Experiments in Fluids 7, 309–317 (1989)

Experiments in Fluids

Absolute instability in variable density round jets

K. R. Sreenivasan, S. Raghu and D. Kyle

Mason Laboratory, Yale University, New Haven, CT 06520, USA

Abstract. This is a brief report on the properties of round jets of different densities issuing into the ambient air. Different densities were obtained by premixing helium and air in various proportions. We show that these jets have two types of behaviour depending on the density ratio between the jet and the ambient fluids, one characterized by very sharp peaks in the power spectral density of the velocity in the near field of the jet, and another with broadened and much less prominant spectral peaks. We examine the possibility that the first state corresponds to absolute instability, and the second to convective instability. It appears that the nature of instability can be changed from absolute to convective by very simple means reminiscent of similar possibilities in low Reynolds number wakes of circular cylinders. Flow visualization reveals that the low-density jets intermittently breakdown, and spread spectacularly, beyond a certain small axial distance.

1 Introduction

The stability properties of fluid flows are of interest for a number of reasons, one of them being the hope that they contain a key to the eventual evolution to a turbulent state; the other expectation is that this knowledge will be helpful in the important area of flow control. We are here considering that class of flows in which stability is lost at some critical value of the control parameter (such as the Reynolds number), and a bifurcation into an observable periodic state ensues. The periodic state is extremely regular in a subclass of flows (whose detailed specification we shall avoid here) while being only roughly so in some others. As an example, we show in Fig. 1 a and b two power spectral densities, one of which was measured in the wake of a circular cylinder, and the other in the unstable shear layer of constant density, isothermal round jet. In both cases, the fluid was air, and the method of measurement and spectral resolutions were identical, but the differences in the nature of spectral densities are immediately clear. While the spectral peak corresponding to the vortex shedding in the case of the wake is about six to eight orders of magnitude above the broadband 'noise', the distinct peaks in the cold jet are hardly an order of magnitude or so above the background. This qualitative difference is important in both contexts mentioned earlier.

Some explanation for this type of difference has been offered in the past (e.g., Betchov and Criminale 1966; Gaster 1968), but more concrete progress has recently been made by Koch (1985), Huerre and Monkewitz (1985), Monkewitz and Sohn (1988), among others. While the stability analysis of fluid flows is ubiquitous, the past analyses examine either the temporal or the spatial growth of perturbations. The principal point of the recent analyses, following the procedure set forth by Briggs (1964) and Bers (1975) in other contexts, is that they examine *transient* perturbations which could grow in *both* space and time; if, for certain conditions of the basic state, the analysis reveals the existence of instability modes of zero group velocity which grow temporally, the resulting bifurcation state is believed to be dominated by self-sustaining large-amplitude oscillations at some single, very pure, frequency. This has been termed absolute instability by Landau (e.g., Briggs 1964). In a rough equivalent sense, the flow is believed to develop global 'resonance' (e.g., Koch 1985) which then imparts this special pure-frequency characteristic to the new state. There is some experimental evidence for absolute instability in the wake of a circular cylinder (Strykowski 1986, Sreenivasan et al. 1987) and in the shear layer between two countercurrent shear flows (Ramshankar 1986), and some numerical evidence in the wake of a flat plate (Hennemann and Oertel 1987).

While, as can be expected from Fig. 1 b and the above description, the isothermal, constant density jets are not absolutely unstable, the analysis of Monkewitz and Sohn (1988) indicates that absolute instability could occur if the jet fluid is heated sufficiently strongly so that the density ratio is below about 0.7. An experimental verification of the basic outcome of the theory would be desirable, but it requires that, for air, the jet fluid be heated to temperatures of the order 500 K. This was thought to be somewhat cumbersome; instead, it was felt that the principal feature of the analysis could be tested by obtaining the required density ratios with the addition of varying amounts of helium to the jet fluid. This paper describes the results of a preliminary investigation. Besides this point of interest, our study has shown another phenomenon relating to a dramatically en-

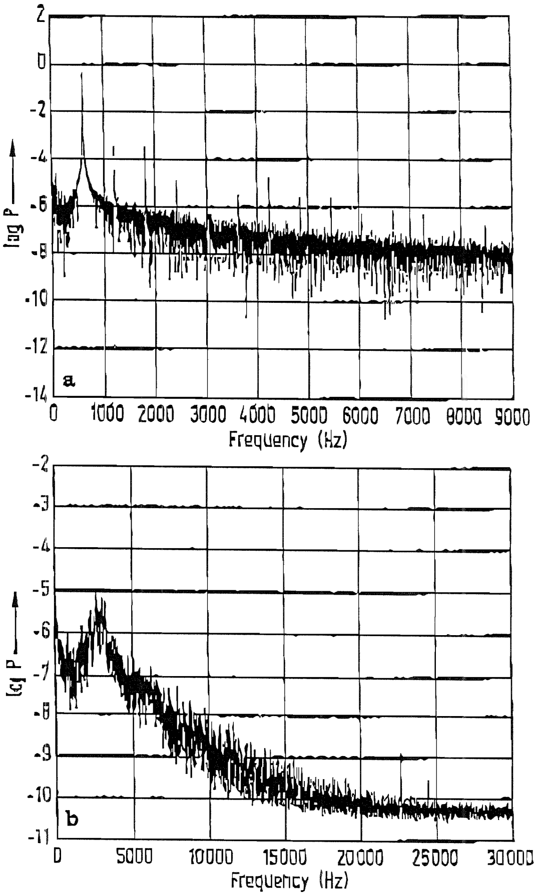

Fig. 1 a and b. Power spectral densities of the streamwise velocity fluctuation a in the shear layer of the wake of a circular cylinder, 10 diameters downstream, $Re \doteq 48$, and b in the shear layer of an axisymmetric cold air jet, $x/D \simeq 2$, $Re = 3000$; the ordinate in these and other spectral densities to be presented below is power in logarithmic units, to base 10

hanced spread, beyond a certain x/D, of low density jets compared to that of air jets. Together, these two form the contents of this paper. So far as we are aware, both observations are new. Another aspect, very briefly explored, is the lock-in and quasiperiodic behavior of low-density jets when subjected to external excitations, and the evolution to chaos at sufficiently large amplitudes of the excitation source.

