Skewness of temperature derivatives in turbulent shear flows

K. R. Sreenivasan and R. A. Antonia

Department of Mechanical Engineering, University of Newcastle, New South Wales 2308, Australia
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The nonzero value of the measured skewness of the streamwise temperature derivative is not necessarily inconsistent with the concept of local isotropy of small scale turbulence, and may be estimated adequately from a ramp-like model for the large scale structure of temperature.

INTRODUCTION

It is generally suggested that local isotropy of small scale turbulence implies that the skewness S of the streamwise derivative θ_x of the temperature fluctuation θ is zero. In the literature,[1–5] considerable concern has therefore been expressed about the measured nonzero magnitude of S in various turbulent flows, both in the laboratory and atmosphere. Freymuth and Uberoi[6,7] suggested that S decreases in magnitude as the Reynolds number increases, and speculated that it might be asymptotic to zero at large Reynolds numbers. However, the data of Gibson *et al.*[1] in the atmosphere, heated jet, heated and cooled wakes, of Mestayer *et al.*[5] in a high Reynolds number boundary layer in a wind-water tunnel, as well as other evidence to be presented here, show that $|S|$ remains at a value of about 0.8 independent of Reynolds number. Wyngaard[3] attempted to explain this nonzero value by examining the contamination of the measured temperature by velocity, because of the velocity sensitivity of the temperature wire, but, as suggested by Gibson *et al.*[1] and Mestayer *et al.*,[5] Wyngaard's correction is always positive, so that the observed negative skewness in some flows cannot be explained in this manner. In any case, there is evidence, summarized by Antonia,[8] that the correction could be about one sixth (and thus about 0.1 in magnitude in most cases) of the magnitude originally suggested by Wyngaard. The correction would thus appear to be small and cannot fully account for the measured value. Freymuth[9] has recently put forward arguments based on dimensional analysis to show that $|S|$ would be a constant independent of Reynolds number if it depends only on small scale turbulence. Unfortunately, the constant cannot be predicted by dimensional arguments, and local isotropy of small scale implies that the constant is zero. When $|S|$ is completely determined by the large scale structure, Freymuth finds, again on dimensional arguments, that $|S|$ varies as R_λ^{-3}, where $R_\lambda \equiv \tilde{u}\lambda/\nu$, $\tilde{u}$, λ, and ν are the rms streamwise velocity fluctuation, the Taylor microscale, and kinematic viscosity of the fluid, respectively. Freymuth argues that $\overline{\theta_x^3}$ is of order $\tilde{\theta}^3/L^3$, where L is an integral scale of turbulence. However, as terms with derivatives inside the averaging operator are of lower order than those with derivatives outside the operator,[10] a more appropriate estimate for $\overline{\theta_x^3}$ is likely to be $\tilde{\theta}^3/\lambda^3$, in analogy with the velocity field. This latter estimate yields a constant $|S|$ independent of Reynolds number; this constant could indeed be nonzero without violating local isotropy. It is possible that a property such as S depends only on the large scale, just as, for example, the magnitude of the turbulent dissipation is controlled exclusively by the large scale motion. We show here that it is possible to obtain a simple estimate for S assuming that only the large scale is responsible for it. In particular, this is consistent with the hypothesis that the small scale structure is locally isotropic.

RESULTS

Figure 1 shows a plot of $|S|$ as measured by different investigators in different flows at different R_λ. (It appears from the study of Gibson *et al.*[1] and Mestayer *et al.*[5] that the sign of S is the same as that of the product of mean vorticity and mean temperature gradient.) Where the relevant experimental information was available, a correction for the velocity sensitivity has been applied to previously uncorrected data. This correction is based on Wyngaard's[3] analysis, but with a more plausible value for the spectral constant n suggested by Antonia.[8] One value from Freymuth and Uberoi's data in the axisymmetric wake[7] has been omitted because the

