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Conditional Probability Densities in a Turbulent Heated Round Jet

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SUMMARY Conditional (turbulent) probability density functions, and high-order moments, of axial (u) and radial (v) components of velocity and temperature (θ), and of the products uv, uθ and vθ, are obtained at several radial positions of an axisymmetric heated turbulent air jet with a co-flowing external stream. The probability density function of θ is used to determine whether the flow is turbulent or not. As the distance from the jet axis increases, the probability density functions of u, v and θ show increasing departures from Gaussianity; for products, the assumption of joint Gaussianity becomes poorer. Departures from Gaussianity are generally larger for the inward (v < 0), than for the outward (v > 0), radial motion. A fourth-order bi-variate Gram-Charlier expansion only marginally improves agreement with measured conditional products.

1 INTRODUCTION

Ribeiro and Whitelaw (1975) measured probability density functions of the axial u and radial v velocity fluctuations in the self-preserving region of a round isothermal jet. Venkataramani *et al.* (1975) extended these measurements to include temperature fluctuations θ in a heated round jet. Although both marginal and joint probability density functions were presented by Venkataramani *et al.* in the intermittent part of the flow, no attempt was made to distinguish between the turbulent and non-turbulent fluctuations. In this paper, use is made of a digital data acquisition system together with a technique developed by Bilger *et al.* (1976) for the determination of turbulent/non-turbulent interface from the probability density function of a passive scalar; the effectiveness of using a passive thermal contaminant for this purpose has been established in the literature (e.g., LaRue and Libby, 1974; Kovasznay and Ali, 1974; Antonia *et al.*, 1975). This technique is then applied to obtain high-order moments and probability density functions of velocity and temperature fluctuations as well as their products, in the turbulent part of the flow. Deviations of conditionally turbulent densities from Gaussianity are found to be smaller for the outward (v > 0) than for the inward (v < 0) radial motion, and are highlighted in the discussion of probability density functions of the fluctuations in the products uv, uθ and vθ in the turbulent part of the flow. These deviations are significant away from the axis, and cannot be fully accounted for by a cumulant discard approach assuming a Gram-Charlier distribution. These calculations produce only marginally better agreement with measurements.

2 DATA ACQUISITION AND EXPERIMENTAL CONDITIONS

A description of the experimental facility is given in Antonia *et al.* (1975). Velocity fluctuations u and v were obtained with an X-wire (5 μm dia. Pt-coated tungsten wires) operated with a DISA 55M01 constant temperature anemometer. The temperature fluctuation θ was measured with a cold wire (1 μm dia. Wollaston wire) located at a distance of about 1 mm from the X-wire.

Signals proportional to u, v (both decontaminated for temperature fluctuations) and θ, were recorded on a Philips Analog-7 FM tape recorder (frequency response flat up to 10 kHz) and digitised through a digitiser (10 bit including sign) and processed on a PDP 11/45 computer. The sampling frequency (real time) was 12 kHz, and the record duration 27.73 s. Probability density functions were generated for different bins varying from 128 to 1024. All measurements were made 59 diameters downstream of the jet nozzle (2.03 cm diameter). the jet velocity was kept constant at 32.0 m s^{-1} whilst the jet to external air velocity ratio was 6.6. The maximum temperature T_0 above ambient was 3.3°C and the maximum velocity above ambient U_0 was 3.1 m s^{-1}. The length scale L_0, the radial distance r from jet axis to position where the mean temperature above ambient is equal to $T_0/2$, was 6.48 cm.

3 THE TECHNIQUE

Conventional probability density functions and high-order moments of u, v and θ, obtained at several radial positions η (≡ r/L_0), are discussed with regard to their accuracy and other similar measurements in Sreenivasan *et al.* (1977) and Antonia and Sreenivasan (1976). Of particular interest here is the characteristic bimodal shape of the conventional probability density function p(θ). In the intermittent region of the flow, p(θ) should ideally exhibit a delta function centered at the free stream ambient temperature. However, fluctuations in the ambient temperature and noise in the electronics produce, by convolution, a smearing of the delta function leading to the smeared-out spike (Figure 1). With increasing distance from the jet axis, the height of this smeared-out spike increases in relation to the peak in p(θ) associated with temperature fluctuations in the fully turbulent part of the flow. The appearance of a distinct peak in p(θ) even on the centre line of the jet (see inset in Figure 1) suggests that the flow is not fully turbulent on the jet axis (γ ≃ 0.998). The distinctive bimodal shape of p(θ) has been observed in other intermittent flows by LaRue and Libby (1974), Venkataramani *et al.* (1975) and Batt (see Alder, 1974).

