of 10%; there may indeed be other sources of errors due to data acquisition.

The second remark concerns the so-called intermittency corrections to the spectral form in the inertial range. There is a general belief[3] (although contested often enough[16–18]) that the spectral exponent gets slightly modified by small-scale intermittency. This modification, even if it exists, is small and cannot be accommodated in a consistent and satisfactory way given other uncertainties in the data; it is therefore ignored uniformly.

As a final remark, we have thought it to be most appropriate—contrary to some previous practices—to plot the Kolmogorov constant against the microscale Reynolds number $R_\lambda \equiv u_1' \lambda/\nu$, where u_1' is the root-mean-square of the velocity fluctuation u_1 in the direction of the mean velocity U and λ is the Taylor microscale; ν is the kinematic viscosity of the fluid. In the literature to be examined below, R_λ is often provided by the authors themselves, who usually obtain λ from the relation

$$\lambda = [u_1'^2 / \langle (\partial u_1/\partial x_1)^2 \rangle]^{1/2}, \tag{3}$$

with $\partial u_1/\partial x_1$ replaced by $-(1/U)(\partial u_1/\partial t)$ according to

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TABLE I. Sources and Reynolds numbers for grid-generated turbulence data.

Source	R_λ	C_κ	Remarks
Comte-Bellot and Corrsin[19]	37	0.50	In these experiments, a secondary contraction was used to render the grid turbulence more isotropic than is otherwise possible
	38	0.40	
	41	0.42	
	41	0.55	
	61	0.46	
	65	0.45	
	72	0.46	
Fan[20]	52	0.53	
Gad-el-Hak and Corrsin[21]	108	0.57	Average value from five realizations; see further comments in the text
Gibson and Schwarz[22]	40–60	0.49	Average value for all experiments; difficult to discuss each experiment separately
Kistler and Vrebalovich[23]	264	0.57	See remarks in the text
	522	0.57	
	535	0.57	
	669	0.57	
Schedvin *et al.*[24]	280	0.48	
Sreenivasan *et al.*[25]	34	0.38	
Stewart and Townsend[26]	28	0.33	See remarks in the text
	29	0.38	
	40	0.36	
Uberoi[27]	70	0.49	
Van Atta and Chen[28]	35	0.53	
	49	0.50	
Warhaft and Lumley[29]	45	0.60	
Yeh and Van Atta[30]	35	0.54	

Taylor's hypothesis. When R_λ is not given by the authors, as is often the case for atmospheric data, it is estimated from other means—and the manner of estimation is indicated.

II. GRID TURBULENCE

Turbulence generated by a grid of rods simply decays downstream and the tendency towards sustained large-scale anisotropy is therefore absent. Even though some residual anisotropy persists downstream, the turbulence field is close to being isotropic—especially if prepared with particular care, as in Comte-Bellot and Corrsin.[19] Table I, which summarizes the sources of data and their microscale Reynolds numbers, also includes some comments; a few additional remarks follow.

(1) The dissipation rate in grid-generated turbulence can be measured relatively accurately from the energy decay behind the grid. This makes less severe demands on the frequency response and spatial resolution of turbulence-measuring sensors. However, a different type of uncertainty may be introduced because energy dissipation is estimated by differentiating empirical power-laws fitted to energy decay. In all the cases examined below, the energy decay is consistently obtained from $\frac{3}{2}(d/dt)\langle u_1^2\rangle = -\frac{3}{2}U_o(d/dx)\langle u_1^2\rangle$, where U_o is the mean velocity of the flow upstream of the grid. Note that Kistler and Vrebalovich[23] obtained their energy dissipation data from $-U_o(d/dx)\langle(u_1^2+2u_2^2)\rangle$, u_2 being the velocity fluctuation in a direction transverse to the

