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Diffusion from a Heated Wall-Cylinder Immersed in a Turbulent Boundary Layer

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SUMMARY Measurements of both mean and fluctuating temperature are presented in the 'intermediate' region within the diffusion layer downstream of a heated circular cylinder placed spanwise on a wind-tunnel floor. The cylinder is completely submerged in the inner layer of the oncoming turbulent boundary layer (ratio of cylinder diameter to boundary layer thickness is 0.036). Mean temperature, root-mean-square temperature and temperature intermittency exhibit an approximate similarity on the same characteristic scales. The characteristic temperature scale varies inversely with the distance, and the rate of growth of the layer is comparable to that of a constant pressure turbulent boundary layer.

1 INTRODUCTION

Turbulent diffusion of heat and matter is of considerable practical interest. Since most sources are close to the surface, a study of diffusion in a turbulent boundary layer is of obvious importance. Poreh and Cermak (1964) studied diffusion in a turbulent boundary layer of a steady line source of ammonia, which was located on the floor level. Plate (1967) extended these measurements by introducing a line fence somewhat downstream of the contaminant source. In practice, however, sources of contaminants are also sources of roughness. It is therefore useful to examine the diffusion of a scalar quantity from a line source of roughness.

Poreh and Cermak's results show that the region downstream of the source can be divided into an 'initial' region close to the source where presumably the geometry of the source and other local effects dominate, a 'final' region far away from the source where the growth of the boundary layer dominates diffusion of the scalar quantity, and an 'intermediate' region in between where initial effects are sensibly absent and diffusion is not influenced by details of boundary layer growth. One can then expect this intermediate region to possess some kind of similarity independent both of the details of the source and of the oncoming boundary layer.

The present work concerns measurements of mean and fluctuating temperature essentially in the intermediate region, downstream of a heated circular cylinder resting on the floor. Of particular interest is the relation among the similarity exhibited among different quantities, the growth rate of the diffusion layer and the rate of decay of maximum temperature in the diffusion layer. It is estimated that at the furthest downstream measuring station, the thickness of the diffusion layer is about one third of that of the momentum boundary layer, and is therefore still within the fully turbulent region.

2 EXPERIMENTAL CONDITIONS

A circular steel cylinder, 1.63mm in diameter, was placed on the floor of a wind-tunnel spanning the entire width of the test section. It was heated to about 23°C above ambient by passing a current of approximately 30 A at approximately 15V A.C. The arrangement is schematically shown in Figure 1.

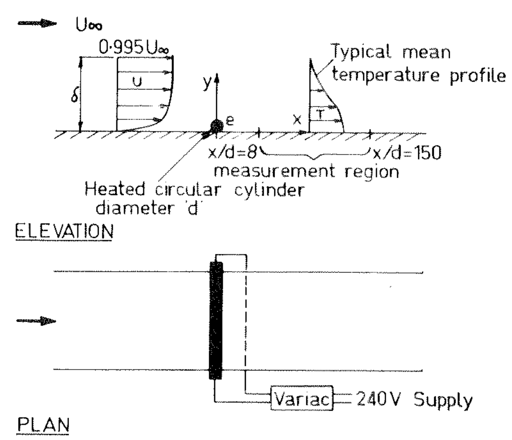

Figure 1 Schematic of experimental arrangement

The tunnel was operated at a nominal free-stream velocity of 9.0 ms^{-1}. The boundary layer just upstream of the cylinder was fully turbulent and had a thickness δ (99.5% of free-stream velocity) of 45mm. The cylinder was fully submerged in the inner layer of the boundary layer.

All temperature measurements were made by 0.6μm diameter Wollaston wires about 0.7mm in length (resistance approximately 580 Ω). They were operated "cold" at a constant current of 0.1mA on a two-channel constant current anemometer described by Stellema et al. (1975). The instrument gives two outputs, one of which (between d.c. and a few

Hz) is at a fixed gain of 1000, and the other (between 0.02 Hz and 20kHz) is at a variable gain of up to 5000. The temperature coefficient of resistance of the wires is found to be $0.0014\,^{\circ}C^{-1}$ by direct calibration in the potential core of an axisymmetric jet heated to about $30^{\circ}C$ above the ambient.