2 Experimental arrangement

The jet facility consists of a stainless steel settling chamber with screens of varying gradation and a capability for attaching matching nozzles of different diameters. The details of the facility are described in Stein (1969). Measurements were made with nozzles of 4 and 6 mm. Each nozzle was contoured to make sure that there was no internal separation; and, at the nozzle exit, both the first and second derivatives of the contour vanished. Most of the data reported here was acquired (unless otherwise specified) with the 6 mm diameter nozzle, for which the contraction ratio was about 100:1. The turbulence level depended somewhat on the nozzle diameter used, but a typical value is 0.15%. Thus the jet cannot be considered a low turbulence facility.

Various densities of the jet fluid were obtained by commercially prepared mixtures of helium and air in different proportions; in some cases, the desired density ratio (that is, the ratio of mixture density to that of ambient air) was obtained by mixing the right amounts of air and helium in a stirrer. On a few random occasions, the proportion of gases in the mixture was tested on a mass spectrometer.

The jet velocity was measured by rotometers calibrated using a Pitot tube in conjunction with a Baratron pressure transducer (Type 370H-10) for low velocities and a mercury U-tube manometer for higher velocities. Hot-wires (5 μm diameter, 0.6 mm active length) operated on constant temperature mode on the DANTEC 55M01 anemometer were used to measure the 'velocity' fluctuations. Since the hot-wire sees species variation also, the hot-wire voltage signals in general do not strictly correspond to the velocity fluctuations alone, but we shall argue in Sect. 3.9 that this is not a serious problem in the present flow. The hot-wire was always positioned along the jet centre-line, but our exploratory measurements have suggested that the important aspects to be discussed below are independent of the radial position. Power spectra were obtained on a single channel HP Spectrum Analyser (model 3561A).

3 Results

3.1 Non-dimensional parameters

There are many parameters in the problem, and a detailed discussion is required for deciding the important ones. The Monkewitz-Sohn theory assumes that two important parameters are the jet Mach number and the density ratio between the nozzle and the ambient fluids. On this basis, we take Mach number M (based on the speed of sound in air) and density ratio S as the two most important parameters. Another important parameter in the theory, namely the difference of the jet center velocity and external velocity divided by the sum of the two, is trivially unity in this case. The buoyancy effects are believed to be small since the gradient Richardson number was typically estimated to be on the order of 10^{-3} at the nozzle exit. The Reynolds number is unimportant in the theory since it is inviscid. We therefore expect *a priori* some deviation from the theory at low Reynolds numbers, but the flow details beyond a certain Reynolds numbers may be independent of it as the theory assumes. It should be pointed out that experiments to be reported here do not allow enough variability in the available parameters to be definitive on this point.

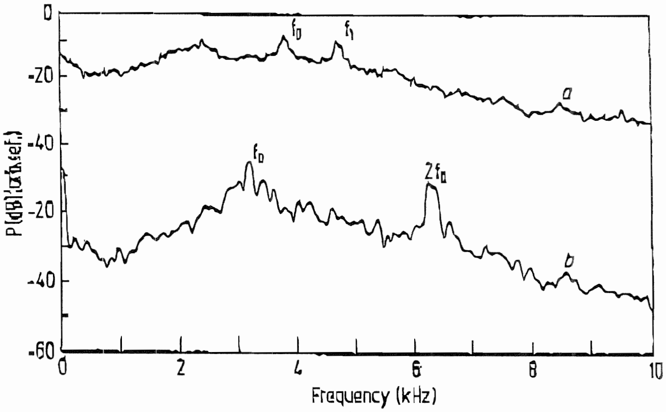

Fig. 2. Two typical power spectral densities in the near field of cold air jets illustrating two different behaviours; the helical mode marked f_1 in the upper figure is absent in the lower figure, where a harmonic appears; x/D in both cases is approximately 2

3.2 *The cold air jet*

To provide a useful contrast, we point out a few salient features of constant-density air jets. As a general rule, distinct but broadband peaks corresponding (as we shall argue in Sect. 3.4) to the axisymmetric and helical modes, are observed; these are identified as f_0 and f_1 respectively in Fig. 2a. The ratio f_0/f_1 is about the same as that observed by Drubka and Nagib (1981). The source of the peak at the lower frequency of about 2,400 Hz is not clear, but it must correspond to some combination of f_0 and f_1. There is no clear pattern on the relative amplitudes of f_0 and f_1. One or the other may be dominant depending on the precise exit velocity profile, distance downstream, turbulence level, etc (Drubka and Nagib 1981). Some flow conditions (such as when there is a mild acoustic excitation) do not favor the presence of the helical mode, and only f_0 and harmonics appear (Fig. 2b). When the subharmonic $f_0/2$ appears, both f_0 and $f_0/2$ components grow with distance downstream (with the subharmonic lagging behind), but decay eventually (with the fundamental decaying earlier). More detailed discussion of the constant-density jet dynamics can be found in Drubka and Nagib (1981) and Ho and Huerre (1985), and in references cited there.