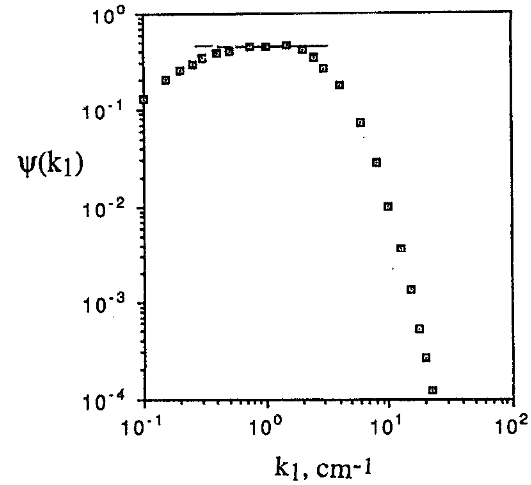

FIG. 1. The compensated one-dimensional spectral density plotted against the wavenumber for one of the grid-generated turbulence experiments of Comte-Bellot and Corrsin,[19] $R_\lambda=61$. The flat region, which is relatively small at low Reynolds numbers, yields the Kolmogorov constant.

mean flow; as already remarked by Pond *et al.*,[4] Gibson,[5] Bradshaw,[6] Schedwin *et al.*[24] and a number of others, this yields a C_κ of 0.68. However, adopting the definition uniformly employed here, one obtains a value of 0.57.

(2) For all cases to be examined below, the compensated spectrum ψ generally showed a flat region when the micro-

TABLE II. Sources and Reynolds numbers for pipe and channel follows, and the fully turbulent part of the boundary layer. They symbol d below refers to pipe radius, channel half-width or the boundary layer thickness, depending on the context.

Flow	Source	Position of measurement	R_λ	C_κ
Pipe	Laufer[34]	$y/d=0.28$	450	0.56
		$y/d=1.0$	230	0.48
	Lawn[35]	$0.1<y/d<1.0$	115–200	0.53[a]
Channel	Anselmet *et al.*[36]	$y/d=0.4$	515	0.55[b]
	Comte-Bellot[37]	$y/d=0.34$	370	0.60[c]
		$y/d=1.0$	220	0.58
	Laufer[38]	$y/d=1.0$	108	0.45
		$y/d=0.7$	137	0.46
		$y/d=0.4$	143	0.43
Boundary layer	Bradshaw[39,40]	log-region	100–400	0.51[d]
	Klebanoff[42]	$y/d=0.2$	230	0.59
	Kailasnath and Sreenivasan[43]	$y/d=0.2$	200	0.55
	Mestayer[44]	$y/d=0.3$	616	0.50
	Saddoughi and Veeravalli[45]	$y/d=0.09$	500	0.49±0.03
		$y/d=0.09$	1400	0.49
		$y/d=0.47$	600	0.49
		$y/d=0.37$	1450	0.49

[a]The error bars provided by the authors are ±8%.
[b]This value was obtained from second-order structure function measurements.
[c]For this flow, Monin and Yaglom[6] estimated C_κ to be about 0.65.
[d]Bradshaw provided different values for the inner and outer layer regions. The value quoted here corresponds to the inner region. The outer intermittency affects C_κ in ways that can be estimated moderately well (see, for example, Kuznetsov *et al.*[41]), but this issue needs a more detailed look.

scale Reynolds number R_λ exceeded about 50: however, a few sets of data with marginally higher R_λ displayed no perceptible flat region, while some others with marginally lower R_λ showed a semblance of a flat region. We believe that some of these differences at the margin are traceable to the manner in which the spectra are obtained (analogue *versus* digital methods) and plotted (discrete points *versus* continuous curves). Where there is no well-defined flat region, the value of C_κ quoted was obtained from the peak of the compensated spectrum. Needless to say, one should place more reliance on those data for which the flat region is at least as prominent as in Fig. 1, but an examination of the entire range of Reynolds numbers covered in available experiments seemed worthwhile: we have omitted only those data of Stewart and Townsend[26] with R_λ less than 20.

(3) The configuration of grids used in various experiments is not identical, but it appears that this is not a sensitive factor for present purposes. For example, Gad-el-Hak and Corrsin[21] produced turbulence behind grids which had the following special feature: holes were drilled into the bars making up the grid through which could emerge co-flowing as well counterflowing jets of air, affecting the drag on the grid and the flow downstream. This feature does not have any effect on the Kolmogorov constant, although it significantly affects the decay rate of turbulence itself. Only the coflow injection case will be examined here, but those of zero injection and counterflow injection are very similar (see Figure 12 of the authors' paper). For the co-injection case, all five experiments (with R_λ between 106 and 112), yielded C_κ between 0.56 and 0.59. The value cited in Table I is the average.

