TURBULENCE AND CHAOTIC PHENOMENA IN FLUIDS
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ON ANALOGIES BETWEEN TURBULENCE IN OPEN FLOWS AND CHAOTIC DYNAMICAL SYSTEMS

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We briefly study turbulence in open flow systems in the context of concepts developed in studies of chaotic dynamical systems. Although several flows have been examined, particular attention will be focussed on the question of transition to turbulence in coiled pipes; some degree of correspondence with the Ruelle-Takens-Newhouse route to chaos is indicated. Using the Grassberger-Procaccia algorithm, the dimension of the attractor for velocity signals during and immediately after transition to turbulence has been computed. Our results, such as they are, indicate that the dimension is relatively low. Brief comments will be made on the difficulties of computing the dimension, as well as on the relevance of strange-attractor theory to fully-developed turbulence.

INTRODUCTION

Recent studies of the dynamics of nonlinear systems with finite (and small) number of degrees of freedom have produced profound results with probable implications to the very notion of chaos — for example, in kinetic theory of gases in the context of the Boltzmann equation — but the interest of fluid dynamicists in these studies stems primarily from the notion of genericity, that is, the expectation that the qualitative properties of the Navier-Stokes equations are shared also by these simpler systems. A related important (and, to our knowledge, as yet untested) expectation is that turbulence, at least not too far away from transition, behaves like a strange-attractor. Without going into details, we may restate the above supposition to mean that turbulence has a manageably small number of 'dynamically significant' degrees of freedom, despite the overwhelming complexity it displays, or that one may be able to extract a finite-dimensional projection out of an infinite-dimensional phase space.

As we know today, three distinct 'scenarios' of chaos have been indentified; more will no doubt be discovered. In the first scenario, chaos sets in abruptly following very few (most probably, three) Hopf bifurcations [1,2]. In the second, the onset of chaos occurs via an infinite cascade of period doubling [3,4,5] with certain well-defined universal characteristics. The third, less-studied, route envisages chaos through gradual merging of decreasingly intermittent chaotic regions [6]. Obviously, these scenarios of chaos have at least qualitative resemblence to transition to turbulence in one or the other of the fluid flows; considerable work [7-10] in the last few years has shown that the correspondence is more than superficial in highly constrained 'closed flow systems', that is, fluid flows which are totally confined within a closed boundary (for example, the narrow-gap Taylor-Couette flow, or convection in a finite box of small dimension).

systems, are more complex than has been visualized in dynamical systems, it is interesting that there are at least a few flows which follow these scenarios fairly closely.

In contrast, so far as we know, the language and concepts of chaos have not penetrated much the domain of 'open flow systems', such as wakes, jets, boundary layers, mixing layers, pipe flows, etc., which are topologically different from the closed flow systems. Devoid of the constraints that seem to be responsible for nudging severely confined flows towards the dynamical-systems-type behavior, it is an open question whether any aspects of chaotic dynamical systems are useful in the description of transition and turbulence in open flows. This paper is a modest beginning of a more ambitious enquiry into this question. In particular, we began with the following questions:

(a) Are there any similarities between the scenarios of chaos and transition to turbulence in open flow systems?

(b) What are the best experimentally accessible measures of strange-attractor behavior?

(c) What is the effect of the inevitably present noise on our perception of (b) above?

(d) What is the eventual significance of dynamical-systems approach to the down-to-earth concerns of a proper turbulence theory based on Navier-Stokes equations?

EXPERIMENTS ON TRANSITION

Consider the flow in helically coiled pipes [11]. Briefly, the arrangement consists of a long straight pipe followed by a coiled section of several turns (more than 5). We examine transition to turbulence by observing the velocity fluctuation u in the axial direction on the pipe centerline at the end of the coil.

