AIAA'85

APPENDIX A

AIAA-85-0540

The Control of Pressure Oscillations in Combustion and Fluid Dynamical Systems

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AIAA Shear Flow Control Conference

March 12-14, 1985/Boulder, Colorado

THE CONTROL OF PRESSURE OSCILLATIONS IN COMBUSTION AND FLUID DYNAMICAL SYSTEMS

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Abstract

This paper is devoted to the control of pressure oscillations. These pressure oscillations could be thermo-acoustic in nature (i.e., sustained by the thermal energy, most often released by combustion), or generated in a variety of circumstances determined by the mechanical and geometrical features of a fluid dynamical system. We provide a discussion of the basic concepts involved, and specifically demonstrate the control of:

(a) the flame-driven longitudinal oscillations in a relatively simple, yet typical, combustion system;
(b) the organ-pipe type resonance set-up by a loudspeaker in a vertical pipe;
(c) the 'whistler-nozzle' phenomenon set-up in a resonance chamber.

I Introduction

In many fluid dynamical systems, acoustic or pressure oscillations are generated by a combination of circumstances determined by the mechanical and geometrical features of the system. These oscillations are most often undesirable as, for instance, in an apparatus used for studying transition to turbulence, where even small-amplitude acoustic oscillations can force certain modes of instability in preference to others.

If the fluid dynamical system includes a source of thermal energy — typical examples being combustion systems such as ramjets, rockets, turbojets, etc. — and if the conditions (as discussed below) are proper, the pressure oscillations build up to levels that are not merely annoying but could be sufficiently violent to alter design conditions, lower efficiency and even damage the structure. (Under certain conditions, it is possible to increase combustion efficiency in the presence of pressure oscillations, if one is prepared to accept the accompanying increase in heat transfer rates and the deterioration of the integrity of the system.) The interaction between the oscillatory pressure and combustion processes — a problem encountered during the development of nearly every advanced propulsion system — has plagued combustion engineers for many years. The phenomenon of unsteady combustion resulting from such interactions is commonly referred to as combustion instability.

Up to now, solutions to combustion instability problems have been based on the concept of avoiding it from occurring. (If unavoidable, they are tolerated — provided the accompanying oscillations are not too severe!) Consequently, extensive research has been devoted to stability analyses in order to ascertain the conditions under which a system becomes unstable, and to evaluating the amplitude of the self-sustained oscillations by an analysis of the nonlinear response of combustion systems. The answers to these questions, although very important and helpful, are unfortunately system-sensitive. As a result, practical solutions to combustion instability problems are still largely based on "experience" and on trial and error methods.* The price one has to pay for avoiding combustion instability is quite often in the form of strong constraints on design as well as the range of operating conditions.

If, on the other hand, there is a mechanism for suppressing the acoustic oscillations actively by thermal means or otherwise (rather than passively by increasing damping), one can clearly extend the range of operation of a combustion system, leading to simpler designs and better efficiencies. Our goal is to develop techniques for the control of combustion instabilities. The basic mechanism is simply one of efficiently converting acoustic energy to thermal energy so that the acoustic oscillations are quenched and the system is stabilized. The idea of contolling instability by thermal means is not altogether new, but it is invariably discussed in the context of redistributing combustion processes in such a way as to avoid the instabilities. The emphasis in our work here is not on avoiding instabilities from occurring, but on suppressing them in situations where they are inevitable. Here, we demonstrate that this can be done in a simple, yet typical, combustion system. Using the same technique, we control (i.e., suppress or amplify as desired) the acoustic oscillations in two other systems, namely, loud-speaker driven organ pipe oscillation as well as the oscillations in the "whistler-nozzle" arising from a coupling between shear layer instability and resonance supply chamber. We believe that this concept of actively quenching pressure oscillations, and its successful demonstration here, will be of practical as well as theoretical interest.

Several months after this work was completed, Professor Ben Zinn of the Georgia Institute of

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* An excellent but brief account on the effort toward solving combustion instability in the past 35 years by one engine manufacturer, Pratt and Whitney Aircraft, is given by Dr. John Chamberlain (ONR/AFOSR Workshop on Mechanics of Instability in Liquid-Fueled Ramjets, CPIA Publ. 375, p.63-73, 1983). A more fundamental approach toward solving practical combustion instability problems is illustrated in the excellent monograph by A.A. Putnam entitled "Combustion-Driven Oscillations in Industry" (Am. Elsevier Publi. Co., N.Y. 1971).

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Technology kindly sent us a copy of a few pages of a chapter he wrote for a book entitled "Advanced Combustion Methods" (ed. Felix Weinberg), to be published in the not-too-distant future. In the book Professor Zinn discussed an example of suppression by thermal means as an illustration of the application of Rayleigh criterion to Rijke tube with two heating coils. However, the possibility of controlling combustion instability and other pressure oscillation by thermal or other external means has never been seriously undertaken and explored.

II The Theoretical Background

Consider a vertically held pipe, typically (as in our experiments to be reported here) about 125 cm long and 10 cm in diameter, open at both ends. Insert an open flame (typically from a burner about 2 cm diameter), say, about 10 cm from the bottom end of the pipe. See figure 1. Almost immediately, a loud, essentially pure-tone, noise can be heard; this noise is generally loud enough to be uncomfortable (and, on sustained exposure, even painful) to hear. It is to a description of conditions leading to the suppression of these oscillations that this section is devoted.

To understand the control mechanism, it is useful first to examine the physical processes responsible for the production and maintenance of the oscillations. When heat is added to a fluid element suddenly, its expansion causes pressure waves to be transmitted into its surroundings. The rate of expansion as well as the strength of the pressure waves depend on the rate of heat addition. In most instances, these waves will be reflected from boundaries of the system (such as the open ends in the example above). If the heat release rate is itself affected by the reflected waves (as is generally the case), new pressure disturbances will be generated which may reinforce or weaken the reflected waves, depending on the phase relationship between them. When there is a sufficiently strong reinforcement, the system will exhibit the phenomena of thermal acoustic instability and thermally-sustained oscillations.