3.3 *Power spectral density in helium-air jets*

With change in density ratio, the character of the spectral density changes. For certain values of M and S, the f_0 mode becomes extraordinarily energetic and sharp; sharp peaks simultaneously appear in the harmonics of f_0. The sharpness of the peaks can be characterized by defining (arbitrarily) a 'quality factor' $Q = f_0/\Delta f$, where Δf is the width of the spectral spike at a power level two orders of magnitude below its peak value at f_0. Q is of the order 1 for the air jet of Fig. 2a and b, but is of the order 10 or higher for the helium jet of Fig. 3.

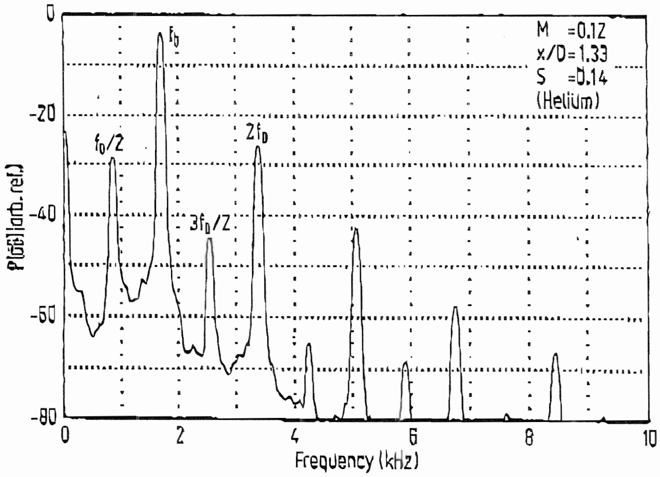

Fig. 3. Power spectral density in helium jet, showing large peaks at f_0, and its harmonics and subharmonic; note the qualitative resemblance to Fig. 1a

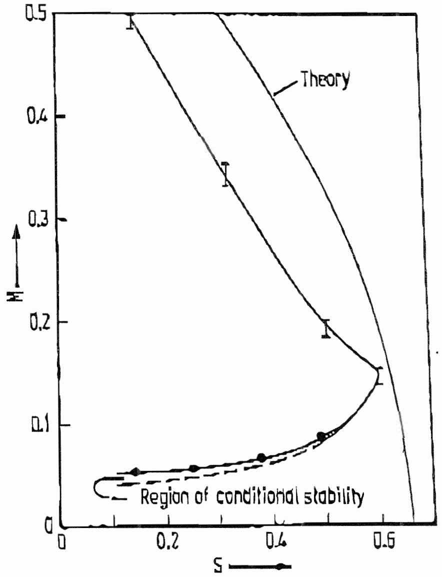

Fig. 4. The region of absolute instability in the Mach number (M)-density ratio (S) plane, compared with the Monkewitz-Sohn theory for a cylindrical vortex sheet; the uncertainty in experimentally marking the boundary is shown; the region of conditional instability is shown at the bottom of the diagram

3.4 *The region of absolute instability in the Mach number-density ratio domain*

The boundary in the M-S plane between the two types of spectra discussed above is quite well defined, and in Fig. 4 we plot that boundary. For these measurements, the hot wire was positioned 1.33 diameters downstream of the nozzle (diameter 6 mm). The possible probe interference effects on the determination of the boundary are discussed in Sect. 3.8. The precise definition of the Q factor is irrelevant for defining the boundary because changes across it are rather sharp. There was some hysterisis effect near the lower boundary; that is, the observed boundary was different depending on whether one approached it from above or below. The transition Mach number was smaller when approached from above, and larger when approached from below. The difference in the two boundaries obviously marks the region of conditional instability.

Figure 4 also shows the theoretically predicted boundary for absolute instability in hot jets by Monkewitz and Sohn (1988). The theory is linear (whereas the perturbation amplitudes are fairly large), does not account for the non-parallel growth of the flow, and ignores viscous and buoyancy effects; the particular set of calculations predicting this curve further assumes that the shear is concentrated in a cylindrical sheet. Although calculations have been made by Monkewitz and Sohn (1988) for distributed vorticity, the essential point can be illustrated by the comparison made in Fig. 4. The trends in experiment and theory are the same, but some differences do exist. The most important qualitative difference occurs in the lower part of the diagram. Our first thought was that this might be a low Reynolds number effect, but subsequent experiments by Kyle (1988) have shown that the corresponding boundary in a 12.7 mm nozzle is substantially lower (if it exists at all). We speculate that the difference could well be related to the small nozzle diameter in the experiment. The reasonable agreement observed in the figure could be interpreted in the most favorable light as being due to the fact that the theory incorporates the essential physics of the problem in spite of the several simplifications it invokes.

As we observe the evolution of the spectral densities while crossing into the region of absolute instability, we have noted that the lower of the two peak frequencies, the one designated as f_1, disappears while the higher frequency peak designated as f_0 strengthens. In a convectively unstable jet, f_1 may rival f_0 or dominate it depending on the precise situation, but the Monkewitz-Sohn theory predicts that in an absolutely unstable jet, the helical mode is significantly more stable than the axisymmetric mode. Noting that the f_1 peaks never appear in the absolutely unstable region, and that the observed f_0 is indeed axisymmetric (as verified in flow visualization, see Sect. 3.8), it appears reasonable to say that the f_1 mode is helical. The transition in the spectral character across the boundary is quite abrupt, as can be seen from Figs. 5 and 6.