(4) We have not made use of the data of Mills *et al.*[31] because they had an unusual (and unexplained) form of energy decay. The data of Lin and Lin,[32] which had a highly unusual grid configuration and did not, in any case, contain enough details for deducing the Kolmogorov constant, have been omitted as well. Sepri's data[33] are identical to those of Yeh and Van Atta[30] and are not considered explicitly.

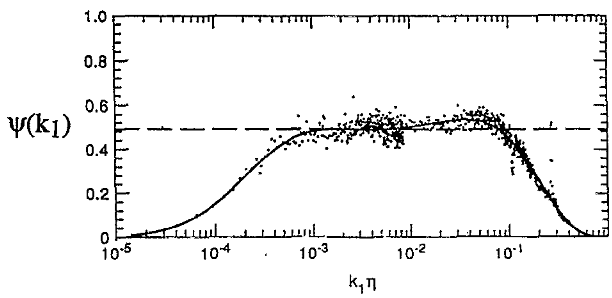

FIG. 2. The compensated one-dimensional spectral density for one of the boundary layer experiments of Saddoughi and Veeravalli,[45] $R_\lambda = 1450$. The abscissa is the wavenumber normalized by the Kolmogorov length scale. Indicated by a horizontal line is the Kolmogorov constant assessed by the authors. The bump apparent at higher wavenumbers is much more pronounced in the spectra of transverse energy components.

III. LABORATORY SHEAR FLOWS

A. Wall-bounded flows

In this section, we examine data from pipe and channel flows as well as the fully turbulent part of turbulent boundary layer. In particular, we have not considered the flow very close to the wall where the effects of viscosity are felt *directly* (say, below y^+ of about 30 where y^+ is the normal distance from the wall normalized by the friction velocity and fluid viscosity), as well as that in the outer intermittent region.

Table II collects data in wall-bounded flows, and provides some commentary. One should perhaps draw special attention to the following facet. In most boundary layer data, the compensated spectrum ψ displays a "bump" as one approaches the dissipative region from the inertial range. This bump is conspicuous to varying degrees in various data sets—most conspicuous, for example, in Mestayer's data—and can affect the perceived values of the Kolmogorov constant. This is illustrated in Fig. 2, reproduced from Saddoughi and Veeravalli;[45] $R_\lambda = 1450$ for this case. Because of this bump, one might infer a slightly larger value for C_κ than one should. The reason for the occurrence of this bump is not entirely clear, but it is often thought[46] to result from small-scale vortex filaments which have the effect of producing a spectral roll-off rate that is less steep than the Kolmogorov form. In a recent study,[47] it has been argued that the combination of the two facts—namely the existence of a constant energy flux across the wavenumber and the rapid damping due to viscosity—leads naturally to this energy pileup near the crossover between inertial and dissipative regions, and has been called the "bottleneck" effect.

TABLE III. Sources and Reynolds numbers for other laboratory shear flows. These sources are not necessarily exhaustive.

Flow	Source	Position	R_λ	C_κ
Cylinder wake	Champagne[48]	Centerline	138	0.55
	Kailasnath and Sreenivasan[43]	Centerline	130	0.51
	Uberoi and Freymuth[49]	Centerline	93–308	0.45
Wake of a sphere	Gibson *et al.*[50]	Axis	224	0.50
	Uberoi and Freymuth[51]	Axis	36–258	0.50
Mixing layer	Champagne *et al.*[52]	$y/x=-0.015$	330	0.46
	Praskovsky and Oncley[7]		2000	0.62
Round jet	Champagne[48]	Centerline	626	0.48
	Gibson[5]	Centerline	780	0.51
		off-center	710	0.53
Plane jet	Bradbury[53]	$y/\delta=0.5^a$	350	0.50
Homogeneous shear flow	Champagne *et al.*[54]	$x/h=10.5^b$	130	0.52
Return channel of wind tunnel	Praskovsky and Oncley[7]		3200	0.58

[a] δ=half-width of the jet.
[b] h=height of the wind-tunnel section in which the shear flow was created.

TABLE IV. Sources and Reynolds numbers for geophysical flows. For atmospheric flows, details of stability conditions are not always known. The effect of stabilty will be examined separately.