A rough understanding of the conditions for reinforcement are known for a long time. Rayleigh[1] suggested that "If heat be given at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged." Using this criterion, Rayleigh satisfactorily explained several thermoacoustic phenomena known at that time. In spite of this success, it is not clear how Rayleigh deduced this apparently general criterion for thermo-acoustic instability, and so, many attempts have been made to verify it. But, it was Putnam and Dennis[2] who first proposed a formula as a mathematical representation of Rayleigh's "hypothesis". The most definitive treatment of the problem is due to Chu.[3] In the part of the analysis relevant to us here, Chu derived an equation for the rate of change of energy of a small amplitude disturbance in a viscous, heat-conducting, compressible medium, and showed that the rate at which energy is transferred to the 'acoustic mode' is given by

$$\int d^3\underset{\sim}{x} \;\; \dot{Q}'T'/\overline{T}(\underset{\sim}{x}), \tag{1}$$

where $\dot{Q}'(\underset{\sim}{x},t)$ and $T'(\underset{\sim}{x},t)$ are respectively the fluctuating components of heat release rate per unit volume and the temperature fluctuation at the location $\underset{\sim}{x}$ and instant t, $\overline{T}(\underset{\sim}{x})$ is the mean temperature at $\underset{\sim}{x}$, and the integration extends over the region where the heat is being released. Whether or not a disturbance in a system will be amplified depends on whether the integral of the above expression over a period, i.e.,

$$\oint dt \int d^3\underset{\sim}{x} \; \dot{Q}'T'/\overline{T}(\underset{\sim}{x}) \tag{2}$$

exceeds the total loss (over a period) of energy through dissipation and acoustic radiation from the system.

The implication from (1) and (2) above is that the condition most conducive for the growth of the disturbance is that the heat release rate fluctuation $\dot{Q}'$ and the temperature fluctuation T' must be in phase. If the effects of entropy fluctuation are negligible, the temperature fluctuation T' and the pressure fluctuation p' are in phase. In such cases the condition for the growth of the acoustic mode depends on the phase relationship between $\dot{Q}'$ and p', and can be written as:

$$\oint dt \int d^3\underset{\sim}{x} \; \frac{\gamma-1}{\gamma} \dot{Q}'p'/\overline{p}(\underset{\sim}{x}) > \text{losses}, \tag{3}$$

where $\overline{p}$ is the mean pressure and γ is the ratio of specific heats at $\underset{\sim}{x}$. Written in this way, the close relationship between Chu's results and Putnam-Dennis' proposed representation of the Rayleigh criterion is obvious.

A corollary of the above discussion is that the energy of the acoustic disturbance will diminish if

$$\oint dt \int d^3\underset{\sim}{x} \; \frac{\gamma-1}{\gamma} \dot{Q}'p'/\overline{p}(\underset{\sim}{x}) < \text{losses}. \tag{4}$$

This condition is conservative in the sense of being sufficient and necessary.

Equation (4) can be viewed essentially as quantifying the second half of Rayleigh's criterion, namely: "If heat be given at the moment of greatest rarefraction, or abstracted at the moment of greatest condensation, the vibration is discouraged."

Returning now to the system shown in figure 1, when heat is imparted from the flame to the surrounding air, two effects are produced. The heated fluid rises in the gravitational field, causing an upward draft through the pipe. The second effect, as already mentioned, is to produce a compression wave which gets reflected from the open ends of the pipe. The boundary conditions constrain the pressure oscillation p' to assume a shape somewhat like that shown in figure 2(b), oscillating back and forth with the maximum as well as minimum occurring alternatively at the mid-section of the pipe.* Associated with the fluctuating pressure is the fluctuating velocity superposed on the mean updraft. Corresponding to the instant when the mid-section

* The presence of the flame in the pipe makes this only an approximate representation, but the differences are not important for a qualitative discussion.

pressure in the pipe is the least (as shown by the dashed line of figure 2(b)), the gas column is in the most rarefied state and the fluctuating velocity u' is everywhere zero (see dashed line in figure 2(c)). As the pressure in the tube starts to rise (for example, to a state shown by the dot-and-dashed line in figure 2(b)), the fluctuating velocity field u' must consist of an inward movement both at the top and bottom ends of the pipe. The inward movement of air at the top is negative in the sign convention shown in figure 2(a), while the inward movement of air at the bottom end is positive. Hence, the velocity fluctuation assumes a mode shape shown by the dot-and-dashed line in figure 2(c), and u' will increase from zero to a positive velocity at the lower end while it will decrease from zero to a negative velocity at the upper end of the tube. The phase relationship between the velocity and pressure is now clear: u' leads p' by $\pi/2$ in the lower half of the tube, u' lags behind p' in the upper half of the tube. The same result could be seen mathematically. To a first approximation, neglecting losses, the equation governing the 'acoustic' field is given by

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} . \qquad (5)$$

If $p' \sim \Sigma(x)\, e^{i\omega t}$, corresponding to the shape shown in figure 2(b), it is easy to see from (5) that

$$u' \sim |\Sigma'(x)| \quad e^{i(\omega t \pm \pi/2)},$$

where the prime indicates derivative with respect to x, and + and - signs hold respectively in the upper and lower halves. Thus, the velocity fluctuation u' leads the pressure fluctuation p' by $\pi/2$ in the lower half of the pipe, while u' lags p' by $\pi/2$ in the upper half. Consequently, the pressure and velocity fluctuations measured as a function of time at typical points in the lower and upper halves of the pipe will look as shown in figure 3.

Let us recall that our interest is in finding the phase relation between the temperature (or pressure) fluctuation and the heat release rate fluctuations. It is easy to see that this latter quantity is influenced by the fluctuating velocity field; for, the area of the flame front and the extension of the flame sheet where combustion occurs are both sensitive to fluid motion. An upward fluctuating velocity will reinforce the mean draft, thus increasing the flame area and the rate of heat release. A downward velocity fluctuation produces the opposite effect. The precise phase lag between the fluctuating velocity and the fluctuating heat release rate cannot be deduced by qualitative arguments, and, in the absence of a full-fledged theory, one may resort to direct measurement of the heat release rate fluctuations. Such measurements are difficult to make, but one can obtain a quantitative appreciation of the details involved by replacing the burner in figure 1 by an electrically maintained heating element (see figure 4); here too a similar acoustically driven instability is observed. This is indeed the classical Rijke tone.[4] Carrier[5] calculated the response of the heater configuration shown in figure 4 to this environment of fluctuating flow field by approximating the heater by a series of parallel plates each of which is placed in an infinite fluid medium. We shall omit all details of Carrier's analysis, but make use of his important result, namely, the heat release rate fluctuation $\dot{Q}'$ lags u' by approximately $3\pi/8$. Combining this result with figure 3, the overall situation is as shown in figure 5. For the heater in the lower half of the pipe, $\dot{Q}'T'$ is positive for most of the cycle so that its average over the cycle is substantially positive. If the heat source is sufficiently strong the integral (2) exceeds the losses, and self-sustained oscillations are set up. On the other hand, if the heater is located in the upper half of the pipe, u' lags p' by $\pi/2$ and so, keeping Carrier's result in mind, we can easily see that $\dot{Q}'T'<0$ for most of the cycle. The situation is thus stabilizing. It is this latter characteristic of the heater that suggests a method of controlling or suppressing the acoustic instability in a combustion system. We conclude that to quench the oscillations created by a heater or a combustion source in the system, it is only necessary to install another heater — a "control heater" — in the upper half of the tube. This is indeed what happens.*