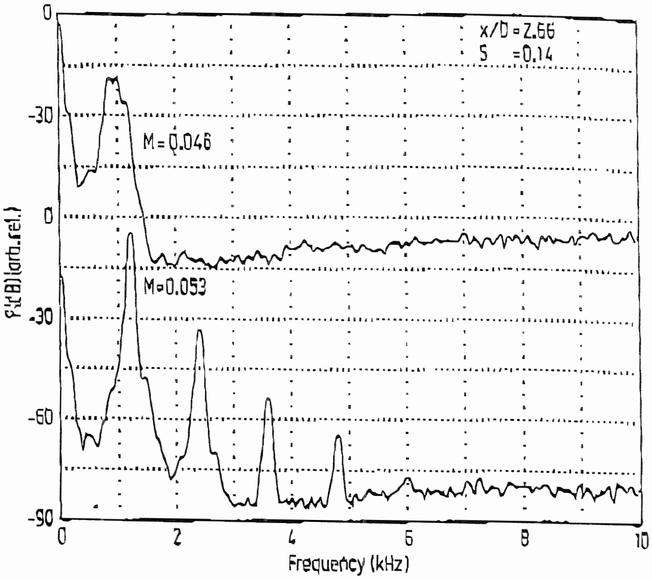

Fig. 5. Changes observed in the character of the spectral density at two different points across the boundary marked in Fig. 4

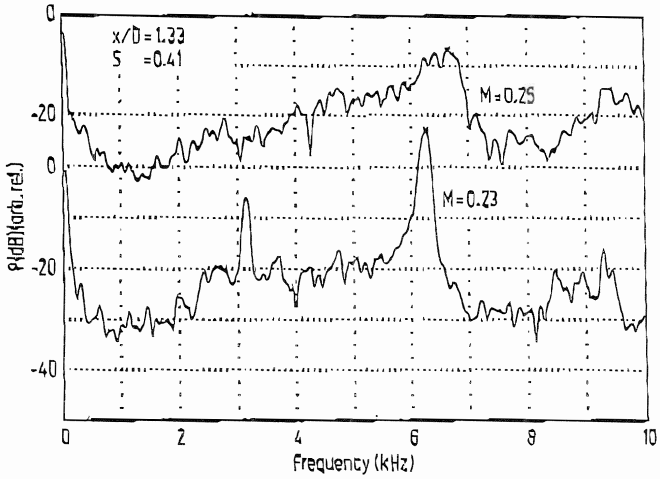

Fig. 6. Changes observed in the character of the spectral density at two different points across the boundary marked in Fig. 4

3.5 Insensitivity to noise

The above remarks show that the observed pure frequency mode roughly corresponds to a state of absolute instability, but it is useful to explore this a bit further. It is obvious from the stability theory that, in a convectively unstable jet, the strength of the passing disturbance is proportional to the initial perturbation amplitude (Freymuth 1966; Morkovin and Paranjape 1971). Hence, if the perturbation to the jet is provided by an acoustic source, the velocity perturbation will be a monotonically increasing function of the acoustic field near the jet.

On the other hand, in an absolutely unstable jet, since the group velocity vanishes for at least one unstable mode, it is to be expected that disturbances grow in place without bound at least until the linear equations are no longer valid. Any upper limit to the growing disturbance must come from the balancing nonlinear effects. This limit is independent of the strength of the initial perturbation caused, for example, by an acoustic source.

This is an important concept: the occurrence of absolute instability bestows upon a system, after the appearance of the first bifurcation, a perfectly periodic state which can be described by a limit cycle to a high degree of accuracy (e.g.,

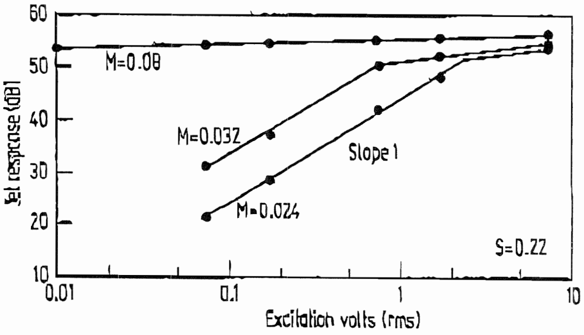

Fig. 7. The response of the jet to external perturbations in the case of absolute (top curve) and convective (bottom two curves) instabilities

Sreenivasan 1985). This is in contrast to the convectively unstable flows where the periodic state is strongly modulated by inherent randomness. In fact, if one uses the usual methods now in practice for characterizing deterministic chaos (such as the dimension of the attractor, Lyapunov exponents – for a description see Eckmann and Ruelle 1985) these measures confer the pedigree of chaos on convectively unstable systems which one knows *a priori* to be essentially periodic – even if noisy in the sense of Fig. 1b. The reason for this is not hard to understand. In a spatially developing flow where the length scales grow with downstream distance, the most amplified disturbance is of slightly different frequencies at different downstream distances, and what one observes at any spatial location is the collection of all those slightly disparate spatially amplified waves. The situation is more like a narrow-band-passed random signal, and is not appropriate for analysis according to the available tools of deterministic chaos.

In Fig. 7, the response amplitude of the jet to external excitation by a loud speaker is plotted as a function of excitation amplitude for three flow conditions. Two of them are in the convectively unstable regime (as per Fig. 4), and one of them (the top-most curve) is in the absolutely unstable regime. In the first two cases, the jet response increases by a factor of 10 (or 20 dB) for each factor of 10 increase in the root-mean-square voltage supplied to the speaker. If the acoustic pressure is proportional to the input voltage to the speaker, we can say that u'/U varies linearly with the acoustic pressure for the convectively unstable jet. As the excitation amplitude is increased further, the response level reaches a ceiling by virtue of nonlinear interaction.

For conditions corresponding to the uppermost curve in Fig. 7, the response of the jet is essentially independent of the excitation amplitude, suggesting (again) that this case corresponds to absolute instability.