The part of p(θ) (1024 bin resolution) which corresponds to the spike is plotted (renormalised by the standard deviation σ of the Gaussian part) in Figure 2 for various values of η. At large η, p(θ) is dominated by the peak, so that the Gaussian curve is a good fit to the data on either side of the peak but, for small η, only one or two points could be includ-

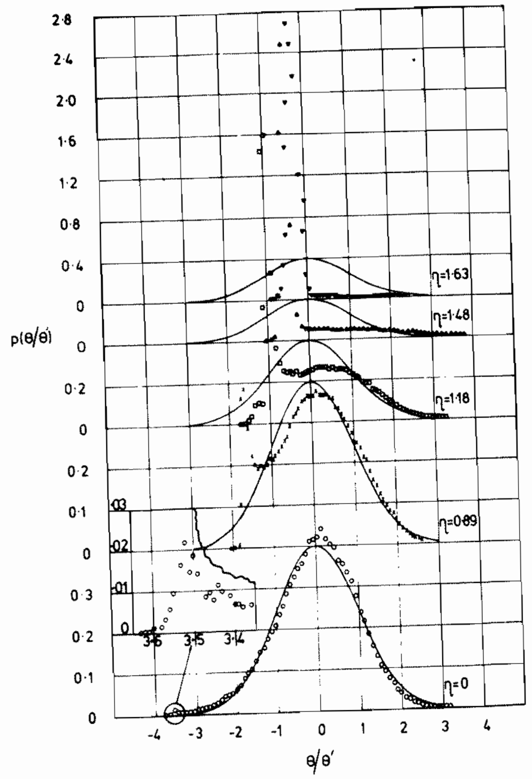

Figure 1 Normalised probability density function of θ

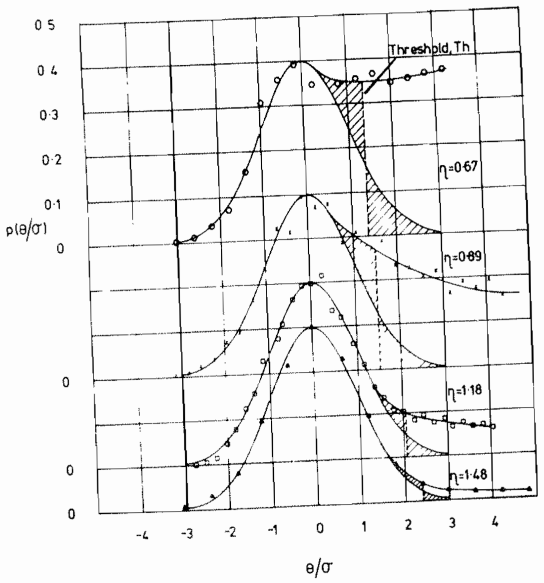

Figure 2 Gaussian fits to the part of $p(\theta)$ associated with noise and free stream fluctuations

ed on the right side of the peak although the points on the left of the peak continue to be adequately fitted by the Gaussian. Using the result of Bilger *et al.* (1976) that the area under the Gaussian represents 1-γ, the threshold Th can be set so that

$$\int_{-\infty}^{Th} (p(\theta) - G)\, d\theta = \int_{Th}^{\infty} G\, d\theta$$

where G is the fitted Gaussian. The threshold settings, chosen so as to satisfy the above equation, are shown in Figure 2 for several values of η. The present values of γ are in good agreement with the values obtained by Antonia *et al.* (1975) using conventional analogue methods. The value of Th is used here to obtain conditional probability density functions corresponding to the state $\theta/\theta' > Th$.

4 CONDITIONAL MOMENTS AND PROBABILITY DENSITY FUNCTIONS OF u, v AND θ

Normalised conditional probability densities of u, v and θ are shown in Figures 3 to 5. The normal-

