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Rapid Distortion of Shear Flows

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INTRODUCTION

It is convenient (whenever possible) to be able to regard turbulence not merely as a characteristic of the flow (which it always is) but as endowing the fluid with certain new characteristics. When so viewed, the description of the response of turbulence to a superimposed deformation, in general similar to that of viscoelastic fluids, assumes considerable importance in the understanding of turbulence structure at large. The ubiquitous eddy-viscosity theories apply in the extreme limit of instantaneous relaxation to equilibrium from a deformed state; in the opposite limit of suddenly applied strains the viscous and the inertial relaxation mechanisms can be expected to be dominant. We may call this latter limit "rapid distortion", borrowing the phrase from a study by Batchelor & Proudman [1] (see also Ribner & Tucker [2]) when the turbulence is homogeneous and initially isotropic. Passage of turbulence through wind tunnel contractions and shear flows in highly favourable or adverse pressure gradients are examples of fairly rapid distortion.

It is useful to examine the relevance of the ideas of rapid distortion theory to shear flows which are acted upon by the inherent rate of shear strain [3, 4]. It turns out, however, that there are some experimental situations in shear flows in which rapid distortion theory plays a more direct role: substantial evidence now exists showing that the outer part of a highly accelerated turbulent boundary layer undergoing relaminarization [5] and rapidly distorted wakes [6] behave in a manner essentially dominated by rapid distortion. However, in this more general class, the turbulence structure is neither isotropic nor homogeneous. It is therefore essential to inquire into the way in which the final results will be affected if the initial state of turbulence departs from isotropy and homogeneity. Recent work [7] on axisymmetric turbulence relaxes to some extent the restriction of isotropy when the departure from it is small, and it is to the complementary problem of examining the effect of small inhomogeneities (to be defined more precisely later) that this note addresses itself. Furthermore, no theory of homogeneous turbulence throws direct light on the behaviour of shear stress during straining.

FORMULATION

Consider a suddenly accelerated two-dimensional turbulent boundary layer. It can be analysed [5] by splitting it into an inviscid but rotational outer layer and a viscosity dominated, and essentially laminar, inner layer, with a matching velocity U_s (slip velocity for the outer layer) between them (Fig. 1). Define

$$\lambda \equiv \frac{U'_\infty \delta}{U_\infty - U_s}, \quad \text{where } U'_\infty \equiv \frac{dU_\infty}{dx},$$

δ is a measure of the overall shear layer thickness and the subscript ∞ indicates free-stream conditions. λ is thus a ratio of the strain rates in the streamwise and transverse directions. We now study the response of turbulence structure to rapid straining under the limit of $\lambda \to \infty$. It is easy to show that the sufficient condition for the boundary layer approximation to be valid is that $L\ (\equiv U_\infty/U'_\infty) >> \delta$, and that it is not inconsistent with the limit $\lambda >> 1$. Then, the equations governing the Reynolds stress tensor τ_{ij} and the orders of magnitude in the outer layer of various terms therein (obtainable by rather general and familiar arguments) can be schematically written in the following form [3]:

$$\left\langle \begin{matrix}\text{Convec-}\\ \text{tion}\\ \text{terms}\end{matrix} \right\rangle = \left\langle \begin{matrix}\text{Defor-}\\ \text{mation}\\ \text{terms}\end{matrix} \right\rangle + \left\langle \begin{matrix}\text{Pressure-}\\ \text{velocity}\\ \text{correlation}\\ \text{terms}\end{matrix} \right\rangle + \left\langle \begin{matrix}\text{Triple}\\ \text{Correlation}\\ \text{terms}\end{matrix} \right\rangle + \left\langle \begin{matrix}\text{Viscous}\\ \text{terms}\end{matrix} \right\rangle$$

1	λ^{-1} or unity	$\lambda^{-1}(\frac{\delta}{l})$
	$\lambda^{-1}(\frac{\delta}{l})$	$\lambda^{-1}\frac{\delta}{l}\frac{\nu}{Ul}$