Mean temperature profiles were obtained from the d.c. output of the anemometer. To avoid the problem of drift of the anemometer, two wires were used simultaneously, one of which was always kept in the free-stream. The difference in the d.c. outputs of these two wires was then directly related to the temperature difference using the static calibration mentioned above.

Fluctuating temperature signals (obtained from the a.c. output of the anemometer) were suitably amplified and low-pass filtered through a DISA 55D26 signal conditioner and recorded on a Philips analogue 7-channel FM tape recorder at a speed of $38.1\,cms^{-1}$. The tape recorder had a frequency response curve which was flat up to 10kHz, a dynamic range of ± 5V and a nominal signal/noise ratio of 40 dB. Root-mean-square (rms) values were read on a DISA 55D35 rms meter.

The recorded signals were later played back at a speed of $2.38\,cms^{-1}$ and, after suitably filtering the signals to avoid aliasing, processed on a PDP11/45 computer at the University of Sydney.

3 RESULTS

Measurements were confined to the region $8 < x/d < 150$, where d is the diameter of the cylinder. No corresponding velocity measurements were made.

3.1 Mean Temperature Profiles

Measurements show that the mean temperature is maximum at the wall and decreases monotonically towards the free-stream value. If the maximum temperature (above ambient) T_{max} and the normal distance λ from the wall where $T = \frac{1}{2} T_{max}$ occurs are chosen as the temperature and length scales respectively, all measurements (except at x/d = 8) exhibit a well-defined similarity (Figure 2).

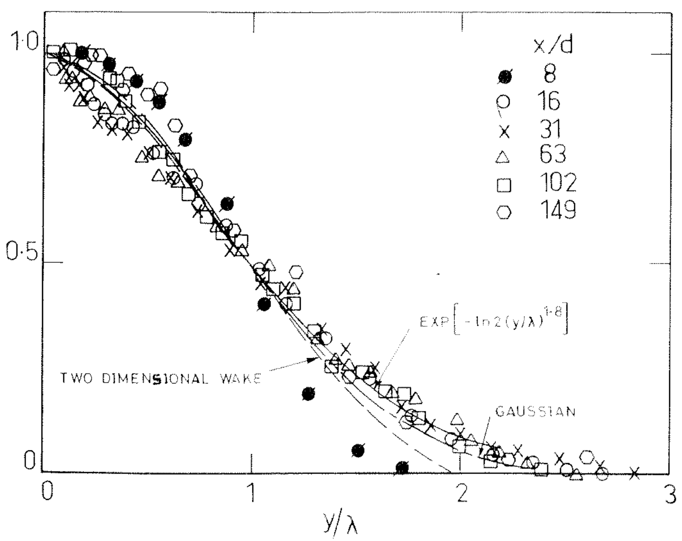

Figure 2 Similarity of mean temperature profiles

When normalized in this manner, the concentration profiles measured by Plate (1967) were closely fitted by the expression

$$\exp \{ - \ell n\, 2\, (y/\lambda)^{\mu} \} \qquad (1)$$

for values of μ between 1.6 and 1.8. Figure 2 shows that (1) is a reasonable approximation to mean temperature profiles as well, except at x/d = 8 and 149. Clearly, at x/d = 8, similarity is not established; at x/d = 149, because the diffusion layer is comparable in thickness to the fully turbulent region of the momentum boundary layer, it is possible that this similarity breaks down. (Note that the scatter in measurements is maximum very close to the wall). Also plotted for comparison are the Gaussian and the mean temperature profiles appropriate to a self-preserving two-dimensional wake (Townsend, 1956).

3.2 Root-mean-square Profiles

RMS temperature also shows a similarity (for x/d > 16) when its maximum value $\tilde{\theta}_{max}$ and the distance λ' from wall of the location of its occurrence are used for the temperature and length scales respectively (Figure 3).