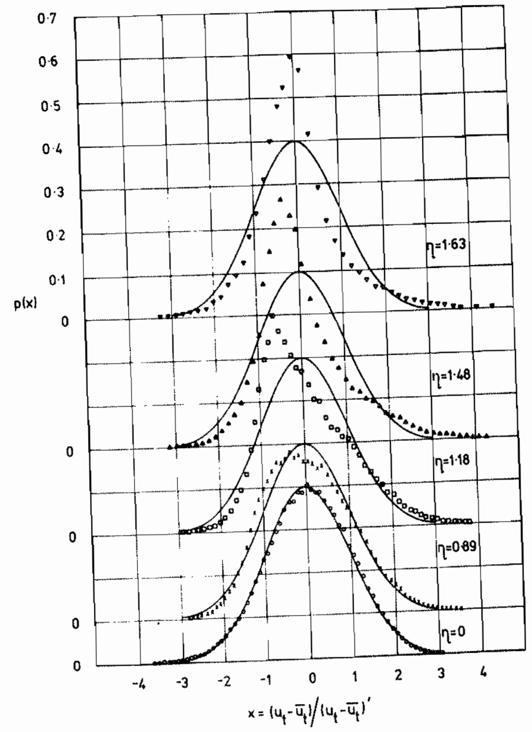

Figure 3 Normalised probability density of $u_t-\overline{u}_t$.

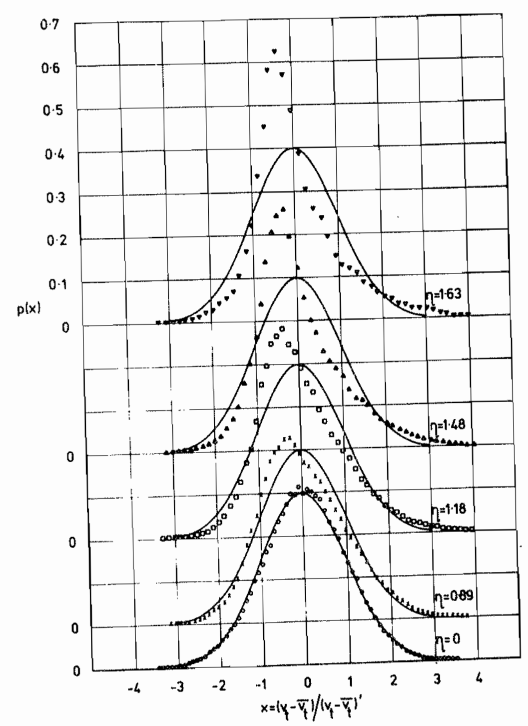

Figure 4 Normalised probability density of $v_t-\overline{v}_t$.

ised conditional moments $\overline{(x_t - \overline{x}_t)^n}/[(x_t - \overline{x}_t)']^n$, derived from the probability density functions of $x_t - \overline{x}_t$ are given in Table I. Here $(x_t - \overline{x}_t)'$ is the conditional rms of x_t measured with respect to the zone average $\overline{x}_t$. On the axis of the jet, the distributions $u_t - \overline{u}_t$ and $v_t - \overline{v}_t$ are quite close to the Gaussian curve, while $p(\theta_t - \overline{\theta}_t)$ is significantly different from the Gaussian.

TABLE I

CONDITIONAL (TURBULENT) MOMENTS OF VELOCITY AND TEMPERATURE FLUCTUATIONS

η	γ	$(u_t-\overline{u}_t)'/u'$	$(u_t - \overline{u}_t)^m/(u_t - \overline{u}_t)'^m$ order of moment, m: 3	4	5	6	$(v_t-\overline{v}_t)'/v'$	$(v_t - \overline{v}_t)^m/(v_t - \overline{v}_t)'^m$ order of moment, m: 3	4	5	6	$(\theta_t-\overline{\theta}_t)'/\theta'$	$(\theta_t - \overline{\theta}_t)^m/(\theta_t - \overline{\theta}_t)'^m$ order of moment, m: 3	4	5	6
0	≈1	1.00	-0.14	3.00	-1.54	15.8	1.00	-0.05	3.16	-0.55	17.6	0.97	-0.30	3.16	-2.66	16.6
0.67	0.95	1.00	0.01	2.79	-0.02	12.9	1.00	0.15	2.91	1.06	14.3	0.95	-0.13	2.52	-0.44	9.04
0.89	0.93	1.00	0.23	2.81	1.73	14.2	1.02	0.34	3.19	2.83	18.4	0.92	0.12	2.40	1.19	8.36
1.19	0.66	1.12	0.54	3.04	3.95	16.8	1.14	0.54	3.62	4.73	24.5	0.85	0.38	2.45	2.88	10.2
1.48	0.29	1.60	0.85	4.26	8.57	35.5	1.58	0.77	4.18	7.55	34.0	1.10	0.57	2.71	4.48	14.3
1.63	0.14	1.84	0.56	5.96	6.50	87.9	1.97	0.91	4.89	10.3	47.5	1.62	0.55	2.42	3.81	11.4