One may be inclined to think that for complete stabilization the power needed for the upper heater must be comparable to that acquired at the lower or primary heater. A little thought will show that the former need only be a small fraction of the latter. For, in a steady-state periodic oscillation, the total energy supplied to the acoustic mode per cycle of oscillation must be equal to the total energy loss per cycle by dissipation and by acoustic radiation. But the acoustic energy accounts for only a very small fraction of the power supplied to the primary heat source, which assumes the additional role of heating up the incoming gas. Even if the secondary heat source extracts only a small amount of acoustic energy per cycle of oscillation, the amplitude of vibration is bound to decrease with time. The time required for quenching the vibration completely will be shorter with an increase of the power supplied to the secondary heat source, because the fluctuating component of heat release is directly proportional to the temperature of the control heater above the surroundings.

IV Results on the Suppression of the Rijke Tone

A primary result of interest is the ratio of power supplied to the control heater to that required to be supplied to the primary heat source for the maintenance of the oscillations. For convenience of obtaining data on this power ratio, both the primary and control heater were maintained electrically in some of our experiments; our estimates show that approximately similar numbers are valid for the flame driven oscillations of figure 1 quenched by an electrical heater. Both heaters are of similar construction, and each consists of a nichrome ribbon folded around a flat thin annular ring of asbestos. The solidity is approximately 40%.

* We are grateful to Professor Ben Zinn for calling our attention to a paper of Collyer and Ayres[6] in which they mentioned a finding of D.H. Edwards and M.J. Beckensall similar to our own. Unfortunately, a literature search failed to yield a published account of this research.

The discussion in the last paragraph of the previous section shows that the power ratio required for contolling the pressure oscillation depends primarily on two factors: (a) the level of the oscillations and the percentage amplitude reduction sought, and (b) the duration between the instants at which the control heater is turned on and at which the desired level of reduction occurs. Let us indicate this as the 'waiting time'; its magnitude depends, among other things, on the heater design as well as its precise location in the pipe. (In our exploratory study, the "waiting time" is relatively long — of the order of 10 sec. This is chiefly a reflection of the large thermal inertia associated with the crude, first-generation heater built for this study. An improved design which is now complete has cut down the waiting time by a factor of order 10.) In figure 6 are shown the power ratio data for a fixed waiting time (arbitrarily chosen to be about 20 sec). The primary heater is located at L/4 from the bottom, while the data are presented for three positions L'/L of 0.25, 0.17 and 0.13 of the control heater; L here is the total length of the pipe and L' is the distance of the control heater from the top end. The inset also shows (for L'/L = 0.17) the oscillograms of the pressure oscillations for different values of the power ratio. For these oscillations, we may note that the half-wave length is equal to the pipe length, which indicates that the heaters do not significantly alter the flow and pressure oscillations.

It is seen that the best position for the control heater, in terms of the least power, is L'/L = 0.13. The power ratio is of the order of 9% for a thousand-fold reduction in acoustic power.

Figure 7 shows the power supplied for complete quenching as a function of waiting time. For waiting times of the order of 80 sec, one requires slightly more than a third of the power required when waiting times are of the order of 10 sec.

One consequence of the insertion of the control heater is the alteration of the temperature distribution of the original set-up. Figure 8 shows a typical temperature distribution in the pipe when both heaters are electrically maintained. The temperature rises nearly discontinuously at both heaters, with the rise at the control heater being small compared with the overall rise. Dashed lines represent an ideal distribution.

Table I supplies some useful data on the power ratio as well as the additional temperature rise in the presence of the control heater. For waiting times of the order of 80 sec (for a particular set-up), one pays the price of a 3% power and about 2% additional temperature rise before achieving total suppression. Impressive though these numbers are, one must note that the power required at the control heater is large compared with the acoustic power itself. Based on measurements made with a calibrated microphone, we estimate that the acoustic power is only about 0.03% of the primary heater power, so that the power required to quench the oscillations completely is at least an order of magnitude higher than the acoustic power. With improvement in heater design, it should be possible to reduce the power ratio required for complete quenching by an order of magnitude or more.

Table I

Waiting time, sec	Power Ratio	Additional temp. rise	Control heater power / acoustic power
20	9%	7%	30
80	3%	2%	10

V Suppression of organ-pipe resonance set-up by a loudspeaker

Once the basic phenomena is understood, it is natural to attempt to extend this control technique to other situations also. Suppose now in the same vertical pipe as in figure 1, we set up a resonance by tuning a loudspeaker in such a way that the half-wave length of the excited oscillation is equal to the pipe length (see figure 9). To suppress this oscillation, all one needs is a heater anywhere in the upper half of the tube. Figure 10 shows the signals with the heater (located at H) off and on respectively. Clearly, the amplitude of the pressure oscillation drops off.

The signals of figure 10 were obtained by means of a condenser microphone mounted at the location c within the pipe. Since the loudspeaker exciting the organ pipe resonance is not perfectly directional, the microphone will naturally pick up the background signal even when the resonance in the pipe is completely suppressed. In fact, the oscillogram of figure 10(b) is very nearly the same in amplitude as this background level, and so, we conclude that the control heater has suppressed the resonance essentially completely.

Although the mechanism for suppression of resonance here is the same as that discussed in section IV, one additional comment must be made. In the previous case, there was a natural updraft set up by the flame or the primary heat source. This draft is essential to introduce the proper phase relation between $\dot{Q}'$ and T' (or p'). For, without a mean flow, the heat release rate from the heater will be a rectified sine wave when the fluctuating velocity is a sine wave, and can be considered to have two cycles for each cycle of the superposed velocity fluctuation. It is the mean flow that sets up the required bias so that the $\dot{Q}'$ follows u' with a phase lag. The control heater will here have to serve the dual purpose of setting up the mean flow, as well as supplying fluctuations in the heat release rate in proper phase with the pressure fluctuations. It is thus clear that the process here is not as efficient in extracting energy from the pressure oscillations as for the system discussed previously, although not much less so.

