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Properties of Wall Shear Stress Fluctuations in a Turbulent Duct Flow

Measurements of wall shear stress fluctuations have been made in a fully developed turbulent duct flow, using a surface heat transfer gauge. Measurements, made over a moderate Reynolds number range, include RMS values, probability density functions, spectra, and zero-crossing frequencies of the wall shear stress fluctuation. The ratio of RMS of the fluctuation to the mean value of the wall shear stress is found to be about 0.25. The zero-crossing frequency computed from the measured spectra using the relation derived by Rice for a Gaussian process is found to be a good approximation to the measured value, although the measured probability density function is not Gaussian. The zero crossing frequency and spectra of wall shear stress fluctuations appear to scale with outer variables for asymptotically large Reynolds numbers.

1 Introduction

The use of surface heat and mass transfer gauges for the study of instantaneous heat, momentum, and mass transfer in different fluids has been well established in the literature, e.g., [1–6].[1] Since the surface heat transfer gauge does not interfere with the flow, it is a convenient device to use for the study of the observed bursting process in the wall region of turbulent boundary layer and pipe or duct flows. In the present investigation we measure a few statistical properties, such as RMS intensities, probability density functions and spectra of the wall shear stress fluctuations, using a commercial heat flux gauge inserted at the wall of a fully developed turbulent duct flow. This particular flow was chosen since the mean wall shear stress, which is determined accurately from the linear static pressure distribution over the fully developed part of the flow, can be readily used for a static calibration of the heat transfer gauge. Measurements of the wall shear stress fluctuations are made over a moderate Reynolds number range and are compared, whenever possible, with other available results.

Some attention is given in Section 5 to the experimental determination of the zero-crossing frequency of the heat gauge signal. This part of the investigation was motivated by a recent study by Badri Narayanan, et al. [7], who found that the zero-crossing frequency of the longitudinal velocity fluctuation, as obtained from a hot wire placed in a turbulent boundary layer, was equal to the frequency associated with the bursting phenomenon. Ueda and Hinze [8] have shown that the bursting frequency measured in the viscous sublayer is approximately one half the value obtained in the "buffer" and the semilogarithmic parts of the layer. It is therefore useful to compare zero-crossing frequencies obtained from surface hot-film probes with those available from hot-wire signals in other parts of the flow. Possible scaling parameters for the zero-crossing frequencies measured in the present investigation are discussed in Section 5.

2 Description of Equipment and Experimental Conditions

The channel used for the present measurements is described in detail in Churchill [9]. Briefly, it has a width of 1.27 cm, a span of 40.6 cm (aspect ratio 32:1) and a length of 183 cm. Upstream of the test section is a contraction, a settling chamber and a diffuser connected via a flexible pipe to a 1 HP motor-fan combination. Three screens, two on the upstream and one on the downstream end of the settling chamber, are provided.

Surface shear stress measurements were made with a DISA miniature 55A93 probe, which is a quartz-coated nickel film deposited on the plane end of a quartz rod. The frequency response for this probe, as quoted by the manufacturer [10] is good up to 30 kHz. The signal from the probe was fed into a DISA 55M10 constant temperature anemometer. The anemometer signal was first linearized (DISA 55D10 linearizer) and then passed through a DISA 55D26 signal conditioner before recording on a Hewlett-Packard 3960A FM recorder at a speed of 38.1 cms^{-1}; its -3 dB upper frequency cutoff was 6.3 kHz. The root-mean-square values were read out on a DISA 55D35

[1] Numbers in brackets designate References at end of paper.

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RMS meter, and the bridge voltage measured with a 55D30 voltmeter. The recorded signals were processed on the digital computer ARCTURUS (in the School of Electrical Engineering at the University of Sydney) to obtain probability densities and spectra with the use of the hardwired FFT processor, described by Gottlieb and De Lorenzo [11]. The tape recorder was played back at 2.32 cms^{-1} and the signal digitized at a sampling frequency of 600 Hz (real time frequency of 9.6 kHz), with appropriate prefiltering to avoid aliasing. The sampling frequency of 9.6 kHz represents twice the Kolmogorov frequency f_η, estimated to be 4.8 kHz (for the highest Reynolds number used here) at the "edge" of the thermal layer associated with the hot film. The number of digital samples used was approximately 5×10^5, corresponding to a real time duration of approximately 52 s.

The zero-crossing frequencies were obtained by first removing the d-c component of the signal with a Krohn-Hite filter (cut off at 0.02 Hz), and then passing the filtered output through a Digital Electronics comparator (with the comparator level grounded) and a digital frequency meter, which displayed the frequency directly. Generally, about 10 readings yielded reasonably steady averages, but about 25 were usually taken.