Flow	Source	Height	R_λ	C_κ
Atmospheric surface layer	Boston and Burling[55]	4 m above tidal mud flat	4400–5500	0.51 ± 0.09^a
	Champagne[48]	4 m above flat land	7000	0.50
		56 m above flat land	13 000	0.55
	Gibson *et al.*[56]	Different heights over water	2620	0.69^b
	Kailasnath and Sreenivasan[43]	6 m over wheat canopy	2000	0.50^c
	Kaimal *et al.*[57]	Different heights	Not specified	0.50 ± 0.05
	Paquin and Pond[68]	A few meters above water	$O(10^3)^d$	0.57 ± 0.11
	Pond *et al.*[4,59]	A few meters above water	$350–1100^e$	0.48 ± 0.06
	Praskovsky and Oncley[7]	7 m over land	2800	0.61
			3300	0.58
			6900	0.55
			7100	0.54
			9200	0.54
			12 700	0.52
	Sheih *et al.*[60]	108 m above ground	2280–5330	0.65^f
	Stewart *et al.*[61]	1.5–2 m above water[g]	3160–5200	0.53
	Williams and Paulson[63]	2 m above rye grass	1200	0.46
			1270	0.49
			1310	0.57
			1410	0.53
			1780	0.56
			2130	0.57
			2250	0.55
			2600	0.54
			3280	0.53
			3960	0.56
			4150	0.54
			4170	0.53
			4280	0.54
	Wyngaard and Cote[64]	Different heights	$1800–10\,000^h$	0.52 ± 0.04
	Wyngaard and Pao[66,i]	Different heights	2400–10 000	0.53 ± 0.02
Tidal channel	Grant *et al.*[67]	50 ft below the water surface	$3000–18\,000^j$	0.47 ± 0.02^k
	Grant *et al.*[68]	Several depths below surface	...	0.47^l

[a]The values quoted correspond to the mean and standard deviation over 19 sets of data, all taken for nomainally the same conditions.
[b]The authors have remarked that this was an "unusually large value."
[c]These authors obtained data for several similar conditions with the same result.
[d]These authors do not provide adequate data for estimating the Reynolds numbers. From the familiarity with similar conditions elsewhere, R_λ can be "guessed" to be of the order of a few thousands. The Kolmogorov constant quoted is the average over 16 runs.
[e]In this paper, the Reynolds number is given as $\langle\epsilon\rangle^{1/4}L^{4/3}/\nu$, which have been converted to R_λ by assuming that the relation $\langle\epsilon\rangle L/u^3=1$ holds.
[f]This is the value quoted by the authors. The inertial range in these experiments was large but, unfortunately, the number of data points spanning the range was few and the scatter was large. It is thus difficult to place too much reliance on this estimate of C_κ.
[g]The Reynolds number range was estimated using the relation $R_\lambda=5(Uz/\nu)^{1/2}$, where z is the height above the water surface and U is the mean wind speed. For a detailed assessment of this point, see Bradley *et al.*[62] The Kolmogorov constant quoted here is not found in Stewart *et al.*, but has been obtained from their structure function data.
[h]This paper does not quote the microscale Reynolds number range covered in these experiments, and this information has been taken from Wyngaard and Tennekes.[65] The authors note that the mean value of 0.52 may be systematically high.
[i]The data analyzed by Wyngaard and Cote[64] and Wyngaard and Pao[66] are subsets of the data analyzed by Kaimal *et al.*,[57] according to this last reference. The data were taken at heights of 5.66, 11.3, and 22.6 m from a 32 m tower and encompassed different stability conditions. We have listed only the data from Wyngaard and Cote as being representative of the entire data. It is reassuring that these different analyses are quite consistent among themselves.
[j]Grant *et al.* obtained data for a range of Reynolds numbers but did not quote them. The R_λ values given here are estimated by using the method of footnote e above.
[k]The value of C_κ quoted is an average over 17 sets of data, and estimated by the authors by drawing straight lines through log–log plots of spectra. Kraichnan[15] has examined these same data in some detail, and remarked that the estimate would be higher (by more than 10%) if one looked, instead, for flat region in plots of $\psi(k_1)$ vs k_1.
[l]These authors do not give any specific values for C_κ but note that their estimates agreed well with those of Grant *et al.*[67] There is, however, a comment in the paper that some of the runs did not agree with previous results, but that the authors did not believe them because the description by a universal curve had received a lot of support.