VI Control of the 'whistler-nozzle' phenomenon

As the next example, consider the whistler-nozzle phenomenon. Pressure oscillations here are set up because of the coupling between the shear layer instability and the resonance chamber. The most recent paper discussing this phenomenon in some detail is by Hussain and Hasan.[7] The particular set up used in our measurements is shown in figure 11. It consists of a 120 cm × 120 cm settling chamber contracting into a 12.5 cm diameter, 150 cm long circular section (to be called the 'test section') to the other end of which is attached a fifth-order axisymmetric nozzle of area ratio 25. Compressed air from large storage tanks enters the test section, and is spread into the settling chamber by means of a thick foam glued and stapled to the side wall. The flow goes through two screens and enters the test section via a 9:1 axisymmetric contraction. The nozzle at the other end of the test section has a small step, which is the crucial factor in establishing the resonance. The whistler-nozzle mechanism, roughly said, is as follows. The shear layer exiting from the nozzle impinges on the step. If the impinging shear layer has already developed a periodic component (by virtue of the most amplified disturbance), the impingement produces periodic pressure oscillations. If the frequency of these pulses is a multiple of one of the resonance modes of the chamber, a well-defined resonance gets established in the chamber.

The particular resonance chamber shown in figure 11 resonates at two speeds at the nozzle exit of 9 m/sec and 5 m/sec. The resonance frequency for the former case is about 160 Hz (the "high frequency mode") and 70 Hz for the latter case (the "low frequency mode"). The resonance is loud enough to be quite audible in the general working area, and is stronger in the low frequency mode. Our object here is to control (suppress or amplify) these pressure oscillations.

In figure 11, the mode shapes for both resonance conditions are also given. For the high frequency mode, the simultaneous measurement of the fluctuating pressure p' and the associated velocity fluctuations u' in the regions a-b (figure 12a) and c-d (figure 12b) shows that u' leads p' by $\approx \pi/2$. (Simple one dimensional analysis of the flow ignoring nonlinearities and losses shows that this phase difference must be exactly $\pi/2$.) The situation is analogous to the set-up in the lower half of the Rijke tube and the combustion device discussed in section IV. If a heat source of sufficient magnitude is located in these regions, from analogy with figure 5, it follows that the condition (3) holds, and the oscillations build up. On the other hand, in the region b-c (see figure 12 (c)), u' lags p' by $\approx \pi/2$ — a situation analogous to that in the upper half of the Rijke chamber. Again by analogy, it follows that a heat source located anywhere in the region b-c must stabilize the flow and suppress the oscillations.

This indeed is what occurs. Let us first consider the case of suppression. Figure 13(a) shows an oscillogram of pressure oscillations measured with a microphone located at some position in the test section; the heater is located at x_o in the region b-c (see figure 11(a)), but is off. Figure 13(b) shows the power spectral density of the signal. Notice that the ordinate in figure 13(b) is the logarithm (to base 10) of the power. The spectral density has a peak around 160 Hz as expected, and has minor harmonic content. The noise level is 5 or 6 orders of magnitude lower than the spectral peak. When the heater is turned on, the microphone output is typically as shown in figure 13(c), with its power spectral density shown in figure 13(d). The peak in figure 13(d) has diminished by approximately five orders of magnitude and is only one or two orders of magnitude above the basic noise level. The background pressure level and its power spectral density in the off-resonance conditions of the chamber are shown in figures 13(e) and (f). Clearly, the signal levels with the heater on are quite comparable to the background conditions of figures 13(c) and (d).

When the heater is moved to any location in the regions a-b or c-d, the oscillations are amplified instead of diminished.

Let us examine the low frequency mode shown in figure 11(c). Anywhere in the region a-d, the phase relationship between u' and p' (u' leads p' by $\approx \pi/2$; see figure 14) is such that a heat source located in the region will enhance the pressure oscillations. For a typical position x_o of the heater, figures 15(a) and (b) represent sample oscillograms. Figure 15(a) gives an oscillogram with the heater off, while figure 15(b) shows that with the heater on. Figure 15(c) represents the power spectral density of the pressure signal with the heater off. To suppress the oscillations, it is necessary to locate the heater outside of the test section, perhaps upstream of the 9:1 contraction, but that has not been attempted.

VII Discussion and conclusions

We have demonstrated the control of flame-driven (or electrically maintained) longitudinal pressure oscillations in a vertical pipe, of organ-pipe resonance maintained by a loudspeaker, as well as of the 'whistler nozzle' oscillations in a resonance chamber. In this paper, we claim no originality with regard to the basic idea itself. It has been in existence since the days of Rayleigh himself and, in a much clearer fashion, since the time this criterion was derived from first principles.[3] The control is effected by the application of the Rayleigh criterion, that is, by controlling the phase between the heat release rate $\dot{Q}'$ and the pressure oscillation p'. To be more exact, the problem reduces to making the integral of Putnam and Dennis[2]

$$\oint dt \int d^3\underset{\sim}{x}\, \dot{Q}'p'$$

sufficiently negative — a fact few will disagree. We have shown that <u>this can be achieved</u> by suitably locating a control <u>heater in a given flow</u> system. The contribution of this paper consists chiefly in the demonstration that combustion instability, or pressure oscillations arising for any other reason, can be <u>controlled</u> simply by the application of the basic idea spelt out by Rayleigh and by Chu.*

To apply the idea for control of pressure oscillations in any new system, one would ideally have to measure the phase relationship between $\dot{Q}'$ and p' (or T'). For many heater configurations, it seems that the phase lag between $\dot{Q}'$ and u' is given at least approximately by Carrier's analysis, and so, it is quite often only necessary to measure the phase relation between p' and u'. Generally speaking, u' measurements are much simpler than $\dot{Q}'$ measurements, but in environments of high temperatures the converse may be true. With such relatively simple measurements, it appears that the

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* Chu obtained a generalized Rayleigh criterion which accounts not only for the energy transfer due to the interaction of pressure fluctuations with a heat source, but also for that resulting from the interaction of entropy fluctuations and the fluctuating heat release rate.

application of the idea to more complex systems — especially those not involving very high temperatures — is relatively straightforward. It is of course true that in many practical situations one is often bedeviled by a variety of constraints, all of which have to be looked into in detail before a particular design of the control device can be selected.

We close our remarks with the observation that there are several other means of active control of pressure oscillations, and that the work in progress is aimed towards developing these newer means.

Acknowledgements

We gratefully acknowledge the financial support of the AFOSR. Thanks are due to Mr. P.J. Strykowski who constructed the whistler-nozzle apparatus used in this study.

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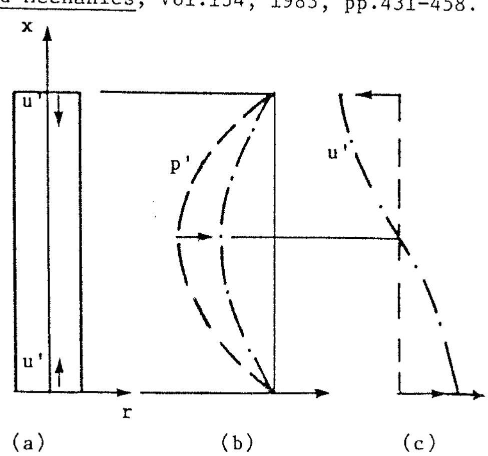

Figure 2: The pressure and velocity fluctuations in an environment of self-sustained oscillations.