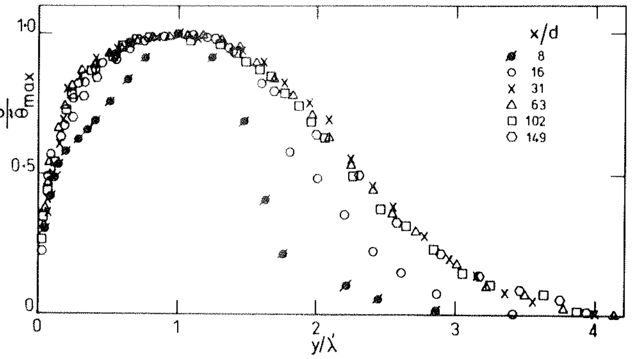

Figure 3 Similarity of rms temperature profiles

It is interesting to note that the rms temperature takes longer to attain similarity than the mean temperature.

3.3 Probability Density

Figure 4 shows a typical probability density of temperature fluctuations at a point x/d = 63, y/d = 3 in the flow where the intermittency factor γ (defined as the fraction of time for which the flow is turbulent) is 0.95.

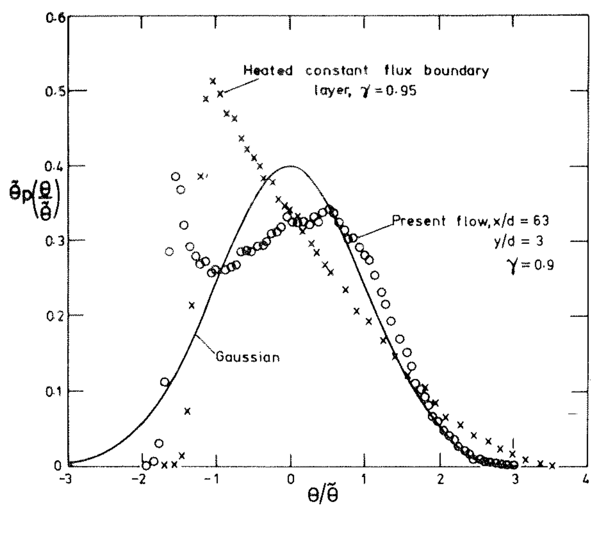

Figure 4 Probability density function of temperature

A characteristic feature is the bimodal nature of the distribution, with a sharp peak centered roughly around the ambient temperature. Bilger et al. (1975) argued that the peak is associated with the 'noise' in ambient conditions and electronic instruments, and showed that they can be fitted rather closely by a Gaussian whose area is equal to $1 - \gamma$. These arguments should in principle be applicable to any passive contaminant in any intermittent shear flow. Indeed, probability density functions of temperature measured in a heated wake (LaRue and Libby, 1974), heated jet (Antonia and Sreenivasan, 1976), and of concentration in mixing layers (Roshko, 1976) clearly show this behaviour in the intermittent region. However, temperature fluctuations in a constant heat flux boundary layer do not show this feature. A probability density function measured in such a boundary layer under conditions (such as Reynolds number, intermittency, signal/noise ratio) comparable to those in the present case, is also plotted in Figure 4.

3.4 Intermittency factor

Intermittency was obtained by two methods: the conventional method of setting an appropriate threshold and by the method of Bilger et al. (1975) mentioned above. The agreement between the two methods was found to be very good. Figure 5 shows that the distribution of γ is very nearly Gaussian.