The two-dimensionality of the flow was checked in [12] with the use of a Preston tube, moved to various spanwise locations in the flow. For a fully developed two-dimensional duct flow, it follows from the equations of motion that the mean static pressure gradient (dp/dx) is constant, and is related to the mean wall shear stress $\bar{\tau}_0$ through the relation

$$\bar{\tau}_0 = D(dp/dx) \tag{1}$$

where D is half width of the duct. The gradient dp/dx was found constant over a substantial region of the flow ($90 < x/D < 190$). The skin-friction coefficient $c_f(=2\bar{\tau}_0/U_c{}^2$, where U_c is the center-line velocity) evaluated from equation (1) was in good agreement with the Preston tube values and the results of Johnston [13] and Dean [14].

It is necessary to discuss briefly the effect on the measurement of wall shear stress fluctuation of the physical dimensions of the hot-film probe, of the overheat ratio at which it is operated and of errors resulting from a possible mismatch between the film surface and the wall of the duct. Spence and Brown [15] showed that the Nusselt number Nu for the film is proportional to $\bar{\tau}_0{}^{1/3}$, provided that

$$\frac{6.6}{\sigma^{1/2}} < \frac{U_* l}{\nu} < 64\,\sigma$$

where σ and ν are the fluid Prandtl number and kinematic viscosity, respectively, U_* is the friction velocity $(=\bar{\tau}_0{}^{1/2})$ and l is the streamwise extent of the film. This condition requires that $0.08 \text{ mm} < l < 0.5$ mm at the highest Reynolds number considered here and $0.15 \text{ mm} < l < 1$ mm at the lowest Reynolds number. The present probe (DISA 55A93, $l = 0.15$ mm) meets this criterion for most of the flow conditions encountered.

The heat transfer from the film must be sufficiently small for the temperature to be considered a passive contaminant of the flow. The ratio $\Delta T/T_0$ (where ΔT is the difference between the film temperature and the adiabatic tunnel wall surface temperature T_0) needs therefore to be as small as can be tolerated by other requirements, such as the signal/noise ratio. The present film was operated at an overheat ratio of 1.2, yielding $\Delta T \simeq 80°$C. Spence and Brown [15] indicated that even higher values of ΔT may be tolerable as Nu falls off extremely rapidly with distance from the surface (the thickness of the thermal boundary layer is about $l/10$). A small mismatch between the film surface and duct wall was found to have little effect on either mean or RMS voltages at the output of the anemometer.[2]

3 RMS Intensity and Probability Density Function

The ratio $\bar{\tau}_0'/\bar{\tau}_0$, where $\bar{\tau}_0'$ is the RMS of the wall shear stress fluctuation τ_0' is plotted in Fig. 1 as a function of Reynolds number. No

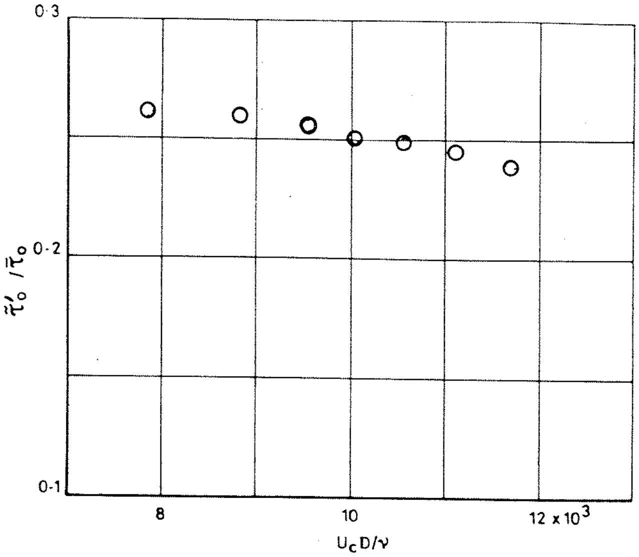

Fig. 1 Root-mean-square wall shear fluctuations as a function of Reynolds number

definite conclusions can be drawn on the Reynolds number dependence of $\tau_0'/\bar{\tau}_0$ partly because the extreme values of this plot are within the reproducibility of measurement (± 5 percent), and partly because the characteristic thermal layer thickness (over which the probe does, in some sense, average) is itself weakly dependent on Reynolds number. It seems more appropriate to assume a constant value of about 0.25 for $\bar{\tau}_0'/\bar{\tau}_0$, which compares very well with the value of 0.24 obtained by Eckelmann [4]. Mitchell and Hanratty [6] and Fortuna and Hanratty [17] have obtained slightly higher values for $\bar{\tau}_0'/\bar{\tau}_0$ of 0.30 and 0.32, respectively, also independent of Reynolds number. It may be of interest to mention here that the RMS value of the Reynolds shear stress fluctuations measured by Antonia [18] and Gupta and Kaplan [19] in the inner region of the turbulent boundary layer, is approximately $2\bar{\tau}_0$, an order of magnitude larger than the level of wall shear stress fluctuations. The ratio of the RMS value of the wall pressure fluctuations to the RMS of the wall shear stress fluctuations is about 10 (using, for example, the result of Willmarth and Roos [20]).