Figure 1 shows the power spectral density of u at several Reynolds numbers; sample time traces are also shown. The first stages of transition are characterized by the appearance of a dominant periodic component f_1 modulated by a low frequency motion f_2; the uppermost set (a) of figure 1, Re = 5940, corresponds to the end of this stage. Clearly, neither peak is really sharp, presumably because of the many non-linear interactions possible between the f_1-band and the f_2-band. For Re = 6360 (figure 1b), the low frequency modulation has grown in relative size but, more interestingly, a third frequency f_3 (also not sharp), barely visible in figure 1a, has grown. When this peak reaches a sufficiently large amplitude, the peak at f_1 diminishes in amplitude and a broad-band spectrum begins to develop (figure 1c); this is followed very quickly by the disappearance altogether of peaks f_3 and f_1 (figure 1d), and eventually also of f_2 (figure 1e); at the same time, the broad band component grows continuously. The key to the transformation from an essentially quasiperiodic state as in (a) to an essentially aperiodic state as in (d) is the appearance of the third frequency f_3 of sizeable magnitude. (We may note that the spectra in figures 1 are actually averaged over several records, and that unaveraged spectra show much sharper peaks.) The situation is reminiscent of the Ruelle-Takens-Newhouse picture of transition, and underscores the possible presence of a strange attractor.

IS TURBULENCE A 'STRANGE ATTRACTOR'?

The most direct attribute of a strange attractor is the sensitivity to initial conditions; while this seems 'obviously' the case in turbulence, it is not easy to quantify it directly. One way of doing this is by evaluating the Lyapunov numbers, but, in practice, this is not viable (at least, it has not so far been possible for us) because of the finite precision with which the required Jacobians can be evaluated from the experimental data. Yet another characteristic is the relatively small dimensionality of the attractor despite the bewildering complexity. The relevant dimension, as has been pointed out by Mandelbrot [12], is the so-called

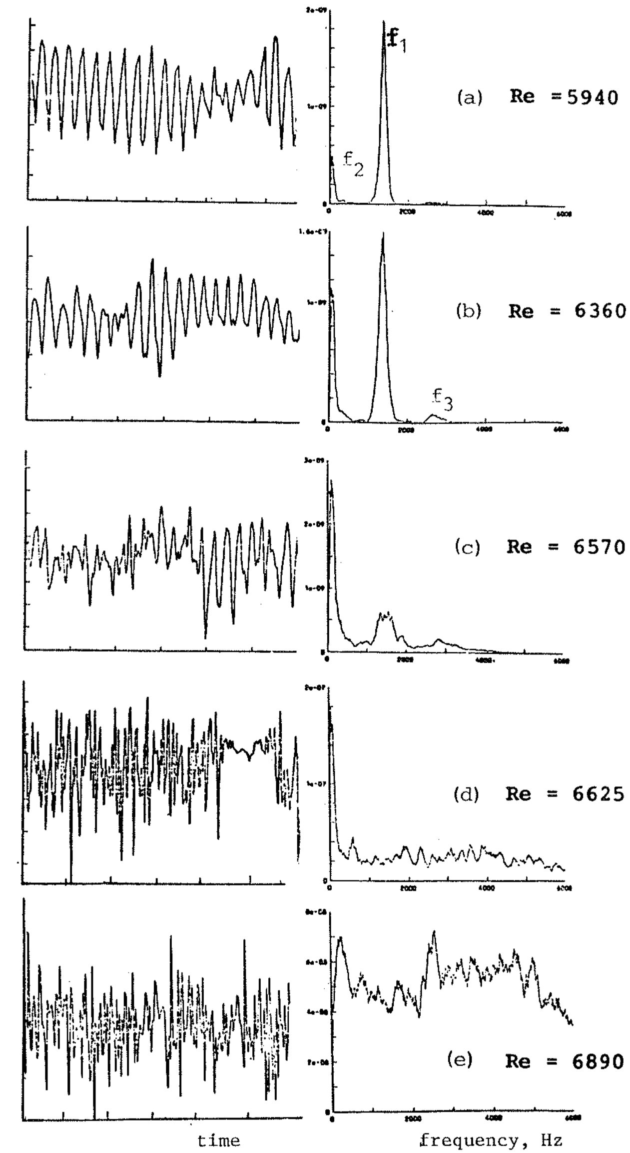

Figure 1. Time traces (duration 15 ms) and power spectral densities of the fluctuating velocity u on the pipe centerline of a coiled pipe; pipe diameter = 3.18 mm, coil radius = 42 mm. Same gain for all cases. Note that the spectral ordinates do not all have the same scale. In general, open systems are characterized by the presence of sizeable noise in the initial conditions, which is why we have chosen to plot the power on a linear scale. Some averaging has been performed on the power spectra.