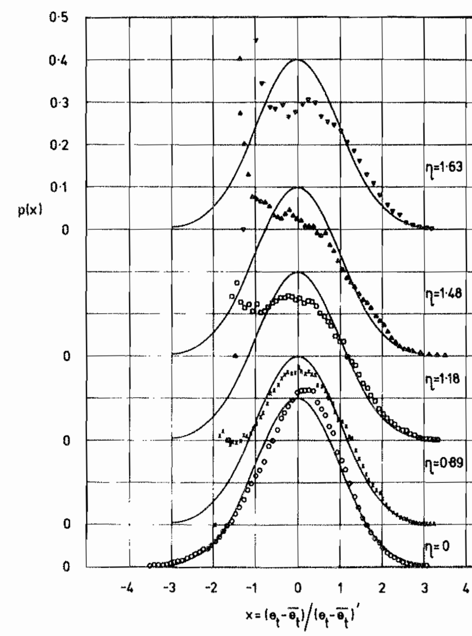

Figure 5 Normalised probability density of $\theta_t-\overline{\theta}_t$.

Although, away from the axis, the distribution of $p(\theta_t - \overline{\theta}_t)$ shows a marked departure from Gaussianity at negative values of $\theta_t - \overline{\theta}_t$, the Gaussian curve is a reasonable approximation for positive values of $\theta_t - \overline{\theta}_t$, (particularly $\theta_t - \overline{\theta}_t > \theta_t'$). At $\eta \simeq 0.89$, the flatness factor ($\simeq 2.4$) and the normalised sixth moment ($\simeq 8.36$) of $\theta_t - \overline{\theta}_t$ are well below the corresponding Gaussian values of 3 and 15 respectively. This considerable reduction is very likely to be a result of the unmistakable presence of ramps in the θ signal at this value of η.

Ribeiro and Whitelaw (1975) indicated that positive v fluctuations are associated with near Gaussian u distributions whereas skewed u distributions result when v is negative. To test this, normalised moments of u and θ are computed (for $\eta = 0.89$) for the further condition that $v > 0$ or $v < 0$ (in the case of conventional fluctuations) and $v_t > \overline{v}_t$ or $v_t < \overline{v}_t$ (in the case of conditional fluctuations), see Table II. These moments are considerably closer to the appropriate Gaussian values when v (or $v_t - \overline{v}_t$) > 0 than when v (or $v_t - \overline{v}_t$) < 0; u is considerably more Gaussian than θ when v (or $v_t - \overline{v}_t$) > 0, but the moments of θ show less departure from their Gaussian values than those of u when v (or $v_t - \overline{v}_t$) < 0. In the turbulent part of the flow, even order moments of u are slightly more Gaussian than the corresponding conventional values, but odd order moments are less Gaussian than their conventional counterpart. The opposite situation holds for θ.

TABLE II

DEPENDENCE ON THE SIGN OF v OF CONVENTIONAL AND CONDITIONAL (TURBULENT) MOMENTS OF u AND θ

x	Conventional				Conditional (Turbulent)			
	$v > 0$		$v < 0$		$v_t > \overline{v}_t$		$v_t < \overline{v}_t$	
	u	θ	u	θ	$u_t-\overline{u}_t$	$\theta_t-\overline{\theta}_t$	$u_t-\overline{u}_t$	$\theta_t-\overline{\theta}_t$
mean+	0.3	-0.1	-0.4	0.8	0.3	-0.2	-0.4	0.7
rms+	1.0	0.9	0.9	0.9	1.0	0.9	0.9	0.9
x^3/x'^3	-0.01	0.2	0.5	-0.3	-0.03	0.1	0.4	-0.3
x^4/x'^4	2.9	2.8	3.3	2.3	2.9	2.7	3.2	2.5
x^5/x'^5	- 0.1	1.4	4.6	-2.3	- 0.2	0.4	3.6	-2.6
x^6/x'^6	13.7	11.7	24.6	8.5	14.0	10.6	22.7	9.9

+These values have been normalised by the appropriate conventional or conditional rms values