3.6 *Helium jets with forced excitation*

As we remarked earlier, in absolutely unstable flows, the periodic state is very 'clean'. We have already remarked on the likelihood of this periodic state undergoing subsequent bifurcation which can be defined rather well. It is also likely that forcing this well-defined periodic state (in some orderly way) will itself yield some useful insight into the flow dynamics. This is done here in the spirit of the work described by Olinger and Sreenivasan (1988) in wakes behind oscillating cylinders; there, analogies between excited wakes and the sine circle map were quantitatively exploited.

A 8.5 cm diameter loudspeaker was placed in the stagnation chamber and driven by a signal generator for the purpose of forced excitation. The jet velocity was kept constant at 26 m/s ($M = 0.08$). The natural frequency (f_0) of the jet at this velocity is 960 Hz. The excitation frequencies (f_e) were chosen such that they were rational fractions of f_0. In the present experiments, ratios (f_e/f_0) in the vicinity of 1, 4/5, 3/4, 2/3, 2/5, and 1/3 were chosen. Figure 8 shows a typical power spectral density of the velocity fluctuations during a typical 'lock-in' phenomenon. The frequency ratio is 2/3 in this case. Note that the subharmonic ($f_0/2$) present in the unforced jet is completely absent in the excited jet but, instead, rational multiples of the excitation frequency (f_e) are seen. The regions of lock-in as a function of the amplitude of the excitation are shown in Fig. 9. The upper limit of the excitation amplitude was imposed by the distortion in the response of the loudspeaker beyond this voltage. The exact dynamics of lock-in and its effects on the structure of the jet are not known at present, but it is clear that chaos (in the form of broadband power spectral density) sets in at large excitation amplitudes. Although further work is clearly necessary to understand this phenomenon, it appears from experience with wakes and analogies of the present situation to them (Sreenivasan et al. 1987; Olinger and Sreenivasan 1988) that the helium/air jet dynamics in the absolutely unstable region has the potential of being modelled simply.

3.7 *Variation with axial distance*

So far, we have discussed measurements made essentially at one downstream distance in the jet, but it is important to establish the dependence, if any, of the flow properties on the spatial distance. As one indicator, we have plotted in Fig. 10 the power spectral densities of the streamwise velocity fluctuations at various distances downstream of the exist for a given Mach number and for a density ratio of 0.4. Note that the harmonics grow stronger for $x/D > 2$, and that the subharmonic evolution occurs beyond about an x/D of 3.5. As the measuring probe gets closer to the nozzle exit, there is a pronounced effect on the power spectral density (Fig. 11), this being analogous to effects observed by Strykowski and Sreenivasan (1985) in wakes. This led us to the study of the suppression of flow instabilities as discussed in the next section.

3.8 *The control of absolute instability*

First, a brief qualitative note on the effects of probe location on the jet instability is in order. For given M and S, once

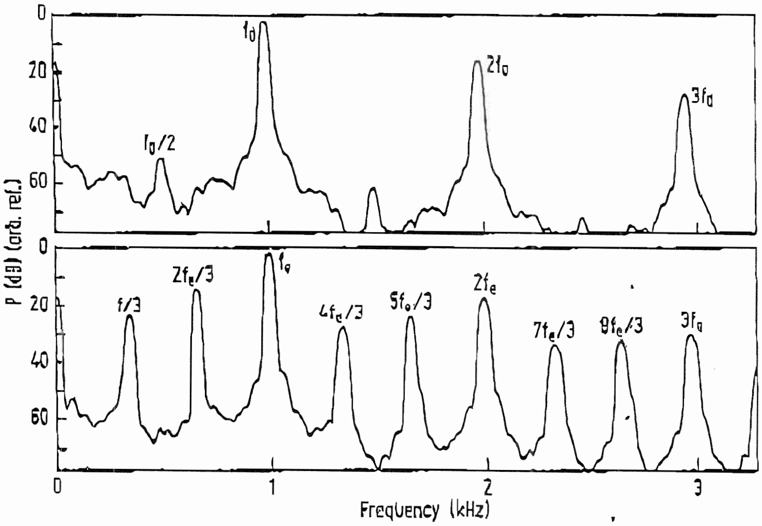

Fig. 8

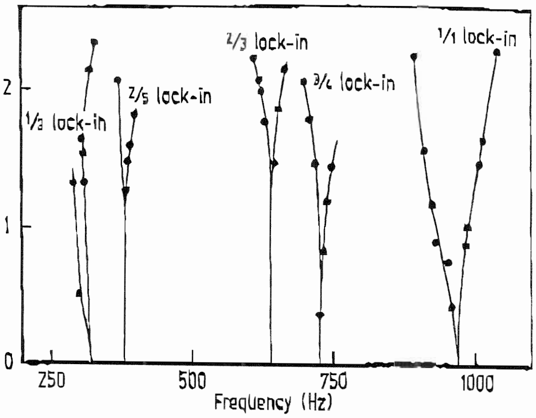

Fig. 9

Fig. 8. A typical power spectral density in the excited jet; note that in the bottom diagram, the only peaks to be seen are the excitation frequency and its rational multiples, while the peaks corresponding to f_0 and its harmonics, seen in the unexcited case (upper diagram), disappear

Fig. 9. Diagram showing various lock-in regions; if the jet is excited *around* any rational multiple of the jet frequency, it locks on exactly to the excitation frequency and its multiples. The various tongues show the lock-in regions for several excitation frequencies; the ordinate is the amplitude of excitation in arbitrary units