fractal dimension D, an adaptation of the Hausdorff dimension. (The fractal dimension may be viewed as a measure of the information necessary to specify the location of a fractal set. For classical cases with self-similarity, it coincides with the usual notion of dimension.) Calculating D using box-counting algorithms is not practical if $D \gtrsim 2$ (see [13]), as is surely the case for turbulence (see below). Another dimension ν, related to the fractal dimension $D (\nu \leq D)$, as well as the information-theoretic entropy, has been proposed [14]. If $\underline{v}$ is the n-dimensional vector in time domain, one computes first the quantity C(r) given by

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j=1}^{N} H\{r - |\underset{\sim}{v}_i - \underset{\sim}{v}_j|\}, \tag{1}$$

where $v_i = \underset{\sim}{v}(i\tau')$, τ' being the sampling interval, and H is the Heaviside step function. For r not too large, it can be shown that $C(r) \sim r^{\nu}$. Grassberger & Procaccia [14] have shown that $\nu = D$ for several chaotic attractors commonly discussed in the literature on dynamical systems, and have argued that, where it is smaller than D, ν is in fact the more appropriate quantity to consider. We shall not discuss this further but only note that D is a quantity related to geometry, while ν has a probablistic content in it. In our computations of ν, we used real-time data of the axial velocity component to construct a multidimensional vector using the delay coordinates $(u_t, u_{t+\tau}, \ldots, u_{t+(d-1)\tau})$ with increasing values of d, and evaluated ν as indicated above; τ is an integral multiple of τ'. Initially, ν increases with d but settles down eventually. It is this asymptotic value

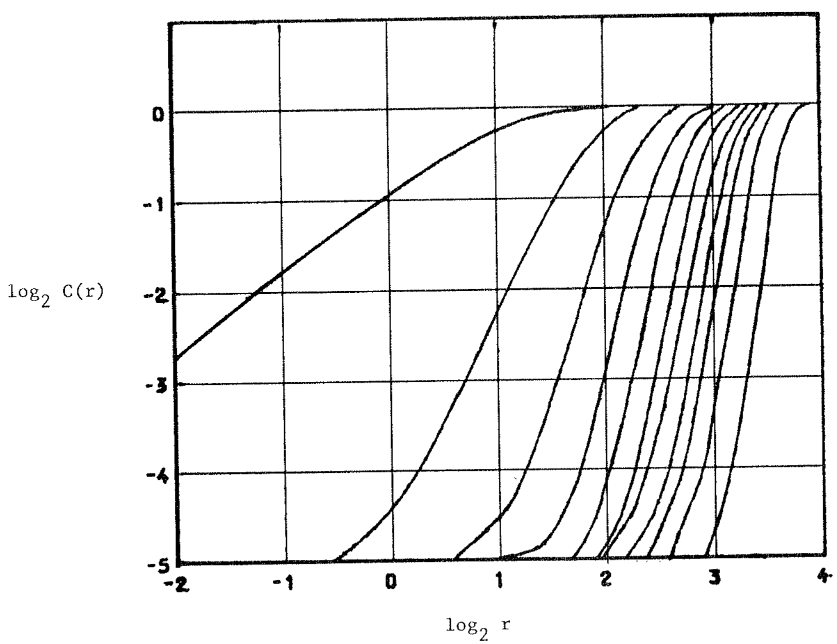

Figure 2. The quantity $\log_2 C(r)$ vs $\log_2 r$, r in arbitrary units. Re = 6625. Different curves correspond to different d. From left to right, d = 1, 5, 10, 15, 20, 25, 30, 20, 50 and 70.

of ν that is of interest to us. If ν is relatively small, the concept of strange attractors may be very useful in turbulence; otherwise, it is hard to assess its significance.