B. Other shear flows

Among the prototypical flows studied in the literature are free shear flows such as wakes, jets and mixing layers, as well homogeneous shear flows. We have not been equally thorough in compiling data from each of these classes of flows. Table III may therefore have omitted some useful data.

IV. GEOPHYSICAL FLOWS

Geophysical turbulence in the atmospheric surface layer above land and oceanic waters is of special interest because of the large Reynolds numbers of these flows. Even though geophysical data possess some uncertainties because the flows are not well controlled, practitioners generally pick

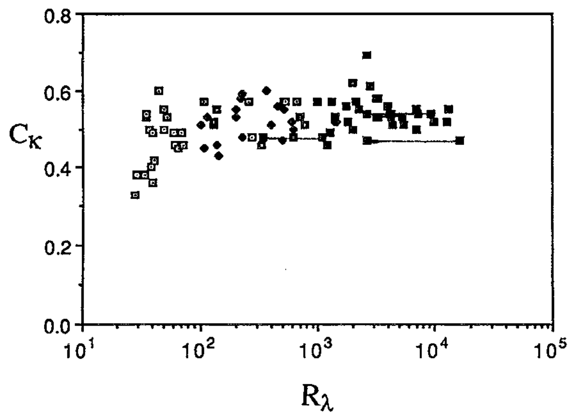

FIG. 3. The Kolmogorov constant C_K *versus* the microscale Reynolds number R_λ for a variety of flows listed in Tables I–IV. A single symbol is used to denote all data from each single table. Ignoring data for $R_\lambda<50$ (where there may be an increasing trend with R_λ), the mean value of all the data is 0.53, with a standard deviation of 0.055.

conditions which are nearly steady—and thus provide valuable high-Reynolds-number data. Following the pioneering measurements of Grant *et al.* in a tidal channel, many sets of spectral data have been obtained in the atmosphere over land as well as water. These are collected in Table IV. The Reynolds number estimates in some cases are somewhat uncertain, but the conclusions to be reached remain unaffected by this artifact.

V. DISCUSSION AND CONCLUSIONS

So far, we have separately tabulated data in various shear flows as well as grid turbulence. In the initial phase of this study, separate plots were prepared for each class of flows. However, a brief examination of those plots showed that the differences among them are not large enough to persist with this treatment. In fact, given that the Reynolds number range for any class of flows is not too large, there are definite advantages in plotting all the data together, which makes the point about universality more unequivocally. It is conceivable, however, that the shear may have some influence on the value of the Kolmogorov constant, but this issue seems to be of secondary importance at least for the non-dimensional shear rates encountered in standard shear flows.

Figure 3 shows all the data tabulated so far, with each symbol representing data from each table. It appears that C_K increases with Reynolds numbers for $R_\lambda<50$, as has already been made by Bradshaw,[6] and is consistent with Sreenivasan's observation[69] that other quantities, such as the normalized dissipation rate, also possess a Reynolds number trend at the low end.

If we agree to ignore the data at the very low end of the R_λ range, our first reaction to the figure is one of wonder: hundreds of experiments made in different flows under different conditions yield *approximately* the same value of the Kolmogorov constant. It is therefore clear, at least for the conditions covered by these experiments, that the Kolmogorov constant is *more or less* universal, *essentially* independent of the flow as well as the Reynolds number (for $R_\lambda>50$ or so). The scatter in the data is undoubtedly large. However, ignoring the outliers, a case can be made that the scatter represents the uncertainty in flow conditions as well as measurement limitations (especially in obtaining the energy dissipation), rather than the non-universality of the Kolmogorov constant. In the former category belong, for instance, aspects such as variability of wind speed and direction in atmospheric flows. In the latter category belong uncertainties relating to spatial and temporal resolutions of the hotwire. As an example, Wyngaard and Cote[64] noted that a large fraction of the scatter in their own measurements was due to hotwire drift, and that the standard deviation of the measurements was halved when they selectively picked records with little drift. It is true that hotwire drift and related issues have undergone significant improvement since the 1960s, but the fact remains that no systematic change exists between the "old data" and the "new data" when taken collectively. Finally, some scatter is undoubtedly due to differences in data processing techniques.