It should be noted (following Eckelmann [4]) that

$$\frac{\bar{\tau}_0'}{\bar{\tau}_0} = \frac{|\overline{(\partial u/\partial y)_0{}^2}|^{1/2}}{(\partial U/\partial y)_0} \approx (\tilde{u}/U)_{y\to 0},$$

where U and u are the local mean and streamwise fluctuating velocities of the flow. There is considerable evidence [4, 21, 22] to show that indeed $\tilde{u}/U \to 0.25$ as $y \to 0$, independent of Reynolds number. The result of Yucel and Graf [23] who find that $\bar{\tau}_0'/\bar{\tau}_0$ decreases from 0.38 at $\overline{U}D/\nu \simeq 10^5$ to 0.12 at $\overline{U}D/\nu \simeq 6 \times 10^5$, seems to be in doubt. Py [24] shows that close to wall, $\tilde{u}/U$ is essentially independent of Reynolds number, but is strongly affected by the geometry of the sensor element.

Fig. 2 shows a normalized probability density plot of the linearized signal; also shown for comparison is a Gaussian distribution. Clearly, the signal is far from being Gaussian and is skewed positively, consistent with the traces recorded by Eckelmann [4], who however gives no probability densities. The probability density function appears to be independent of Reynolds number (at least over the range covered here). Partial support for this statement is provided by the skewness and flatness factors listed in Table 1.

The present values of skewness and flatness factors are comparable with, but distinctly less than, the values by Kreplin and quoted by Eckelmann [4] (see Table 1). Ignoring this discrepancy for the moment, it is interesting to note that Kreplin's measurements show mild peaks in both $S(\simeq 1.1)$ and $F(\simeq 4.4)$ of u signals (at $y^+ \equiv yU_*/\nu \simeq 1.5$), and that the corresponding values obtained "at" the wall are substantially lower. This suggests that a possible explanation for the

[2] Quantitative measurements due to the effect of mismatch may be found in Wetzel and Killen [16].