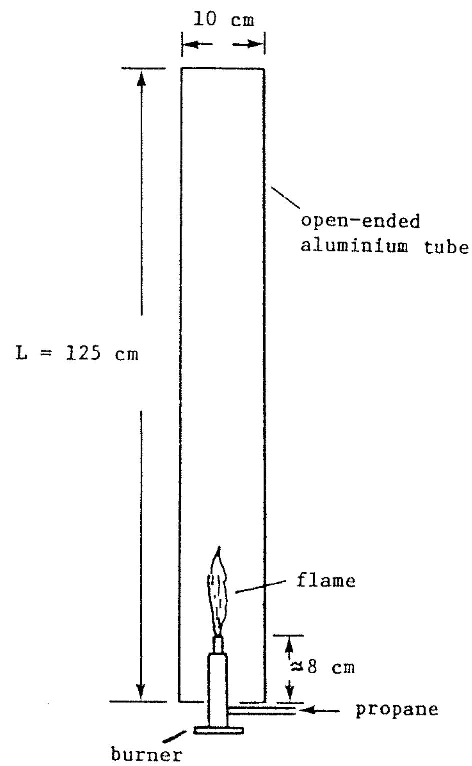

Figure 1: Schematic of the experimental apparatus.

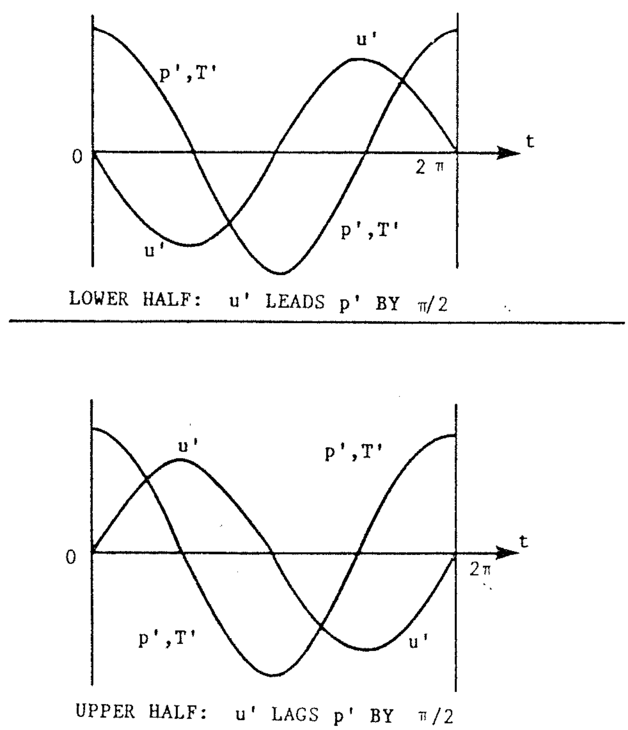

Figure 3: Schematic of variations with respect to time of velocity, pressure and temperature fluctuations at some typical positions in pipe.

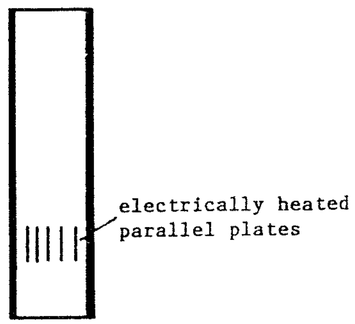

Figure 4: Schematic of configuration analyzed by carrier[5].

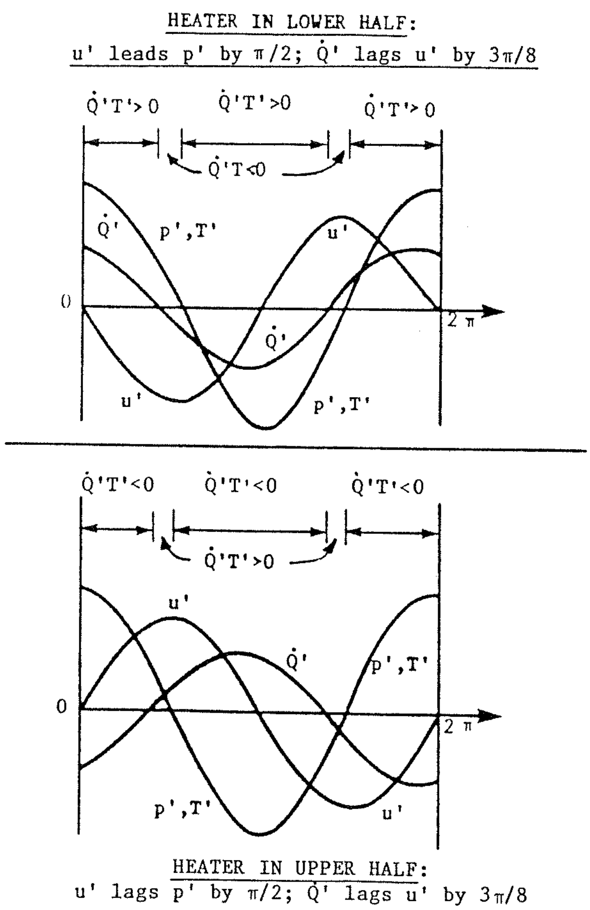

Figure 5: The phase relations between velocity, pressure (or temperature) and heat release rate fluctuations.

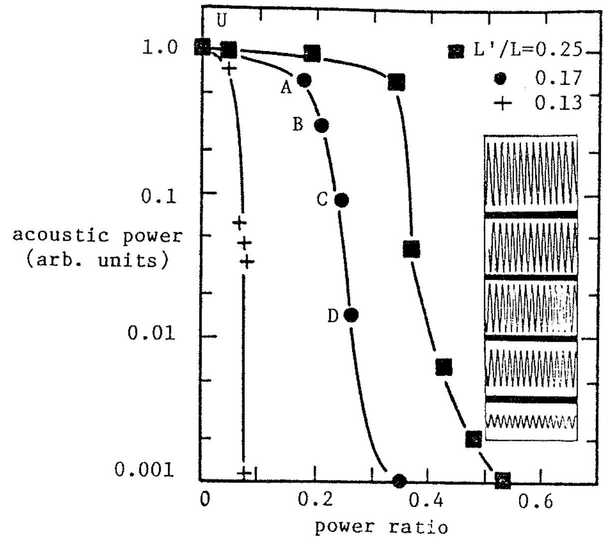

Figure 6: The reduction of the acoustic power attained as a function of the power ratio. Inset shows oscillograms (for L'/L = 0.17) of measured pressure fluctuations in the pipe. From top to bottom, they correspond respectively to U, A, B, C and D. The bottom three signals have a gain of 2 compared with the top two.