It also appears that the data do not support the existence of a trend with Reynolds number; no trend is apparent even if one examines (as indeed we have) data for each individual classes of flows separately. It is clear that any trend that may exist, if at all, must be weak enough to be hidden in the scatter exhibited by the data. To be certain about the existence or otherwise of such a trend, one has to cover a wide range of Reynolds numbers in a single, well-controlled flow, and use instrumentation whose resolving power and quality remains equally good in the entire range. Further, one has to be aware that certain data processing quirks could artificially introduce weak trends. Such experiments and efforts are not yet on the horizon at present; in their absence, the best that is possible is precisely what has been done here.

Two remarks may be useful. First, as already mentioned, the data collected here come nearly entirely from the longitudinal spectra. In shear flows, the behavior of the transverse spectra at all but high Reynolds numbers is quite complex: as has already been pointed out in Ref. 70, the spectral roll-off rates at low R_λ seem to be less steep than 5/3, up to an R_λ of 1000 or so. The few data sets of transverse spectra available at higher Reynolds numbers also yield the same C_K. The difference is that the meaning of "high enough" Reynolds number has to be upgraded from an R_λ of 50 or so for the longitudinal spectra to one that is perhaps as high as 1000 for the transverse spectra.[71]

The second point concerns the effects of the stability of the atmospheric flows on the value of the Kolmogorov constant. It may be recalled that we did not pay special attention in Table IV to whether or not the atmospheric surface layer was stable, neutrally stratified or unstable. While an extremely stable atmosphere inhibits turbulence altogether, all available data (see Fig. 4, taken from Ref. 64) suggest that there is little effect on C_K whether the atmosphere is strongly unstable or stable. It appears that the Kolmogorov constant is remarkably robust.

In summary, for "high enough" Reynolds numbers, the average value of the Kolmogorov constant from Fig. 3 is 0.53 with a standard deviation of about 0.055. However, it should be recalled that this value is based on the as-

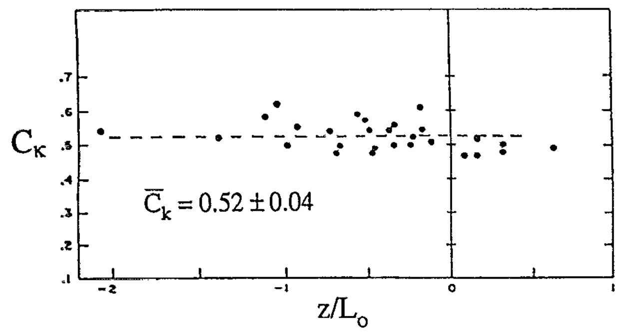

FIG. 4. The Kolmogorov constant as a function of the stability conditions of the atmospheric surface layer, from Wyngaard and Cote.[64] Here L_o is the so-called Monin–Obukhov scale and z is the height from the ground. Negative values of z/L_o imply unstable conditions and *vice versa*. Varying stability conditions do not seem to affect the value of C_κ.

sumption of local isotropy which implies that $\langle\epsilon\rangle = 15\nu\langle(\partial u_1/\partial x_1)^2\rangle$. This is at best an asymptotically valid result, and a *full* assessment of its validity in shear flows at finite Reynolds numbers has not been made. One can take some guidance from measurements in homogeneous shear flows[54,72–76] in which full energy dissipation has been measured by energy balance and compared with the local isotropy estimates. These data have been compiled by Sreenivasan[77] who noted that the local isotropy estimates are always smaller than the full dissipation. Although the ratio presumably tends to unity at very high Reynolds numbers, it was noted that the approach to unity is very slow in Reynolds number. As can be seen from Eq. (2), an underestimation of the energy dissipation overestimates the Kolmogorov constant. Thus, even if the local isotropy estimate is low by about 10%, it is clear that the mean value will have to be revised to something like 0.5. We think that this is about the best estimate possible today for the Kolmogorov constant.

Note added in proof. Akiva Yaglom brought to this author's attention papers by A. M. Yaglom[78] and B. A. Kader[79] whose conclusions about the Kolmogorov constant are completely consistent with the present.

ACKNOWLEDGMENTS

I am grateful to Robert Kraichnan and Detlef Lohse for some useful comments on the draft, and to AFOSR for financial support.

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