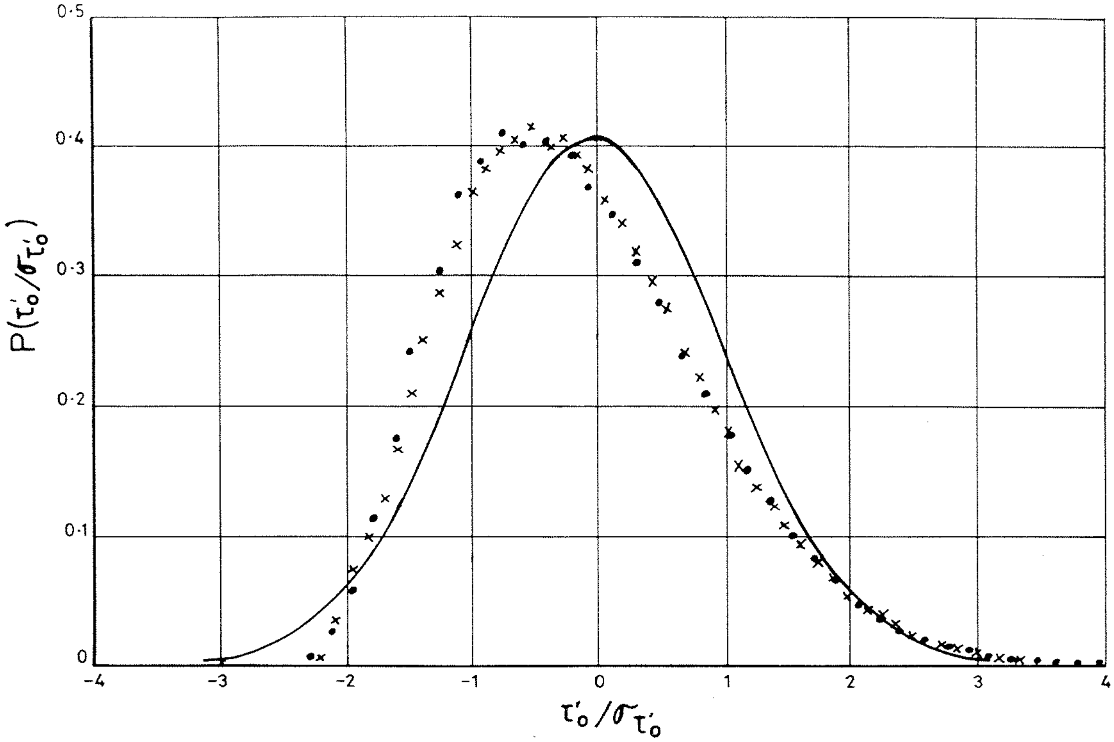

Fig. 2 Probability density of the wall shear stress fluctuations at two different Reynolds number. ●, $U_c D/\nu$ = 6.05 × 10^3; x, $U_c D/\nu$ = 10.34 × 10^3; —, Gaussian

Table 1 Skewness (S) and flatness (F) of wall shear stress fluctuations

	$U_c D/\nu$	S	F
Present	11.78 × 10^3	0.58	3.05
	10.34	0.52	3.04
	9.14	0.51	3.30
	8.21	0.55	3.02
	7.04	0.53	3.19
	6.05	0.53	3.10
Kreplin (as given in [4])	8.20 × 10^3	0.75	3.70

difference[3] between the present values and Kreplin's values at the wall is the thicker thermal boundary layer in the latter ($y^+ \simeq 2$, as against 1.4 in the present case), because, as mentioned previously, the surface probe tends to average u over a normal distance of the order of the thermal boundary layer. However, both the present and Kreplin's measurements are consistent in that the skewness and flatness factors of $\partial u/\partial y$ at the wall are significantly lower than the maximum values attained by the skewness and flatness factors of u in the vicinity of the wall.

Armistead and Keyes [5] also measured the probability density using a hot-film gauge, but found that it was more nearly Gaussian. We find a similar result if the signal is not linearized, and it is not quite clear whether Armistead and Keyes linearized their signals. In fact, Comte-Bellot [25] and Frenkiel and Klebanoff [27] find that linearization has considerable effect especially on the skewness of u close to the wall.

[3] Unfortunately the agreement among different workers of the value of skewness and flatness factors of the longitudinal velocity fluctuation u is not much better close to the wall. For example, at $y^+ = 2$, Kreplin's values are 0.9 and 3.4 for S and F, respectively, while Comte-Bellot's [25] corresponding values are 0.5 and 2.8, respectively. Zaric [26], on the other hand, reports values which are even higher than Kreplin's. This departure is even worse when one considers different flows; the corresponding values obtained by Gupta and Kaplan [19] are 1.5 and 7, respectively, in a boundary layer.

4 Power Spectral Density

Fig. 3 shows the normalized spectral density ϕ of τ_0'. It would appear that when nondimensionalized using the outer variables, namely, the velocity U_c and the duct half width D, all the spectra plot close to one another over the present Reynolds number range. A representative set of data of Kutateladze, et al. [28], agrees with the present measurements. When nondimensionalized with U_* and ν, the spectra do not exhibit any universality. It should be noted that, at a given Reynolds number, the spectral density of Reynolds shear stress fluctuations reported by Antonia and Van Atta [29] scale with outer variables over a large extent of the boundary layer.

Close to the wall, the behavior of u should follow closely that of the wall shear fluctuations. Eckelmann [4], for example, notes that the wall shear stress fluctuations and the logitudinal velocity fluctuations close to the wall ($y^+ = 0.8$) are almost identical except for a small phase difference. It is therefore to be expected that the u spectrum of Bakewell and Lumley [21] at $y^+ = 1.25$, at a Reynolds number of 5450, should agree with the present wall shear stress spectra at a comparable Reynolds number. This is supported by Fig. 3.