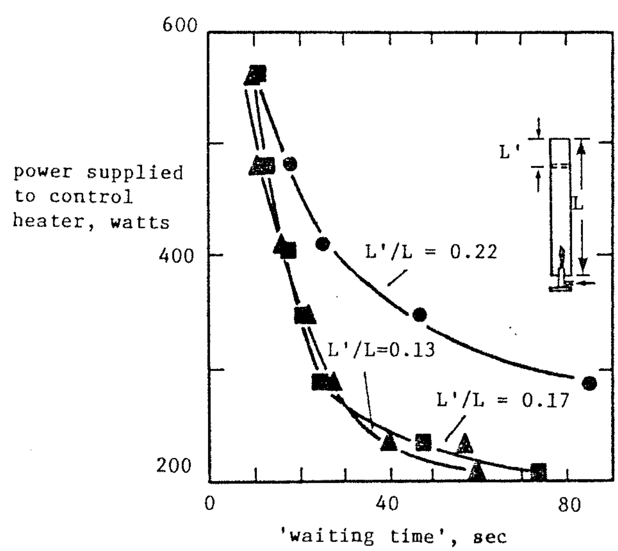

Figure 7: The 'waiting time' (that is, the time required for 'complete' quenching of oscillations) as a function of control heater power. Data are for flame-driven oscillations.

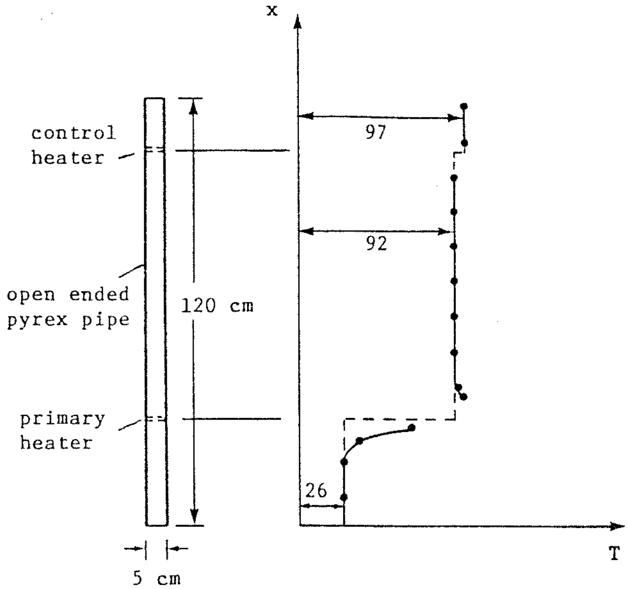

Figure 8: Measured temperature distributions (in °C) for a case of total control. Both heaters were maintained electrically. Dashed lines correspond to ideal distribution.

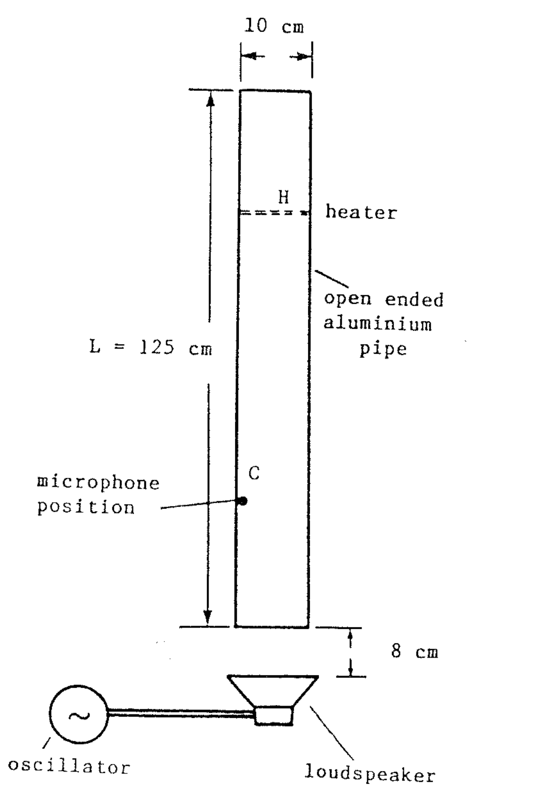

Figure 9: Schematic of set-up with loudspeaker-driven resonance oscillations. When the heater at H is turned on, the oscillations die out; see figure 10.

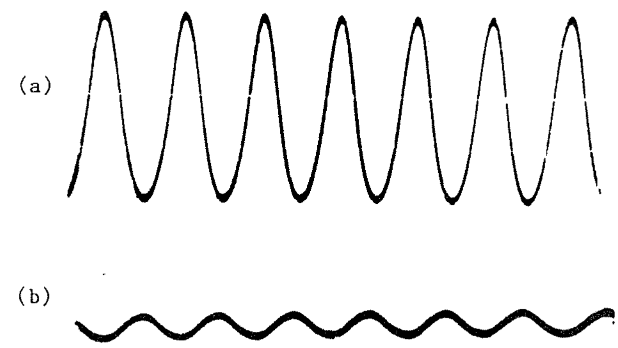

Figure 10: Pressure signals measured with a condenser microphone at C in figure 9: (a) with the heater off, and (b) with the heater on. The signal measured with the heater on is comparable to the background level outside the tube.