Expanding the Reynolds stress components (with u^2, v^2, w^2 denoting the normal stresses and τ denoting the shear stress: see, e.g. Townsend [3] for notation) in powers of λ^{-1}, one gets to order λ^o

$$L\overline{u_o^2} = -2\overline{u_o^2}\,U' \qquad (2.1)$$

$$L\overline{v_o^2} = 2\overline{v_o^2}\,U' \qquad (2.2)$$

$$L\overline{w_o^2} = 0 \qquad (2.3)$$

$$L\,\tau = 0 \qquad (2.4)$$

where L is the operator $U\frac{\partial}{\partial s}$, with suffix o indicating the lowest order terms, and s indicating distance along a streamline. Here, we have replaced the velocity along a streamline by its component along the direction of the free-stream. To the next order λ^{-1},

$$L\,\overline{u^2} = -2\,\overline{u^2}\,U' + 2\,\tau_o\frac{\partial U}{\partial y} \qquad (2.5)$$

$$L\,\overline{v^2} = 2\,\overline{v^2}\,U' \qquad (2.6)$$

$$L\,\overline{w^2} = 0 \qquad (2.7)$$

$$L\,\tau = \overline{v_o^2}\frac{\partial U}{\partial y} \qquad (2.8)$$

where no suffix is shown in the above equations, it implies total quantities to order λ^{-1}; e.g. $\tau = \tau_o + \lambda^{-1}\tau_1$, etc. Clearly, further order-relations (e.g. between l/δ and λ) need to be postulated to obtain equations of motion to higher order in λ^{-1}. We note that exactly the same analysis is valid for accelerated wakes also if we define $\lambda = U_\infty\delta/w_o$, with w_o as the maximum velocity defect (see Fig. 1).

SOLUTION

If $\partial U/\partial x > 0$, it is seen that to the lowest order, $\overline{u_o^2}$ decreases and $\overline{v_o^2}$ increases (the variation in each case depending nonlinearly on its own value). Further, $\overline{w_o^2}$, τ_o and $\overline{u_o^2}\cdot\overline{v_o^2}$ remain constant along streamlines, thus implying that to this order the correlation coefficient K_o also remains unaffected. This simple analysis explains the important observation made of 'stress-freezing' during rapid straining of shear flow [8]. Comparison with experiments [9, 10] for the quantity τ has been made in Fig. 2 for some typical streamlines in the outer flow. Fig. 3 gives a similar comparison for the quantity $\overline{w^2}$ from the experiments of Blackwelder & Kovasznay [10].

Further, for small total strain ratios, it is possible to replace L by $U\frac{\partial}{\partial x}$, so that Eqns. (2.1) and (2.2) integrate to

$$\overline{u_o^2} = \text{Const. } U^{-2}, \quad \overline{v_o^2} = \text{Const. } U^2 \qquad (3.1)$$

These results are in fact identical with those obtainable by Prandtl's [11] arguments to homogeneous two-dimensional flows, and also with those of the Batchelor-Proudman theory in the limit of zero wave-number, $k \to 0$. It is therefore a reasonable extrapolation to expect that the actual behaviour of shear flow turbulence will be quantitatively similar to the 'integrated' result over all Fourier components, valid for homogeneous turbulence.

Therefore, it appears logical to use (for normal stresses at least) results from the appropriate complete theory for homogeneous turbulence; e.g. if initially turbulence is approximately axisymmetric with a given value for $R\,(\equiv \overline{u^2}/\overline{v^2})_{s_i}$, it follows [7] that

$$\overline{v_o^2}(s) \simeq (1-R/4)\,e_1\,\overline{v_o^2}(s_i), \qquad (3.2)$$

where $e_1 = U(s)/U(s_i)$, and the suffix i on s indicates the initial station on a given streamline.

Using Eqn. (3.2), it is easy to integrate Eqn. (2.8) to give

$$\tau(s) = \tau(s_i) + \frac{(1-R/4)}{(R+2)}\left(\frac{\overline{q_o^2}}{U}\right)_{s_i}\int_{s_i}^{s}\left(\frac{\partial U}{\partial y}\right)_{s'}ds'$$

$$\simeq \tau(s_i) + (1-R/4)\,\alpha_i\,(s-s_i) \qquad (3.3)$$

$$\text{with}\quad \alpha_i \equiv \frac{1}{R+2}\left(\frac{\overline{q_o^2}}{U}\cdot\frac{\partial U}{\partial y}\right)_{s_i} \qquad (3.4)$$

following the result that the mean vorticity is convected without change along a streamline in the outer layer [5]. Here $\overline{q_o^2} = \overline{u_o^2} + \overline{v_o^2} + \overline{w_o^2}$. Fig. 2 shows typical comparisons with the experimental result of Blackwelder & Kovasznay [10].