The similarity of wall shear stress spectra when scaled on outer variables has been noted by several workers, e.g., Mitchell and Hanratty [6], Armistead and Keyes [5], Kutateladze, et al [28], and Wetzel and Killen [16]. However, we suggest that the similarity alluded to (except possibly for Wetzel and Killen's data) is only a consequence of the narrow Reynolds number range covered by each worker cited in the foregoing, as well as the scatter in the data which, unfortunately, masks the existing differences. The differences show up clearly when the data obtained by various investigators are all plotted in the same figure (Fig. 3). We see that the present spectra which agree with those of Bakewell and Lumley [21] and Kutateladze, et al. [28], are in fact obtained for a narrow Reynolds number range (5.45 × 10^3 < $U_c D/\nu$ < 11.78 × 10^3). On the other hand, the spectra of Wetzel and Killen [16], obtained in the range 1.32 × 10^5 < $\overline{U}D/\nu$ < 2.66 × 10^5, are clearly different from those obtained at much lower Reynolds numbers although they are in good agreement among themselves. The wall shear stress spectra of Mitchell and Hanratty [6], obtained at an intermediate Reynolds number ($\overline{U}D/\nu \simeq$ 2.29 × 10^4) and the u spectrum of

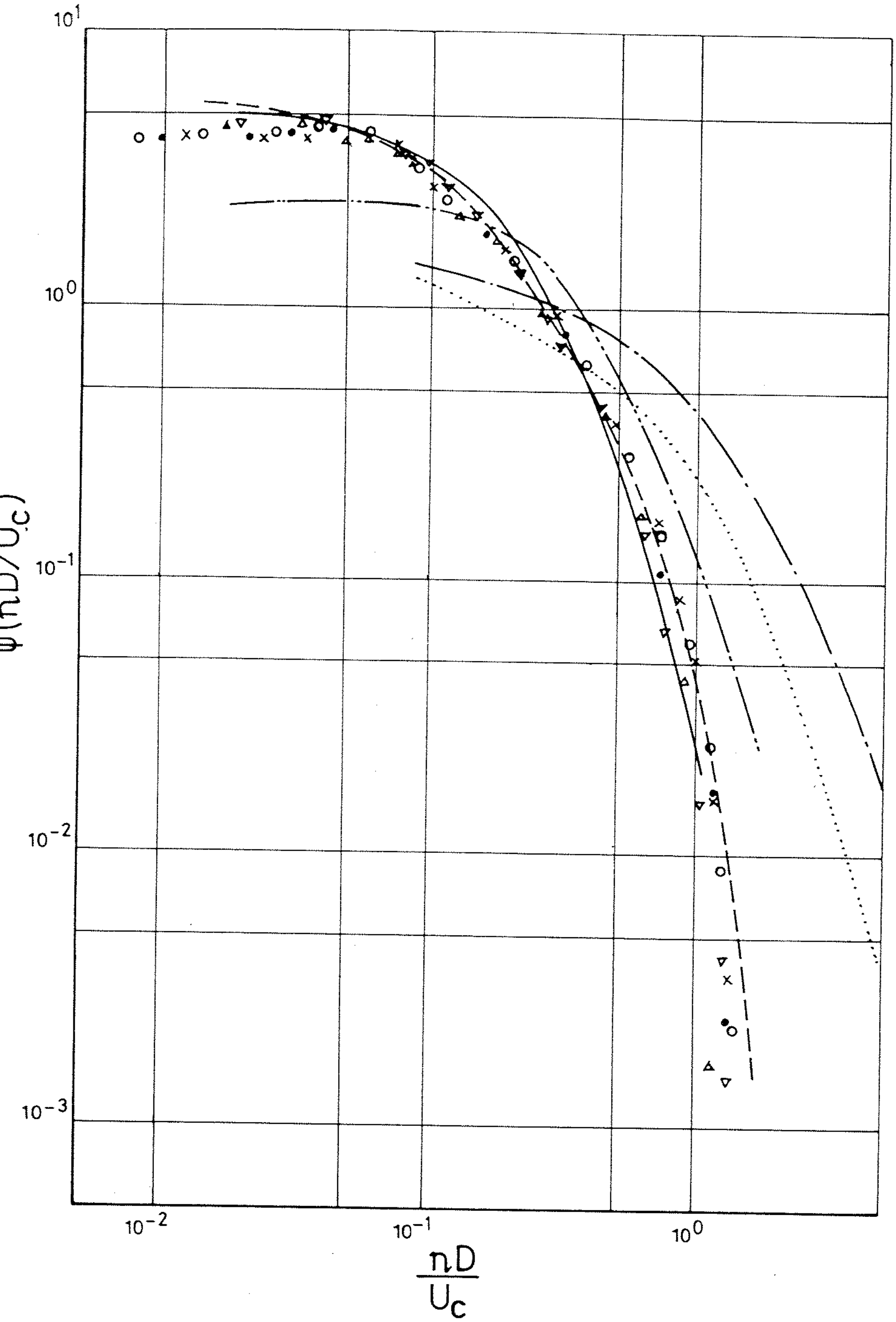

Fig. 3 Nondimensional and normalized power spectral density of τ_0' at different Reynolds numbers. ○, $U_cD/\nu = 11.78 \times 10^3$; x, $U_cD/\nu = 9.14 \times 10^3$; ●, $U_cD/\nu = 8.21 \times 10^3$; Δ, $U_cD/\nu = 7.04 \times 10^3$; ∇, $U_cD/\nu = 6.05 \times 10^3$, - - -, Kutateladze et al. $UD/\nu = 4015$; —··—, Mitchell and Hanratty; $UD/\nu = 2.29 \times 10^4$; —·—, Wetzel and Killen, $UD/\nu > 1.32 \times 10^5$; —, Bakewell and Lumley, u spectrum at $y^+ = 1.25$, $UD/\nu = 4350$; ··· Klebanoff, u spectrum at $y^+ \simeq 3.13$, $U_\infty\delta/\nu \simeq 8 \times 10^4$.

Klebanoff [30] ($y^+ \simeq 3.13$) obtained at $U_\infty\delta/\nu \simeq 8 \times 10^4$ lie somewhere between the present spectra and those of Wetzel and Killen [16]. We therefore suggest that this type of scaling is, at best, valid only at large Reynolds numbers.

5 Zero-Crossing Frequency

The number of times a turbulent signal crosses its mean value appears to be of some importance in relation to the fine structure of the turbulence (Badri Narayanan, et al. [7], Antonia, et al. [31]). Fig. 4 shows that the positive (or negative) zero-crossing frequency N_0, nondimensionalized with either wall or outer variables, is not constant over the Reynolds number range covered here.