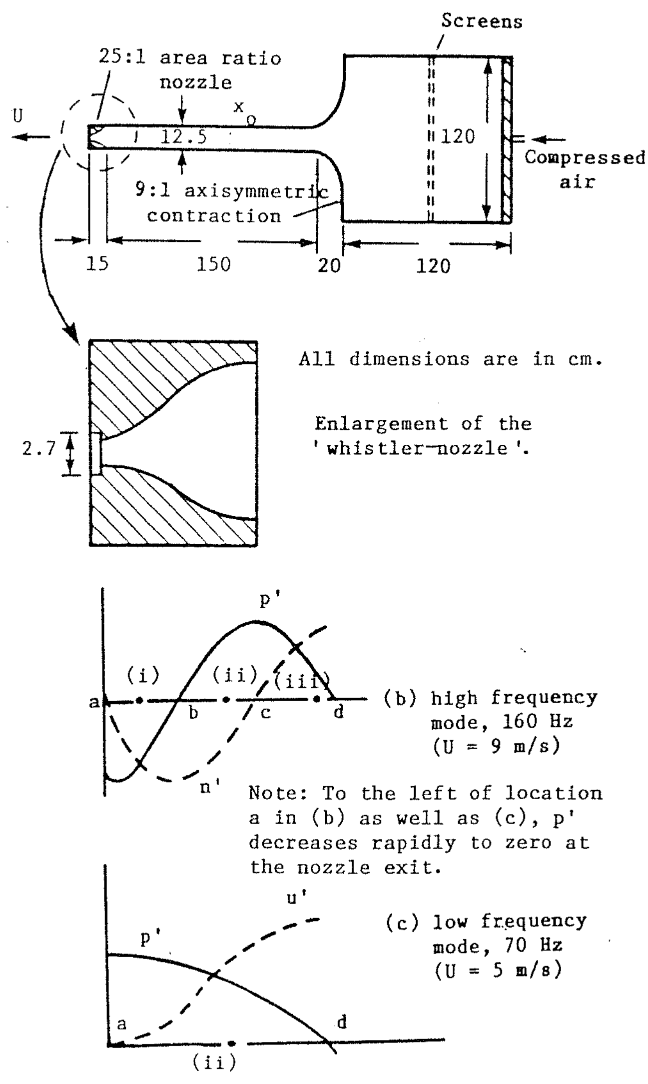

Figure 11: (a) The "whistler-nozzle' apparatus; (b) and (c) are the measured mode shape of velocity and pressure oscillations. The location a in the figures (b) and (c) corresponds roughly to the upstream end of the 25:1 nozzle.

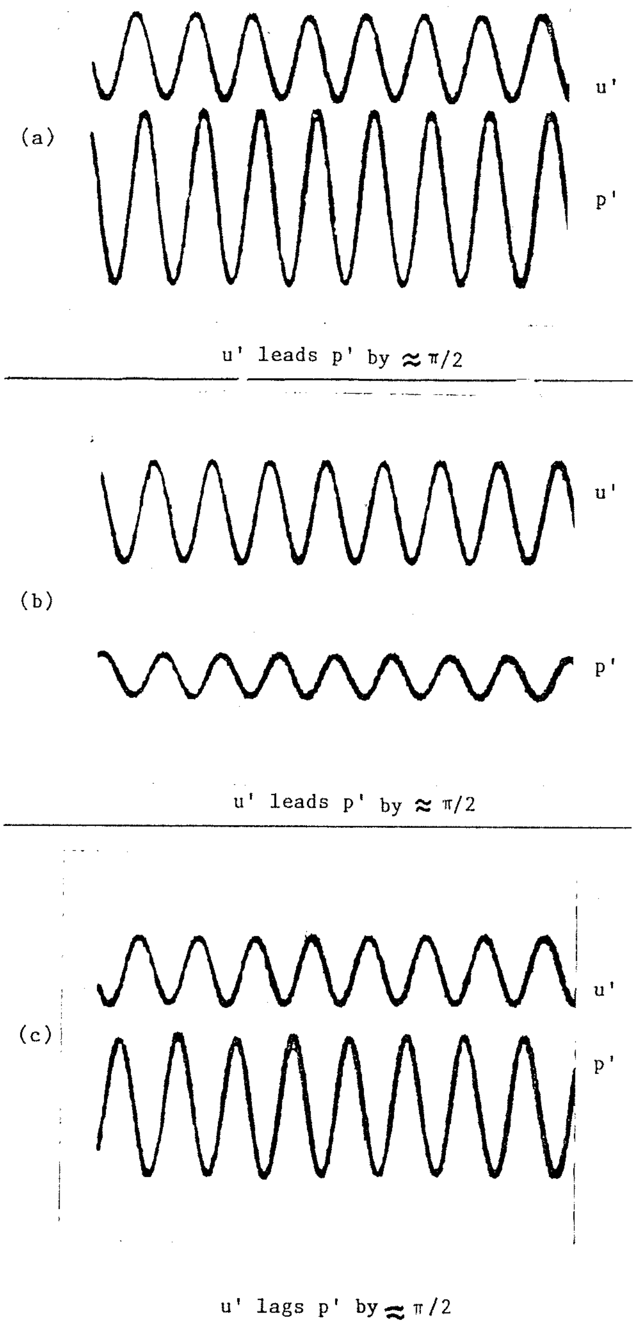

Figure 12: Velocity and pressure oscillations for the high frequency mode measured simultaneously in the test-section of the 'whistler-nozzle' apparatus of figure 11. Signals in figure 12(a) were obtained at station (i) of figure 11 (b), while signals of 12(b) were obtained at station (iii) of figure 11(b). Heater anywhere in the regions a-b or c-d of figure 11(b) amplifies the pressure oscillations. Figure 12(c) corresponds to measurements at (ii) of figure 11(b). Heater anywhere in the region b-c of figure 11(b) suppresses oscillations; see figure 13.