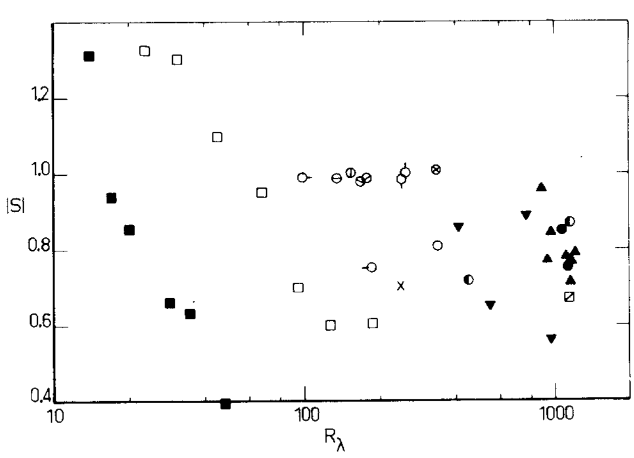

FIG. 1. Variation of $|S|$ with R_λ. Gibson *et al.*[1]: ●, atmospheric boundary layer; ◐, heated jet; ○, heated wake; ⊗, cooled wake. ▲, Mestayer *et al.*,[5] boundary layer. ▼, Gibson *et al.*,[2] atmospheric boundary layer (corrected for velocity sensitivity); (×), Antonia and Van Atta,[4] heated jet. Freymuth and Uberoi: □, heated two-dimensional wake[6]; ■, heated axisymmetric wake.[7] ⊖, ⦶, ⊘, ⦸, Sreenivasan *et al.*,[13] heated boundary layer. ⧄, unpublished data, atmospheric surface layer. -o, ǫ, ọ, o-, unpublished data, axisymmetric heated jet, $r/L_0 = 0$, 0.89, 1.15, and 1.63, respectively.

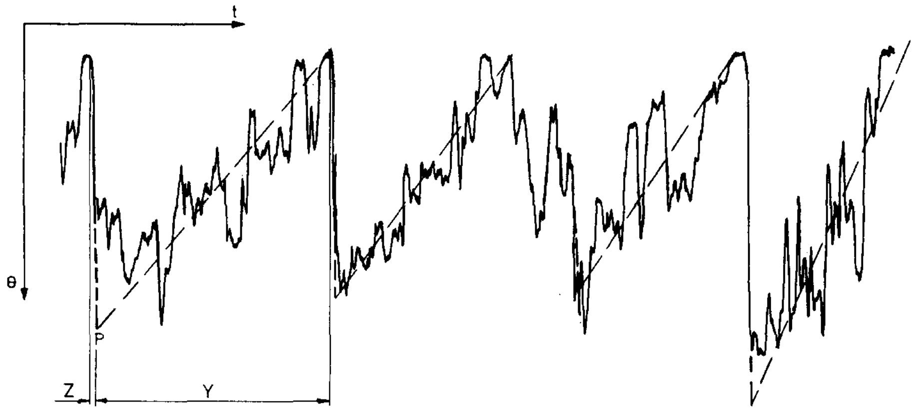

FIG. 2. Typical temperature signal in a jet and the ramp model. $\beta = Y/Z$.

wake was not self-preserving at the corresponding measuring station of $x/D = 60$. Although the data fall in a rather broad band, they suggest that $|S|$ decreases with R_λ for $R_\lambda \lesssim 50$, and is constant (mean value $= 0.82$, and standard deviation $= 0.16$) for $R_\lambda \gtrsim 50$.

It is assumed here that θ (Fig. 2) consists of a ramp-like large scale structure, with a relatively sharp leading edge, and of a small scale structure superimposed on this ramp. A variation of this model has been used by Antonia and Atkinson[11] to explain some features of higher order moments of θ. Antonia and Van Atta[12] have recently shown that nonzero high odd-order moments of temperature structure functions are consistent with the height and repetition rate of temperature ramps observed in both laboratory and atmosphere. The models of Antonia and Atkinson, and Antonia and Van Atta, which assumed that the leading edge of the ramp is sharp, leads to an infinite value of $|S|$ and is avoided here by allowing a more realistic finite rise time for the leading edge of the ramp. The model of Fig. 2 yields a value of S (using Taylor's hypothesis that $\Delta x = -\Delta t\, U$, where U is the local mean velocity) given by

$$S = -[(\beta - 1)/\beta^{1/2}]/(1+\alpha)^{3/2}, \quad (1)$$

where β is the ratio of the distance between the position P of the apex of the ramp and the trailing edge to the distance between P and the leading edge of the ramp (see Fig. 2), and α is the ratio of the contributions of the small scale and ramp, respectively, to $\overline{\theta_x^2}$. The flatness factor F may be written as

$$F = [\beta^{-1}(\beta^2 - \beta + 1) + F_s \alpha^2](1+\alpha)^{-2}, \quad (2)$$

where F_s is the flatness factor associated with the small scale contribution to θ_x.

Clearly, the magnitude of S (and F) depends on α and β. A record (of about 2 sec) of temperature θ obtained in the region of maximum turbulent energy production in a heated axisymmetric jet with a co-flowing free stream at ambient temperature, and $R_\lambda \simeq 200$, was examined by eye to determine the ratio β. The result is $\beta \simeq 23 \pm 10$ over the number of samples (approximately 50) contained in the record. This value was also confirmed by ensemble averaging all large scale structures of a given length (in practice, to within $\pm 10\%$ about the mean). The large standard deviation reflects the uncertainties in this estimate, because of the problem of precisely identifying the ramp-like structures.