Accurate determination of the zero-crossing frequency is not straightforward. It is conceivable that a "noisy" signal will give too high a value for N_0. Alternatively, if the high frequency content of a random signal is filtered off, N_0 can be expected to be much lower. The effect of the cutoff frequency on N_0 is shown in Fig. 5. The upper part of the figure shows the effect of the low cutoff, f_l; it is seen that there is no significant effect on N_0 below about 1 Hz (for a signal with a nonzero mean, one can however expect a sudden jump in N_0 at d-c).

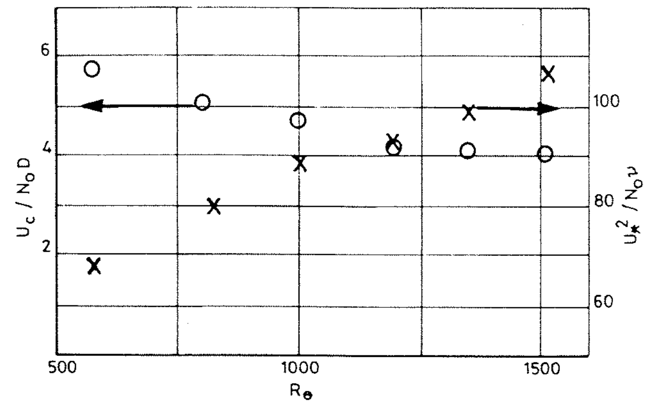

Fig. 4 Variation with Reynolds number of the nondimensional zero-crossing frequency; ($R_\theta U_c \theta/\nu$, θ is the momentum thickness)

The lower part of the figure shows, for different stream velocities, the dependence of N_0 on the high cutoff frequency, f_h. N_0 increases with f_h (in a linear fashion for white noise) until, at lower speeds, it levels off at some f_h; but if the filter is set to higher and higher values of f_h (i.e., more and more of the electronic noise is allowed to contaminate the signal), N_0 increases sharply again. For lower values of Reynolds number a reasonable plateau appears to exist in N_0, in which the precise filter setting is not critical. As U_cD/ν increases, the plateau decreases significantly both in definition and extent, and disappears completely at the highest speed. It is likely that this highest speed at which there is no clear demarcation between the zero-crossings due to signal and those due to noise (presumably because the highest signal containing frequency, typically the Kolmogorov frequency, overlaps the noise limit of the electronic system) will depend on the signal/noise of the setup. It would seem that the appropriate filter setting is the Kolmogorov frequency. This is what was used for the present determination of N_0.

The figure also shows curves corresponding to the zero-crossing frequency computed using the relation

$$N_0(f_h) = \left[\frac{\int_0^{f_h} n^2 \phi(n)\, dn}{\int_0^{f_h} \phi(n)\, dn} \right]^{1/2} \tag{2}$$

where $\phi(n)$ is the measured spectral density. It is clear that the calculations are reasonably close to the measurements approximately up to a filter setting f_h of f_η.