5 PRODUCTS uv, uθ AND vθ

The probability density function p_{xy} of the product xy of two Gaussian variables x and y is given by (see, e.g., Antonia and Atkinson, 1973)

$$p_{xy} = \frac{\exp(r_{xy}\, t)\, K_o(|t|)}{\pi\, \alpha^{\frac{1}{2}}} \qquad (1)$$

where $t = xy/\alpha$, $\alpha = 1 - r_{xy}^2$, r_{xy} is the correlation coefficient $\overline{xy}/(x'y')$, and K_o is the zeroth order modified Bessel function of the second kind. Near the axis of the jet, products uv, uθ and vθ closely follow expression (1) (Antonia and Sreenivasan, 1976), when the measured value of r_{xy} is used. Away from the axis, (1) is a poor approximation to the conventional densities of products (Figure 6). Conditional (turbulent) densities are considerably closer to (1), although differences still persist (see Figure 7). These differences appear small enough to justify the replacement of a Gaussian p(x, y) by the assumption of a Gram-Charlier expansion for p(x, y) where only the first few cumulants are retained. The result of discarding all but the first four cumulants is shown in Figure 7. There is a slight improvement for negative values of the product, but the improvement does not seem to justify the extra effort involved in obtaining the required cumulants.

6 SUMMARY OF RESULTS

The probability density of temperature fluctuations in the axisymmetric heated jet reveals a spike which is centered near the ambient temperature of the external co-flowing stream; this applies even on the axis of the jet. The spike is closely fitted by a Gaussian throughout the jet, and an estimate of the area under the Gaussian can be used to determine the intermittency factor and the threshold required for conditional measurements.

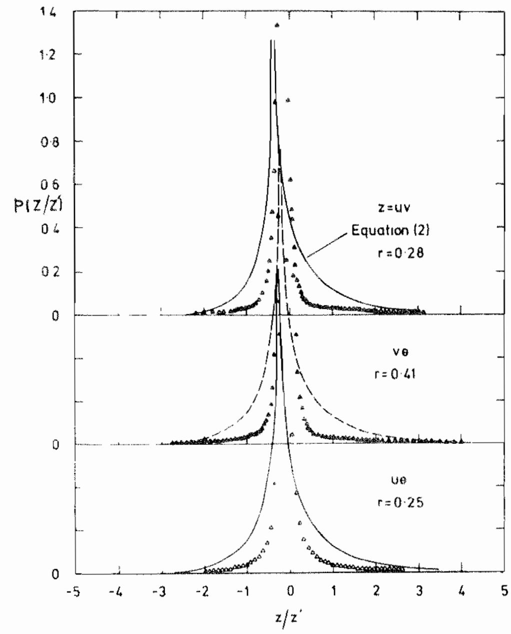

Figure 6 Normalised probability density function of conventional products at η = 1.48

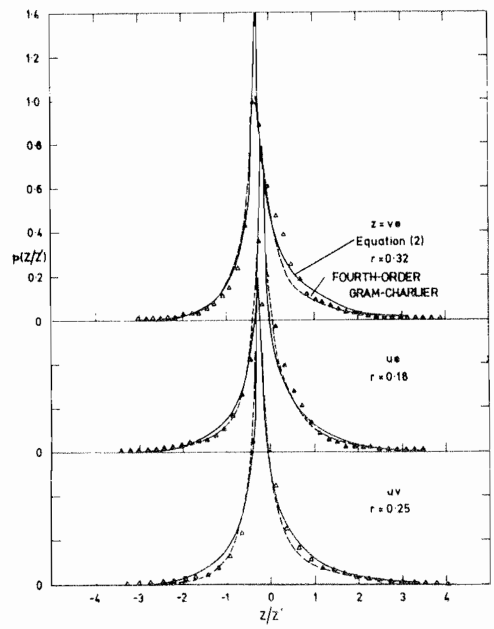

Figure 7 Normalised probability density function of conditional products at η = 1.48

Probability densities of velocity fluctuations in the turbulent part of the flow are closer to Gaussianity near the jet axis than those away from the axis. Conditional densities of θ and u are considerably more Gaussian for v > 0 than for v < 0, suggesting that the turbulence associated with the outward radial motion is nearly Gaussian at least in the region near the jet axis. The inward motion is distinctly non-Gaussian, probably as a result of recently entrained ambient fluid from the co-flowing stream.

On the axis of the jet, probability density functions of the products uv, uθ and vθ are closely approximated by joint Gaussian shapes of p(u, v), p(u, θ) and p(v, θ). Away from the axis, probability densities of conventional products are poorly represented by Gaussian calculations, while these latter calculations are relatively better approximations for the turbulent part only of the flow. The remaining discrepancies in the case of conditional products cannot be fully accounted for by a fourth-order Gram-Charlier expansion.

7 ACKNOWLEDGMENTS

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