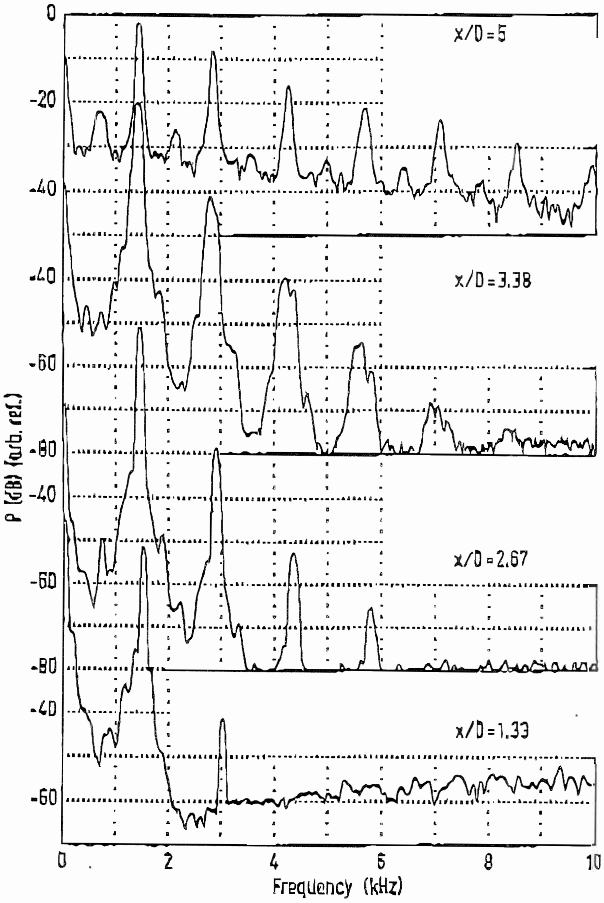

Fig. 10. Spectral density variation with respect to axial distance in a helium-air jet

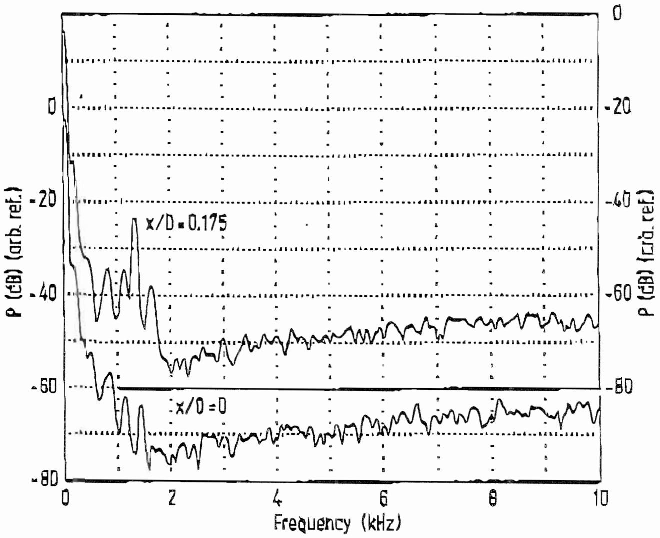

Fig. 11. Effect of the probe in the near field of the jet; note that this illustration corresponds to the extreme case of the probe at the nozzle exit

absolute instability was established, the frequency was virtually unaffected by probe location. For the combination of the nozzle diameter and probe size used, this statement is applicable only for $x/D > 1$. However, probe location in the region $x/D < 1$ resulted in large frequency changes. Further, for a given S, the value of the M corresponding to the transition between the absolute and convective instabilities depended on the streamwise location of the probe. In order

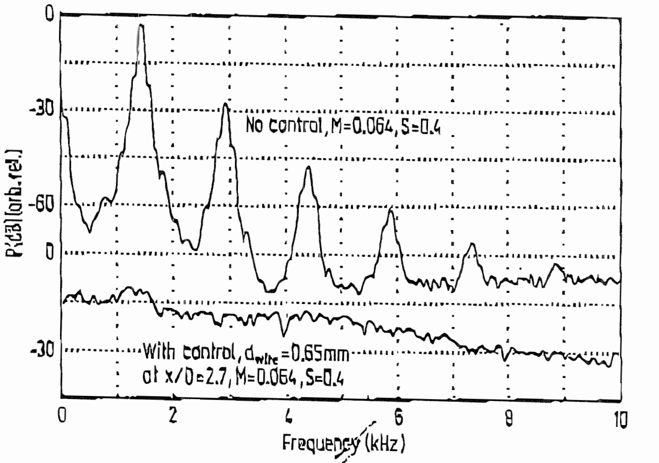

Fig. 12. Suppression or 'control' of strong spectral peaks, brought about by inserting diametrically at the nozzle exit a control wire of 0.65 mm

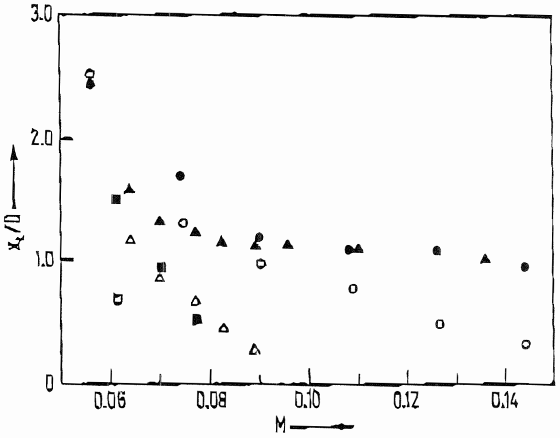

Fig. 13. The 'critical' (i. e. minimum) suppression distance for various combinations of density ratios and control wire diameters: □ $S = 0.14$, $d_{wire} = 0.38$ mm; ■ $S = 0.14$, $d_{wire} = 0.65$ mm; △ $S = 0.3$, $d_{wire} = 0.38$ mm; ▲ $S = 0.3$, $d_{wire} = 0.65$ mm; ○ $S = 0.4$, $d_{wire} = 0.38$ mm; ● $S = 0.4$, $d_{wire} = 0.65$ mm

to determine the marginal curve in the M-S plane without the probe effects, the following experiment was performed. The probe was brought in from very far downstream while repeatedly confirming the Mach number M_{tr} of transition from absolute to convective instability. The value of M_{tr} at which it starts to change is noted to be the appropriate one for that density ratio. This elaborate procedure yielded results that were not substantially different from those of Fig. 4, obtained at the fixed x/D of 1.33. In general, the smallest probe distance x/D for which M_{tr} is unaffected at a given S tends to be larger for smaller S.