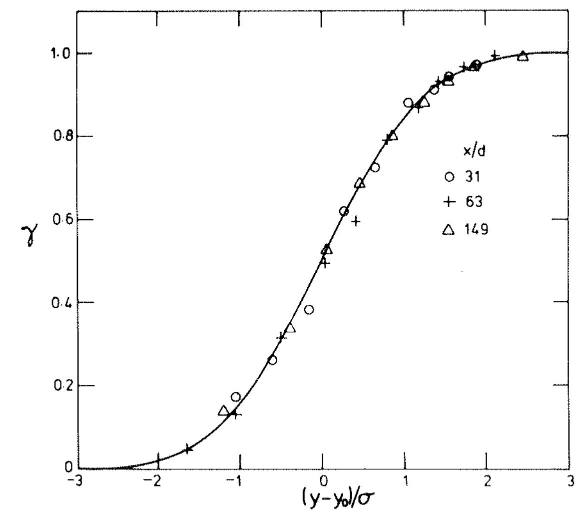

Figure 5 Intermittency Profiles

4 DISCUSSION

4.1 Similarity

It can be argued that any bounded, monotonically decreasing function will exhibit an approximate similarity when normalized in the manner described in Section 3.1. A more sensitive check on the existence of similarity is to calculate the moments of the temperature profile and examine their variation with x (Shlien & Corrsin, 1976). Here, we have computed only the quantities

$$\bar{T} = \int (T/T_{max})\ d\ (y/\lambda),$$

$$\bar{Y} = \int (T/T_{max})\ (y/\lambda)\ d\ (y/\lambda)$$

and $\overline{(y - \bar{Y})^2} = \int (T/T_{max})\ (\ (y - \bar{Y})/\lambda)^2\ d\ (y/\lambda)$, where $\bar{T}$ can be thought of as the total enthalpy at a section, $\bar{Y}$ is the centre of gravity of the similarity profile and $\overline{(y - \bar{Y})^2}$ represents its variance.

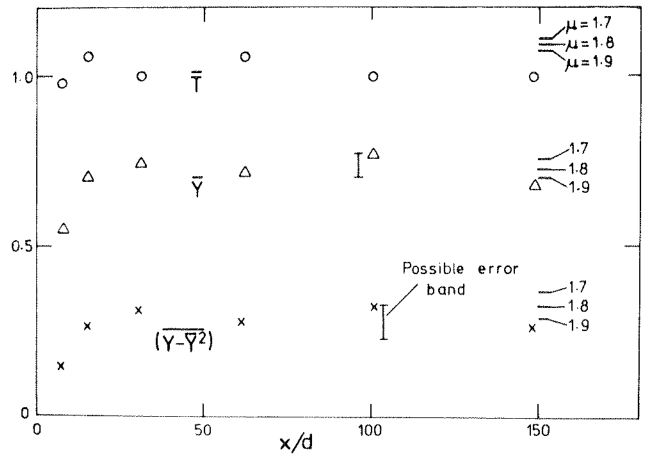

Figure 6 Total enthalpy at a station, centre of gravity of the mean temperature profile and its standard deviation

Figure 6 shows that, for x/d > 30, within the scatter of the data, all three quantities are sensibly invariant with x, which suggests that similarity is well established in this region. Also indicated on Figure 6 are the corresponding quantities evaluated using (1), for μ = 1.7, 1.8 and 1.9. The agreement with measurement is reasonably good in this range of μ, although no single value is consistently good for all the three quantities. This of course is not surprising, as (1) does not fit the measurements uniformly well for a given value of μ; for example, small departures between measurement and (1) with μ = 1.8 are noticeable in Figure 2 for $y/\lambda < 1$. Note incidentally that a moment of given order m can be computed for (1) from

$$\overline{(y - \bar{Y})^m} = (\mu\bar{T})^{-1} \sum_{r=0}^{m} {}^mC_r\, \bar{Y}^{-r}\, (\ell n 2)^{(-m+r-1)/\mu} \times \Gamma\,[(m-r+1)/\mu]$$

where the Gamma function is defined as

$$\Gamma(a) = \int_0^{\infty} e^{-x} x^{a-1} dx.$$

4.2 Relation among different scales

It was shown previously that both mean and rms temperature profiles and the intermittency profile all exhibit similarity when non-dimensionalised by the appropriately chosen scales. The relation among the different scales is shown on Figure 7. Here y_m and σ are respectively the position of the mean interface and its standard deviation.