As a check on our computational procedure, we may note that ν was found to be 1 for a sine wave and 0.63 for a Cantor set, as expected. Since a purely random signal, such as the output of a white-noise generator, has a space-filling attractor, $\nu \approx d$ for all d.

Figure 2 shows several curves of log C(r) vs log r, computed with increasing values of d, from the velocity data for Re = 6625 just after the onset of the broadband spectral behavior. Typically, these curves have a linear region; the levelling off of the curves for large r is the result of the finiteness of the attractor, while deviation from linearity towards the very low end of the curves arises from resolution problems. The slope of the linear region increases with d initially but appears to settle down to a constant beyond a certain d. This can be seen more directly from figure 3. The asymptotic value of ν is around 6.

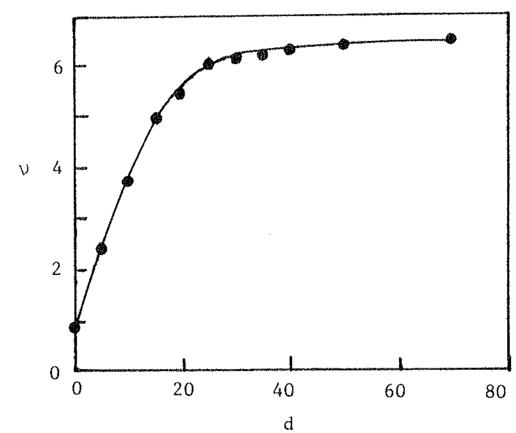

Figure 3: The slope ν of the straight regions of curves in figure 2, vs the dimension of the phase space, d. The asymptotic value is around 6.

The data presented in figures 2 and 3 are typical of our computations, which extend to Reynolds numbers on either side of 6625. However, they are not sufficiently systematic at this point to be included here as conclusive results. This is so chiefly because we have not yet made the various sensitivity tests on ν. First, before the signal is digitized, some low-pass filtering is necessary; we have not investigated the effect of varying this cut-off on ν. We have also not investigated very thoroughly the effect of varying τ on ν. Typically, however, this latter effect is not significant over a fairly wide range of τ. With these reservations noted, we may mention that, for Re < 6625, the value of ν is less than 6, while being a rather strongly increasing function of Re at higher Re; in fact, at the highest Reynolds number of our computations, we have not yet seen ν settle down even for d as large as 100. (Our initial results presented at the meeting in Kyoto were necessarily at lower Reynolds numbers than 6625.)

DISCUSSION AND CONCLUSIONS

The results of the previous two sections represent only a small part of a largely unyielding investigation. In relation to transition and the scenarios of chaos, our experience is that none of the above-mentioned routes to chaos occurs during transition to turbulence in open systems like jets or wakes. While it is of course possible that more than one of the above scenarios operate simultaneously, it looks certain that turbulence, unless constrained severely, does not behave like a simple dynamical system. On the other hand, we would like to make a specific mention of the fact that our initial experience with coiled pipes was disappointing too; it was only after some modifications of the flow were made, primarily in the form of a smoother inlet to the upstream straight section, that we could observe the evolution discussed earlier. Can we then make the sweeping generalization that, by making 'appropriate' changes to the flow, perhaps by way of restricting initial conditions to a suitable (but unknown) 'basin of attraction', we can nudge transition to follow some well-defined scenario of chaos?

What specifically has our work shown in relation to fully developed, or, at least, 'nascent' turbulence? While much work needs to be done, it suggests that, at least at Reynolds numbers not too far above the transition value, the attractor for turbulent signals is relatively low-dimensional. It may thus justify attempts at extracting for the Navier-Stokes equations a finite-dimensional projection out of the seemingly infinite-dimensional phase space. We should, however, note that the dynamical systems approach will at best represent a small part of the total picture in turbulence unless the spatial chaos and order, as well as the relation between these latter characteristics and temporal behavior, are discussed.

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