Since the nature of instability could be affected by the presence of a foreign body, it seemed reasonable to examine this issue in a controlled manner. (The sensitivity of absolute instability to changes in geometry, and therefore in the mean velocity field, should not be confused with its insensitivity to external noise discussed in Sect. 3.5.) Two thin wires – called control wire hereafter – of diameters 0.65 and 0.38 mm were used for the purpose. A wire was placed (at some fixed streamwise position) in the center of the jet perpendicular to its axis. Figure 12 shows an example of the effect of control wire on the velocity spectrum. As can be seen, the resonance is completely eliminated. For given M and S, control (by which is roughly meant that one could alter the state of instability from absolute to convective type) could be achieved by placing the control wire within a definite region downstream of the exit. Figure 13 shows a plot of the maximum distance (x_c) at which control can be achieved for 3 different density ratios (0.14, 0.3 and 0.4) and two wire sizes (0.38 and 0.65 mm). A few salient features are worth mentioning here For a given Mach number and density ratio, control can be achieved at larger downstream distances using a larger control wire. The region within which control can be achieved decreases with decreasing density ratio for a given M – meaning that the instability is stronger at lower densities. Further, for all S the region of control decreases with increasing M and, as can be seen, beyond a certain Mach number control is not possible with the wire diameters experimented here.

3.9 Flow visualisation

The helium jet was seeded with fine droplets of Uranine solution in ethanol. A TSI 6-jet atomiser was used for generating the droplets. The droplets fluoresce when energized by light. The jet was illuminated by a thin sheet of laser light using a *Nd: YAG* laser. The light sheet thickness was on the order of 250 µm, and the pulse width was about 10 ns.

Photographs of laser induced fluorescence (Fig. 14a and b) show a sudden breakdown after a few diameters downstream. This abrupt breakdown, beyond which the jet spread is rather spectacular, occurs intermittently in the form of jetlets, and its details are not understood at present. We infer from a visual correlation of the pictures with the previous data that the region up to the point of abrupt spreading could be the region of absolute instability.

A closer look at the jet in the near-field region is shown in Fig. 15. It is seen that the instability mode is indeed axisymmetric as dicussed in Sect. 3.4. The scale of the vortices just before breakdown is a sizeable fraction of the jet radius. In fact, this picture suggests the breakdown occurs as soon as the size of the vortices gets comparable to the jet radius.

Kyle (1988) has measured the centerline concentration of helium as a function of axial distance from the nozzle. He has found that the concentration is essentially unchanged until the breakdown point, suggesting that the hot-wire located in this region responds *primarily* to velocity fluctuations, and not to concentration fluctuations. This statement is not correct for positions beyond the point of breakdown.

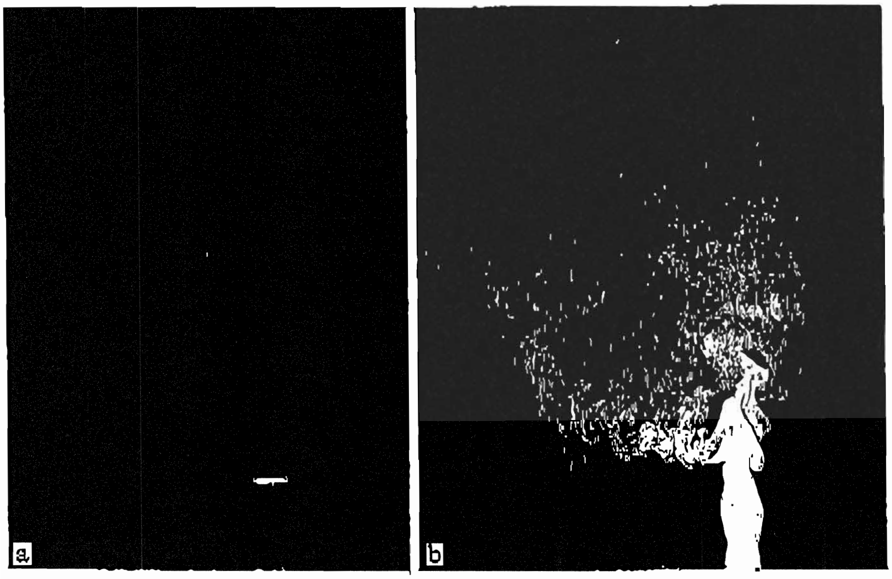

Fig. 14 a and b

Fig. 15

Fig. 14 a and b. Flow visualization pictures of the helium jet issuing from the nozzle into the ambient air; the growth rate beyond the break-up point is much larger than that of air jets; M = 0.064, S = 0.14, and Re = 1400

Fig. 15. A closer view of low density jets in the near field. Note that the scale of the axisymmetric vortex structures is comparable to the jet radius itself; M = 0.087, S = 0.3, Re = 7400, gradient Richardson number at the nozzle exit = 3.6×10^{-4}

4 Discussion and conclusions

We have investigated some features of jets of variable density produced by premixing various combinations of helium and air. The margin of absolute instability seems to follow the theory of Monkewitz and Sohn although there are differences between measurement and the theoretical prediction. We believe that the existence of the pure tone, as in wakes behind cylinders, enables us to study the initial development of bifurcation sequence *via* the dynamical systems approach. Some beginnings have been made.