Again, using Eqn. (3.2), one gets

$$(\overline{u^2}\cdot\overline{v^2})_s = (\overline{u^2}\cdot\overline{v^2})_{s_i} + 2\,\alpha_i(s-s_i) \qquad (3.5)$$

Eqns. (3.3) and (3.5) can be looked upon as a two term Taylor expansion for the quantities τ and $(\overline{u^2} - \overline{v^2})$. A similar expansion for the square of the correlation coefficient can be shown to be

$$K^2 = K_o^2 \left[1 + \frac{2(1 - \frac{R}{4} - K_o^2)}{(s_i)} \alpha_i (s - s_i)\right] \qquad (3.6)$$

for small relative increments in τ .

In the outer part of a relaminarizing boundary layer, further details regarding the comparison with experiment of the behaviour of the normal stresses, as predicted by the complete rapid distortion theory, can be found elsewhere [5].

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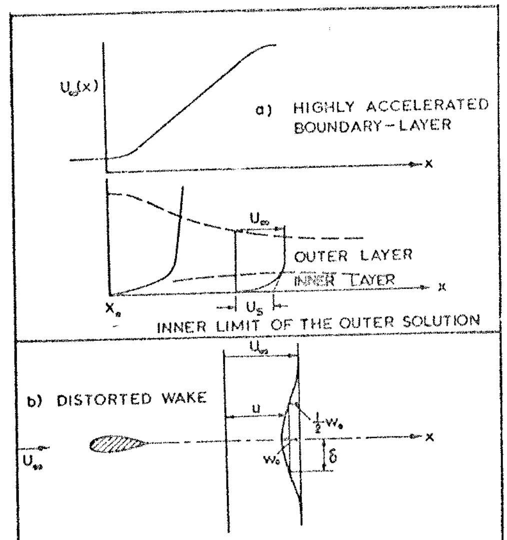

FIG. 1. FLOW CONFIGURATION AND NOTATION

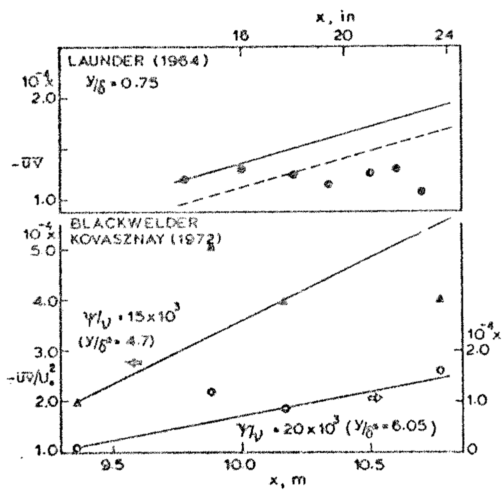

FIG. 2. VARIATION OF REYNOLDS SHEAR STRESS ALONG STREAMLINES DURING RELAMINARISATION. FULL LINES ARE THEORY.

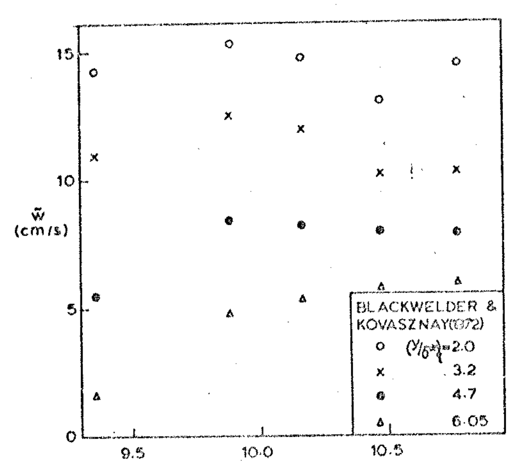

FIG. 3. VARIATION OF $(\overline{w^2})^{1/2}$ ALONG STREAMLINES THEORY PREDICTS CONSTANT $\tilde{w}$.

ERRATA:

p.2.11. Col.1, ll. 14-15 should read:

" ......mechanisms can be expected not to be dominant....."

p.2.11. Col.2, 14 from below, insert after τij:

" (see, e.g. [3])".

p.2.12. Col.1, 1 1, omit: "[3]" .

p.2.12. Col.1, ll 8-9, omit:

" : see, e.g. Townsend [3] for notation"

p.2.12. Col.2, ll 8-10 should read:

" Comparison with experiment [9] for the quantity τ has been made in the upper part of Fig.2 for a typical streamline......."

p.2.14. Upper part of fig.2, omit:

(a) the lines

and (b) $y/\delta = 0.75$ in the inset.

The ordinate should be $-\overline{uv}/U_0^2$.