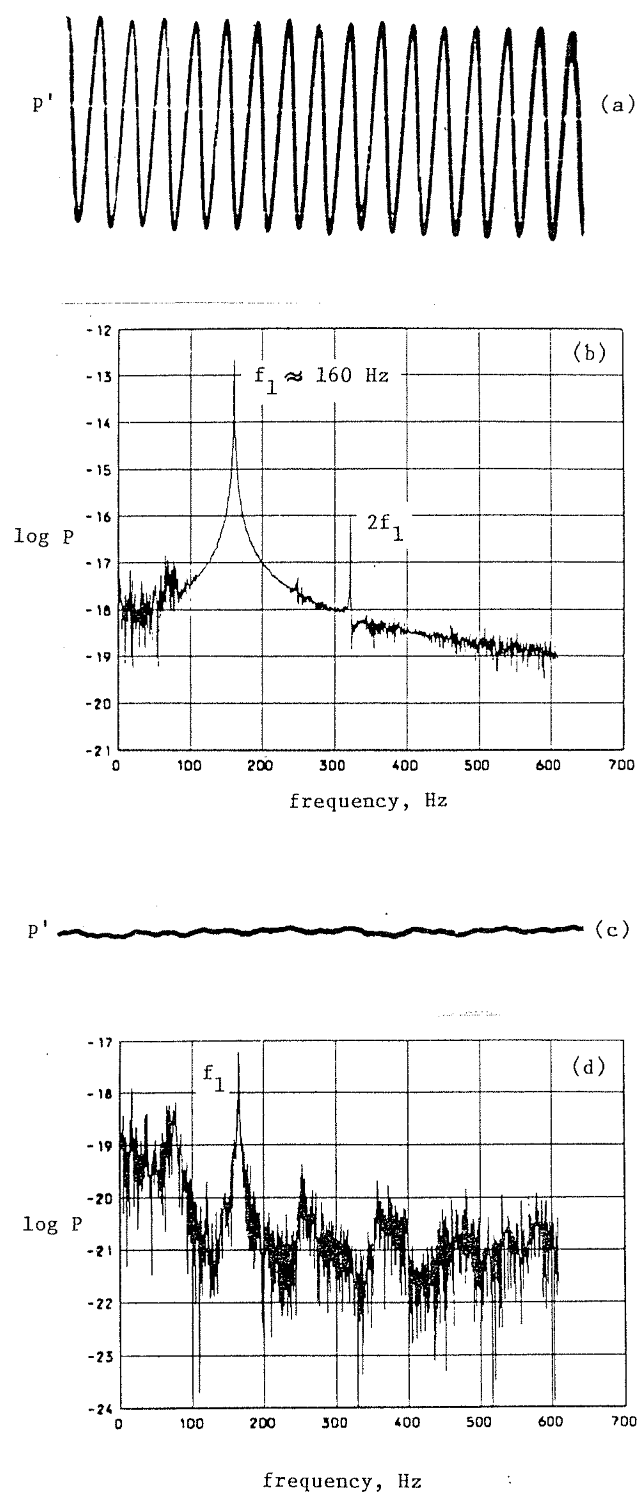

Figure 13 is continued on page 10.

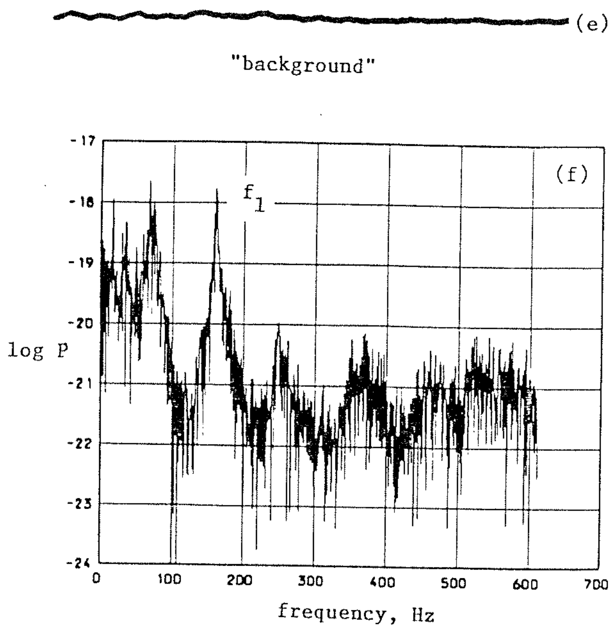

Figure 13: Oscillograms and spectral densities of pressure oscillations for the high frequency mode in the 'whistler nozzle'. (a) and (b) correspond to no suppression, (c) and (d) correspond to suppression with heater located at x_o in figure 11 (a); (e) and (f) are background fluctuation levels.

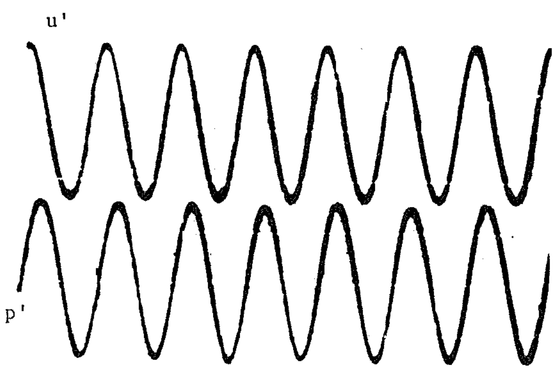

Figure 14: The velocity and pressure oscillations in the 'whistler-nozzle' for the low frequency mode. Measurements correspond to the station (ii) of figure 11(c). Note that u' leads p' by $\approx \pi/2$. Thus, heater located anywhere in the region a-d of figure 11(c) leads to amplification of oscillations; see figure 15.

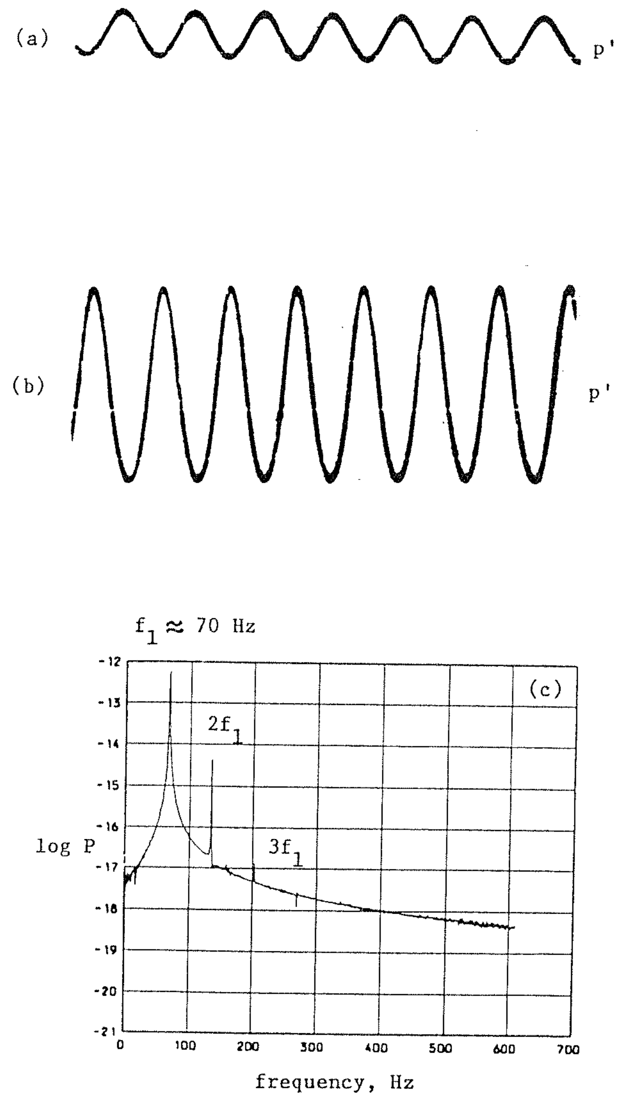

Figure 15: The amplification of pressure oscilla- for the low frequency mode in the 'whistler nozzle' when the heater is located at x_o in figure 11(a). Figure 15(a) is obtained with heater off, 15(b) with heater on; 15(c) is the power spectral density of the signal shown in figure 15(a).