The pure tone oscillations arising from this instability could be suppressed by means of a control wire placed in the near-field of the jet. The suppression of oscillations lies in the capability of the control wire to prevent the 'resonance' or 'feedback' in the near field of the jet. Since the strength of the absolute instability (as can be seen from the intensity of the velocity peak compared to the background) increases with decreasing density ratio, the efficiency of the control wire in suppressing the oscillations also decreases. Beyond a certain Mach number, flow oscillations cannot be suppressed by a given size control wire. However, the general idea of control has been established. We plan to explore other methods of control based on the present work.

Flow visualisation pictures show a drastically different structure of helium-air jets compared to normal constant-density jets. It appears from a visual correlation of these pictures and the spectral data that the organized roll-ups in helium jets break down rather dramatically just beyond the region of absolute instability. We believe that this sudden breakdown has to do primarily with the significantly larger energy stored in the roll-ups in the absolutely unstable jets as compared to the convectively unstable case.

Finally, we may note that Corrsin and Uberoi (1950) studied the spread of heated jets, and found that the asymptotic spread rate increases with increasing temperature (decreasing density); for $S = 0.51$, the ratio of the spread rate to that of the cold jets was about 1.7. But the 'spectacular' spread we refer to occurs at the end of the transition region, and is a local phenomenon related to the stability characteristics of the near field rather than the dynamics of the turbulent region – the latter having been the primary subject of Corrsin and Uberoi (1950).

Acknowledgements

These observations were first made in the summer of 1986, and were reported in Kyle (1986); we are thankful to Peter Monkewitz for a helpful discussion at the time. Mark Morkovin kindly commented on the draft version of the manuscript. A brief report of these results was made in the 1987 APS meeting in Eugene/OR (Kyle et al. 1987), for which we thank C. Meneveau. Financial assistance for this work was provided by AFOSR.

References

Bers, A. 1975: Linear waves and instabilities. In: Physique des plasmas (eds. Dewitt, C.; Peyraud, J.), pp. 117–213. New York: Gordon and Breach

Betchov, R.; Criminale, W. O. 1966: Spatial instability of the inviscid jet and wake. Phys. Fluids 9, 359–362

Briggs, R. J. 1964: Electron-stream interaction with plasmas (Research Monograph no. 29). Cambridge/MA: MIT Press

Corrsin, S.; Uberoi, M. S. 1950: Further experiments on the flow and heat transfer in a heated turbulent air jet. Washington/DC, NACA Rep. 998

Drubka, R. E.; Nagib, H. M. 1981: Instabilities in near field of turbulent jets and their dependence on initial conditions and Reynolds number. Illinois Institute of Technology, Chicago/IL, Tech. Rep. AFOSR-TR-82

Eckmann, J.-P.; Ruelle, D. 1985: Ergodic theory of chaos and strange attractors. Rev. Mod. Phys. 57, 617–656

Freymuth, P. 1966: On transition in a separated laminar boundary layer. J. Fluid Mech. 25, 683–704

Gaster, M. 1968: Growth of disturbances in both space and time. Phys. Fluids 11, 723–727

Hennemann, K.; Oertel, H. 1987: Numerical simulation of the absolutely and convectively unstable wake. J. Fluid Mech. (in press)

Ho, C.-M.; Huerre, P. 1985: Perturbed free shear layers. Annu. Rev. Fluid Mech. 16, 365–424

Huerre, P.; Monkewitz, P. A. 1985: Absolute and convective instabilities in free shear flows. J. Fluid Mech. 159, 151–168

Koch, W. 1985: Local instability chracteristics and frequency determination of self-excited shear flows. J. Sound Vibr. 99, 53–83

Kyle, D. 1986: Absolute instability in variable density jets. Dept. of Mechanical Engineering, Yale University, New Haven/CT, Rep. 86/FM6

Kyle, D. 1988: Laser-induced fluorescence images of helium/nitrogen jets. Dept. of Mechanical Engineering, Yale University, New Haven/CT, Rep. 88FMDK1

Kyle, D.; Raghu, S.; Sreenivasan, K. R. 1987: Periodic and aperiodic phenomena in variable density jets. Amer. Phys. Soc. 40th Annu. Meeting, paper FD4

Monkewitz, P. A.; Sohn, K. D. 1988: Absolute instability in hot jets. To appear in A.I.A.A.J.

Morkovin, M. V.; Paranjape, S. V. 1971: On acoustic excitation of shear layers. Z. Flugwiss. 19, 328–335

Olinger, D. J.; Sreenivasan, K. R. 1988: Nonlinear dynamics of the wake of an oscillating cylinder. Phys. Rev. Lett. 60, 797–800

Ramshankar, R. 1986: Experiments on a temporal analog of spatially developing free shear flows. Yale University, New Haven/CT, Rep. 86FM5

Sreenivasan, K. R. 1985: Transition and turbulence in fluid flows and low dimensional chaos. In: Frontiers of fluid mechanics (eds. Davis, S. H.; Lumley, J. L.), pp. 41–67. Berlin, Heidelberg, New York: Springer

Sreenivasan, K. R.; Strykowski, P. J.; Olinger, D. J. 1987: Hopf bifurcation, Landau equation, and vortex shedding behind circular cylinders. In: Forum on unsteady flow separation, (ed. Ghia, K. N.), pp. 1–13. ASME publication FED, vol. 52

Stein, G. D. 1969: Design of a multipurpose wind tunnel. Rev. Sci. Instrum. 40, 1058–1061

Strykowski, P. J. 1986: The control of absolutely and convectively unstable shear flows. Ph.D. Thesis, Yale University, New Haven/CT

Strykowski, P. J.; Sreenivasan, K. R. 1985: The control of transitional flows. AIAA Paper no. 85-0555

Received September 19, 1988