To a reasonable approximation, different length scales are in constant ratio to each other. The same conclusion holds also for the ratio of temperature scales used in mean and rms profiles. This is strong evidence for genuine similarity of the flow and it therefore follows that T_{max}

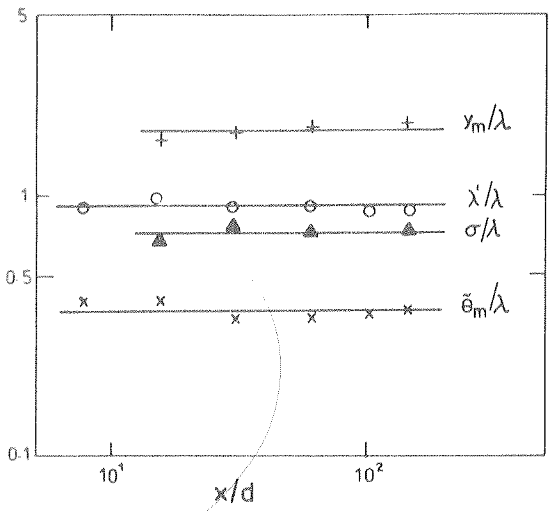

Figure 7 Relation among different scales

and λ may be used as universal temperature and length scales of similarity respectively.

4.3 Rate of Growth of Scales

Figure 8a shows the streamwise variations of the similarity scales T_{max} and λ.

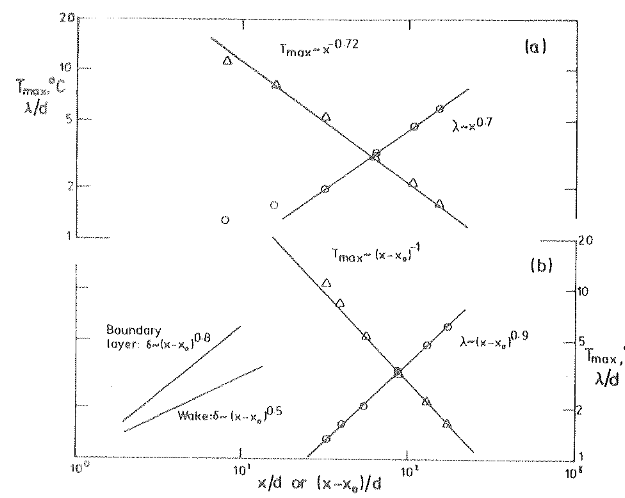

Figure 8 Streamwise variation of length and temperature scales

Over the region of present measurements, the variation is very nearly according to

$$T_{max} \sim x^{-0.72}, \quad \lambda \sim x^{0.7}$$

To compare the growth rate with those of other flows (such as a boundary layer or wake) the effect of virtual origin should be considered. Virtual origin is here determined by extrapolating to zero the straight line drawn through $\lambda^{1/0.7}$ on a linear plot with x as the abscissa. The results (Figure 8) show that

$$T_{max} \sim (x - x_o)^{-1} \text{ and } \lambda \sim (x - x_o)^{0.9}.$$

The growth rate of the diffusion layer is thus comparable to that of a turbulent boundary layer and is much higher than that of a two-dimensional thermal wake. The rate of decay of maximum temperature follows very closely Townsend's (1965) prediction for a line source of heat immersed within an oncoming turbulent boundary layer.

5 CONCLUSIONS

It is shown here that, in the intermediate region, the use of the same length and temperature scales produces similarity in both mean and rms temperature profiles. The length scale is proportional to the mean position of the interface or its standard deviation. Further, the similarity profile for the mean temperature is essentially the same as that of concentration profiles of ammonia, although, in the latter case, the diffusion layer develops from somewhat different initial conditions. The rate of spread of the diffusion layer is comparable to that of a constant pressure turbulent boundary layer.

6 ACKNOWLEDGMENTS

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