Equation (2) is strictly valid [32] only for Gaussian processes but the present results suggest that it is at least approximately true for a non-Gaussian signal such as the wall shear stress fluctuation. This conclusion is consistent with observations by Wetzel and Killen [16] and Antonia, et al. [31], but in disagreement with the finding by Badri Narayanan, et al. [7], that in turbulent shear flows, equation (2) overestimates the measured N_0 by a factor of 3 to 4.

It is of interest to note the variation with distance from the wall of the zero crossing frequency of the streamwise velocity fluctuation u. Fig. 6 shows that in the pipe flow of Bakewell and Lumley [21], N_0 computed from equation (2) (with f_h set to ∞) is constant for $y^+ \lesssim 5$ (or essentially in the viscous sublayer). This is also true for the boundary layer as is indicated by calculations of the zero crossing frequency from the data of Ueda and Hinze [8]. (Here U_∞ is the free-stream velocity and δ is the boundary-layer thickness). This appears therefore to be a common feature of all wall flows. Because N_0 computed anywhere in the sublayer ($y^+ \lesssim 5$) is constant and, very close to wall, $u(y) \propto \tau_0'$, it follows that the zero-crossing frequency of shear stress fluctuation is equal to that of $u(y)$ for $y^+ \lesssim 5$. Thus one can compute the zero-crossing frequency of τ_0' by measuring the zero-crossing frequency of u (for $y^+ \lesssim 5$), or alternatively from the spectral density of u (for $y^+ \lesssim 5$) by the use of equation (2).

It is interesting to note that the so called "bursting" frequency of high frequency pulses also shows a similar variation with distance from the wall [8], which suggests a close relation between this bursting

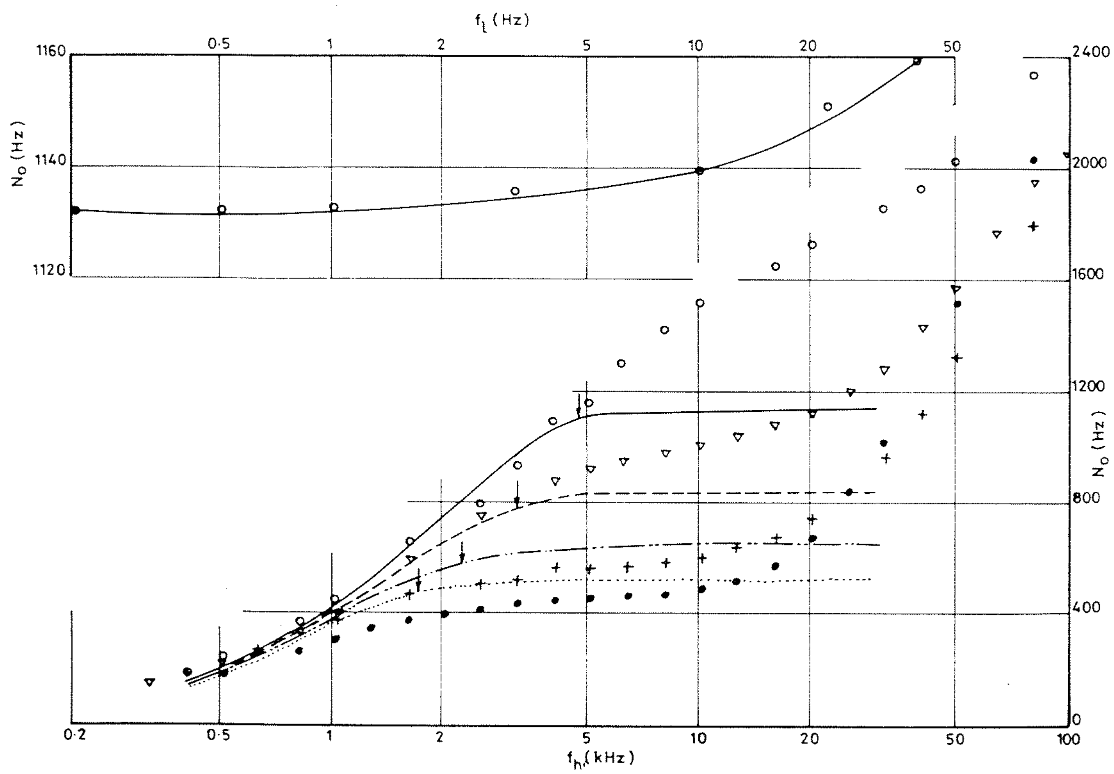

Fig. 5 Variation with the low and high cutoff filter frequencies of the zero-crossing frequencies. Curves are calculated values according to eq. (2). Top part of figure: upper cut off at 5 kHz, lower cutoff variable, U_cD/ν = 11.53 × 10^3. Lower part of figure: lower cutoff at 0.02 Hz, upper cutoff variable. ●, ..., U_cD/ν = 5.81 × 10^3; +, — • • —, U_cD/ν = 7.25 × 10^3; ▽, - - -, U_cD/ν = 8.98 × 10^3; ○, —, U_cD/ν = 11.53 × 10^3. ↓ indicates estimated f_η.