If the large scale is responsible for the nonzero value of the skewness, the skewness of bandpass limited θ_x should approach zero as the high-pass filter cutoff f is moved toward higher and higher frequencies. The result of such an experiment (suggested by Freymuth[9]), with the low-pass filter setting set at the Kolmogoroff frequency, is shown in Fig. 3. As the measured values of $\overline{\theta_x^3}$ and $\tilde{\theta}_x$ were obtained by analogoue techniques, it is possible that small values of $|S|$ are unreliable, but it is quite clear that the contribution to θ_x from frequencies in the range $fL_0/U \gtrsim 10$ (where L_0 is the radial distance from the jet axis to the point where the mean temperature is equal to the average of the ambient and the maximum jet temperature) is nearly zero; it emphasizes the importance of the contribution to $|S|$ from the ramp-like structures. Figure 3 also suggests a possible means of distinguishing between "small" and "large" scale structures in the present context. From the power spectral density of θ_x, α may be obtained as the ratio of $\overline{\theta_x^2}$ from the range $fL_0/U > 10$ to that from $fL_0/U < 10$. We find that $\alpha \simeq 2$. (This value of α is likely to be somewhat over-estimated because the contribution from the sharp corners of the ramp is incorrectly included in this definition of "small" scale.) For $\alpha = 2$ and $\beta = 23$, Eq. (1) yields $S \simeq -0.88$. Corresponding to the previously mentioned standard deviation of ± 10 on β, $|S|$ may vary between -0.64 and -1.07, a range which is in satisfactory agreement with measured values (see Fig. 1).

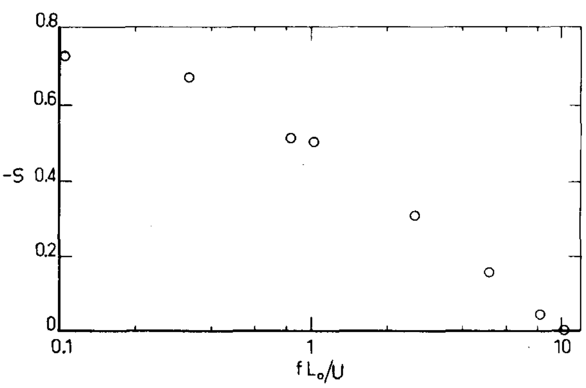

FIG. 3. Variation of S with high-pass cutoff frequency.

TABLE I. Skewness of temperature derivatives in a turbulent boundary layer.

	Skewness of		
$\frac{y}{\delta}$	θ_x	θ_y	θ_z
0.06 ($R_\lambda \simeq 134$)	0.96	−0.96	0.07
0.12 ($R_\lambda \simeq 153$)	1.00	−1.25	−0.02
0.18 ($R_\lambda \simeq 169$)	0.90	−1.03	0.03
0.24 ($R_\lambda \simeq 175$)	0.94	−1.09	−0.11

It should be emphasized that the aim of this article is not to determine the precise value of $|S|$, but rather to indicate that reasonable estimates can be obtained from a simple model for large scale, using plausible values of α and β. This simple model cannot be used to assess the dependence of $|S|$ on R_λ, presumably because α and β both vary with R_λ, but judging by the trend of Fig. 1, it would seem that (for $R_\lambda \gtrsim 50$) an increase in β would be accompanied by a (proportionately larger) increase in α as R_λ increases. The R_λ trend of $|S|$ for $R_\lambda \lesssim 50$ seems consistent with the departure from local isotropy of the properties of small scale structures at these low Reynolds numbers, and can be accounted for by modifying (1) as

$$S = -\left(\frac{\beta+1}{\beta^{1/2}} + \overline{s^3}\right)(1+\alpha)^{-3/2} ,$$

where $\overline{s^3}$ is the third moment of the small scale content of θ_x.

If the large scale motion occurs randomly in the spanwise direction z, the derivative θ_z can, on average, be expected to contain the effects only of the small scale motion, and hence possess zero skewness by local isotropy. In contrast, the derivative θ_y of θ, in the direction y normal to the surface, will have nonzero skewness of either sign (possibly depending on the mean temperature gradient). Data from the four-wire probe measurements of Sreenivasan *et al.*[13] on the skewness of θ_x, θ_y, and θ_z (see Table I) confirm this conclusion.

Improved deterministic models for the large-scale structure (such as the exponential ramp used by Antonia and Atkinson[11]) result in similar estimates for $|S|$. However, to obtain reasonable estimates of higher order moments of θ_x, it appears that the probabilistic behavior of both the height and duration of a ramp must be considered.

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