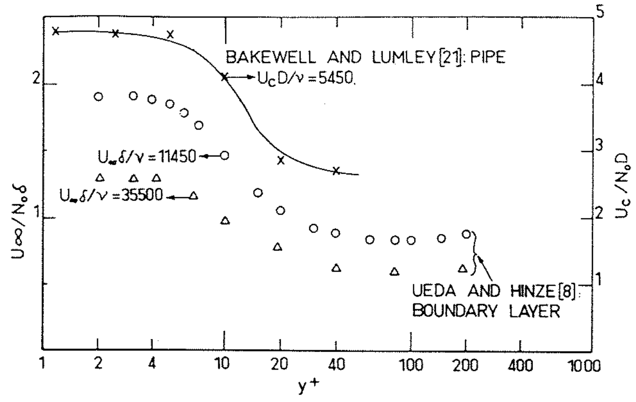

Fig. 6 Variation with normal distance in the wall region of the quantity U_c/N_0D or $U_\infty/N_0\delta$

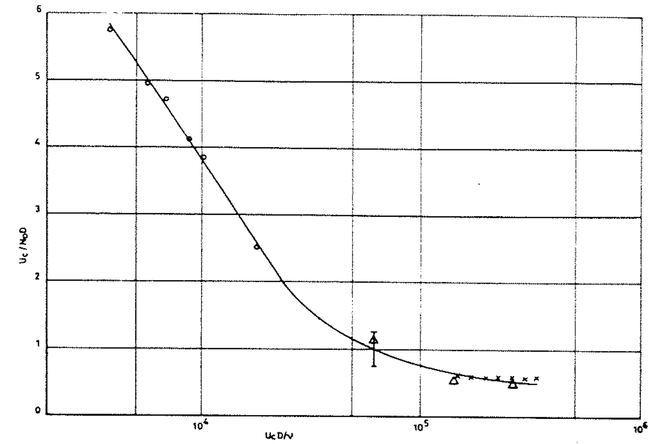

Fig. 7 Variation with Reynolds number of the quantity U_c/N_0D. O, present data; x, Wetzel and Killen, N_0 measured in both cases. Δ, Comte-Bellot: Here N_0 was estimated from the measured spectral densities using (2). The vertical bar indicates the possible uncertainty in the estimates.

frequency and zero-crossing frequency. Badri Narayanan, et al. [7], found that the ratio of these two frequencies is independent of Reynolds number and is approximately unity. This, coupled with the observed outer scaling of the bursting frequency (Rao, et al. [33], Badri Narayanan, et al. [34], Laufer and Badri Narayanan [35], Ueda and Hinze [8]), implies that the zero-crossing frequency should also scale with outer variables. We have however observed earlier that this is not true, but the uncertainties associated with the determination of N_0 (Fig. 5) and the narrow range of Reynolds number covered, make this conclusion somewhat uncertain. Data from Wetzel and Killen and the present experiments (together covering a Reynolds number of about two decades) are reexamined in Fig. 7, where U_c/N_0D is plotted against the Reynolds number U_cD/ν. Also plotted in the figure are data from Comte-Bellot [25] at intermediate Reynolds numbers assuming, as discussed above, the validity of equation (2) and that U_c/N_0D is independent of y^+ for $y^+ \lesssim 5$. It appears that U_c/N_0D might at best asymptotically approach a constant value, which suggests that the universal scaling of spectra with outer variables is also asymptotic. The evidence presented here shows that the two results, namely, the independence on Reynolds number of the ratio of bursting frequency to zero-crossing frequency and that the scaling of bursting frequency with outer variables, are not self-consistent. In fact, Antonia, et al. [31], have found that the ratio of zero-crossing frequency to the bursting frequency is Reynolds number dependent.

6 Conclusion

For the Reynolds number range considered here, the RMS value of the wall shear stress fluctuation is roughly 0.25 times the mean wall shear stress, and does not seem to depend significantly on Reynolds number; the probability density is also independent of Reynolds number and is skewed positively. The zero-crossing frequency and spectra of wall shear stress fluctuations may scale with outer variables for asymptotically large values of the Reynolds number. Further, the zero-crossing frequency can be computed approximately from Rice's result, which is strictly valid only for a Gaussian process.

Acknowledgments

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